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Channeled Spectropolarimetry Using Iterative Reconstruction

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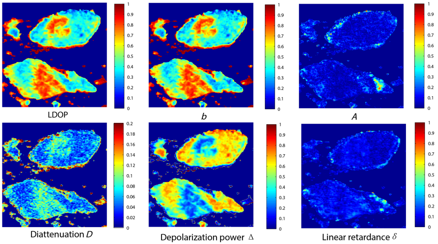
Outline

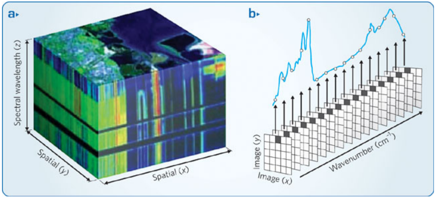
1. Overview of CSP
2. Review of Fourier reconstruction (FR)
3. Introduce new iterative reconstruction (IR) technique
4. Compare FR and IR techniques
5. Summary

1. Overview of CSP

CSP combines polarimetry and spectroscopy

- Polarimetry helps distinguish targets from background, evaluate stress bifringence, characterize biological tissues
- Spectroscopy provides insights in biomedical imaging and remote sensing
- CSP combines these capabilities to study the polarimetric and ellipsometric properties of dispersive materials

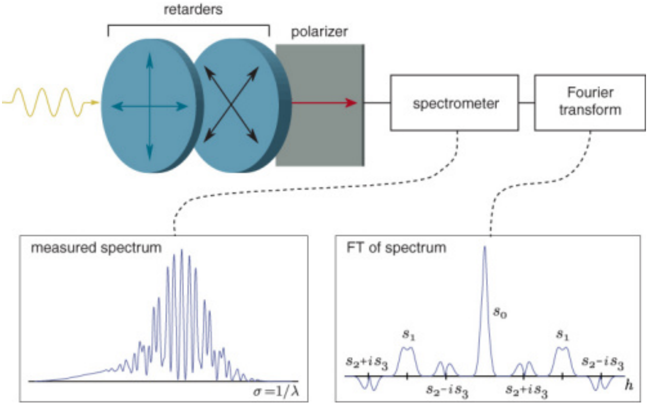




Du, et. al., J. Biomed. Opt. 19(7) (2014)
Bannon, N. Photonics 3, 627-629 (2009).

Fourier processing recovers the Stokes parameters

- Fourier transform the CSP spectrum, then filter the channels—easy procedure but it has drawbacks
- It suffers from noise due to vibration, thermal fluctations, imperfect sampling
- It assumes the channels are band-limited; cross-talk is bad
- Fourier transform requires uniformly spaced samples

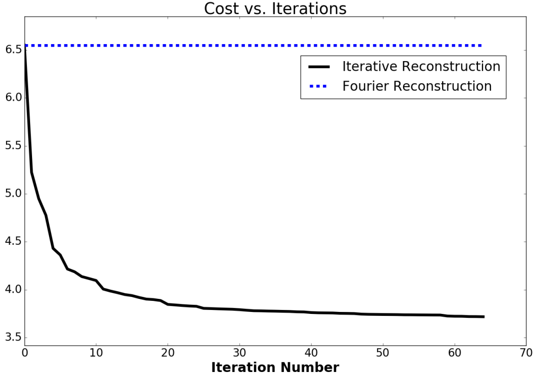


Sabatke, et. al., Opt. Express 11(22) (2015).

Iterative reconstruction (IR) improves on Fourier reconstruction (FR)

Advantages of IR:

- Reduce noise from vibrations and thermal fluctuations
- Reconstruct signals with more bandwidth, a significant advancement over FR
- Process measurements that are non-uniformly spaced in wavenumber
- Quantify reconstruction improvement through minimization of a cost function



The CSP system measures a spectrum

- Let's consider a system with a QWP, 45° retarder, analyzer, and spectrometer
- Mathematically,

$$\mathbf{S}_{\text{out}}(\sigma) = \mathbf{M}_{\text{pol}}\ \mathbf{M}_{45^\circ}(\sigma)\ \mathbf{M}_{\text{QWP}}\ \mathbf{S}_{\text{in}}(\sigma),$$

  where

$$\mathbf{M}_{\text{pol}} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{M}_{45^\circ}(\sigma) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\phi) & 0 & -\sin(\phi) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(\phi) & 0 & \cos(\phi) \end{bmatrix},$$

$$\mathbf{M}_{\text{QWP}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

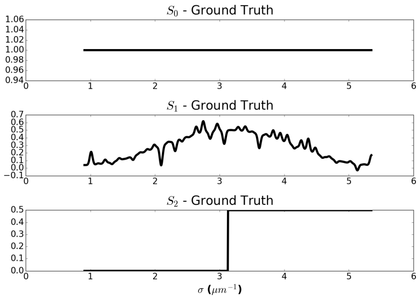
- After some manipulation, the measured intensity is

$$I(\sigma) = \frac{1}{2}\left\{S_0(\sigma) + S_1(\sigma)\cos[\phi(\sigma)] + S_2(\sigma)\sin[\phi(\sigma)]\right\}$$

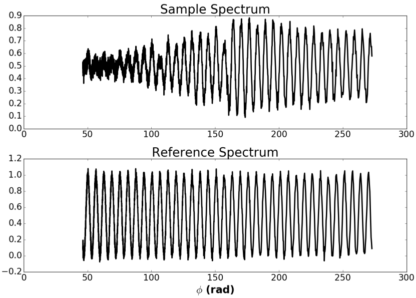
2. Review of Fourier reconstruction (FR)

Simulate a spectrum using truth Stokes parameters

- For simulation, we set values for the Stokes parameters



- Simulate the spectrum from these parameters



- Add noise to spectrum:

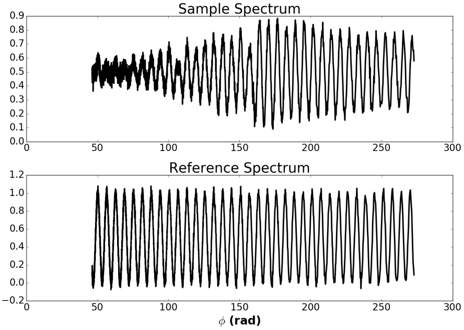
$$I(\sigma) = I_{\text{model}}(\sigma) + n.$$

where

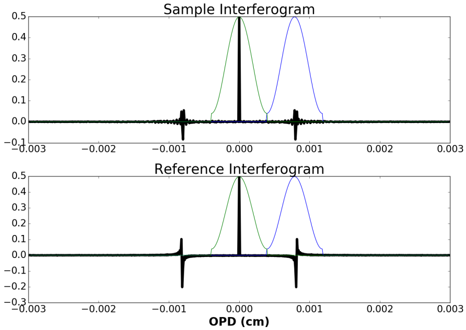
$$n \sim \mathcal{N}\left(\mu, \sigma_n^2\right),$$

Step 1: Inverse FT the spectra to get interferograms

- First measure the sample and reference spectra.

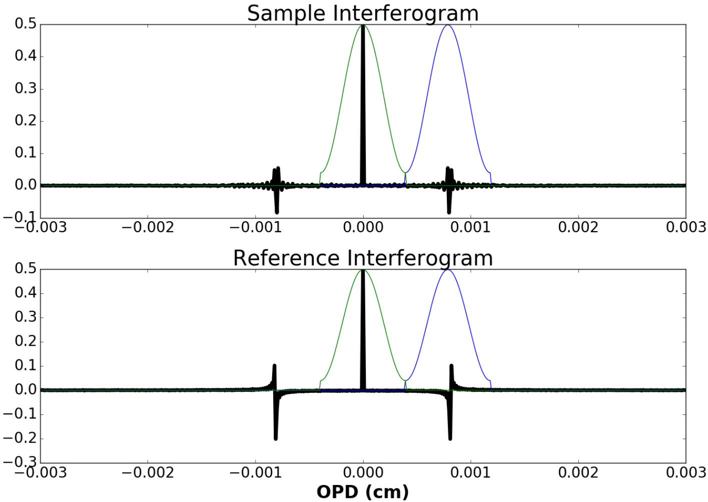


- Inverse FT the spectra to get interferograms.



Step 2: Separate the interferogram into channels

- From the sample interferogram, extract $C_0(d)$ and $C_1(d)$ (sample channels 0 and 1) by applying Hamming filters
- From the reference interferogram, extract $\tilde{C}_0(d)$ and $\tilde{C}_1(d)$ (reference channels 0 and 1)



Step 3: Fourier transform each channel

- Calculate S_0 from channel 0:

$$S_0(\sigma) = \left|\mathcal{F}\left[C_0(d)\right](\sigma)\right|.$$

- Calculate phase correction from the reference channel:

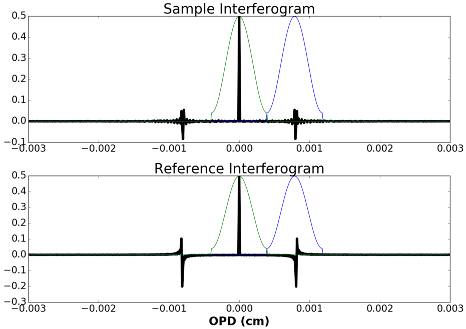
$$\tilde{\phi}_1(\sigma) = \arg\left\{\mathcal{F}\left[\tilde{C}_1(d)\right](\sigma)\right\}.$$

- Calculate S_1 from channel 1 with phase correction:

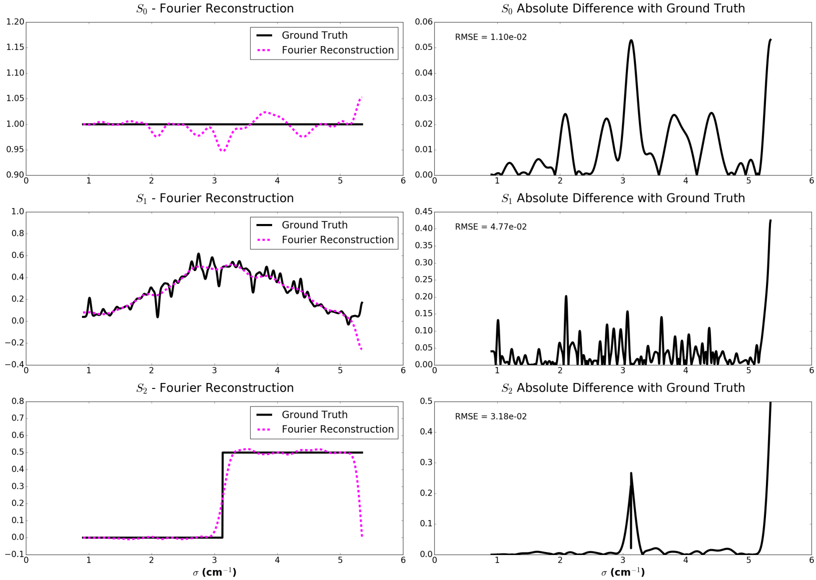
$$S_1(\sigma) = \mathrm{Re}\left\{\mathcal{F}\left[C_1(d)\right](\sigma)\cdot e^{-i\tilde{\phi}_1(\sigma)}\right\}$$

- Calculate S_2 from channel 2 with phase correction:

$$S_2(\sigma) = \mathrm{Im}\left\{\mathcal{F}\left[C_1(d)\right](\sigma)\cdot e^{-i\tilde{\phi}_1(\sigma)}\right\}.$$



Compare ground truth and Fourier reconstruction



3. Introduce new iterative reconstruction (IR) technique

Model the measured spectrum

- Model the measurements as

$$I_{\text{model}}(\sigma) = \frac{1}{2}\left\{S_0(\sigma) + S_1(\sigma)\cos[\phi(\sigma)] + S_2(\sigma)\sin[\phi(\sigma)]\right\}$$

- Actual measurements have noise:

$$I(\sigma) = I_{\text{model}}(\sigma) + n.$$

- Minimize the difference between model and measurements:

$$L(S_0, S_1, S_2) = \frac{1}{2}\int_{\mathbb{R}}\left[\frac{1}{2}\left\{S_0(\sigma) + S_1(\sigma)\cos[\phi(\sigma)] + S_2(\sigma)\sin[\phi(\sigma)]\right\} - I(\sigma)\right]^2 d\sigma$$

Minimize a cost function to find the Stokes parameters

- Derivatives represent how much the signal changes:

$$R_0(S_0) = \int_{\mathbb{R}} \sqrt{|\nabla S_0(\sigma)|^2 + \epsilon}\, d\sigma,$$

- Cost combines the likelihood and regularizer functions:

$$c(S_0, S_1, S_2) = L(S_0, S_1, S_2) + \beta_0 R_0(S_0) + \beta_1 R_1(S_1) + \beta_2 R_2(S_2)$$

- Find Stokes parameters by minimizing cost:

$$\left(\hat{\mathbf{S}}_0, \hat{\mathbf{S}}_1, \hat{\mathbf{S}}_2\right) = \underset{\mathbf{S}_0, \mathbf{S}_1, \mathbf{S}_2 \in \mathbb{R}^N}{\operatorname{argmin}} c(\mathbf{S}_0, \mathbf{S}_1, \mathbf{S}_2).$$

- Minimize each parameter successively:

$$\mathbf{S}_0^{(n+1)} = \underset{\mathbf{S}_0 \in \mathbb{R}^N}{\operatorname{argmin}}\, c\left(\mathbf{S}_0,\ \mathbf{S}_1^{(n)},\ \mathbf{S}_2^{(n)}\right),$$

$$\mathbf{S}_1^{(n+1)} = \underset{\mathbf{S}_1 \in \mathbb{R}^N}{\operatorname{argmin}}\, c\left(\mathbf{S}_0^{(n+1)},\ \mathbf{S}_1,\ \mathbf{S}_2^{(n)}\right),$$

$$\mathbf{S}_2^{(n+1)} = \underset{\mathbf{S}_2 \in \mathbb{R}^N}{\operatorname{argmin}}\, c\left(\mathbf{S}_0^{(n+1)},\ \mathbf{S}_1^{(n+1)},\ \mathbf{S}_2\right).$$

Find S_0 using gradient descent

- Update S_0 by minimizing cost:

$$\mathbf{S}_0 = \underset{\mathbf{S}_0 \in \mathbb{R}^N}{\operatorname{argmin}}\, c\left(\mathbf{S}_0, \mathbf{S}_1, \mathbf{S}_2\right)$$

- Updates for S_1 and S_2 follow a similar procedure

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function UPDATE_S_0 ($\mathbf{S}_0^{\text{in}}, \mathbf{S}_1, \mathbf{S}_2$)

$\quad \mathbf{S}_0^{(0)} \leftarrow \mathbf{S}_0^{\text{in}}$

$\quad i \leftarrow 0$

$\quad$ **while** $c\left(\mathbf{S}_0^{(i)}, \mathbf{S}_1, \mathbf{S}_2\right) > \epsilon$ **do**

$$\mathbf{d} \leftarrow -\nabla_{\mathbf{S}_0} c\left(\mathbf{S}_0, \mathbf{S}_1, \mathbf{S}_2\right)\Big|_{\mathbf{S}_0=\mathbf{S}_0^{(i)}}$$

$$= -\nabla_{\mathbf{S}_0} L\left(\mathbf{S}_0, \mathbf{S}_1, \mathbf{S}_2\right)\Big|_{\mathbf{S}_0=\mathbf{S}_0^{(i)}} - \beta_0 \nabla_{\mathbf{S}_0} R_0\left(\mathbf{S}_0\right)\Big|_{\mathbf{S}_0=\mathbf{S}_0^{(i)}}$$

$\quad\quad \alpha^* \leftarrow \operatorname{argmin}_{\alpha \in \mathbb{R}}\ c\left(\mathbf{S}_0^{(i)} + \alpha\mathbf{d}, \mathbf{S}_1, \mathbf{S}_2\right)$

$\quad\quad \mathbf{S}_0^{(i+1)} \leftarrow \mathbf{S}_0^{(i)} + \alpha^*\mathbf{d}$

$\quad\quad i \leftarrow i + 1$

$\quad$ **end while**

$\quad \mathbf{S}_0^{\text{out}} \leftarrow \mathbf{S}_0^{(i)}$

$\quad$ **return** $\mathbf{S}_0^{\text{out}}$

end function

Optimize for S_0, S_1, and S_2

- Minimize each parameter successively:

$$\mathbf{S}_0^{(n+1)} = \underset{\mathbf{S}_0 \in \mathbb{R}^N}{\operatorname{argmin}}\, c\left(\mathbf{S}_0,\ \mathbf{S}_1^{(n)},\ \mathbf{S}_2^{(n)}\right),$$

$$\mathbf{S}_1^{(n+1)} = \underset{\mathbf{S}_1 \in \mathbb{R}^N}{\operatorname{argmin}}\, c\left(\mathbf{S}_0^{(n+1)},\ \mathbf{S}_1,\ \mathbf{S}_2^{(n)}\right),$$

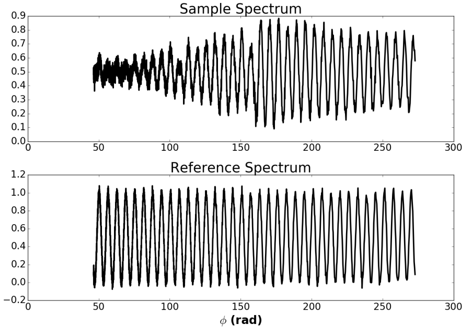
$$\mathbf{S}_2^{(n+1)} = \underset{\mathbf{S}_2 \in \mathbb{R}^N}{\operatorname{argmin}}\, c\left(\mathbf{S}_0^{(n+1)},\ \mathbf{S}_1^{(n+1)},\ \mathbf{S}_2\right).$$

Initialize $\mathbf{S}_0^{(0)}$, $\mathbf{S}_1^{(0)}$, $\mathbf{S}_2^{(0)}$ by Fourier reconstruction.
$n \leftarrow 0$
while $c\left(\mathbf{S}_0^{(n)}, \mathbf{S}_1^{(n)}, \mathbf{S}_2^{(n)}\right) > \epsilon$ **do**
    $\mathbf{S}_0^{(n+1)} \leftarrow$ UPDATE_S_0 ($\mathbf{S}_0^{(n)}, \mathbf{S}_1^{(n)}, \mathbf{S}_2^{(n)}$)
    $\mathbf{S}_1^{(n+1)} \leftarrow$ UPDATE_S_1 ($\mathbf{S}_0^{(n+1)}, \mathbf{S}_1^{(n)}, \mathbf{S}_2^{(n)}$)
    $\mathbf{S}_2^{(n+1)} \leftarrow$ UPDATE_S_2 ($\mathbf{S}_0^{(n+1)}, \mathbf{S}_1^{(n+1)}, \mathbf{S}_2^{(n)}$)
    $n \leftarrow n + 1$
end while
$\mathbf{S}_0 \leftarrow \mathbf{S}_0^{(n)}$
$\mathbf{S}_1 \leftarrow \mathbf{S}_1^{(n)}$
$\mathbf{S}_2 \leftarrow \mathbf{S}_2^{(n)}$
return $\mathbf{S}_0, \mathbf{S}_1, \mathbf{S}_2$

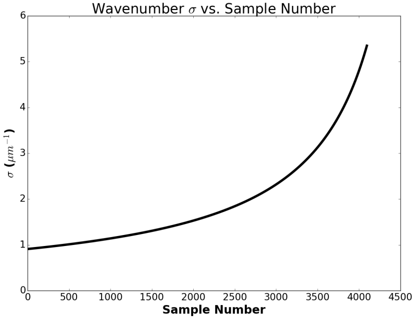
4. Compare FR and IR techniques

Input simulated spectrum into the IR algorithm

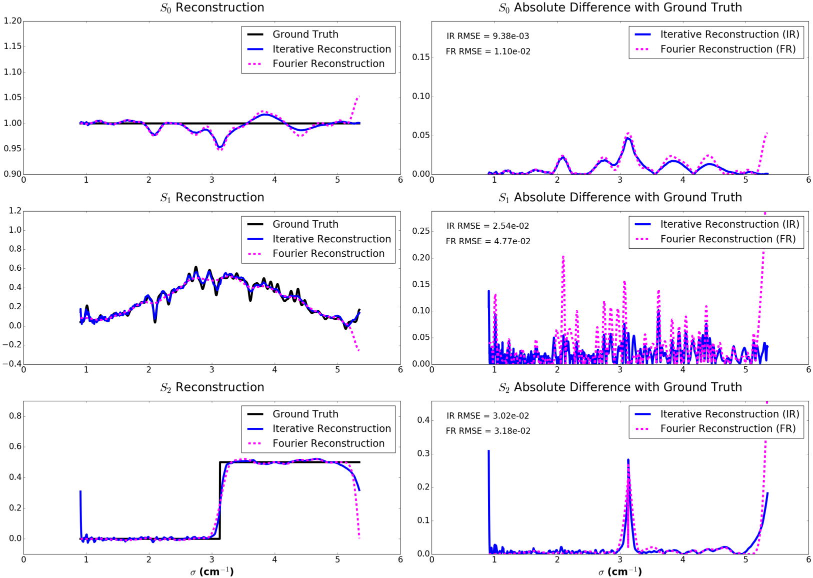
- Calculate S_0, S_1, S_2 from the simulated spectrum



- Input spectrum is non-uniformly spaced in wavenumber

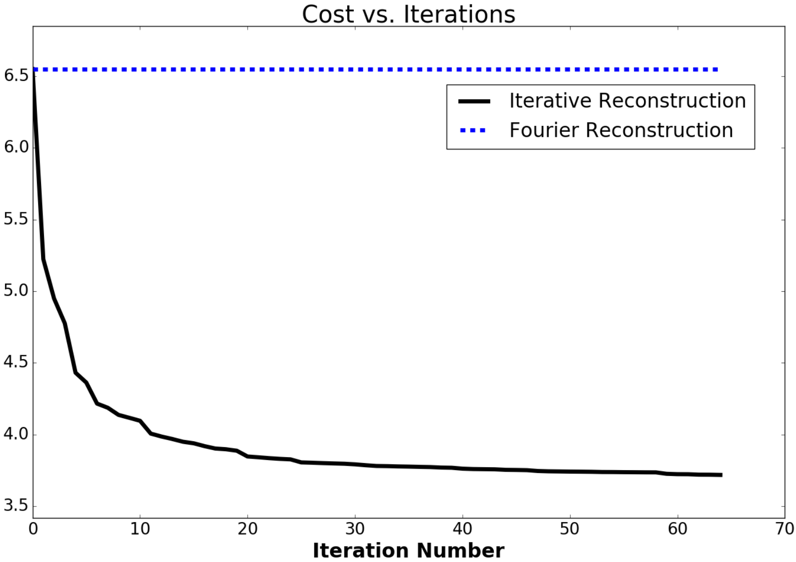


IR is closer to ground truth



Cost shows convergence

- Measure cost after each iteration

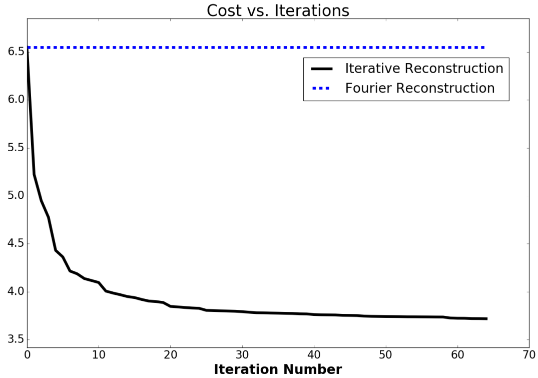


5. Summary

Iterative reconstruction offers many advantages

Advantages:

- Reconstruct signals with more bandwidth
- Reduce errors from vibration noise and thermal fluctuations
- Process samples that are non-uniformly spaced in wavenumber
- Track progress through minimization of a cost function



Acknowledgements

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