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Reliable analog quantum simulation

or: how I learnt to stop worrying and love parameter space compression

Mohan Sarovar
Digital & Quantum Information Systems
Sandia National Laboratories, Livermore

Quantum and Nano Control Workshop, Institute for Mathematics and Its Applications, April 2016

Reliability of analog quantum simulation,
M. Sarovar, J. Zhang, L. Zeng. arXiv:1603.09283

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Analog quantum simulation

- Emulate one quantum system by engineering another to mimic it
- Feynman's original idea for *quantum supremacy*
  Feynman, Int. J. Theor. Phys. **21**, 467 (1982)
- Already an experimental reality, especially using AMO systems

nature physics ARTICLES
PUBLISHED ONLINE: 19 FEBRUARY 2012 | DOI: 10.1038/NPHYS2232

Probing the relaxation towards equilibrium in an isolated strongly correlated one-dimensional Bose gas

S. Trotzky[1,2,3]★, Y-A. Chen[1,2,3], A. Flesch[4]★, I. P. McCulloch[5], U. Schollwöck[1,6], J. Eisert[6,7,8] and I. Bloch[1,2,3]

ARTICLE
doi:10.1038/nature09994

Quantum simulation of antiferromagnetic spin chains in an optical lattice

Jonathan Simon[1], Waseem S. Bakr[1], Ruichao Ma[1], M. Eric Tai[1], Philipp M. Preiss[1] & Markus Greiner[1]

nature Vol 465|3 June 2010|doi:10.1038/nature09071
LETTERS

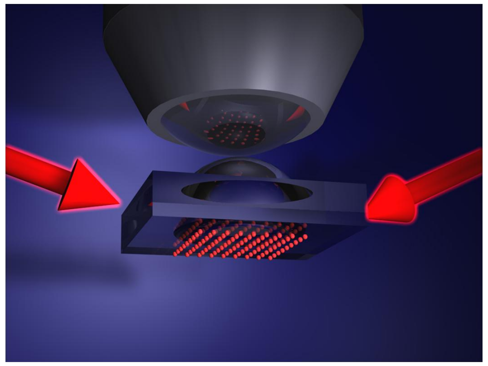
Quantum simulation of frustrated Ising spins with trapped ions

K. Kim[1], M.-S. Chang[1], S. Korenblit[1], R. Islam[1], E. E. Edwards[1], J. K. Freericks[2], G.-D. Lin[3], L.-M. Duan[3] & C. Monroe[1]

New Journal of Physics
The open-access journal for physics

Quantum simulation of the transverse Ising model with trapped ions

K Kim[1,5,6], S Korenblit[1], R Islam[1], E E Edwards[1], M-S Chang[1,7], C Noh[2,8], H Carmichael[2], G-D Lin[3,9], L-M Duan[3], C C Joseph Wang[4], J K Freericks[4] and C Monroe[1]



Greiner Lab, Harvard

Analog quantum simulation

Analog quantum simulation has no natural notion of error correction

Rep. Prog. Phys. **75** (2012) 082401 (18pp) doi:10.1088/0034-4885/75/8/082401

Can one trust quantum simulators?

**Philipp Hauke[1], Fernando M Cucchietti[1,2], Luca Tagliacozzo[1],
Ivan Deutsch[3,4] and Maciej Lewenstein[1,5]**

How does one assess when an analog quantum simulation result is robust to noise/imperfections?

Idealized analog quantum simulation

e.g. Simulation of static properties

$$H_{\text{model}} \qquad \sum_i B_i \sigma_x^i + J_i \sigma_z^i \sigma_z^{i+1}$$

$$H_{\text{exp}} \qquad \sum_i B_i \sigma_x^i + J_i \sigma_z^i \sigma_z^{i+1}$$

Ground state properties

$$|\Psi_0\rangle$$

$$\langle O \rangle_0 = \langle \Psi_0 | O | \Psi_0 \rangle$$

Thermal properties

$$\rho_{\text{th}}(T)$$

$$\rho_{\text{th}}(T) = \frac{e^{\beta H_{\text{exp}}}}{\mathcal{Z}}$$

$$\beta = \frac{1}{k_B T}$$

$$\langle O \rangle_T = \text{tr}\left(O \rho_{\text{th}}(T)\right)$$

Realistic analog quantum simulation

e.g. Simulation of static properties

$$H_{\text{model}} \qquad \sum_i B_i \sigma_x^i + J_i \sigma_z^i \sigma_z^{i+1}$$

$$H_{\text{exp}} \qquad \sum_i B_i \sigma_x^i + J_i \sigma_z^i \sigma_z^{i+1} + \sum_i \epsilon_i \sigma_x^i + \sum_{ij} \delta J_{ij} \sigma_z^i \sigma_z^j$$

PHYSICAL REVIEW E **74**, 031123 (2006)

Ground state overlap and quantum phase transitions

Paolo Zanardi and Nikola Paunković*

Ground state properties

$$|\Psi_0\rangle \neq |\Phi_0\rangle$$

$$\langle O \rangle_0 + \langle \delta O \rangle$$

Thermal properties

$$\sigma_{\text{th}}(T) \neq \rho_{\text{th}}(T)$$

$$\langle O \rangle_T + \langle \delta O \rangle$$

Realistic analog quantum simulation

e.g. Simulation of static properties

$\langle \delta O \rangle$ small?

Physicist Intuition: for many questions we "care about" (coarse-grained observables) microscopic imperfections in the model (Hamiltonian) should not matter.

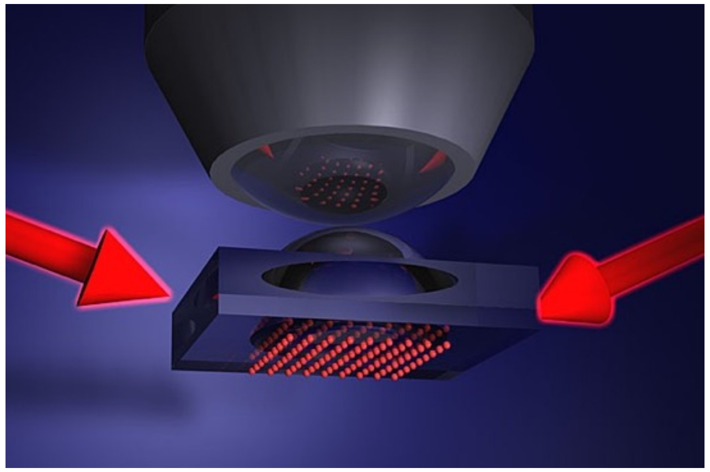
Intuition comes from condensed matter physics: renormalization flow, universality classes, etc.

Aims:

1. To formalize this intuition
2. To develop tools to identify when this intuition is correct and when it is not

Quantifying parameter sensitivity

A quantum simulator produces parameterized probability distributions



Greiner Lab, Harvard

$$\{p_m\}_m$$

Ideal many-body Hamiltonian dependent on K parameters

$$H(\lambda) = \sum_{k=1}^{K} \lambda_k H_k \qquad \lambda = (\lambda_1, \lambda_2, ..., \lambda_K)$$

Observable of interest

$$O = \sum_m \theta_m P_m$$

Produces a parameterized probability distribution

$$p_m(\lambda) = \mathrm{tr}\left(P_m \rho_{\mathrm{th}}(T)\right)$$

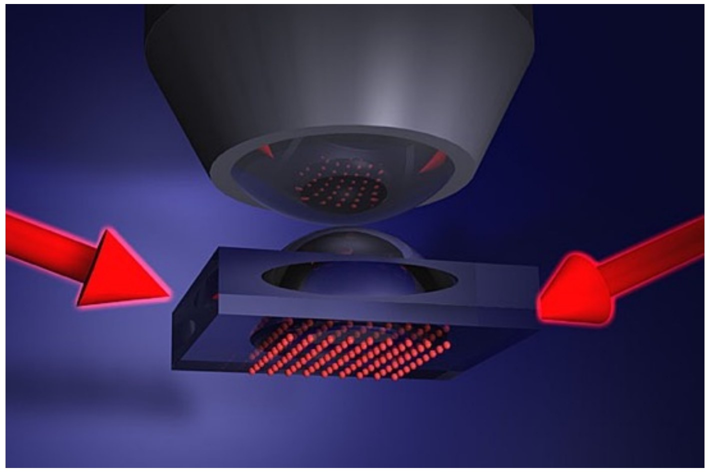
Or ground state

$$\rho_{\mathrm{th}}(T) = \frac{e^{\beta H(\lambda)}}{\mathcal{Z}}$$

$$\beta = \frac{1}{k_B T}$$

Quantifying parameter sensitivity

A quantum simulator produces parameterized probability distributions



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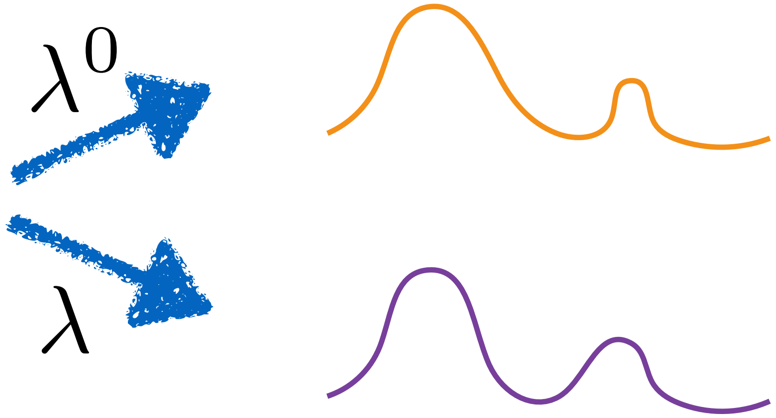
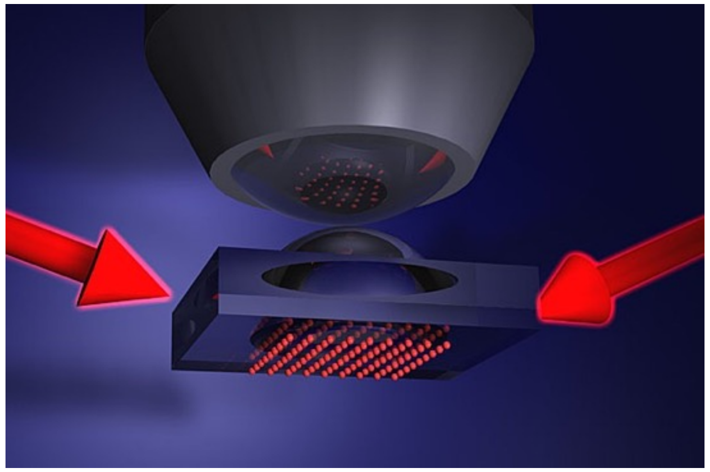


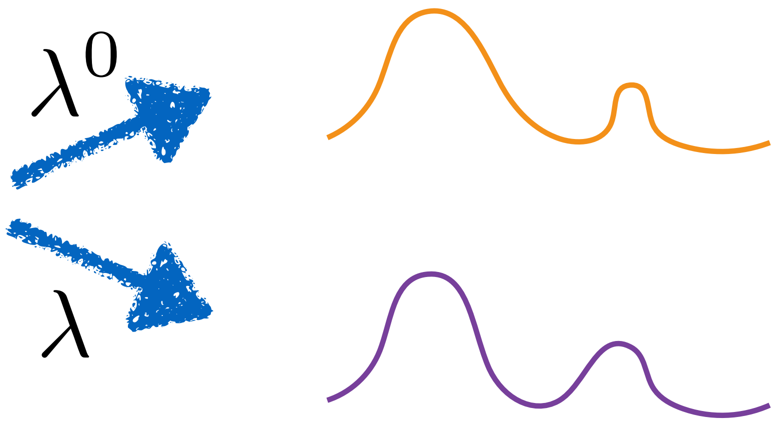
Robustness: how different are the output distributions under perturbations of the Hamiltonian parameters?

Quantifying parameter sensitivity

A quantum simulator produces parameterized probability distributions



Greiner Lab, Harvard



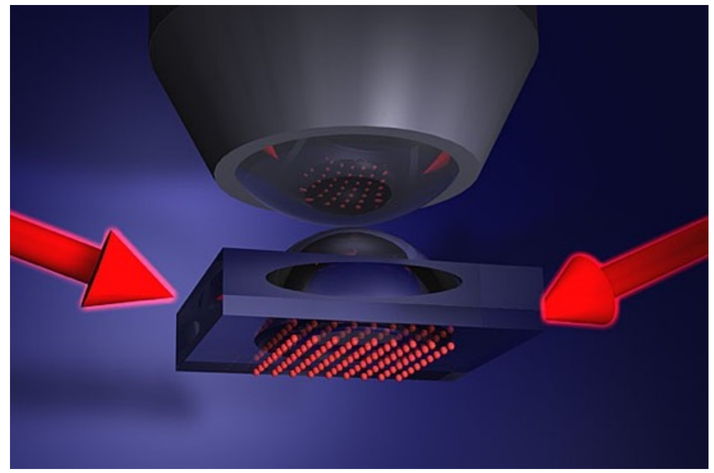
Robustness: how different are the output distributions under perturbations of the Hamiltonian parameters?

Kullback-Leibler divergence: a measure of difference between probability distributions

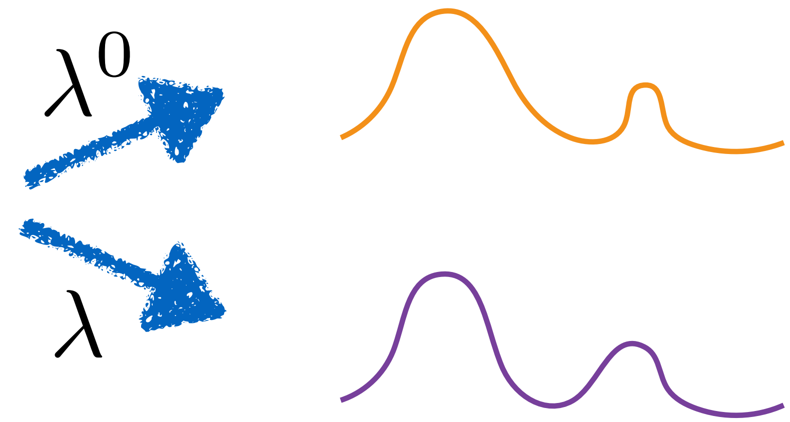
$$D_{\mathrm{KL}}(p(\lambda)||p(\lambda^0)) = \sum_m p_m(\lambda) \log \frac{p_m(\lambda)}{p_m(\lambda^0)}$$

Quantifying parameter sensitivity

A quantum simulator produces parameterized probability distributions



Greiner Lab, Harvard



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$$D_{\mathrm{KL}}(p(\lambda)||p(\lambda^0)) = \sum_m p_m(\lambda) \log \frac{p_m(\lambda)}{p_m(\lambda^0)}$$

$$= \frac{1}{2}\Delta\lambda^{\mathsf{T}} F(\lambda^0)\Delta\lambda + \mathcal{O}(\|\Delta\lambda\|^3)$$ (small deviations)

Fisher information matrix (FIM)

$$\Delta\lambda = \lambda - \lambda^0$$

Quantifying parameter sensitivity

Fisher Information matrix (FIM)

$$F_{ij}(\lambda^0) = \sum_{m=1}^{M} \frac{1}{p_m(\lambda)} \frac{\partial p_m(\lambda)}{\partial \lambda_i} \frac{\partial p_m(\lambda)}{\partial \lambda_j} \Bigg|_{\lambda=\lambda^0}$$

Quantifies influence of parameters on observations

Quantifying parameter sensitivity

Fisher Information matrix (FIM)

$$F_{ij}(\lambda^0) = \sum_{m=1}^{M} \frac{1}{p_m(\lambda)} \frac{\partial p_m(\lambda)}{\partial \lambda_i} \frac{\partial p_m(\lambda)}{\partial \lambda_j}\bigg|_{\lambda=\lambda^0}$$

Quantifies influence of parameters on observations

Spectral analysis of the FIM

$$F = \sum_{k=1}^{K} \zeta_k v_k v_k^\dagger$$

Eigendecomposition of FIM

$$D_{\mathrm{KL}}(p(\lambda)||p(\lambda^0) \approx \frac{1}{2}\Delta\lambda^{\mathsf{T}} F(\lambda^0)\Delta\lambda \;= \sum_{k=1}^{K} \frac{\zeta_k}{2} \|v_k^\dagger \Delta\lambda\|^2$$

Quantum simulation of this model is *trivially robust* if all $\zeta_k \approx 0$

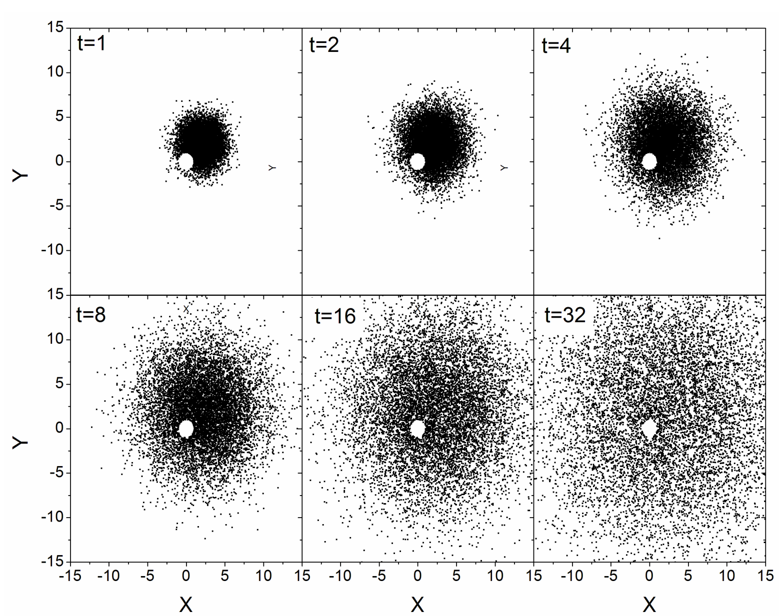
Parameter space compression (PSC)

Sethna group, Cornell University

Why the unreasonable effectiveness of simple models in physics?

Parameter Space Compression Underlies Emergent Theories and Predictive Models
Benjamin B. Machta *et al.*
Science **342**, 604 (2013);
DOI: 10.1126/science.1238723

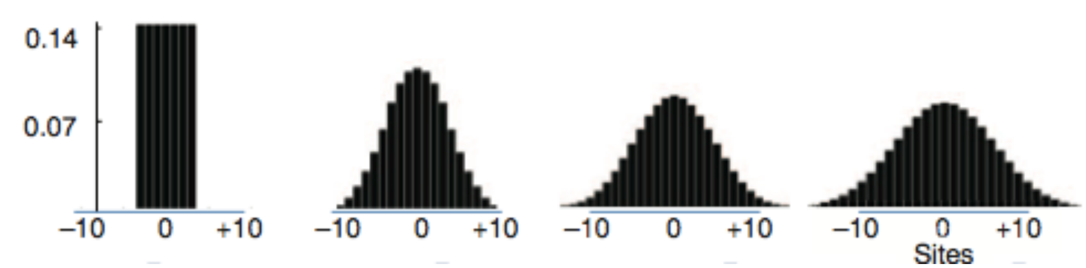
e.g. particle diffusion



Plante & Cucinotta

Equation for particle density

$$\frac{\partial}{\partial t}\rho(x,t) = D\frac{\partial^2}{\partial x^2}\rho(x,t) - V\frac{\partial}{\partial x}\rho(x,t) + R\rho(x,t)$$



Only 3 parameters!

PSC and Fisher Information

Model dependent on K parameters $\lambda = (\lambda_1, \lambda_2, ..., \lambda_K)$

Parametrized probability distribution over measurement outcomes $\{p_m(\lambda)\}$

Fisher Information matrix (FIM)

$$F_{ij}(\lambda^0) = \sum_{m=1}^{M} \frac{1}{p_m(\lambda)} \frac{\partial p_m(\lambda)}{\partial \lambda_i} \frac{\partial p_m(\lambda)}{\partial \lambda_j}\Bigg|_{\lambda=\lambda^0}$$

$$F = \sum_{k=1}^{K} \zeta_k v_k v_k^\dagger$$

Key observation:

Eigenvalues of FIM prescribe an ordering of parameter influence, *i.e.* if ζ_k is large, then the parameter

$$\sum_j \lambda_j v_k^j$$

has a large influence on observations

PSC and Fisher Information

Model dependent on K parameters $\lambda = (\lambda_1, \lambda_2, ..., \lambda_K)$

Parametrized probability distribution over measurement outcomes $\{p_m(\lambda)\}$

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$$F_{ij}(\lambda^0) = \sum_{m=1}^{M} \frac{1}{p_m(\lambda)} \frac{\partial p_m(\lambda)}{\partial \lambda_i} \frac{\partial p_m(\lambda)}{\partial \lambda_j} \Big|_{\lambda=\lambda^0}$$

$$F = \sum_{k=1}^{K} \zeta_k v_k v_k^\dagger$$

Key observation:

Another way to think about this:

Sensitivity analysis — FIM is the expected value of the Hessian:

$$F_{ij} = -\left\langle \frac{\partial^2 \log(p_m(\lambda))}{\partial \lambda_i \partial \lambda_j} \right\rangle$$

Parameter space compression

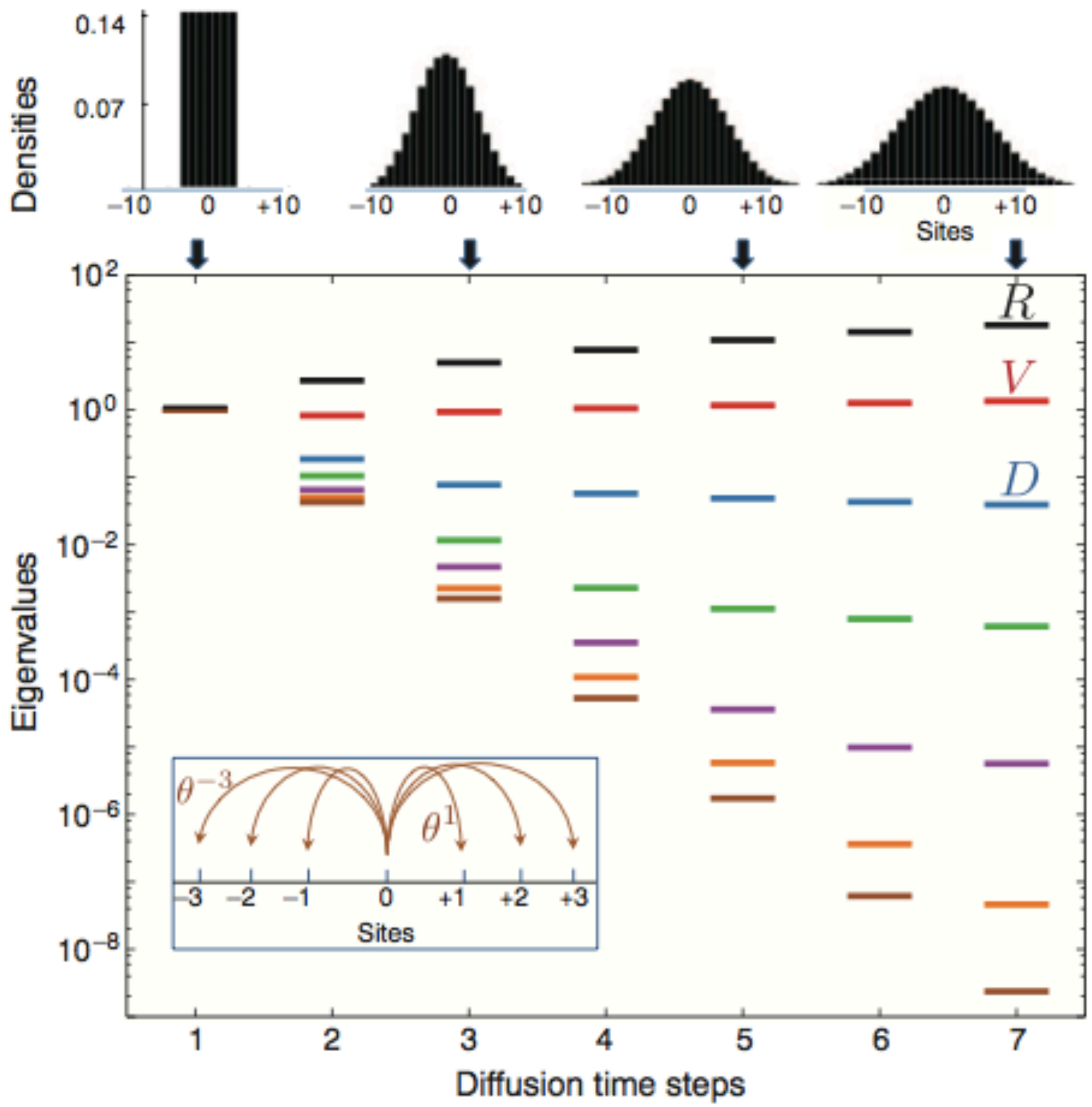
e.g. particle diffusion

Machta et al. Science, 342, 604 (2013)

Equation for particle density

$$\frac{\partial}{\partial t}\rho(x,t) = D\frac{\partial^2}{\partial x^2}\rho(x,t) - V\frac{\partial}{\partial x}\rho(x,t) + R\rho(x,t)$$

Discrete time/space diffusion in 1D



Observable here is the full particle density.

If we ask what the particle density is at every time step, then eigenvalues are clustered.

But if we only ask every few time steps, then eigenvalues separate, and a few important parameters emerge, and these are R, V, D.

Parameter space compression

e.g. particle diffusion

Machta et al. Science, 342, 604 (2013)

Equation for particle density

$$\frac{\partial}{\partial t}\rho(x,t) = D\frac{\partial^2}{\partial x^2}\rho(x,t) - V\frac{\partial}{\partial x}\rho(x,t) + R\rho(x,t)$$

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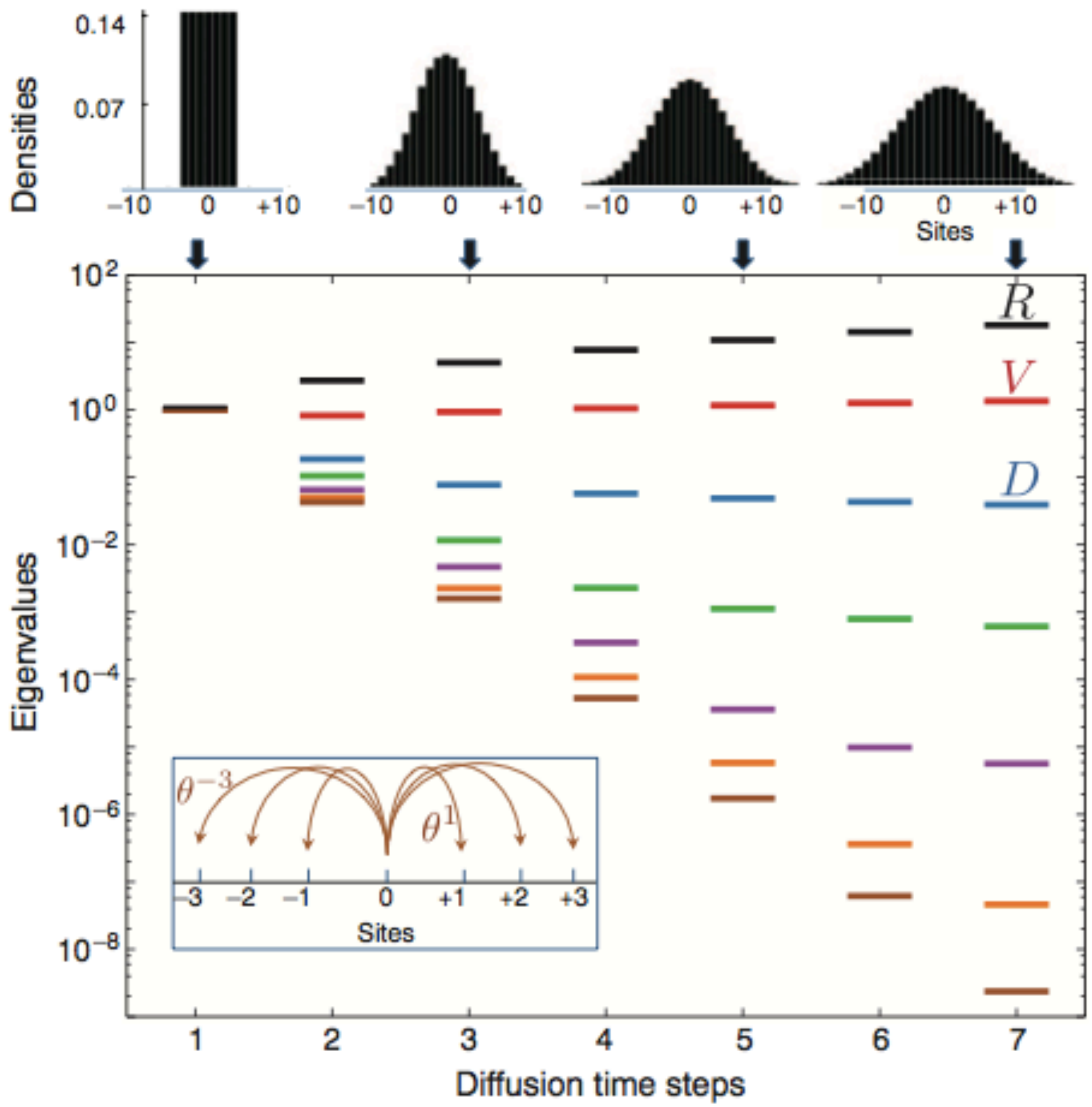


Some terminology

Stiff parameters

Sloppy parameters

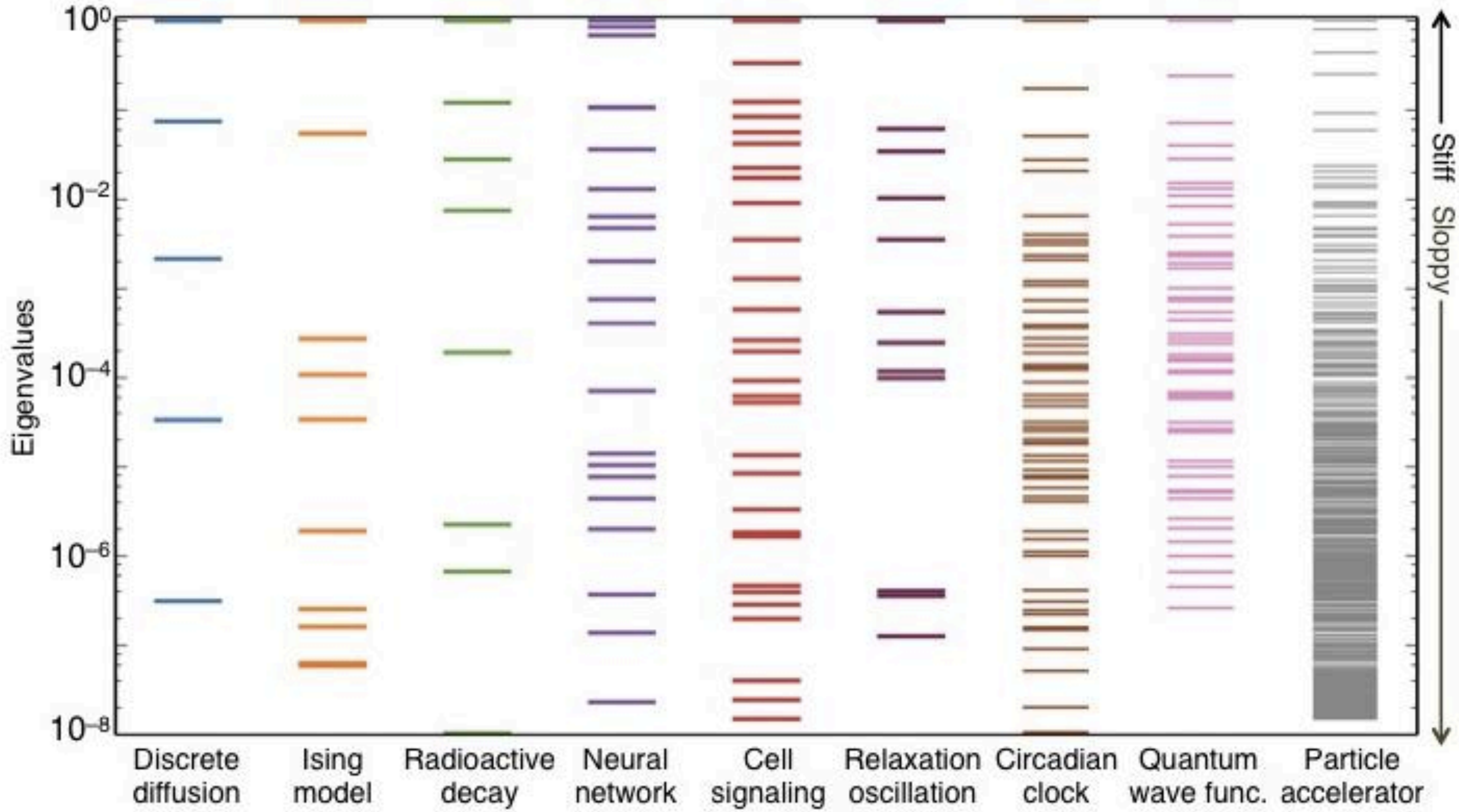
Diffusion is a sloppy model

Parameter space compression

Sloppiness abounds!

This phenomenon has been demonstrated for many models in physics and biophysics.

Models of nature tend to be sloppy, and this is why science is even possible



From the Sethna group website

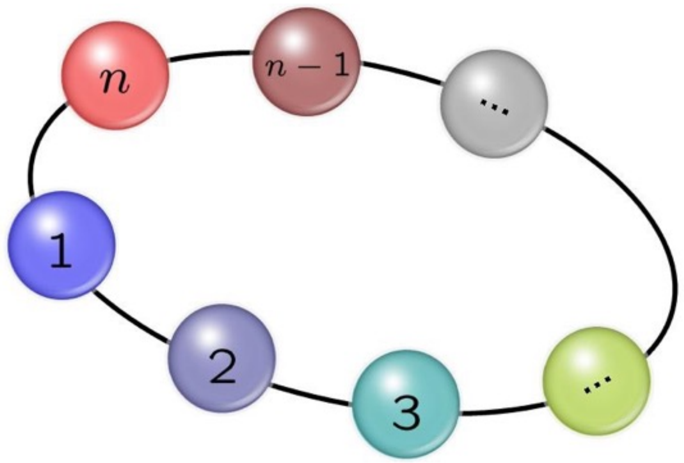
PSC and quantum simulation

Main point

PSC and model sloppiness are prerequisites for *non-trivial* robustness of quantum simulation

Are the quantum many-body models that people are interested in simulating sloppy?

Transverse-field Ising model



$$H_{\text{1D Ising}} = \sum_{i=1}^{n} B_i \sigma_x^i + \sum_{i=1}^{n-1} J_i \sigma_z^i \sigma_z^{i+1} + J_n \sigma_z^n \sigma_z^1$$

Uniform antiferromagnetic model parameter regime, so we want:

$$B_i = B^0 \qquad \forall i$$

$$J_i = J^0 > 0 \qquad \forall i$$

Quantum critical point at

$$B^0 = \frac{J^0}{2}$$

Observables of interest

$$S_z = \sum_i \langle \sigma_z^i \rangle$$

Net magnetization

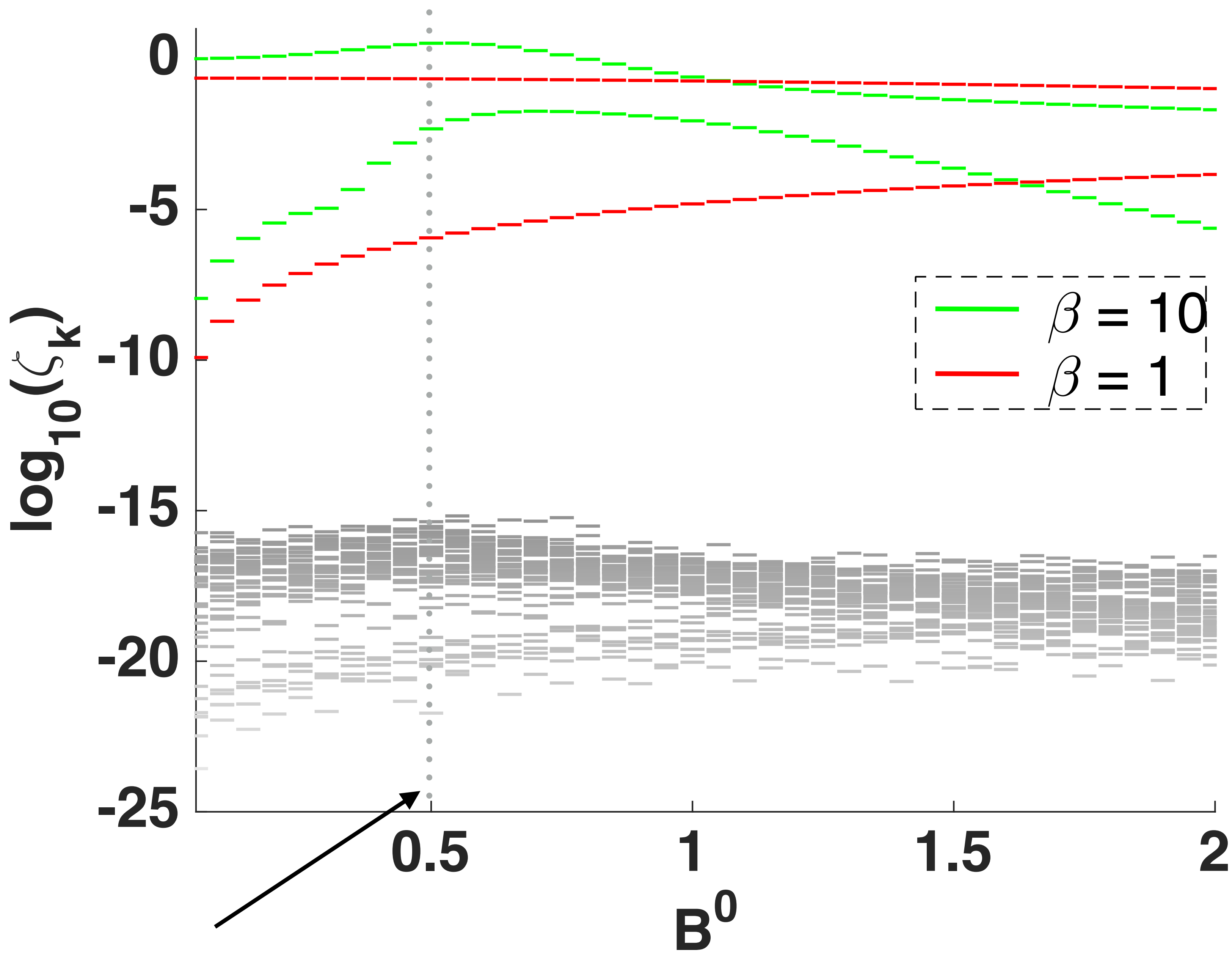
$$C_z(i,j) = \langle \sigma_z^i \sigma_z^j \rangle$$

Correlation functions

Transverse-field Ising model

Net magnetization observable

$n = 10, J^0 = 1$



Stiff parameters

Sloppy parameters

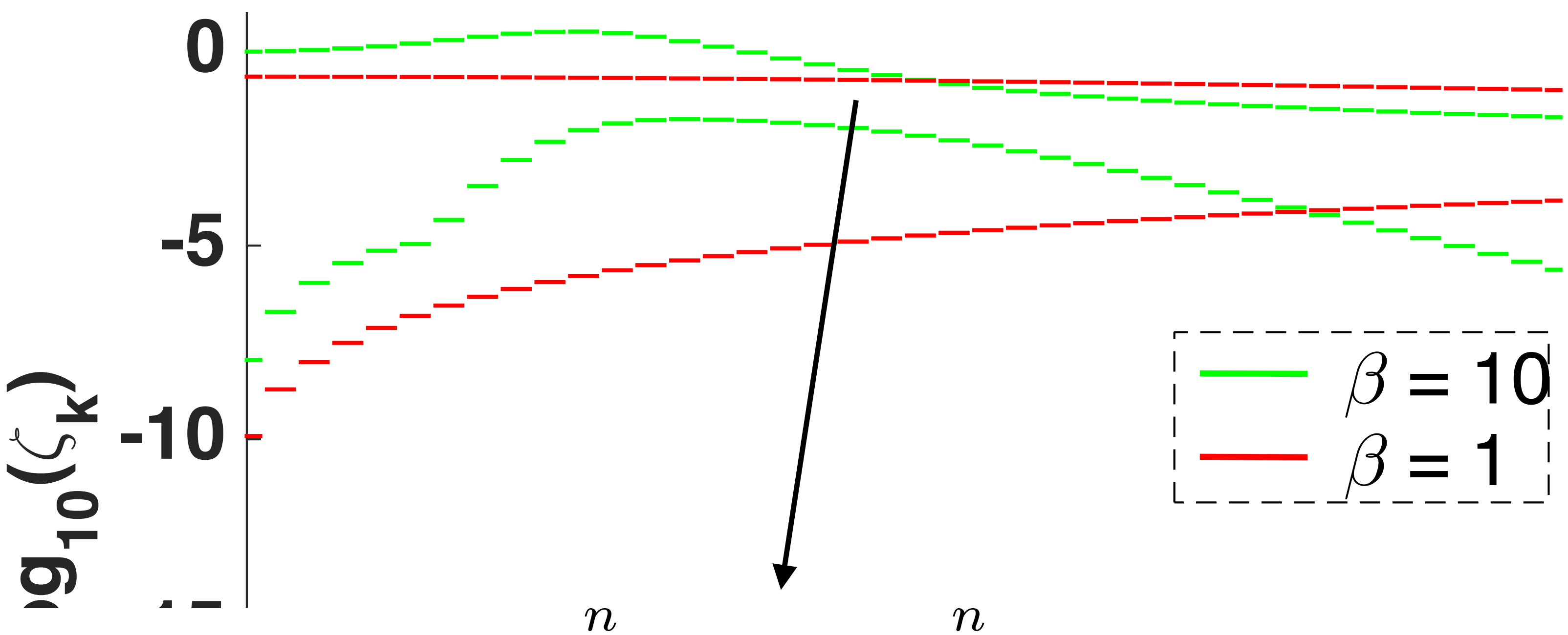
Quantum critical point

Transverse-field Ising model

Net magnetization observable

$n = 10, J^0 = 1$

What is the dominant stiff parameter?



Stiff parameters

$$a_1 \sum_{i=1}^{n} B_i + a_2 \sum_{i=1}^{n} J_i$$

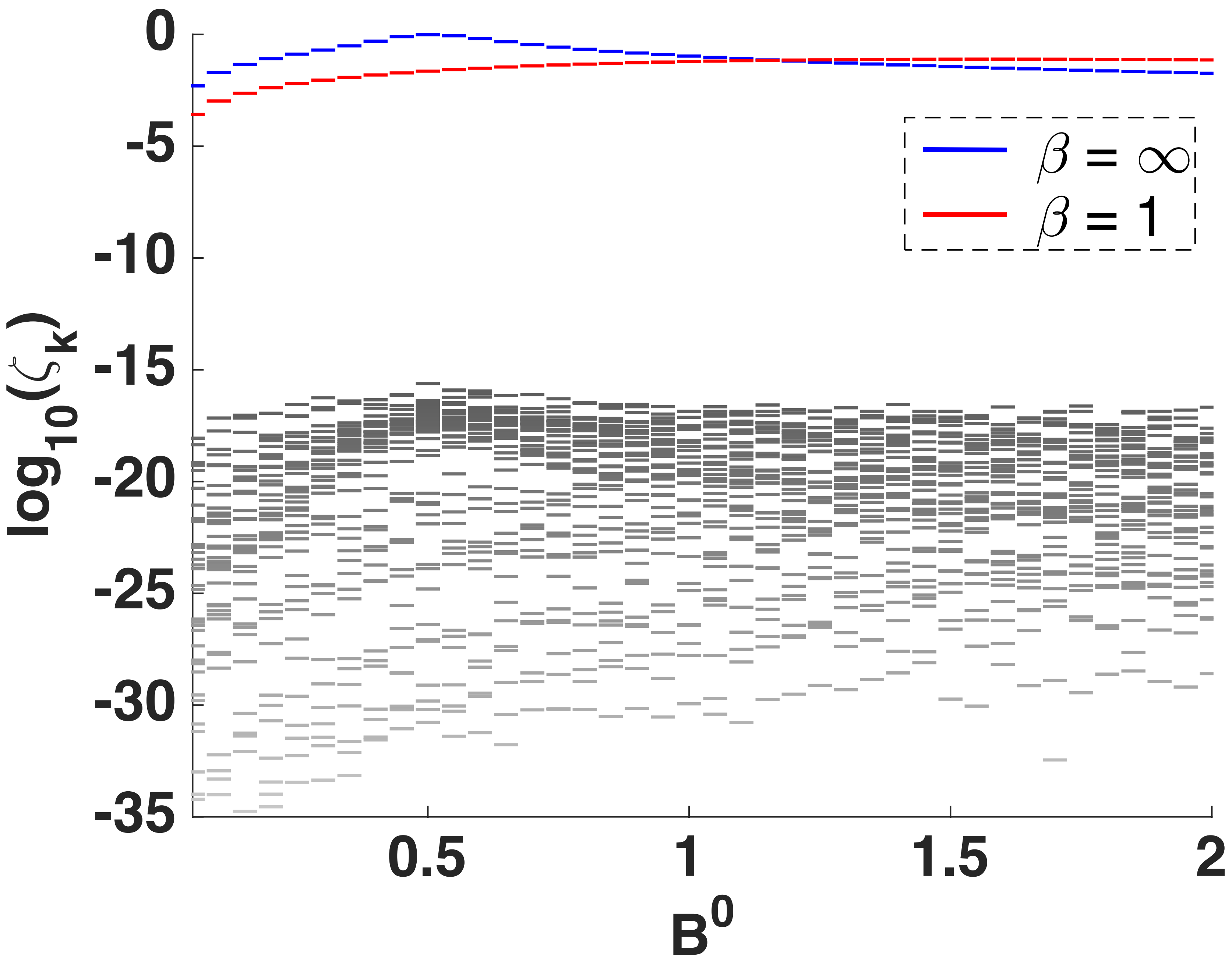
Fluctuations in B_i and J_i do not matter, as long as these (spatially) average to zero

Transverse-field Ising model

Correlation function observable

$C_z(2,6)$

$n = 10, J^0 = 1$



Stiff parameter

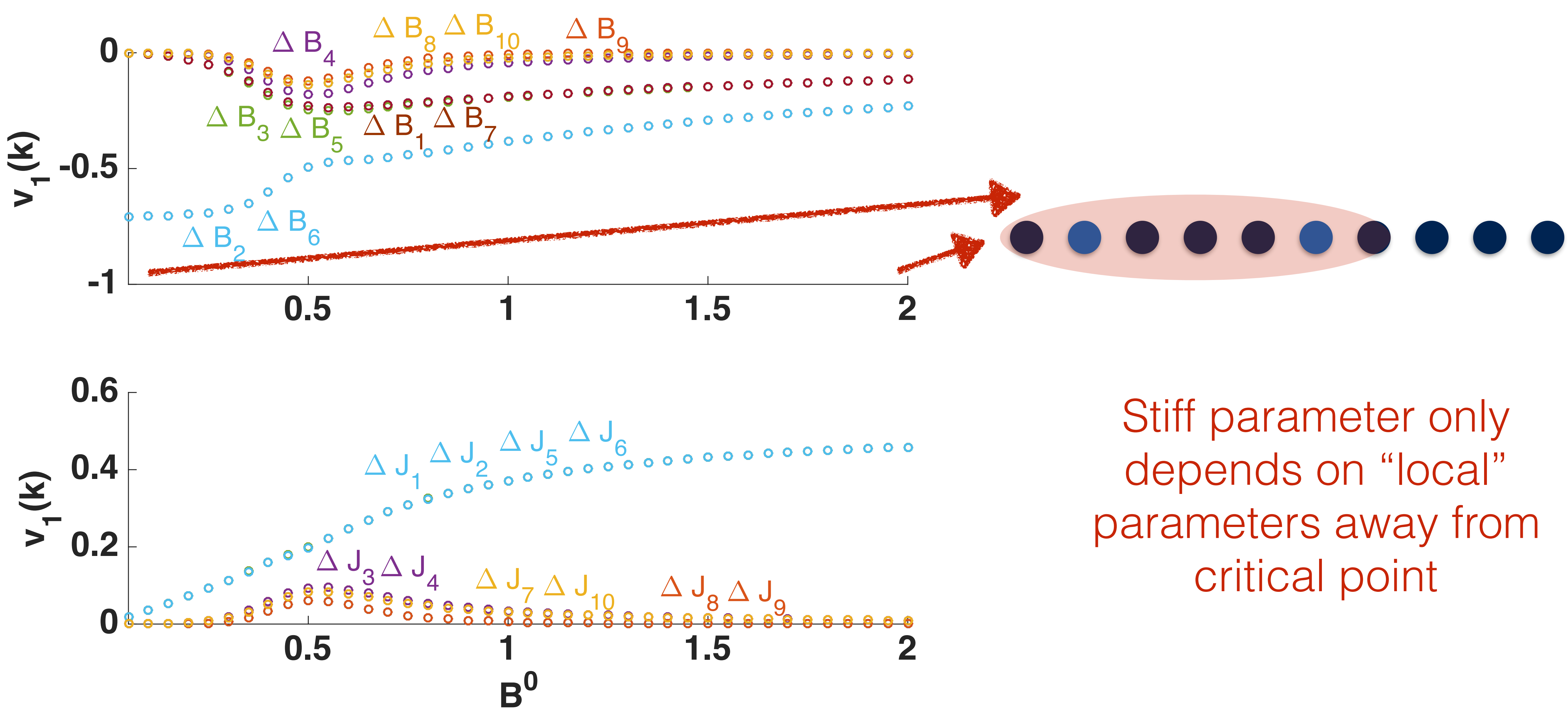
Sloppy parameters

Transverse-field Ising model

Correlation function observable

$$C_z(2,6) \qquad n = 10, J^0 = 1$$

Composition of stiff parameter



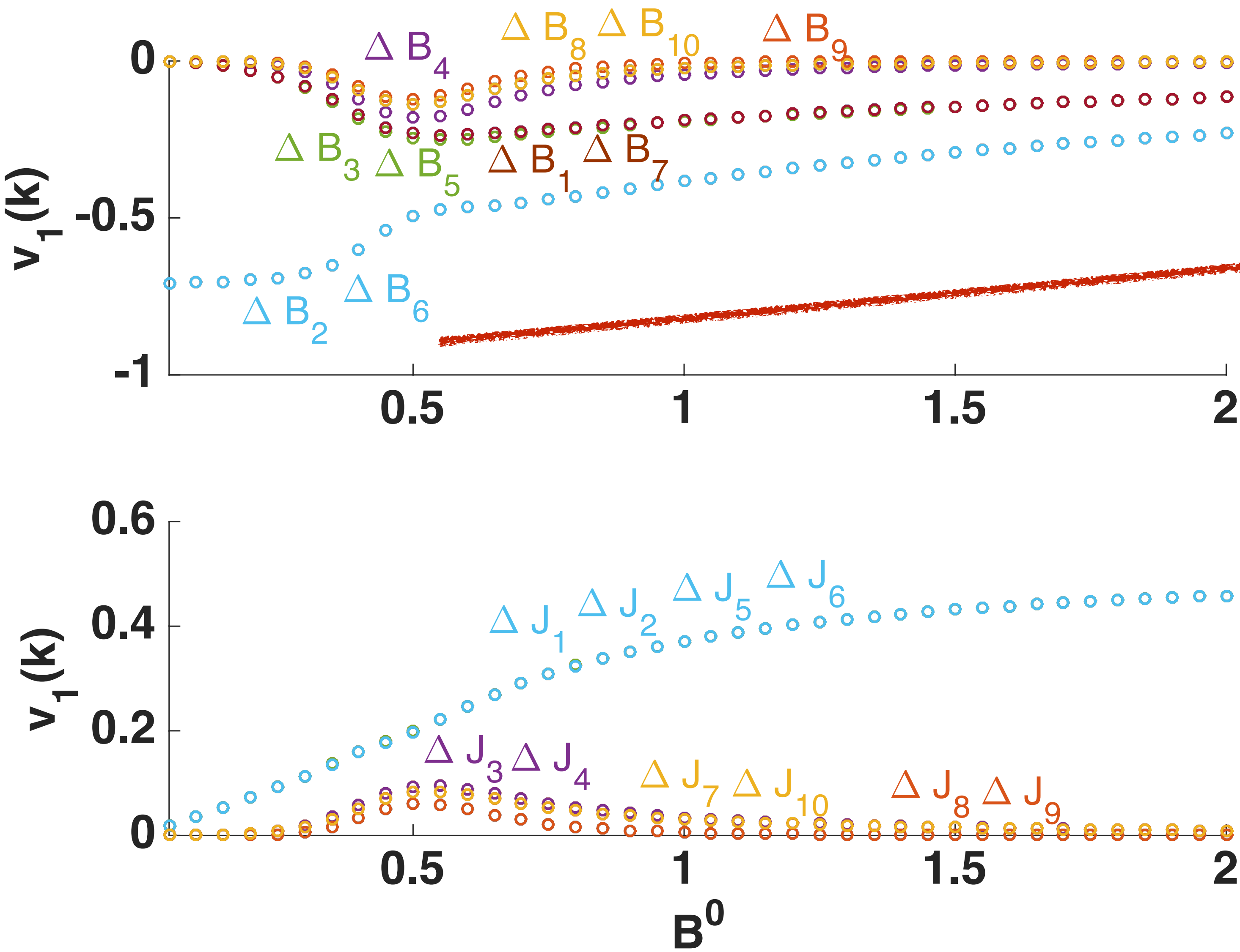
Stiff parameter only depends on "local" parameters away from critical point

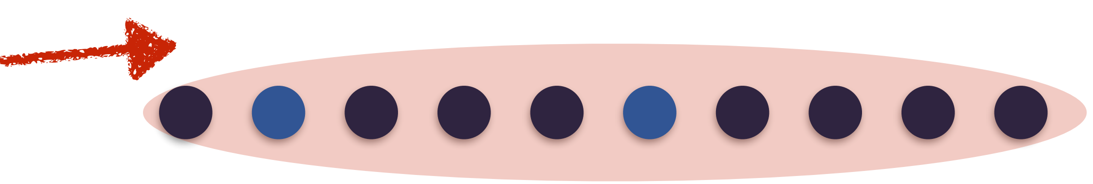
Transverse-field Ising model

Correlation function observable

$C_z(2,6)$ $\quad n = 10, J^0 = 1$

Composition of stiff parameter





Around the critical point, the stiff parameter is a complex linear combination of all the parameters (c.f. collective phenomena, correlations depend on the whole system)

Why so sloppy?

Fisher Information matrix (FIM)

$$F_{ij}(\lambda^0) = \sum_{m=1}^{M} \frac{1}{p_m(\lambda)} \frac{\partial p_m(\lambda)}{\partial \lambda_i} \frac{\partial p_m(\lambda)}{\partial \lambda_j}\bigg|_{\lambda=\lambda^0}$$

Write as:

$$F = V \Lambda^{-1} V^{\mathsf{T}}$$

$$\underset{K \times M}{V} = \begin{pmatrix} \frac{\partial p_1(\lambda)}{\partial \lambda_1} & \frac{\partial p_2(\lambda)}{\partial \lambda_1} & \cdots & \frac{\partial p_M(\lambda)}{\partial \lambda_1} \\ \vdots & & & \vdots \\ \frac{\partial p_1(\lambda)}{\partial \lambda_K} & & \cdots & \frac{\partial p_M(\lambda)}{\partial \lambda_K} \end{pmatrix}$$

$$\underset{M \times M}{\Lambda} = \begin{pmatrix} p_1(\lambda) & & \\ & \ddots & \\ & & p_M(\lambda) \end{pmatrix}$$

Why so sloppy?

Fisher Information matrix (FIM)

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Use rank as a proxy for sloppiness

Λ is full-rank, therefore $\mathrm{rank}(F) \leq \mathrm{rank}(V)$

Why so sloppy?

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What dictates the rank of V?

Quantum simulation model symmetries

Let G be the group of symmetries of the quantum simulation model, *i.e.*

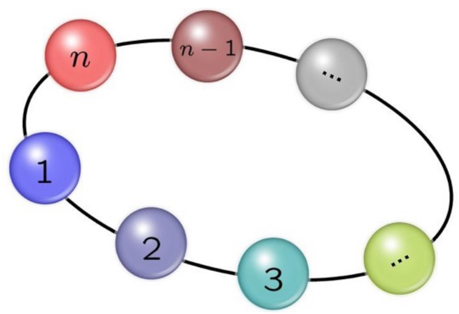
$$[U_g, H(\lambda)] = 0$$

$$[U_g, O] = 0 \qquad \forall g$$

$\{U_g\}_g$ a faithful unitary representation of G

e.g.

Translational invariance



Suppose

$$U_g H_k U_g^\dagger = H_j$$

e.g.

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Recall

$$H(\lambda) = \sum_{k=1}^{K} \lambda_k H_k$$

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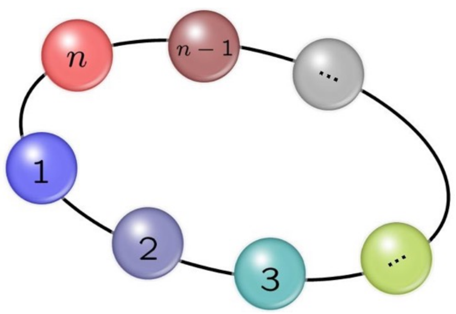

Suppose

$$U_g H_k U_g^\dagger = H_j$$

Then

$$\frac{\partial p_m(\lambda)}{\partial \lambda_k} = \frac{\partial p_m(\lambda)}{\partial \lambda_j} \qquad \forall m$$

Recall

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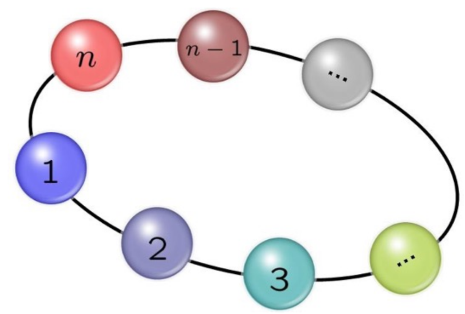

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$$\frac{\partial p_m(\lambda)}{\partial \lambda_k} = \frac{\partial p_m(\lambda)}{\partial \lambda_j} \qquad \forall m$$

$$\begin{pmatrix} \frac{\partial p_1(\lambda)}{\partial \lambda_1} & \frac{\partial p_2(\lambda)}{\partial \lambda_1} & \cdots & \frac{\partial p_M(\lambda)}{\partial \lambda_1} \\ \vdots & & & \vdots \\ \frac{\partial p_1(\lambda)}{\partial \lambda_K} & & \cdots & \frac{\partial p_M(\lambda)}{\partial \lambda_K} \end{pmatrix}$$

Repeated rows in V

Quantum simulation model symmetries

Model symmetries reduce rank of V, and hence of F

Algorithm for computing an upper bound on the rank of F:

1. For all 1<= k <= K, compute the orbit of H_k under the symmetry group

Orbit: $\{U_g H_k U_g^\dagger | g \in G\}$

Recall

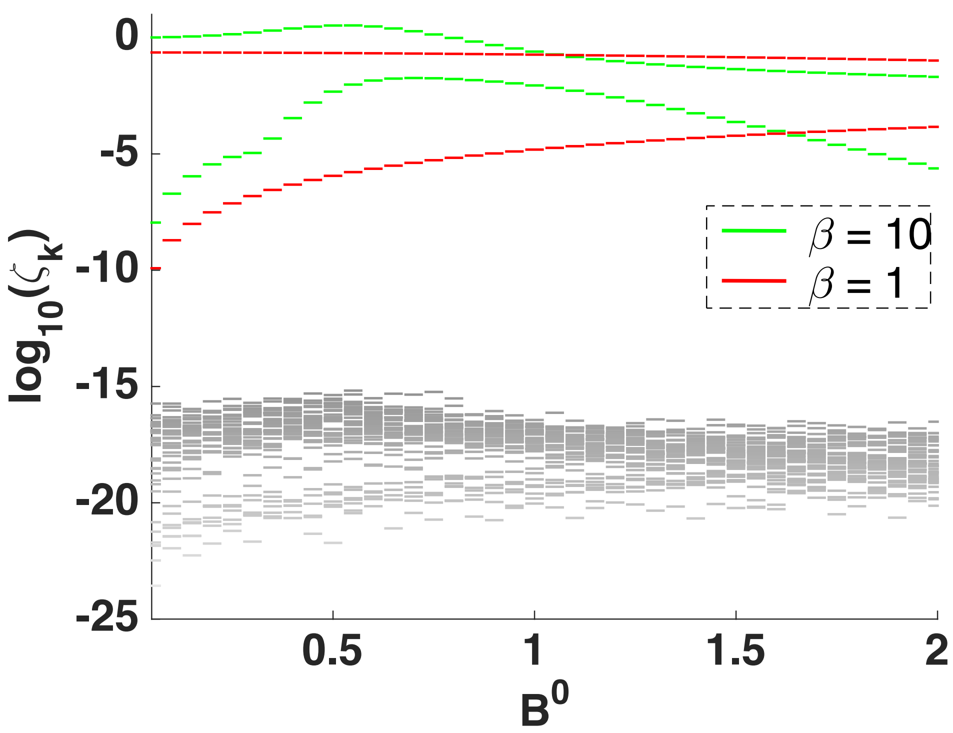
$$H(\lambda) = \sum_{k=1}^{K} \lambda_k H_k$$

2. Number of distinct orbits will be the number of unique rows in V, and therefore a bound on the rank of V and F

Such a symmetry analysis allows us to extract the form of the stiff/influential parameter as well

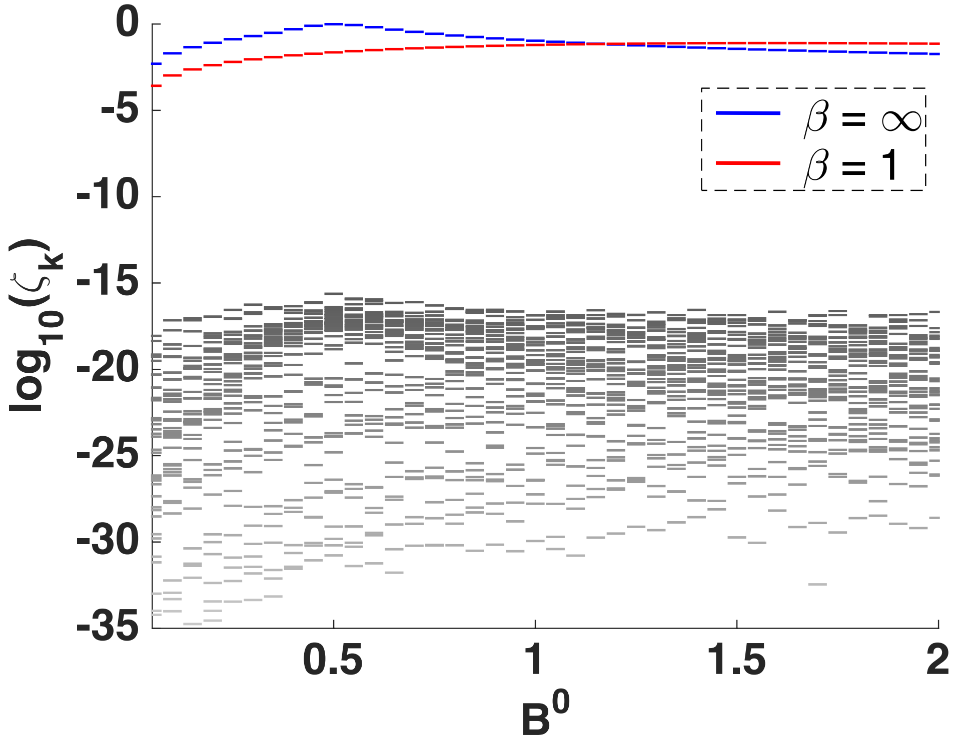
Transverse-field Ising model *revisited*

Net magnetization observable



Rank(F) <= 2

Correlation function observable



Rank(F) <= 1

For any value of the model parameters, temperature, and any number of spins

<u>This is a very sloppy model, and hence robustly simulatable.</u>

Other models

We have studied many other quantum-many body models of interest:

1. 1D and 2D transverse-field Ising models with varying boundary conditions
2. Heisenberg model
3. Fermi-Hubbard model
4. J1-J2-anti-ferromagnetic Heisenberg model (non-nearest neighbor interactions)
5. Random/disordered transverse-field Ising model

See:
Reliability of analog quantum simulation,
M. Sarovar, J. Zhang, L. Zeng. arXiv:1603.09283

Scaling the analysis

The true value of a quantum simulator is to extract properties of models that are not classically tractable.

FIM can only be explicitly calculated for small versions of a model.

We propose the following strategies for scaling this analysis technique:

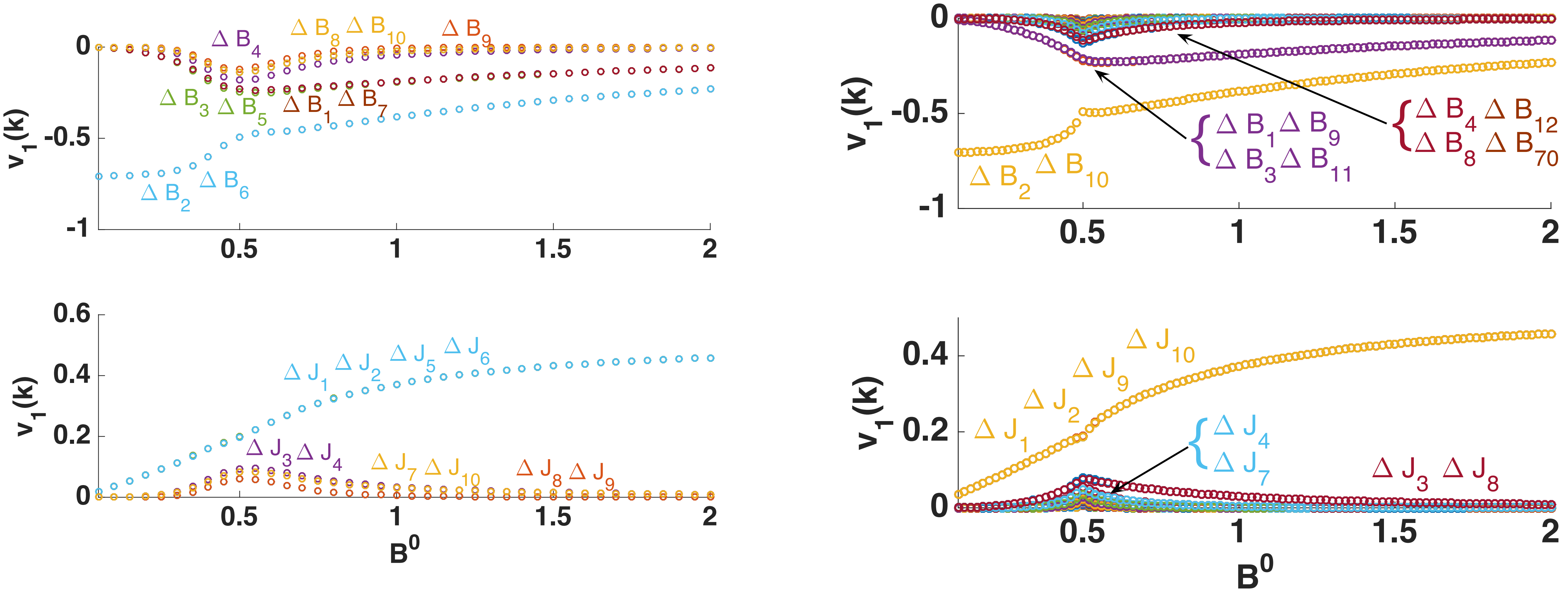
1. Symmetry analysis can be done analytically, independent of system size.
   e.g. translational invariance a powerful symmetry that implies sensitivity to collective parameters for any model size.
2. Sloppiness and form of stiff parameters carries over from small-scale models to large-scale versions.
   e.g. TFIM with correlation function

See:
Reliability of analog quantum simulation,
M. Sarovar, J. Zhang, L. Zeng. arXiv:1603.09283

Transverse field Ising model

Correlation function observable

$n = 10, J^0 = 1$ $\qquad$ $n = 70, J^0 = 1$



Qualitatively the same

Scaling the analysis

The true value of a quantum simulator is to extract properties of models that are not classically tractable.

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   e.g. translational invariance a powerful symmetry that implies sensitivity to collective parameters for any model size.
2. Sloppiness and form of stiff parameters carries over from small-scale models to large-scale versions.

   e.g. TFIM with correlation function
3. Conjecture:

   If a small-scale model is sloppy, then its large-scale version will also be sloppy.

See:
Reliability of analog quantum simulation,
M. Sarovar, J. Zhang, L. Zeng. arXiv:1603.09283

Thanks!

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Reliability of analog quantum simulation,
M. Sarovar, J. Zhang, L. Zeng. arXiv:1603.09283

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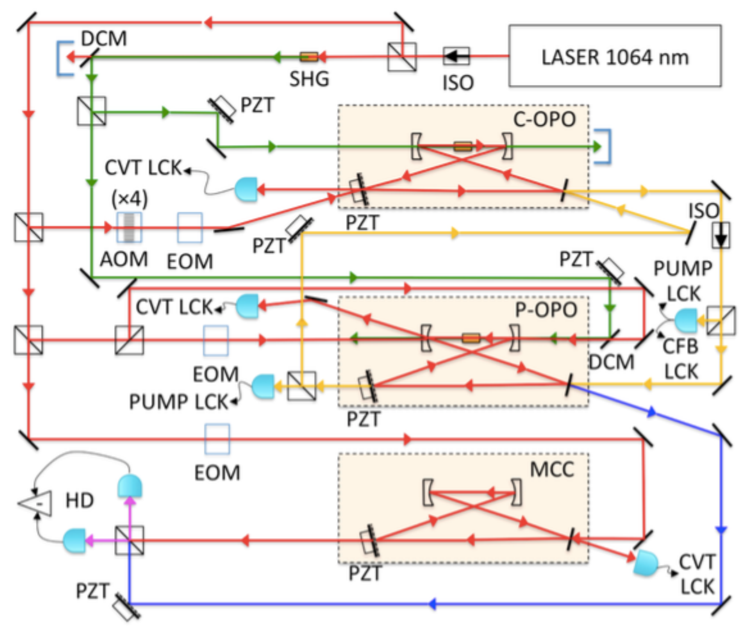
Poster preview

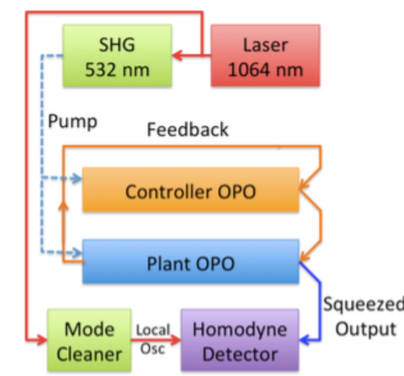
Optimizing coherent quantum feedback network for squeezed-light generation

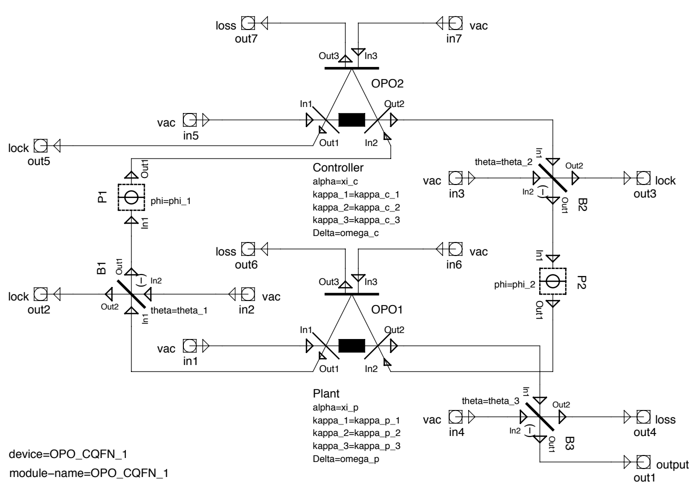
Constantin Brif *et al.*

O. Crisafulli *et al.* Optics Exp. **21**, 18371 (2013)

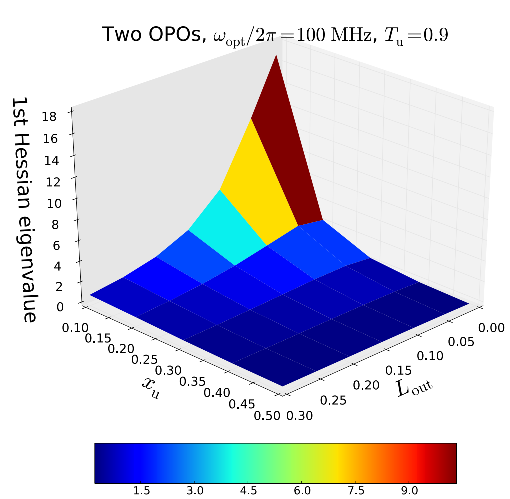
Proposed and implemented two OPOs coupled in feedback configuration to produce enhanced squeezing

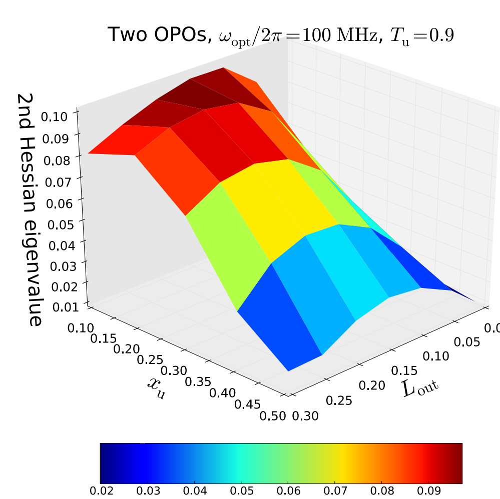






QNET model





We perform an optimization study of this network to determine optimal parameter regimes and limits of performance (maximum squeezing, maximum squeezing bandwidth, etc.).

In addition, we determine the robustness of the feedback loop to parameter fluctuations.