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Large-Scale Inverse Problems for Vibration and Acoustics Applications

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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What is an Inverse Problem?

What is an Inverse Problem?

Inverse problems arise when we have partial information and indirect observations of a system and need to infer (hidden) quantities of interest of the system.

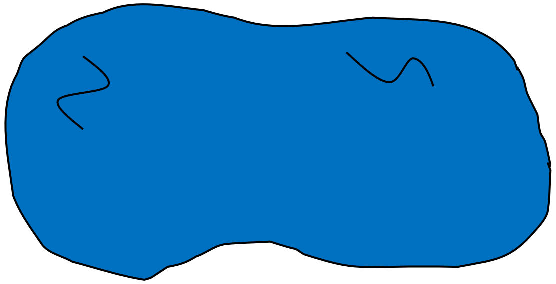
An inverse problem can be viewed as a quest for information that is not directly available from observations or measurements.

The pursuit of a solution to an inverse problem calls for a balance synergy between analysis and experimentation.

Inverse Problems: Observing the Unobservable

Suppose we have a “black box” system in the *as-manufactured* state that has only partially known parameters

Question: can we *non-destructivel*y interrogate the system to “see what is inside”?



Typical unknown parameters:

- Material properties
- Loads
- Boundary conditions
- Residual stresses
- Size/shape/location of inclusions (e.g. composite materials)

Example applications:

- Seismic imaging
- Medical imaging
- Non-destructive evaluation

Categories of Inverse Problems

- Imaging
  - Ultrasound medical
  - seismic
- Calibration of material models
  - Structural material properties, circuits, thermal properties, etc.
- Optimal Experimental Design
  - Best placement of sensors, test fixture setups
- Shape reconstruction (inverse scattering)
  - E.g. finding a buried land mine using EM/acoustic waves
- Information mining
  - Using physics-based models to interrogate satellite data, etc.
- Design of materials
  - E.g. Cloaking, camouflage, noise suppression, etc

Why Do We Care About Inverse Problems?

- Cost savings by minimizing testing on actual systems
- Improved understanding of *as-built* systems
  - Material property distribution, metrology, crack detection, etc.
- Decreased uncertainty in simulation-based forecasting of system performance stemming from robust model calibration.
- Inverse problems provide a natural path to V&V by marrying experiments and simulation.

Inverse Problems – Some Use Cases at Sandia

1. Source inversion
2. Material inversion
3. Boundary condition characterization
4. Crack detection
5. Optimal Experimental Design (OED)
6. Residual stress characterization
7. Vibration and noise control
8. Non-invasive measurements (response at non-instrumented locations)
9. Nonproliferation (combined material/source)

Abstract Optimization Formulation

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Abstract optimization formulation

Objective function / **PDE constraint**

$$\underset{\boldsymbol{u},\boldsymbol{p}}{\text{minimize}} \quad J(\boldsymbol{u},\boldsymbol{p})$$

$$\text{subject to} \quad \boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \boldsymbol{0}$$

Lagrangian

$$\mathcal{L}(\boldsymbol{u},\boldsymbol{p},\boldsymbol{w}) := J + \boldsymbol{w}^T\boldsymbol{g}$$

First order optimality conditions

$$\begin{Bmatrix} \mathcal{L}_u \\ \mathcal{L}_p \\ \mathcal{L}_w \end{Bmatrix} = \begin{Bmatrix} J_u + \boldsymbol{g}_u^T \boldsymbol{w} \\ J_p + \boldsymbol{g}_p^T \boldsymbol{w} \\ \boldsymbol{g} \end{Bmatrix} = \{0\}$$

Newton iteration

$$\begin{bmatrix} \mathcal{L}_{uu} & \mathcal{L}_{up} & \boldsymbol{g}_u^T \\ \mathcal{L}_{pu} & \mathcal{L}_{pp} & \boldsymbol{g}_p^T \\ \boldsymbol{g}_u & \boldsymbol{g}_p & 0 \end{bmatrix} \begin{Bmatrix} \delta\boldsymbol{u} \\ \delta\boldsymbol{p} \\ \boldsymbol{w}^\star \end{Bmatrix} = - \begin{Bmatrix} J_u \\ J_p \\ \boldsymbol{g} \end{Bmatrix}$$

Hessian calculation

$$\boldsymbol{W}\,\Delta\boldsymbol{p} = -\hat{J}',$$

$$\boldsymbol{W} = \boldsymbol{g}_p^T \boldsymbol{g}_u^{-T} (\mathcal{L}_{uu}\boldsymbol{g}_u^{-1}\boldsymbol{g}_p - \mathcal{L}_{up}) - \mathcal{L}_{pu}\boldsymbol{g}_u^{-1}\boldsymbol{g}_p + \mathcal{L}_{pp}$$

Statement of Inverse Problem

Minimize objective function

$$J(\{\boldsymbol{u}\},\{\boldsymbol{p}\}) = \frac{\kappa}{2}\Big(\{\boldsymbol{u}\}-\{\boldsymbol{u}_m\}\Big)^{\mathrm{T}}[Q]\Big(\{\boldsymbol{u}\}-\{\boldsymbol{u}_m\}\Big)+\mathscr{R}(\{\boldsymbol{p}\}),$$

$\{\boldsymbol{u}\}$ State variables (displacement, pressure)

$\{\boldsymbol{u}_m\}$ Measured data (displacement, pressure)

$\{\boldsymbol{p}\}$ Unknown parameters (loads, material parameters)

$[Q]$ Weight matrix

Subject to equations of motion

$$[M]\boldsymbol{a}(t)+[C]\boldsymbol{v}(t)+[K]\boldsymbol{u}(t)=\boldsymbol{f}(t)$$

Inverse Problems – PDE choices

Three modalities can be considered:

Time-domain

$$\boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}(\boldsymbol{p})\dot{\boldsymbol{u}} + \boldsymbol{K}(\boldsymbol{p})\boldsymbol{u} - \boldsymbol{f}$$

- Stiffness, damping parameters
- Linear or nonlinear
- Force/Material identification

Frequency-domain

$$\boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \left[\boldsymbol{K}(\boldsymbol{p}) + i\omega\boldsymbol{C}(\boldsymbol{p}) - \omega^2\boldsymbol{M}\right]\boldsymbol{u} - \boldsymbol{f}$$

- Stiffness, damping, parameters
- Linear only
- Force/material identification

Eigenvalue (modal)

$$\boldsymbol{g}_i = \boldsymbol{g}(\boldsymbol{u}_i,\lambda_i,\boldsymbol{p}) = \boldsymbol{K}(\boldsymbol{p})\boldsymbol{u}_i - \lambda_i\boldsymbol{M}\boldsymbol{u}_i = \boldsymbol{0}$$

- Stiffness parameters
- Linear only
- Material identification

Operator-Based Solution Strategy

The Newton Step equations:

$$\begin{bmatrix} \mathcal{L}_{uu} & \mathcal{L}_{up} & g_u^T \\ \mathcal{L}_{pu} & \mathcal{L}_{pp} & g_p^T \\ g_u & g_p & 0 \end{bmatrix} \begin{Bmatrix} \delta u \\ \delta p \\ w^\star \end{Bmatrix} = - \begin{Bmatrix} J_u \\ J_p \\ g \end{Bmatrix} \quad (1)$$

Reduced-space approach:
Static condensation of δu and $w^\star$

$$W \Delta p = -\hat{J}',$$

$$W = g_p^T g_u^{-T} (\mathcal{L}_{uu} g_u^{-1} g_p - \mathcal{L}_{up}) - \mathcal{L}_{pu} g_u^{-1} g_p + \mathcal{L}_{pp}$$

Full-space approach:
Solve equations (1) as is

Key concept: If we can define the action of each of the operators in equation (1) on a vector or vectors, then

1. Software structure can be modularized
2. We can have access to all optimization methods through a single interface[1]

[1]Heinkenschloss and Vicente, "An Interface between Optimization and Application for the Numerical Solution of Optimal Control Problems", ACM Transactions on Mathematical Software, Vol. 25, No. 2, June 1999, Pages 157–190.

Material Inversion – Modal Approach

Objective function based on eigenvalues/eigenvectors

$$\underset{\{\lambda_i\},\{\boldsymbol{u}_i\},\boldsymbol{p}}{\text{minimize}} \quad J(\{\lambda_i\},\{\boldsymbol{u}_i\},\boldsymbol{p})$$
$$\text{subject to} \quad \boldsymbol{g}_i(\lambda_i,\boldsymbol{u}_i,\boldsymbol{p}) = \boldsymbol{0}$$
$$b_i = 0$$

$$J(\{\lambda_i\},\{\boldsymbol{u}_i\},\boldsymbol{p}) := \frac{\beta}{2}\|\{r_i\}\|^2 + \frac{\kappa}{2}\mathscr{G}(\{\boldsymbol{u}_i\},\{\boldsymbol{u}_{mi}\}) + \mathscr{R}(\boldsymbol{p})$$

$$r_i = \frac{\lambda_i - \lambda_{mi}}{\lambda_{mi}}$$

$$\boldsymbol{g}_i = \boldsymbol{g}(\boldsymbol{u}_i,\lambda_i,\boldsymbol{p}) = \boldsymbol{K}(\boldsymbol{p})\boldsymbol{u}_i - \lambda_i \boldsymbol{M}\boldsymbol{u}_i = \boldsymbol{0}$$

$$b_i = b(\boldsymbol{u}_i) = \boldsymbol{u}_i^T \boldsymbol{M}\boldsymbol{u}_i - 1 = 0$$

$$\mathscr{L}(\boldsymbol{u},\boldsymbol{p},\boldsymbol{w},\boldsymbol{\eta}) := J + \boldsymbol{w}^T\boldsymbol{g} + \sum_{i=i}^{n}\eta_i b_i$$

$$\begin{Bmatrix} \mathscr{L}_{\lambda_i} \\ \mathscr{L}_{\boldsymbol{u}_i} \\ \mathscr{L}_{\boldsymbol{p}} \\ \mathscr{L}_{\boldsymbol{w}_i} \\ \mathscr{L}_{\eta_i} \end{Bmatrix} = \begin{Bmatrix} J_{\lambda_i} - \boldsymbol{u}_i^T \boldsymbol{M}\boldsymbol{w}_i \\ J_{\boldsymbol{u}_i} + \boldsymbol{g}_{\boldsymbol{u}_i}{}^T \boldsymbol{w}_i + b_{\boldsymbol{u}_i}\eta_i \\ J_{\boldsymbol{p}} + \boldsymbol{g}_{\boldsymbol{p}}^T \boldsymbol{w} \\ \boldsymbol{g}_i \\ b_{\boldsymbol{u}_i} \end{Bmatrix} = \{\boldsymbol{0}\}$$

Dissipative Material Inversion

Generic formulation based on complex modulus

$$\underset{\boldsymbol{u},\boldsymbol{p}}{\text{minimize}} \quad J(\boldsymbol{u},\boldsymbol{p})$$ Objective function

$$\text{subject to} \quad \boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \boldsymbol{0}$$ PDE constraint

$$\mathcal{L}(\boldsymbol{u},\boldsymbol{p},\boldsymbol{w}) := J + \boldsymbol{w}_R^T \boldsymbol{g}_R + \boldsymbol{w}_I^T \boldsymbol{g}_I = J + \Re(\boldsymbol{w}^h \boldsymbol{g})$$ Lagrangian

$$\boldsymbol{\sigma}(\omega) = \boldsymbol{D}(\omega)\boldsymbol{\epsilon} = (b(\omega)\boldsymbol{D}_b + G(\omega)\boldsymbol{D}_G)\boldsymbol{\epsilon}(\omega)$$ Constitutive Law

Viscoelasticity

$$b(\omega) = b_R(\omega) + i b_I(\omega)$$
$$G(\omega) = G_R(\omega) + i G_I(\omega)$$

Block Proportional Damping

$$b(\omega) = b + i\omega\beta b$$
$$G(\omega) = G + i\omega\beta G$$

Dashpots

$$E_R = 0$$
$$E_I = \omega c$$

Material Inversion – Combined Approach

Eigenvalue-based approach

$$\begin{aligned} \underset{\{\lambda_i\},\{\boldsymbol{u}_i\},\boldsymbol{p}}{\text{minimize}} \quad & J(\{\lambda_i\},\{\boldsymbol{u}_i\},\boldsymbol{p}) \\ \text{subject to} \quad & \boldsymbol{g}_i(\lambda_i,\boldsymbol{u}_i,\boldsymbol{p}) = \boldsymbol{0} \\ & b_i = 0 \end{aligned}$$

$$\boldsymbol{g}_i = \boldsymbol{g}(\boldsymbol{u}_i,\lambda_i,\boldsymbol{p}) = \boldsymbol{K}(\boldsymbol{p})\boldsymbol{u}_i - \lambda_i \boldsymbol{M}\boldsymbol{u}_i = \boldsymbol{0}$$

$$b_i = b(\boldsymbol{u}_i) = \boldsymbol{u}_i^T \boldsymbol{M}\boldsymbol{u}_i - 1 = 0$$

$$\mathscr{L}(\boldsymbol{u},\boldsymbol{p},\boldsymbol{w},\boldsymbol{\eta}) := J + \boldsymbol{w}^T\boldsymbol{g} + \sum_{i=i}^{n} \eta_i b_i$$

Applicability: stiffness parameters (springs, elastic materials)

Helmholtz-based approach

$$\begin{aligned} \underset{\boldsymbol{u},\boldsymbol{p}}{\text{minimize}} \quad & J(\boldsymbol{u},\boldsymbol{p}) \\ \text{subject to} \quad & \boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = 0 \end{aligned}$$

$$\boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \boldsymbol{K}(\boldsymbol{p})\boldsymbol{u} + i\omega \boldsymbol{C}(\boldsymbol{p})\boldsymbol{u} - \omega^2 \boldsymbol{M}\boldsymbol{u} - \boldsymbol{f}$$

$$\mathscr{L}(\boldsymbol{u},\boldsymbol{p},\boldsymbol{w}) := J + \boldsymbol{w}^T\boldsymbol{g}$$

$$\boldsymbol{\sigma}(\omega) = \boldsymbol{D}(\omega)\boldsymbol{\epsilon} = (b(\omega)\boldsymbol{D}_b + G(\omega)\boldsymbol{D}_G)\boldsymbol{\epsilon}(\omega)$$

Applicability: stiffness, mass and damping parameters

Strategy: eigenvalue-based inversion followed by Helmholtz-based inversion

Structural Acoustic Equations of Motion

acoustics

$$\nabla^2\phi = \frac{1}{c^2}\ddot{\phi}, \;\; \text{in } \Omega_f \times (0,T)$$

$$\nabla\phi \cdot \boldsymbol{n}_f = -\rho_f \ddot{u}_n, \;\; \text{on } \partial\Omega_f^N \times [0,T]$$

$$\phi = 0, \;\; \text{on } \; \partial\Omega_f^D \times [0,T]$$

$$\phi(0,T) = 0, \;\; \text{in } \Omega_f$$

$$\dot{\phi}(0,T) = 0, \;\; \text{in } \Omega_f$$

solid mechanics

$$\nabla \cdot \boldsymbol{\sigma} = \rho \ddot{\boldsymbol{u}}, \;\; \text{in } \Omega \times (0,T)$$

$$\boldsymbol{\sigma n} = \boldsymbol{h}, \; \text{on } \partial\Omega^N \times [0,T]$$

$$\boldsymbol{\sigma} = \boldsymbol{D} : \boldsymbol{\nabla u}, \;\; \text{in } \Omega \times [0,T]$$

$$\boldsymbol{u} = \boldsymbol{0}, \;\; \text{on } \; \partial\Omega^D \times [0,T]$$

$$\boldsymbol{u}(0,T) = \boldsymbol{0}, \;\; \text{in } \Omega$$

$$\dot{\boldsymbol{u}}(0,T) = \boldsymbol{0}, \;\; \text{in } \Omega$$

Time domain

$$[M]\boldsymbol{a}(t) + [C]\boldsymbol{v}(t) + [K]\boldsymbol{u}(t) = \boldsymbol{f}(t)$$

Frequency domain (Helmholtz)

$$[H(\omega)]\boldsymbol{z}(\omega) = \boldsymbol{F}(\omega)$$

$$[H(\omega)] = -\omega^2[M] + i\omega[C] + [K]$$

Structural Acoustic Equations of Motion

Fully coupled formulation

$$\begin{bmatrix} M_s & 0 \\ 0 & M_a \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} C_s & L^T \\ -L & C_a \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{bmatrix} f_s \\ f_a \end{bmatrix}$$

Condensed notation

$$[M]\boldsymbol{a}(t) + [C]\boldsymbol{v}(t) + [K]\boldsymbol{u}(t) = \boldsymbol{f}(t)$$

We will use the condensed notation in following slides

Statement of Inverse Problem (2)

The Lagrangian

$$\mathscr{L}(\{\boldsymbol{d}\},\{\hat{\boldsymbol{d}}\},\{\boldsymbol{p}\}) = \tilde{J}(\{\boldsymbol{p}\}) + \hat{\boldsymbol{u}}_0^{\mathrm{T}}\Big([M]\boldsymbol{a}_0 + [C]\boldsymbol{v}_0 + [K]\boldsymbol{u}_0 - \boldsymbol{f}_0(\{\boldsymbol{p}\})\Big)$$
$$+ \sum_{k=1}^{N}\Big\{\hat{\boldsymbol{u}}_k^{\mathrm{T}}\Big([M]\boldsymbol{a}_k + [C]\boldsymbol{v}_k + [K]\boldsymbol{u}_k - \boldsymbol{f}_k(\{\boldsymbol{p}\})\Big)$$
$$+ \hat{\boldsymbol{v}}_k^{\mathrm{T}}[M]\Big(\boldsymbol{v}_k - \boldsymbol{v}_{k-1} - \Delta t\left[(1-\gamma)\boldsymbol{a}_{k-1} + \gamma\boldsymbol{a}_k\right]\Big)$$
$$+ \hat{\boldsymbol{a}}_k^{\mathrm{T}}[M]\Big(\boldsymbol{u}_k - \boldsymbol{u}_{k-1} - \Delta t\,\boldsymbol{v}_{k-1} - \frac{\Delta t^2}{2}\left[(1-2\beta)\boldsymbol{a}_{k-1} + 2\beta\boldsymbol{a}_k\right]\Big)\Big\}$$

where

$$\{\boldsymbol{d}(\{\boldsymbol{p}\})\} = \Big\{\{\boldsymbol{u}\},\{\boldsymbol{v}\},\{\boldsymbol{a}\}\Big\}$$

Optimality conditions

- Optimality is obtained by setting derivatives of Lagrangian to zero
- We will adopt a reduced space approach where we derive reduced gradients from full space approach
- Reduced space approach can be derived from full space

Statement of Inverse Problem (3)

Gateaux derivatives of the Lagrangian with respect to adjoint variables

$$
\begin{aligned}
\nabla_{\boldsymbol{a}_0}\mathscr{L}\cdot\delta\boldsymbol{a}_0 &= \delta\boldsymbol{a}_0^{\mathrm{T}}\Big([M]\hat{\boldsymbol{u}}_0 - \frac{\Delta t^2}{2}(1-2\beta)[M]\hat{\boldsymbol{a}}_1 - \Delta t(1-\gamma)[M]\hat{\boldsymbol{v}}_1\Big),\\
\nabla_{\boldsymbol{u}_k}\mathscr{L}\cdot\delta\boldsymbol{u}_k &= \delta\boldsymbol{u}_k^{\mathrm{T}}\Big([M](\hat{\boldsymbol{a}}_k - \hat{\boldsymbol{a}}_{k+1}) + [K]\hat{\boldsymbol{u}}_k + \kappa[Q](\boldsymbol{u}_k - \boldsymbol{u}_{m_k}),\Big),\\
\nabla_{\boldsymbol{v}_k}\mathscr{L}\cdot\delta\boldsymbol{v}_k &= \delta\boldsymbol{v}_k^{\mathrm{T}}\Big([C]\hat{\boldsymbol{u}}_k - \Delta t[M]\hat{\boldsymbol{a}}_{k+1} + [M]\hat{\boldsymbol{v}}_k - [M]\hat{\boldsymbol{v}}_{k+1}\Big),\\
\nabla_{\boldsymbol{a}_k}\mathscr{L}\cdot\delta\boldsymbol{a}_k &= \delta\boldsymbol{a}_k^{\mathrm{T}}\Big([M]\hat{\boldsymbol{u}}_k - \beta\Delta t^2[M]\hat{\boldsymbol{a}}_k - \frac{\Delta t^2}{2}[M](1-2\beta)\hat{\boldsymbol{a}}_{k+1},\\
&\quad -\Delta t[M](\gamma\hat{\boldsymbol{v}}_k + (1-\gamma)\hat{\boldsymbol{v}}_{k+1})\Big),\\
\nabla_{\boldsymbol{u}_N}\mathscr{L}\cdot\delta\boldsymbol{u}_N &= \delta\boldsymbol{u}_N^{\mathrm{T}}\Big([M]\hat{\boldsymbol{a}}_N + [K]\hat{\boldsymbol{u}}_N + \kappa[Q](\boldsymbol{u}_N - \boldsymbol{u}_{m_N})\Big),\\
\nabla_{\boldsymbol{v}_N}\mathscr{L}\cdot\delta\boldsymbol{v}_N &= \delta\boldsymbol{v}_N^{\mathrm{T}}\Big([C]\hat{\boldsymbol{u}}_N + [M]\hat{\boldsymbol{v}}_N\Big),\\
\nabla_{\boldsymbol{a}_N}\mathscr{L}\cdot\delta\boldsymbol{a}_N &= \delta\boldsymbol{a}_N^{\mathrm{T}}\Big([M]\hat{\boldsymbol{u}}_N - \Delta t^2\beta[M]\hat{\boldsymbol{a}}_N - \Delta t\gamma[M]\hat{\boldsymbol{v}}_N\Big).
\end{aligned}
$$

Statement of Inverse Problem (4)

(i) Final conditions

$$
\begin{aligned}
[C]\,\hat{\boldsymbol{u}}_N + [M]\,\hat{\boldsymbol{v}}_N &= \boldsymbol{0} \\
\hat{\boldsymbol{u}}_N &= \Delta t^2 \beta\, \hat{\boldsymbol{a}}_N + \Delta t\, \gamma \hat{\boldsymbol{v}}_N \\
[M]\,\hat{\boldsymbol{a}}_N + [K]\hat{\boldsymbol{u}}_N &= \kappa [Q](\boldsymbol{u}_{m_N} - \boldsymbol{u}_N)
\end{aligned}
$$

(ii) Backward transition equations

$$
\begin{aligned}
\hat{\boldsymbol{u}}_k - \beta \Delta t^2 \hat{\boldsymbol{a}}_k - \Delta t\, \gamma \hat{\boldsymbol{v}}_k &= \frac{\Delta t^2}{2}(1-2\beta)\hat{\boldsymbol{a}}_{k+1} + \Delta t(1-\gamma)\hat{\boldsymbol{v}}_{k+1} \\
[C]\,\hat{\boldsymbol{u}}_k + [M](\hat{\boldsymbol{v}}_k - \Delta t\, \hat{\boldsymbol{a}}_{k+1} - \hat{\boldsymbol{v}}_{k+1}) &= \boldsymbol{0} \\
[M]\,\hat{\boldsymbol{a}}_k + [K]\hat{\boldsymbol{u}}_k &= [M]\hat{\boldsymbol{a}}_{k+1} + \kappa [Q](\boldsymbol{u}_{m_k} - \boldsymbol{u}_k)
\end{aligned}
$$

(iii) Last transition equation

$$
\hat{\boldsymbol{u}}_0 = \frac{\Delta t^2}{2}(1-2\beta)\hat{\boldsymbol{a}}_1 + \Delta t(1-\gamma)\hat{\boldsymbol{v}}_1
$$

Solution of Inverse Problem

Do until tolerance < eps

{

Solve forward problem

↓

Solve adjoint problem

↓

Compute gradients, Hessians

↓

Optimization step

↓

Receive design variable updates from optimization solver

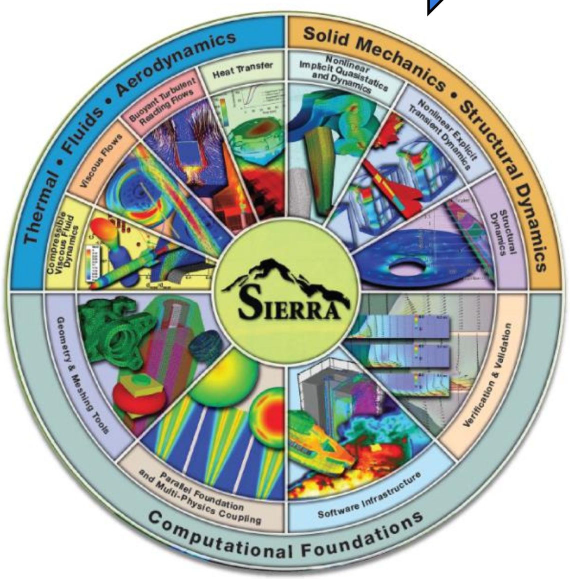
}

end

Beyond Inverse Problems – Calibration vs Prediction

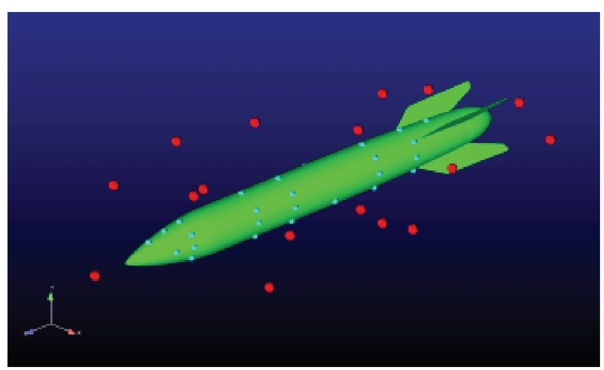
Even if an inverse problem is solved exactly, what can we say about predictive capability at non-measured locations?

Forward Problem	Inverse Problem	Prediction Problem



$$\underset{u,p}{\text{minimize}} \quad J(u,p)$$

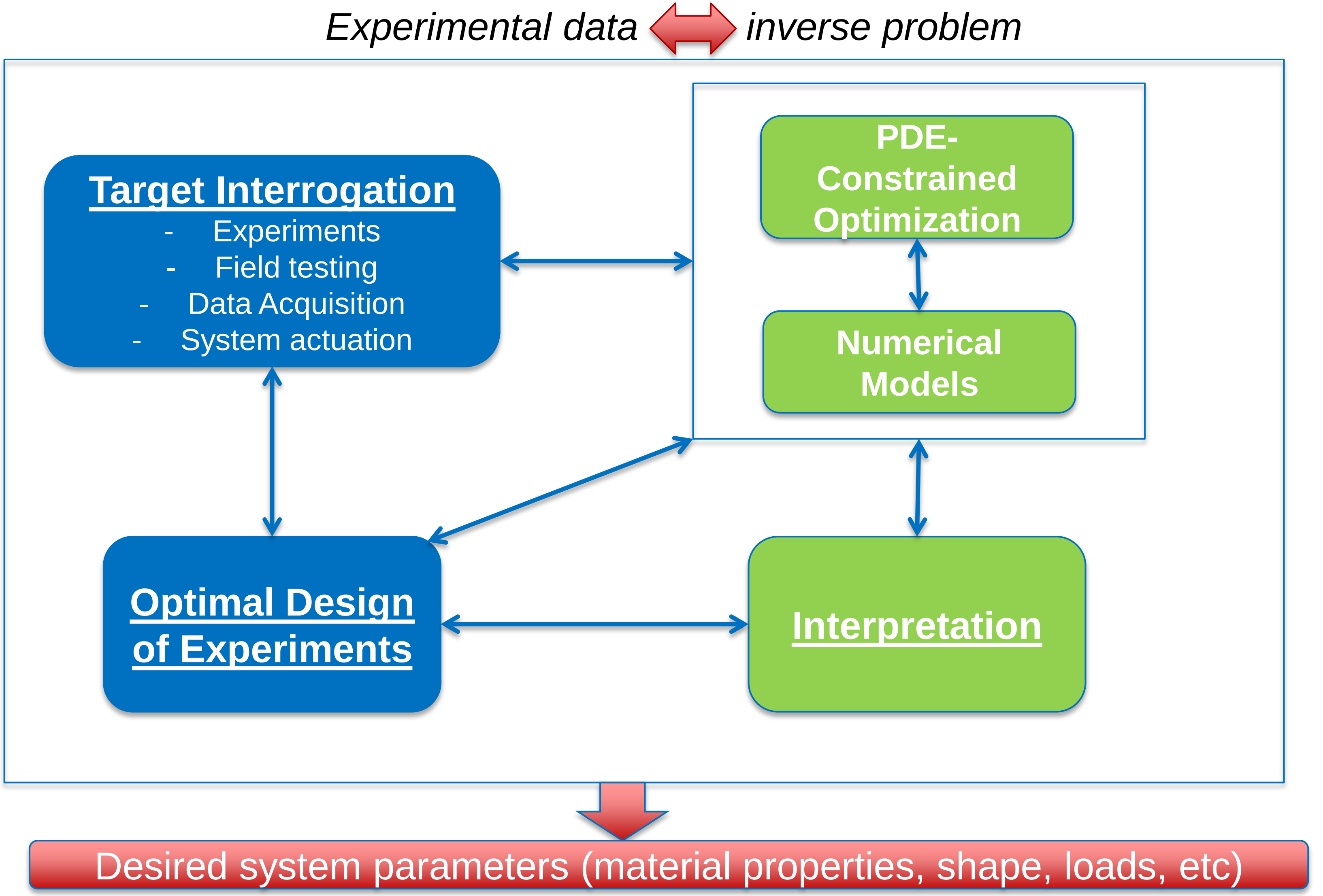
$$\text{subject to} \quad g(u,p) = 0$$



Accuracy of non-instrumented locations?

Moser et at, 2014

Inverse Problems - The Interaction of Experiments and Simulation



Source Inversion: Flight Environment Characterization

- Emulate flight test conditions with ground-based acoustic test (to reduce costs)
  - Given flight test data (e.g. accelerometers), design inputs in ground-based acoustic test
  - We may also be given target acoustic pressure levels from CFD

Source Inversion in Sierra-SD

- Existing capability in Sierra-SD/ROL for both time and frequency domain
- Important for many physics (thermal, EM, nonlinear structures, etc)
- Additional areas of research
  - Nonlinear problems
  - How to maximize information from sparse datasets
  - How to design experimental setups that collect optimal data for inverse problem
  - Extend to other physics (i.e. thermal)

Source Inversion in Sierra-SD

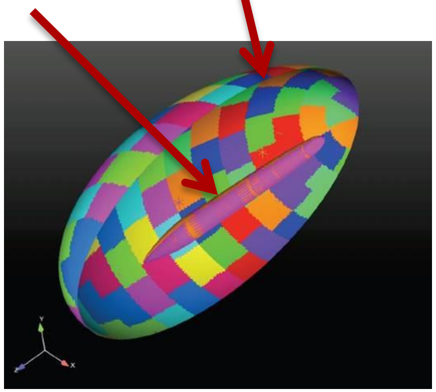
Goal:

Solve inverse problem to obtain acoustic patch inputs that produce the given microphone measurements.

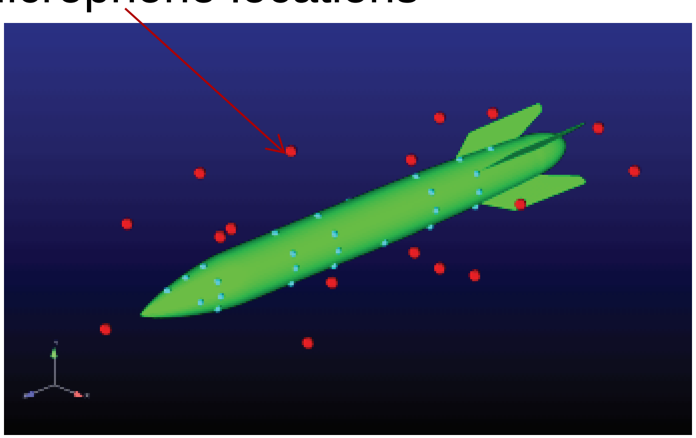
2 approaches:

1. Frequency domain
   - broadband frequency sweep
2. Time domain
   - implicit time integration that covers frequency range of interest

Surface with 172 acoustic patches

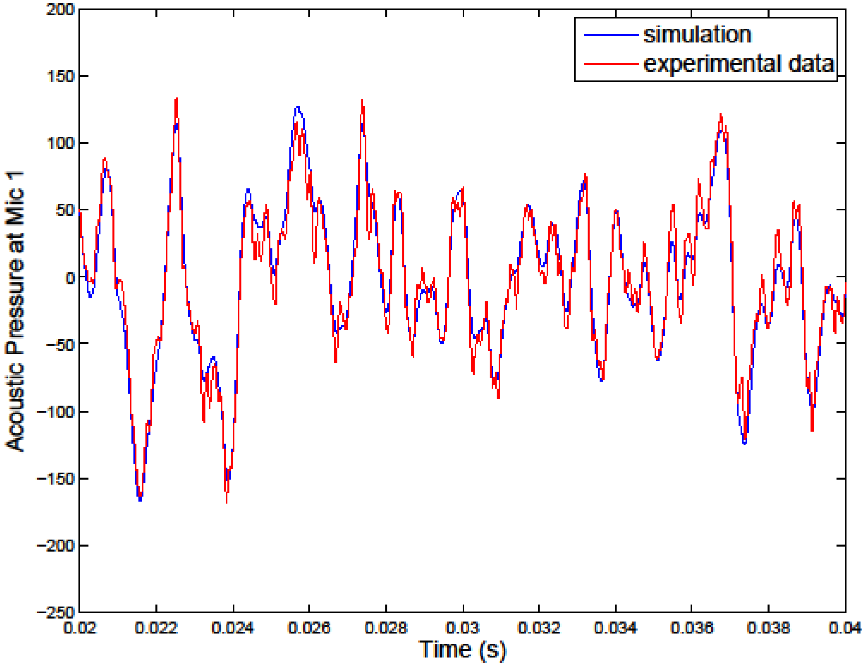


Microphone locations

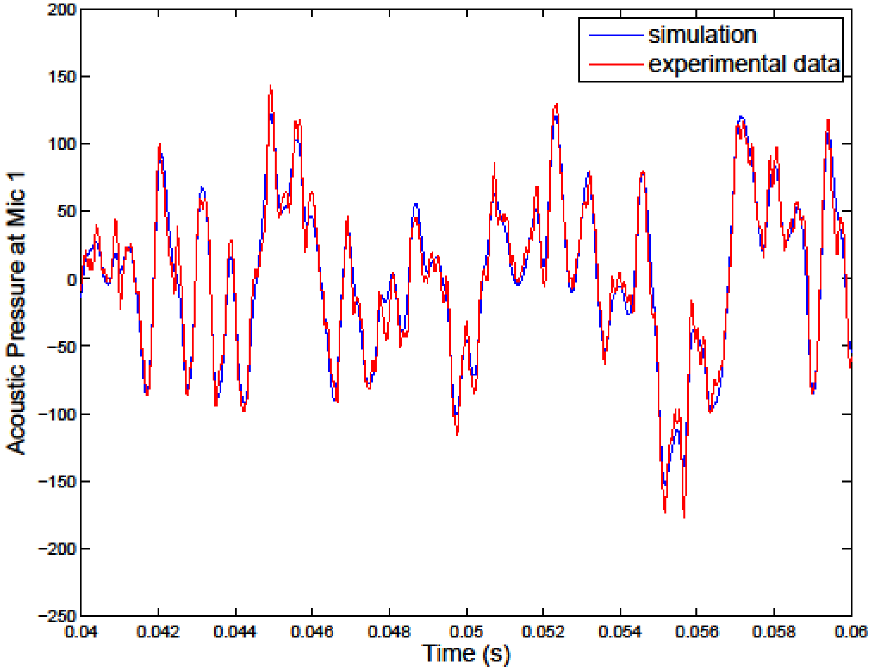


Time Domain Source Inversion Using Sierra-SD/ROL

A Comparison of Experimental Data and Inverse Simulation for Microphone 1



0.02(s) < t < 0.04(s)

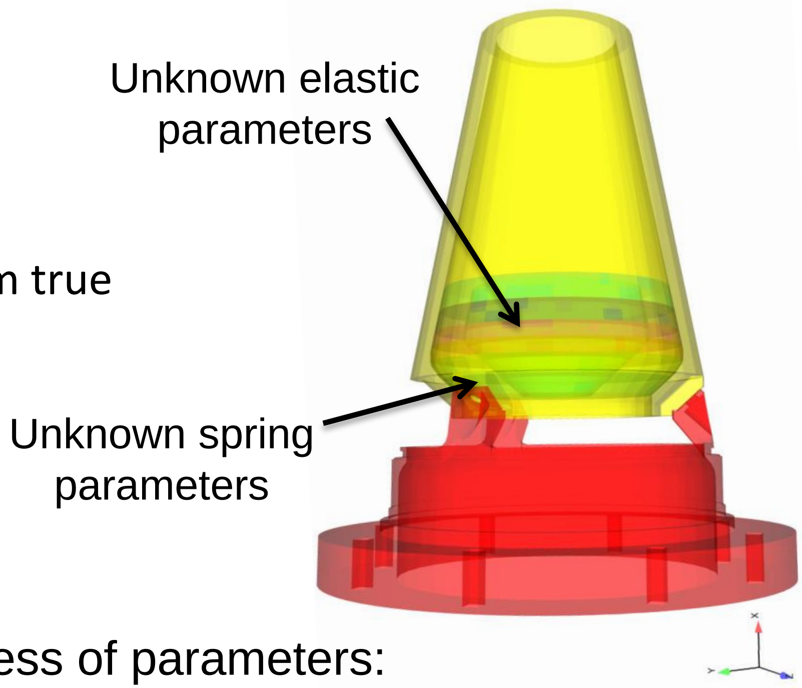


0.04(s) < t < 0.06(s)

Eigenvalue-Based Material Inversion

- Spring/foam calibration on mass-mock
- Goal: Match synthetic modes to computed modes
- Synthetic modal data used as input
- Initial guess of parameters a factor of 10 away from true values

Mode number	Initial Guess	Results from inversion	Exact modes
1	2.759e6	1.240e6	1.240e6
2	2.957e6	1.358e6	1.3607e6
3	1.157e7	7.071e6	7.052e6
4	1.186e7	1.041e7	1.043e7
5	1.266e7	1.102e7	1.103e7
6	1.695e7	1.262e7	1.256e7
7	2.041e8	2.025e8	2.025e8



Initial guess of parameters:

Table 1. Joint2G parameters for joint 1 of LFU model.

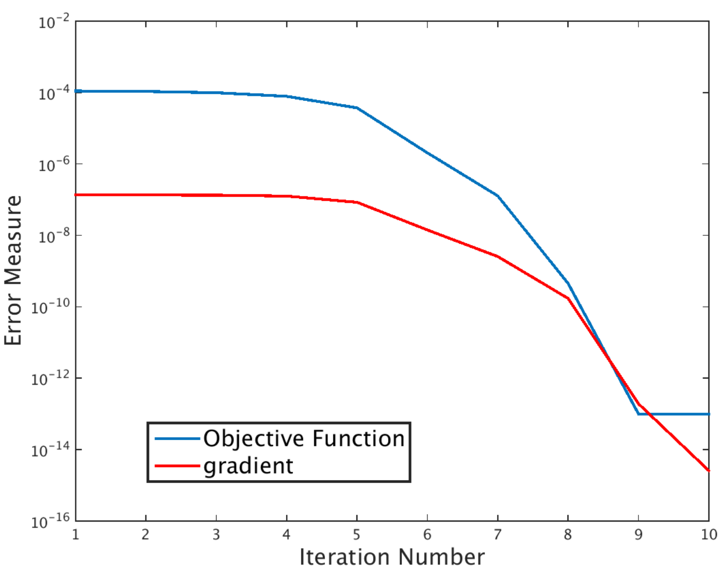
	kx	ky	kz	krz	kry	krz
exact	2.46e6	2.0e8	2.0e8	N/A	N/A	N/A
computed	2.47e6	2.0e8	2.0e8	N/A	N/A	N/A
initial guess	2.46e5	2.0e8	2.0e8	N/A	N/A	N/A

Table 4. Elastic foam parameters for LFU model.

	shear modulus (G)	bulk modulus (K)
exact	1.585e4	4.134e4
computed	1.585e4	4.134e4
initial guess	1.585e3	4.134e3

Frequency-Domain Material Inversion

- Dashpot/foam calibration on mass-mock
- Full Newton with adjoint-based Hessians
- Measured displacements on foam block
- Stiffness parameters from previous slide
- Initial guess: zero damping



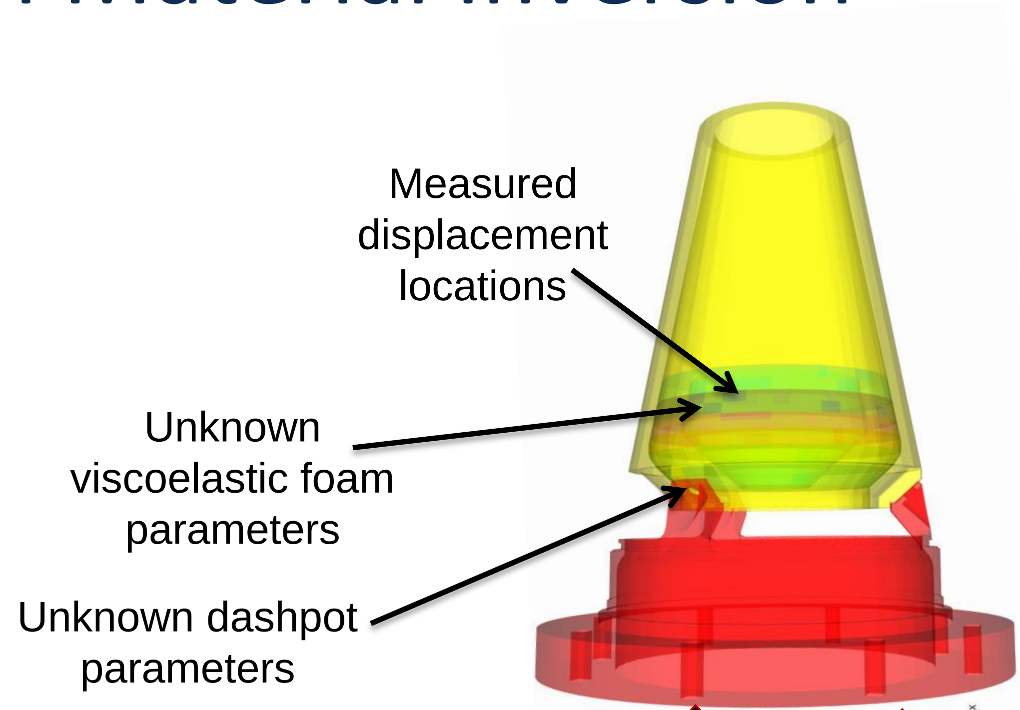


Table 3. Dashpot parameters for mass-mock model.

	cx	cy	cz
exact	1000.0	N/A	N/A
computed	1000.0	N/A	N/A
initial guess	0	N/A	N/A

Table 4. Viscoelastic foam parameters for mass-mock model.

	Imaginary part of G	Imaginary part of K
exact	362.4	785.2
computed	362.3	785.0
initial guess	0	0

The Cloaking Problem

Can we use an inverse method to design an undetectable object?

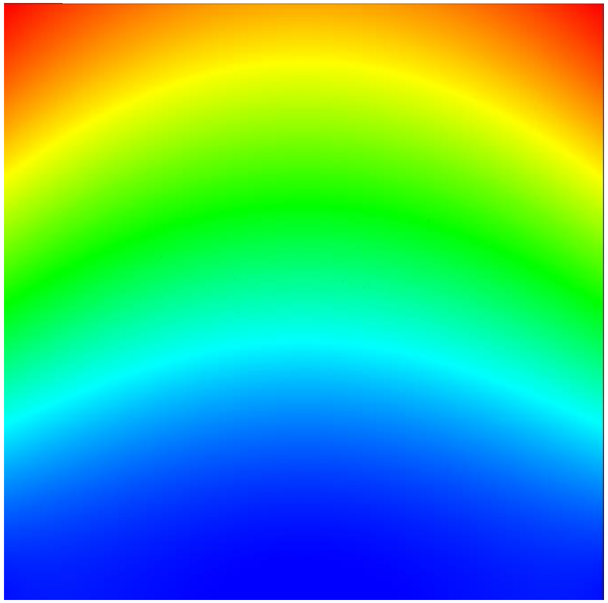
- Frequency-domain material optimization using Sierra-SD/ROL
- Acoustic scattering on viscoelastic structure submerged in water
- Coupled structural-acoustic forward problem

$$\underset{\boldsymbol{u},\boldsymbol{p}}{\text{minimize}} \quad J(\boldsymbol{u},\boldsymbol{p})$$

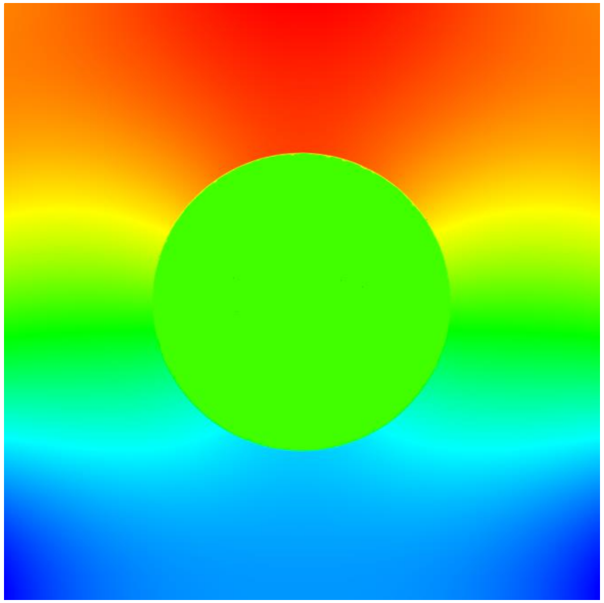
$$\text{subject to} \quad \boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = 0$$

PDE constraint (Helmholtz)

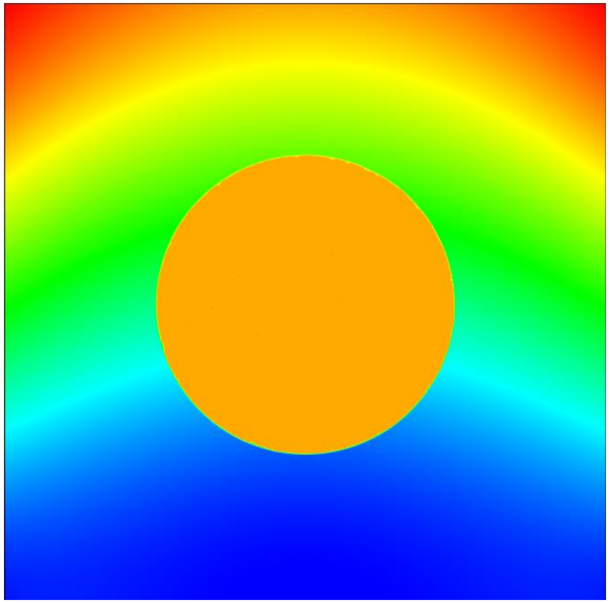
$$\boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \boldsymbol{K}(\boldsymbol{p})\boldsymbol{u} + i\omega\boldsymbol{C}(\boldsymbol{p})\boldsymbol{u} - \omega^2\boldsymbol{M}\boldsymbol{u} - \boldsymbol{f}$$



Pressure field with no scatterer



Pressure field with scatterer

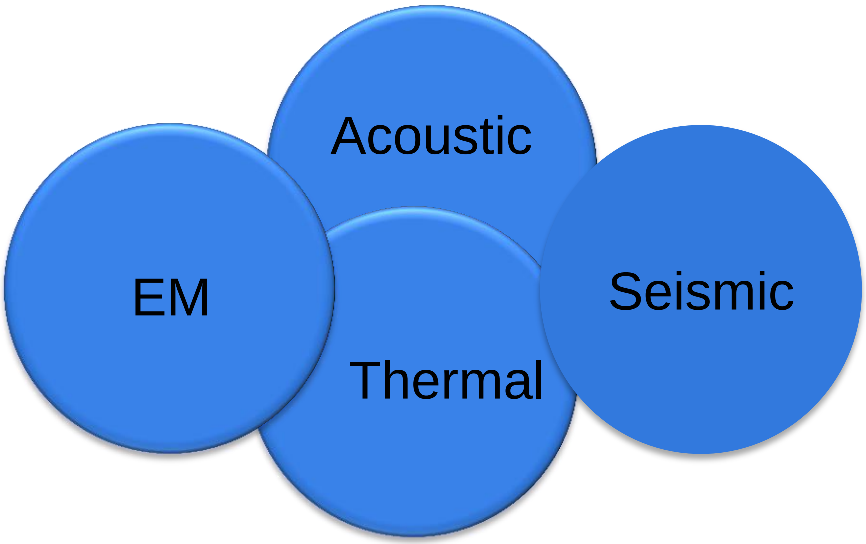


Inverse solution

Inverse Problems - Multiphysics

- Multiphysics inverse problems offer advantages in detection technology compared to single-physics approaches
- Data fusion from various physics -> better inversions
- The detection community is going to multiphysics sensing– thus the data is there to support multiphysics inversions.
- Thermal/acoustics/electromagnetic ↔ mine detection, medical imaging

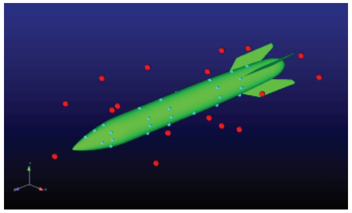
Research at Purdue:
Jie Shen, Yang Yang, Peijin Li, Plamen Stefanov



Inverse Problems – Designing Experiments

- Optimal Experimental Design
  - How to we collect experimental data from system that maximizes information?
  - Accelerometer, microphone, thermometer placements
  - Source/input locations
  - Test fixture setups

Can inverse problems help design optimal experiments?

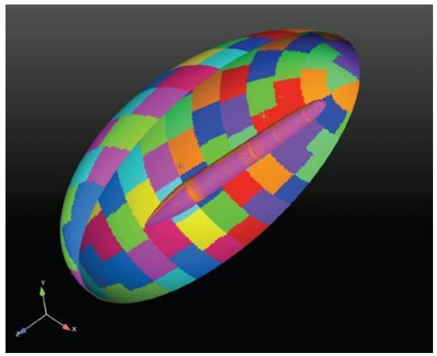
Inverse Problem ⟷  ⟷ Experiment

e.g. how to optimally place sensors in an acoustic test

Generality of Operator-Based Optimization

Sandia National Laboratories

Applications



operators

$$\begin{bmatrix} \mathcal{L}_{uu} & \mathcal{L}_{up} & \boldsymbol{g}_u^T \\ \mathcal{L}_{pu} & \mathcal{L}_{pp} & \boldsymbol{g}_p^T \\ \boldsymbol{g}_u & \boldsymbol{g}_p & 0 \end{bmatrix} \begin{Bmatrix} \delta \boldsymbol{u} \\ \delta \boldsymbol{p} \\ \boldsymbol{w}^\star \end{Bmatrix} = - \begin{Bmatrix} J_u \\ J_p \\ \boldsymbol{g} \end{Bmatrix}$$

$$\boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \left[\boldsymbol{K}(g'(\omega), k'(\omega)) + \boldsymbol{K}_G(\boldsymbol{\sigma}) + i\omega \boldsymbol{C}(\boldsymbol{\eta}(\omega), g''(\omega), k''(\omega)) - \omega^2 \boldsymbol{M}(\boldsymbol{\rho})\right] \boldsymbol{u} - \boldsymbol{f}(\boldsymbol{p}(\omega))$$

Elastic parameters

Residual Stress fields

Impedance boundaries

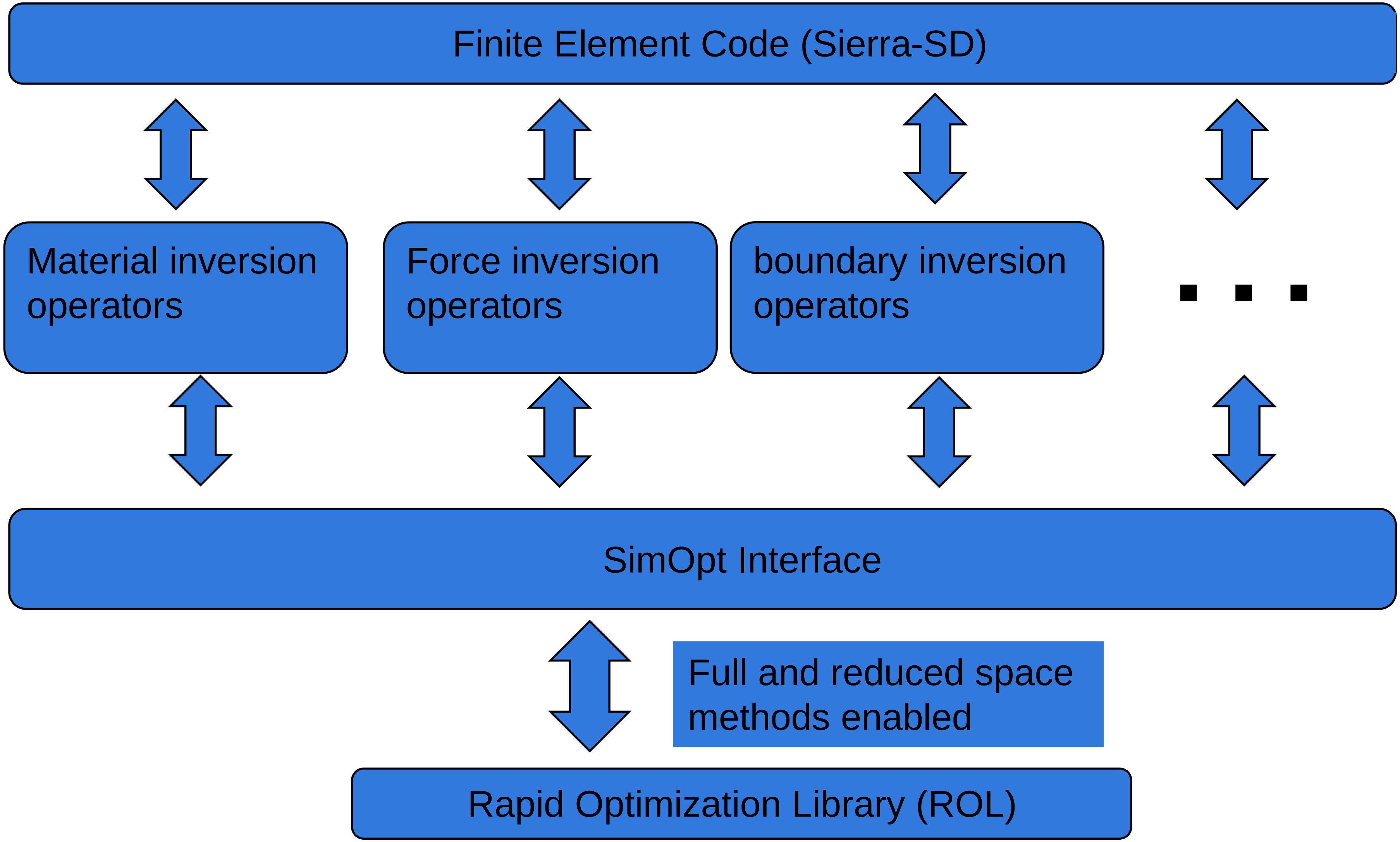
Viscoelastic parameters

density

Loads/ forces

Applications → operators → optimization

Operator-Based Inverse Problems



Conclusions

- Wide-range of potential applications for inverse problems
- Massively parallel structural dynamics and acoustics (Sierra-SD) and optimization software(ROL) have been loosely coupled through SimOpt interface for the solution of a variety of inverse problems.
- Operator-based approach for both mathematical formulation and software development allows for:
  - Modular software infrastructure
  - Allows various optimization methods to be accessed through a single interface