SAND2016-3343C

Assessment of a UQ Approach for Handling Sparse Samples of Discrete Random Functions (Material Stress-Strain Curves)[1]

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* Sandia is a multiprogram laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

[1]Sandia National Laboratories document SAND2016-YYYYC (unlimited release)

2016 SAE World Congress

April 12-14, Detroit

NNSA National Nuclear Security Administration

VERIFICATION VALIDATION™

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The Issue:
Material Response Variability in Stress-Strain Curves from replicate Tension Tests

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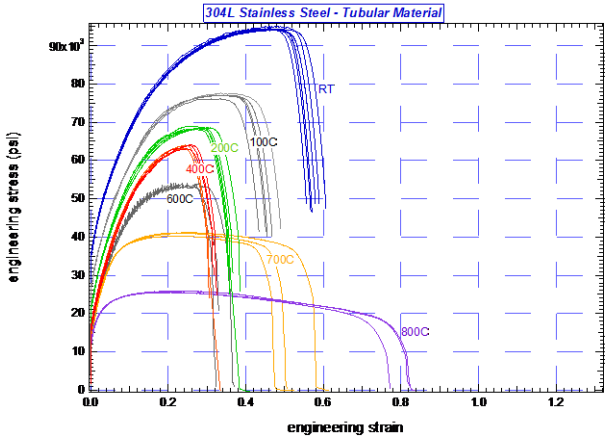
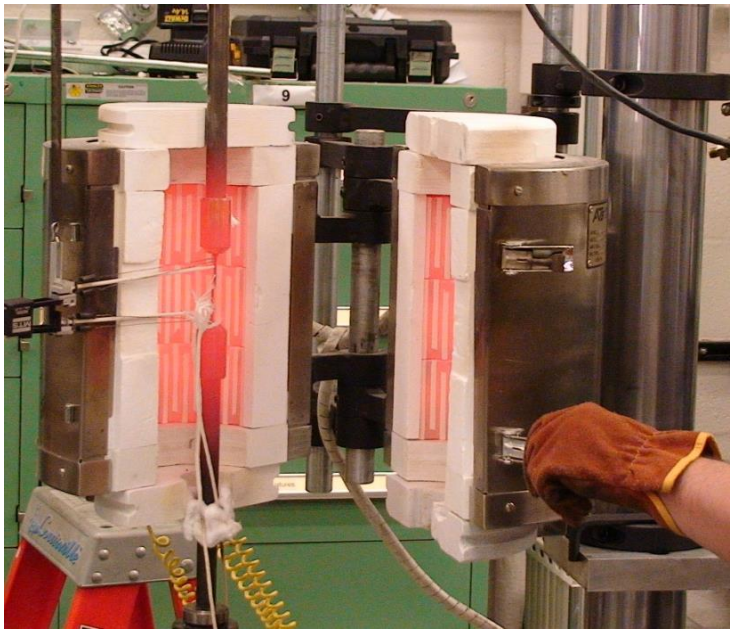
- **Results at various temperatures**





cylinder Tension-test specimen

The Issue:
Variability of Predicted Response due to Material Curve Variability

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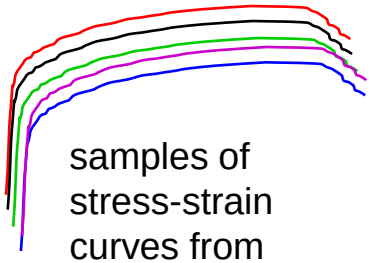
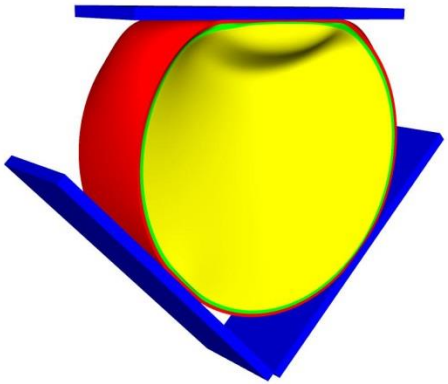


samples of stress-strain curves from material testing

run model with the various material curves



variability of predicted response (e.g. displacement, stress, failure pressure)

The Issue:
Error/Uncertainty of Inferred Population of Responses from propagating relatively few Material Curves

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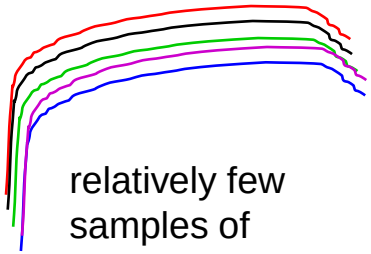
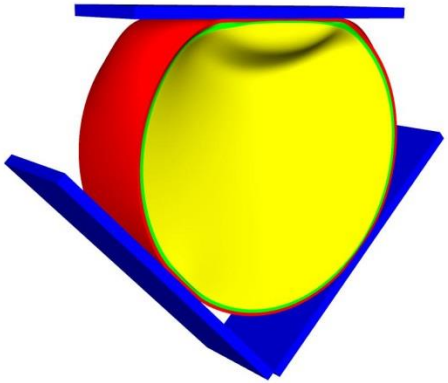
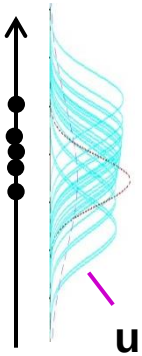


relatively few samples of stress-strain curves from material testing

run mode with the various material curves



predicted variability of response



uncertainty (figurative) of ∞ population of responses inferred from 5 samples

How do we get a handle on the *epistemic* uncertainty due to sparseness of sampling of the *aleatory* variability (the random inputs and outputs) of the system?

Simplified UQ Objective

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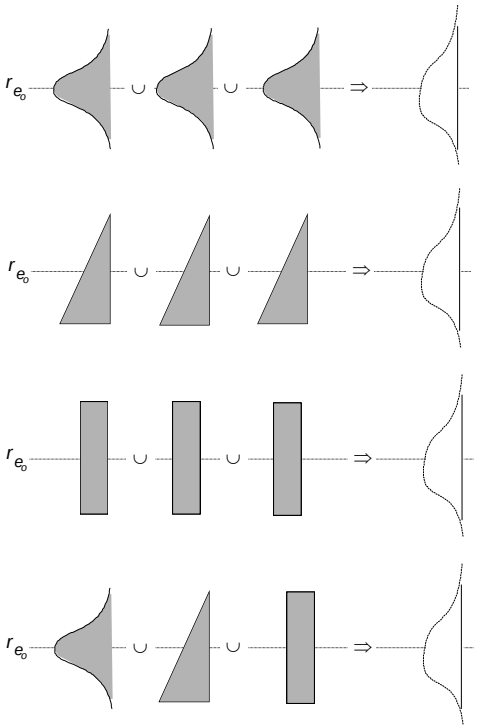
- **A view is taken that:**
  - One should not try to do the impossible (accurate estimation of the PDF from which the sparse samples come)
  - Rather, a *pragmatic goal* is that the uncertainty representation should be conservative, e.g. bound the .025 – 0.975 percentile range of the PDF from which the samples come
  - An *opposing goal* (making this a difficult MiniMax problem) is that the uncertainty representation should not be overly conservative—i.e., should minimally over-estimate the desired percentile range of the true PDF.

Related Problem previously studied:
– Sparse Samples from popular PDF types, Assess Sparse-Data UQ Methods

- 5 methods assessed on 21 test problems



- **1000 trials of each method for fitting sample data from each of:**
  - normal PDF
  - right-triangular PDF
  - uniform PDF
  - 4 different convolutions of these PDF types (figure at left) acting as *three equally dominant* sources of random uncertainty in a linear system.
    - ➢Answer question: does the presence of multiple sources of uncertainty smooth or mitigate the errors in representing the individual PDFs?

- **Fit the data for three sets of sample sizes: *n* = 2, 8, 32 for each PDF**

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A Comparison of Methods for Representing Sparsely Sampled Random Quantities – More Results

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Soc. Automotive Engineers 2013 World Congress

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* Sandia is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

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Classical statistical Tolerance Intervals found to offer the best balance of ease and effectiveness

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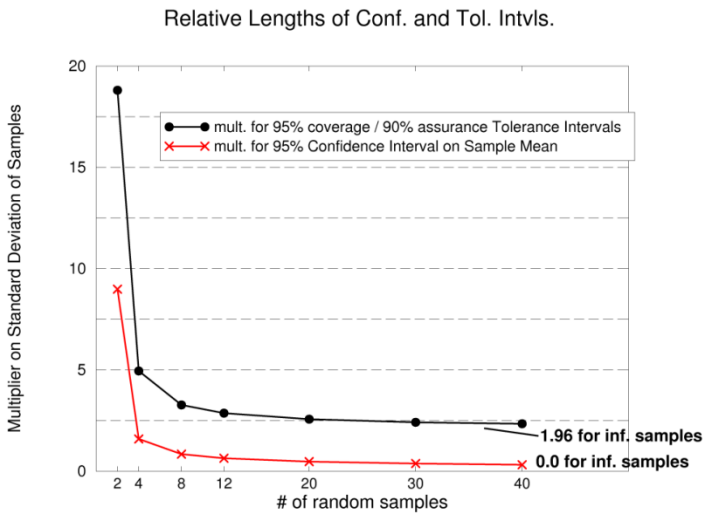
- **Tolerance Interval Approach**
  - calculate the standard deviation $\boldsymbol{\sigma}_i$ of the data
  - multiply $\boldsymbol{\sigma}_i$ by appropriate factor $\boldsymbol{f}$ from statistical tables
  - create interval bars of extent $\boldsymbol{f\sigma}_i$ about the mean $\boldsymbol{\mu}_i$ of the data: $\boldsymbol{\mu}_i \pm \boldsymbol{f\sigma}_i$
  - 0.90/0.95 Tolerance Intervals — the factors $\boldsymbol{f}$ here correspond to approximate 90% reliability or odds that the produced tolerance interval encompasses the central 95-percentile range between the 0.025 and 0.975 percentiles of the true PDF

# samples	$f_{0.9/0.95}$
2	18.80
4	4.94
8	3.26
12	2.86
20	2.56
30	2.41
40	2.33
∞	1.96



- **Very simple to use in practice**

Current Question:
How well do Tolerance Intervals work for highly nonlinear Solid Mechanics and 4 input Stress-Strain curves from mtl. tests?

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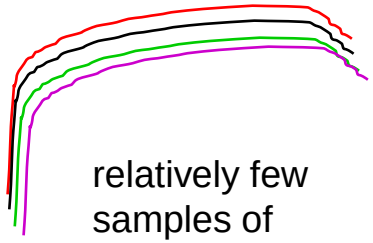
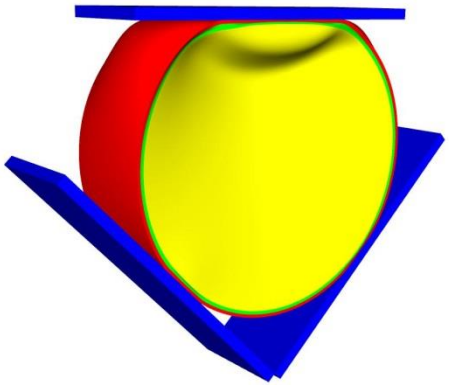


relatively few samples of stress-strain curves from material testing

run mode with the various material curves



predicted variability of response

Tolerance Interval on samples of output response from propagated material curves

Can-Crush Model and Fixturing/Loading

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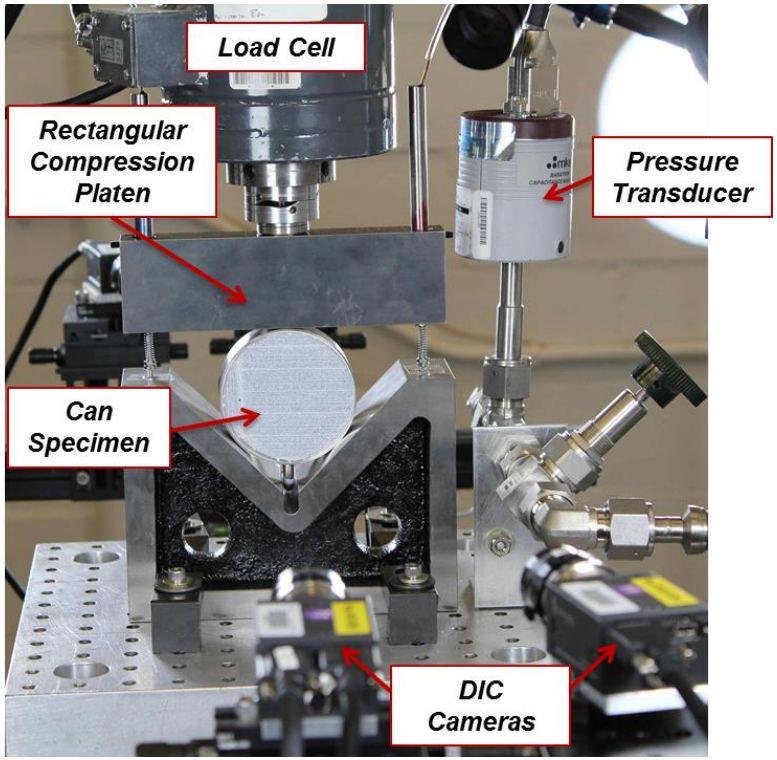
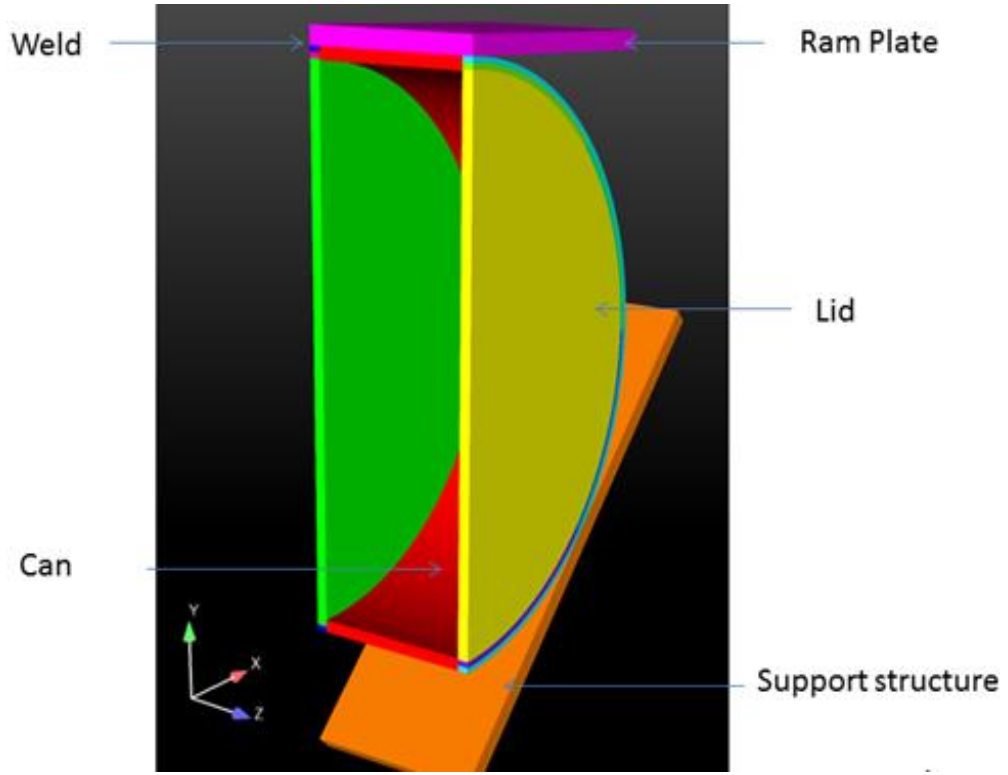


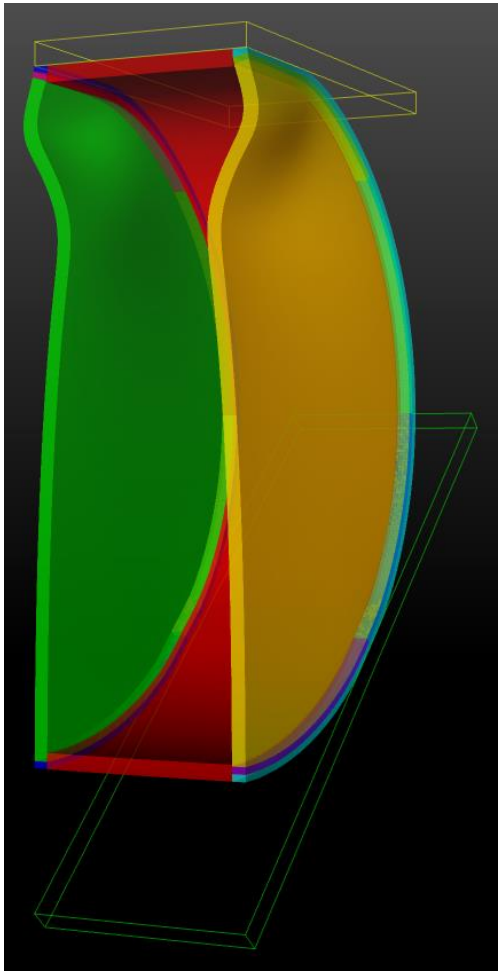
Modified Can Crush Test Setup

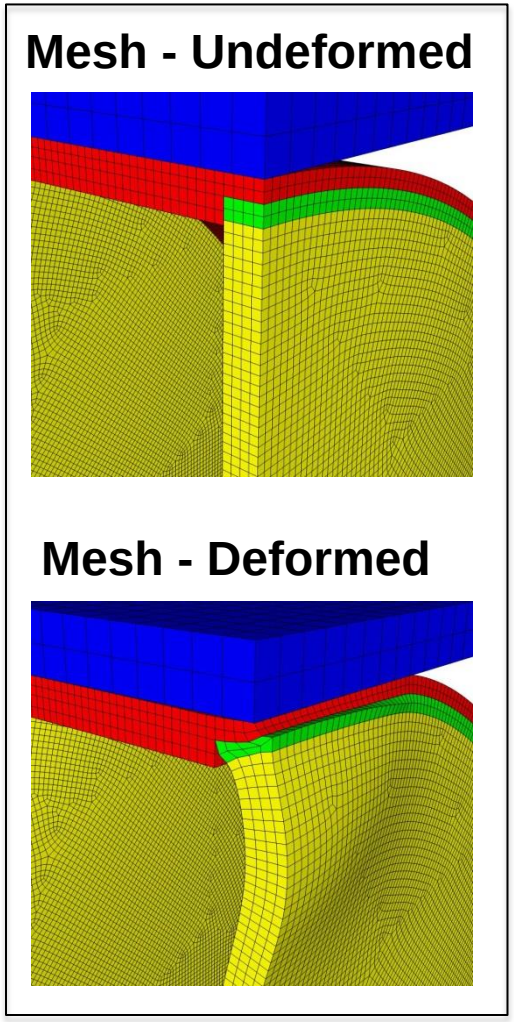


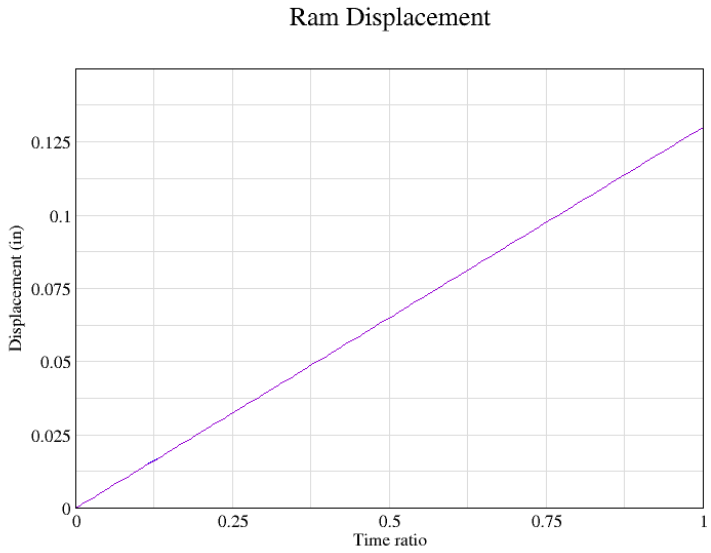
For present UQ study assign all parts the same steel material with synthetic σ-ϵ curves

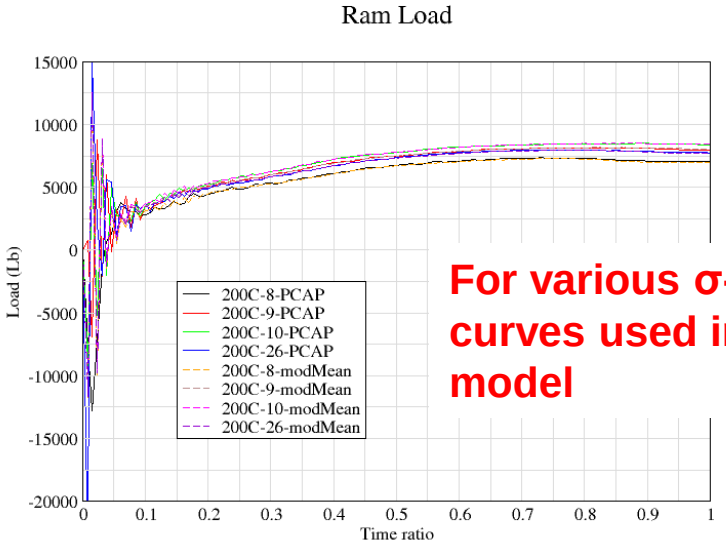
Component	Material	Dimensions	No. Elements	No. thru thickness
Ram Plate	Rigid	1” wide x 0.1” tk	1200	2
Wedge	Rigid	1” wide x 0.1” tk	2400	2
Lid	304L SSTL	3” dia x .062” tk	117264	4
Can	304L SSTL	3” dia x 1” wide x .067” tk	74400	4
Weld	304L SSTL	3” dia. x .03” wide x .062” tk	4800	2x4

Crushing-Ram Constant Advance Rate and Resulting Resistance from Can



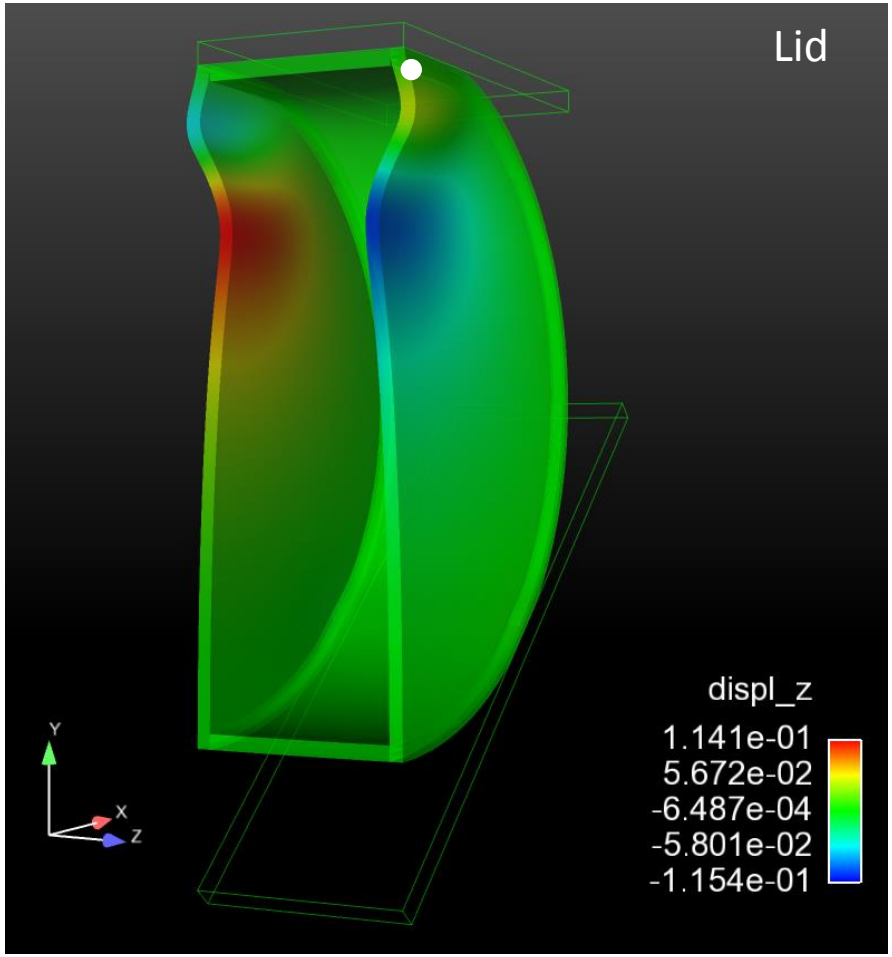


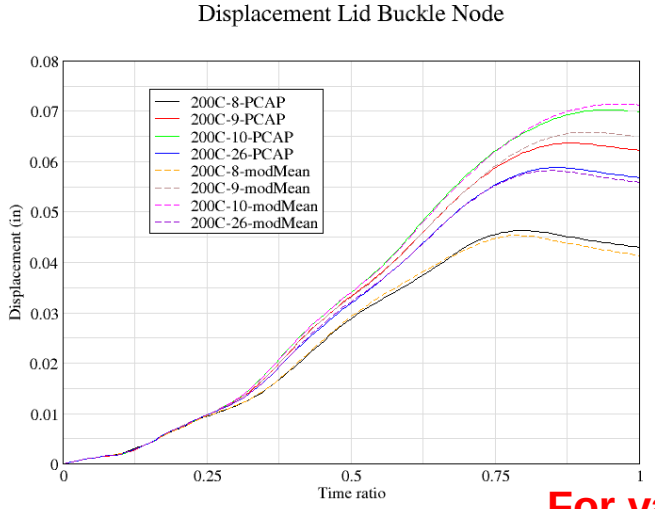




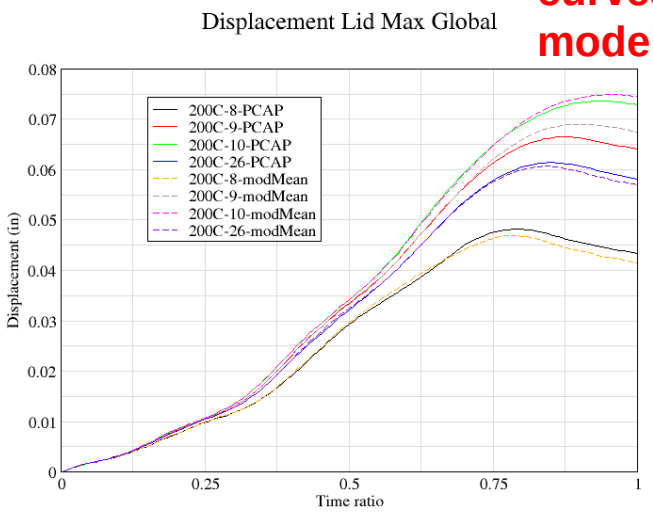
For various σ-ϵ curves used in model

Displacement output locations

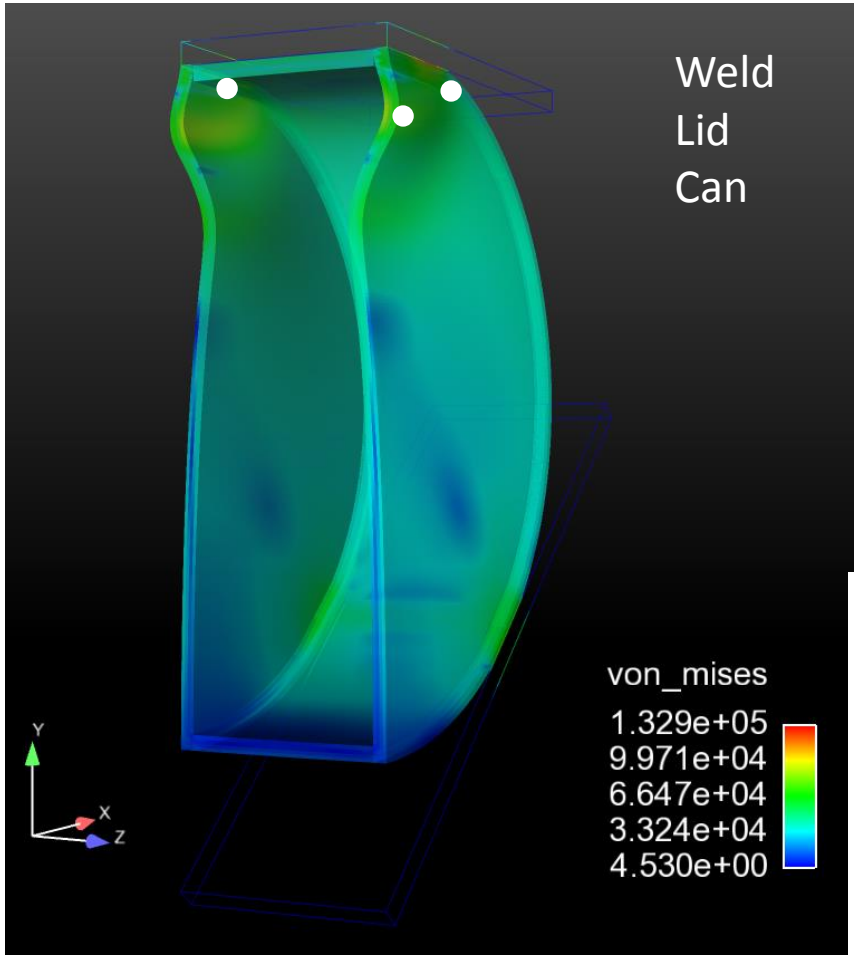


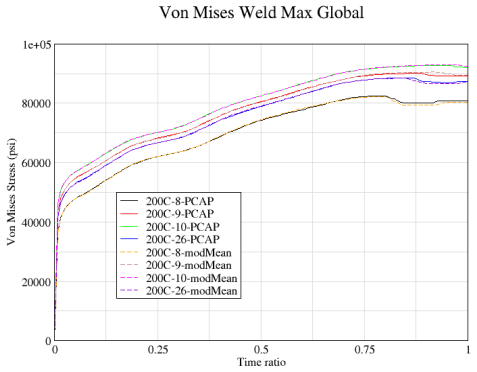


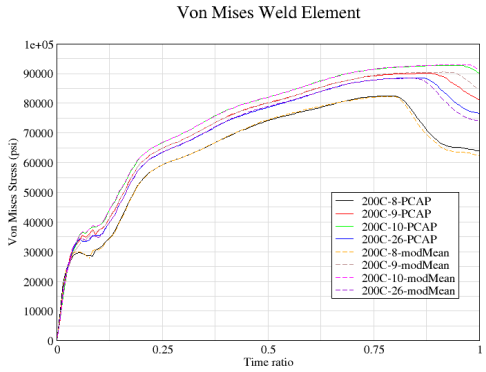
For various σ-ϵ curves used in model



Von Mises Stress output locations







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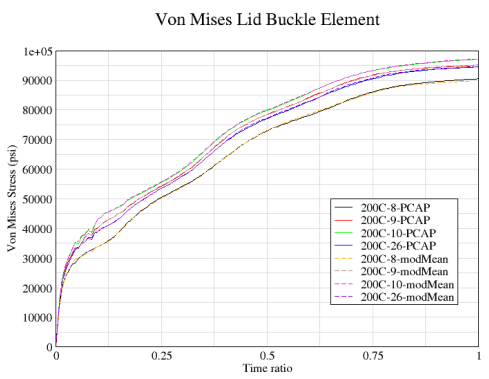
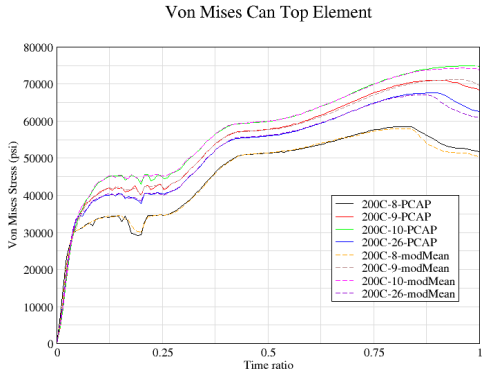
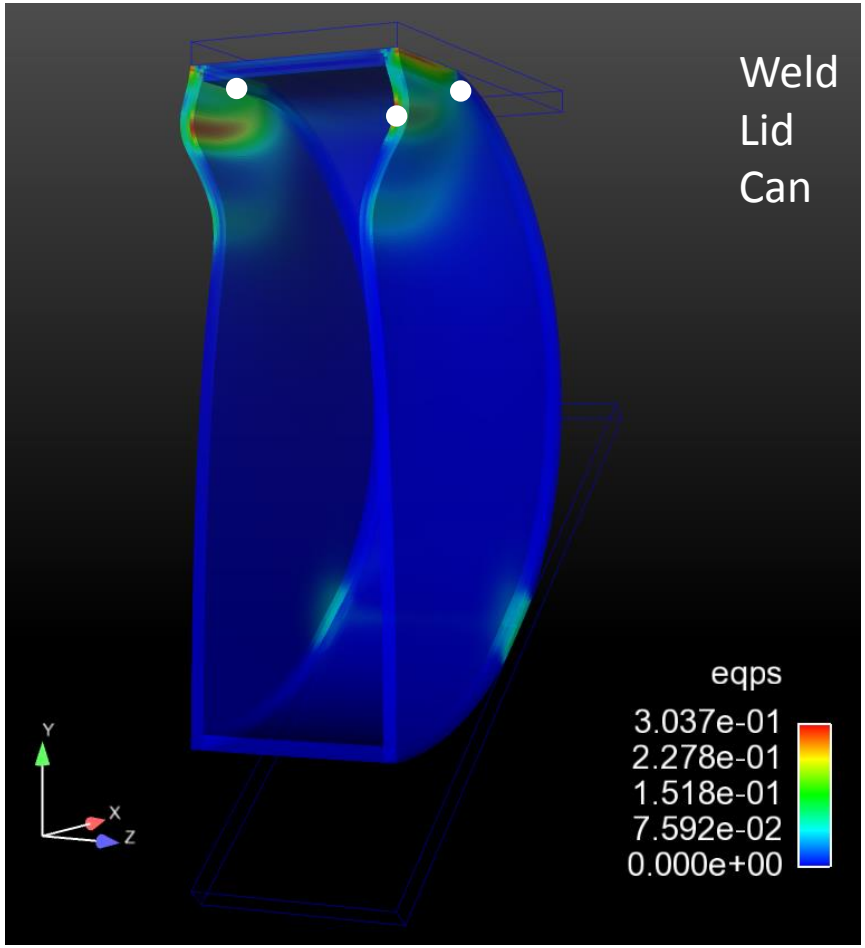
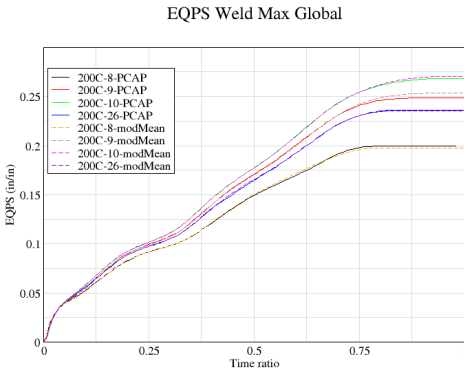
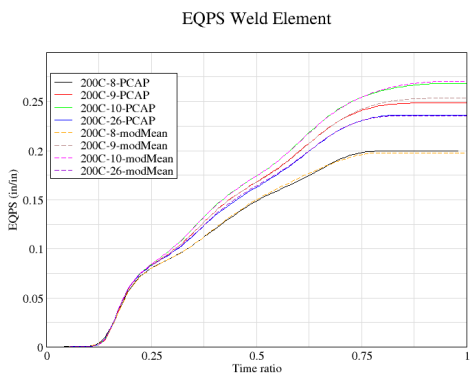




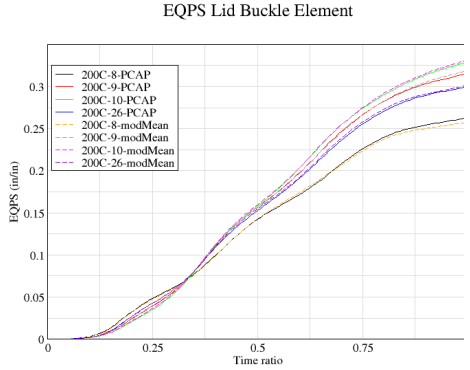
Equivalent Plastic Strain (EQPS) Output Locations

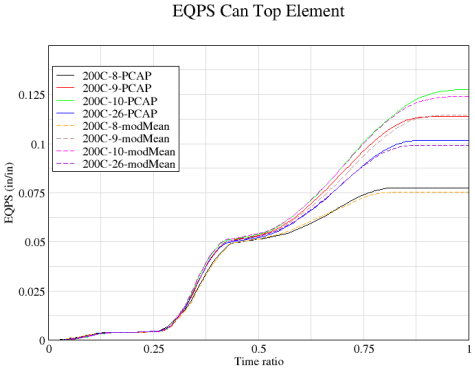




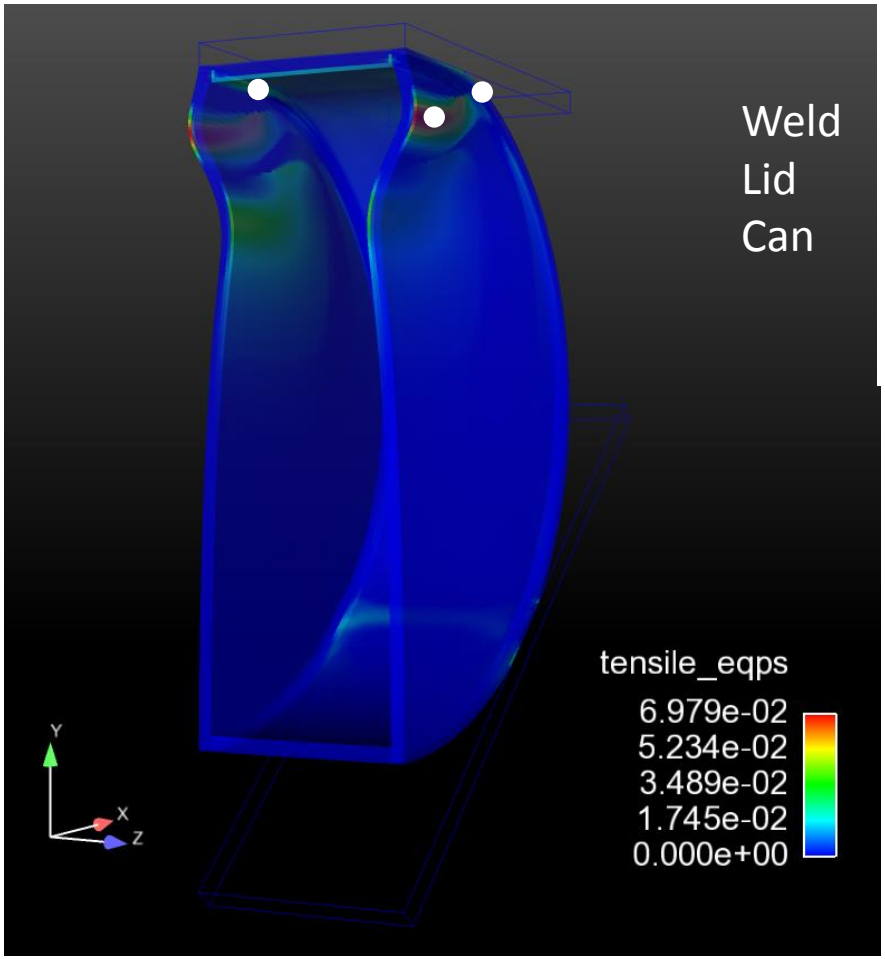


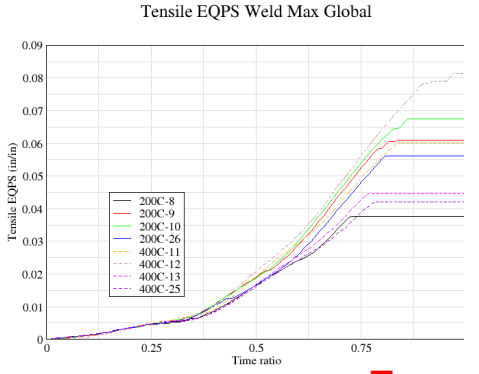
For various σ-ϵ curves used in model

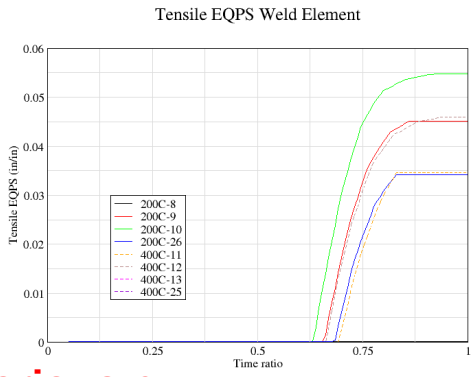




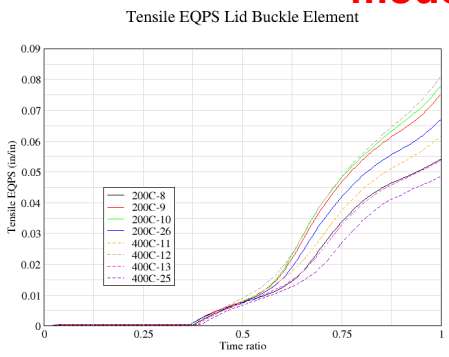
Tensile EQPS output locations

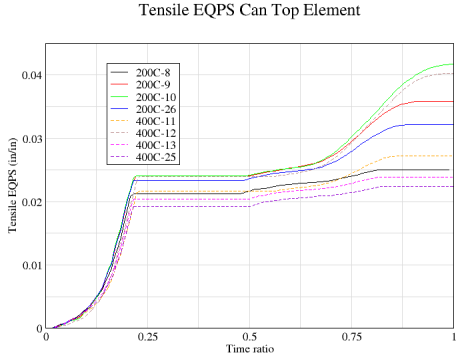




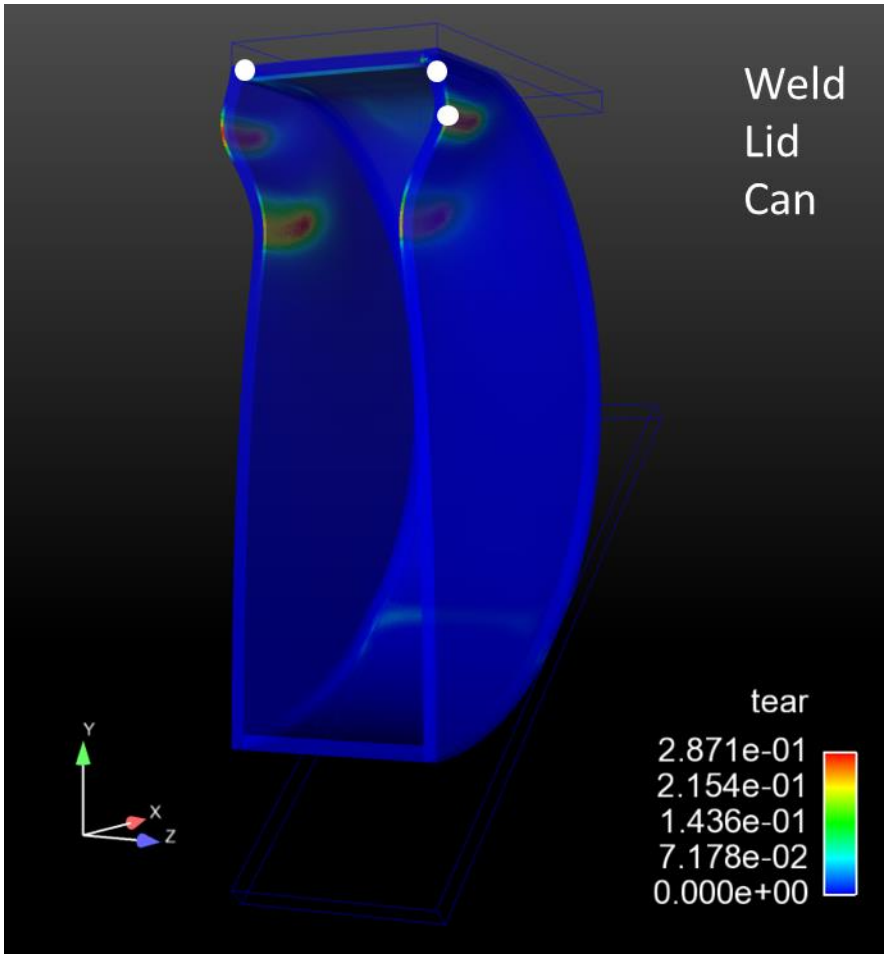


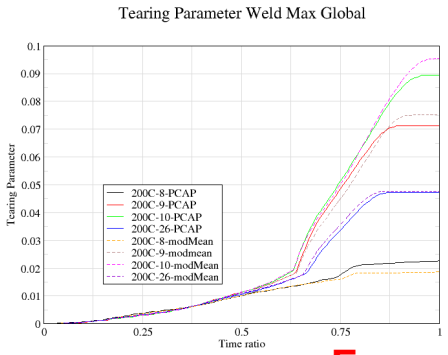
For various σ-ϵ curves used in model

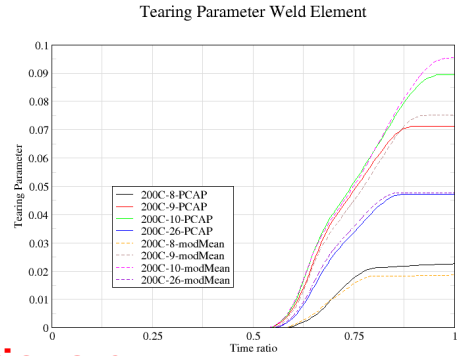




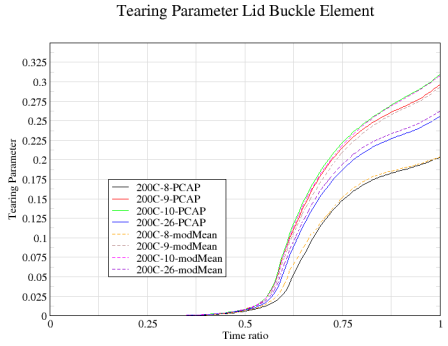
Tearing Parameter output locations

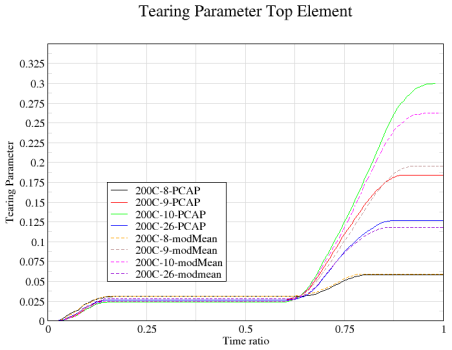






For various σ-ϵ curves used in model





Computational Experiments
(PDFs of Response are Non-Standard and Unknown A-Priori)

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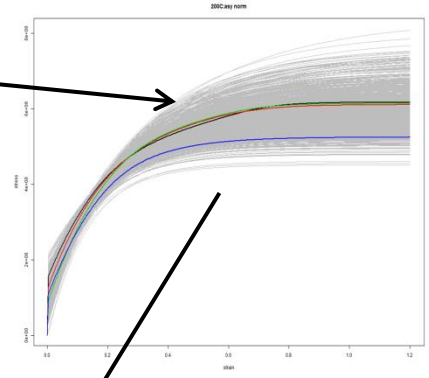
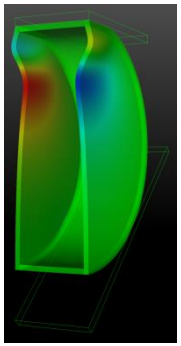
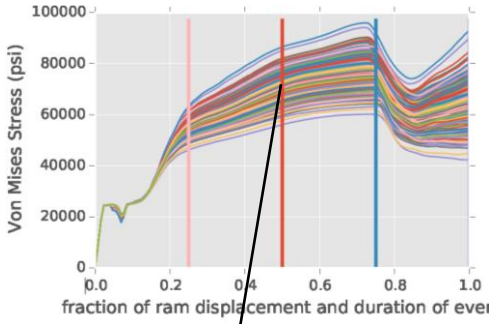
1) Generate 1000 synthetic stress-strain curves from Bayesian UQ on 2-parameter power-law fits to 4 actual stress-strain curves from tests



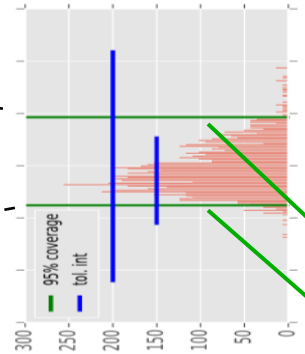
2) Run model for all 1000 synthetic stress-strain curves



output Q.O.I. (1000 realizations)

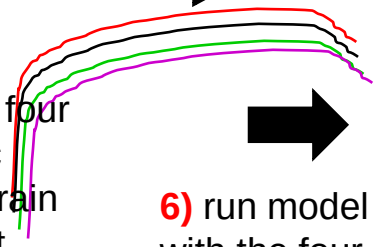


3) **Histogram of 1000 results at time-fraction 0.5**

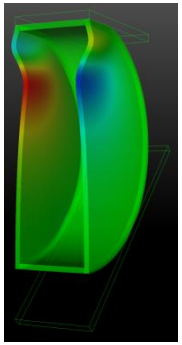


4) **0.025 and 0.975 "exact" percentiles of response from 1000 results**

5) select four synthetic stress-strain curves at random



6) run model with the four synthetic curves

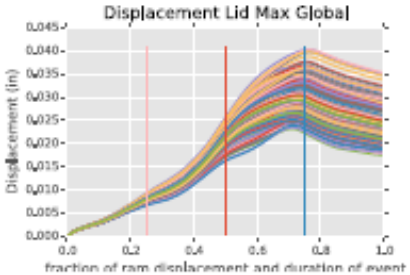


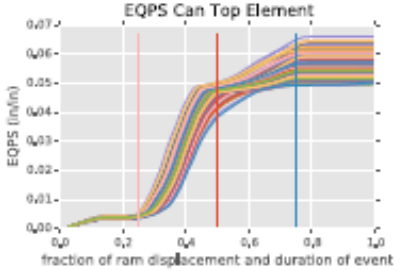
7) **create 0.90/0.95 Tol. Intvl. from model output results**

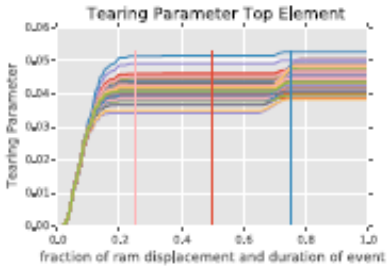
8) **compare TI endpoints against exact percentiles**

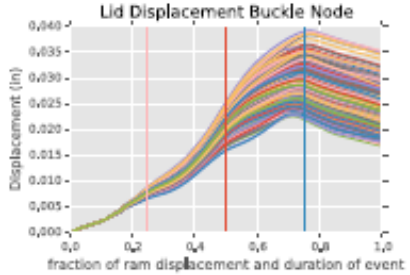
9) do steps 5 – 8 many times (10,000 random trials) to establish reliability rate of TI success in encompassing true 0.025 - 0.975 percentile range

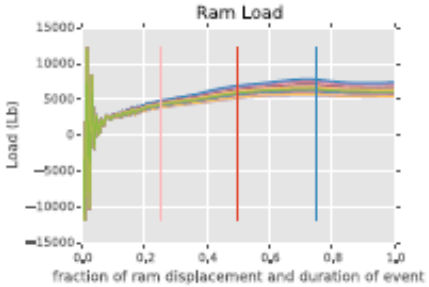
- **18 output quantities**
- **4 time stations each**
- **= 72 QOIs (quant. of int.)**

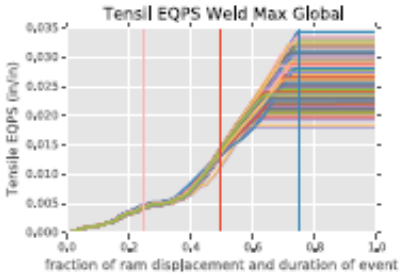


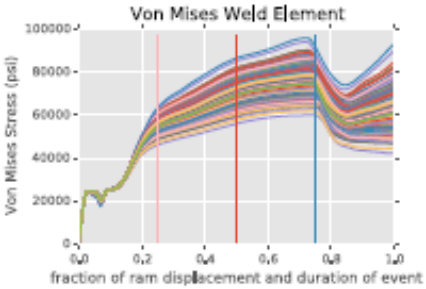


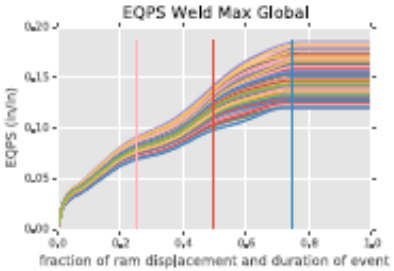


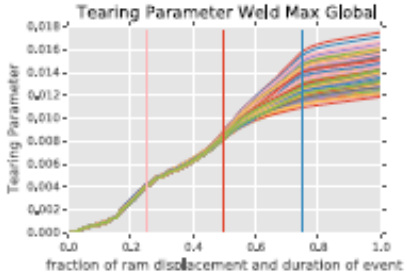


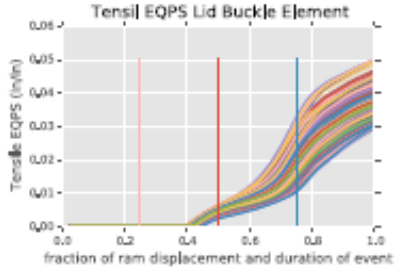


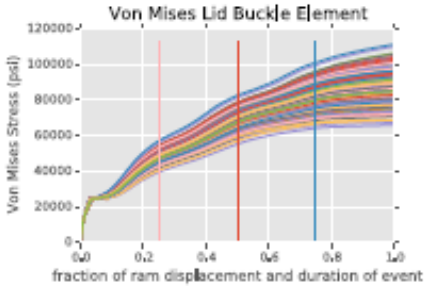


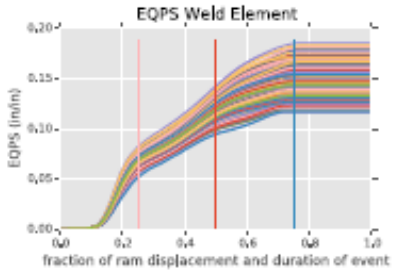


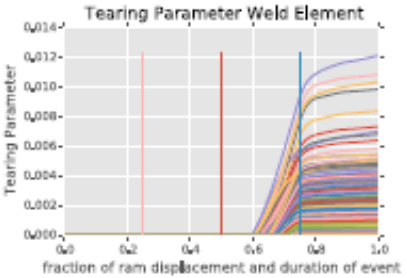


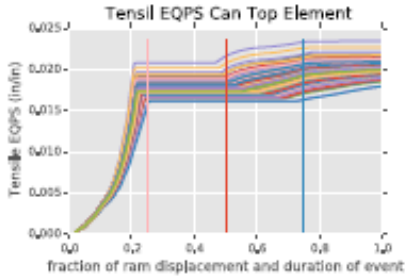


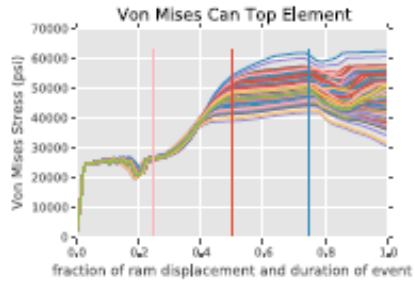


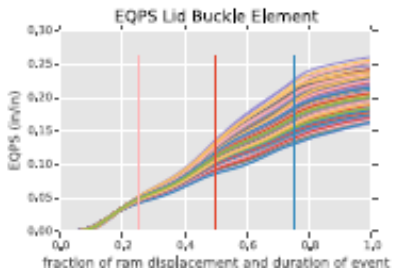


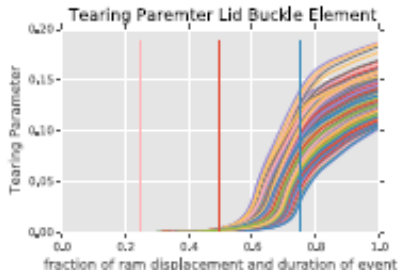


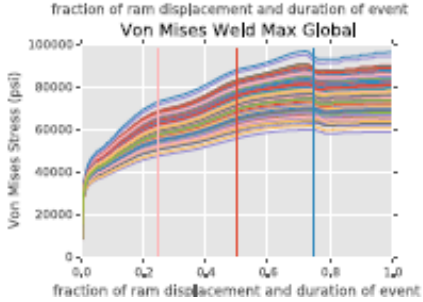






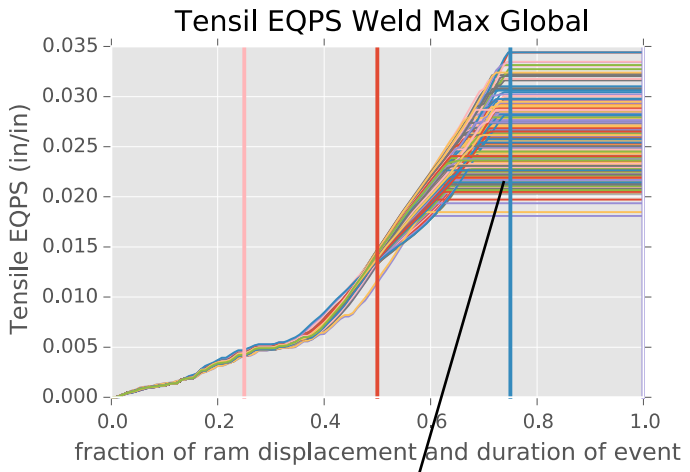


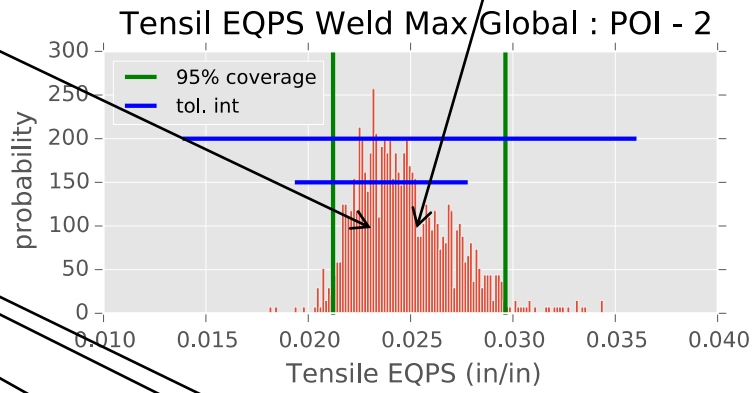


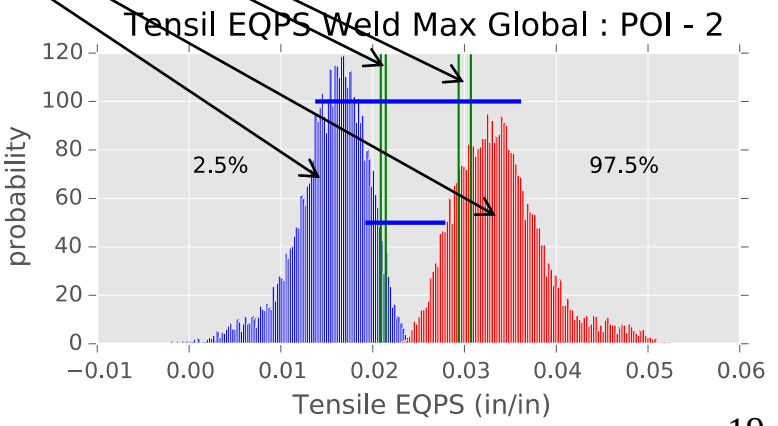


Tol. Intvl. Performance Characterization

- For each QOI the 1,000 results are characterized as a histogram
- The 0.025 and 0.975 quantiles of the histogram are identified as reference “true” values (within small error, see last bullet)
- TI success rate determined from 10K trials of 4 randomly selected input σ-ϵ curves and their QOI results & corresponding 0.9/0.95 TIs. How often do the constructed TIs cover “true” 0.025 - 0.975 range?
- Uncertainty on 95% coverage for 1,000 instances comprising the ensemble determined with non-parametric interval estimates $n\,p \pm z\sqrt{n\,p\,(1-p)}$







Tolerance Interval Empirical Reliability (given four σ-ϵ material curves randomly selected)

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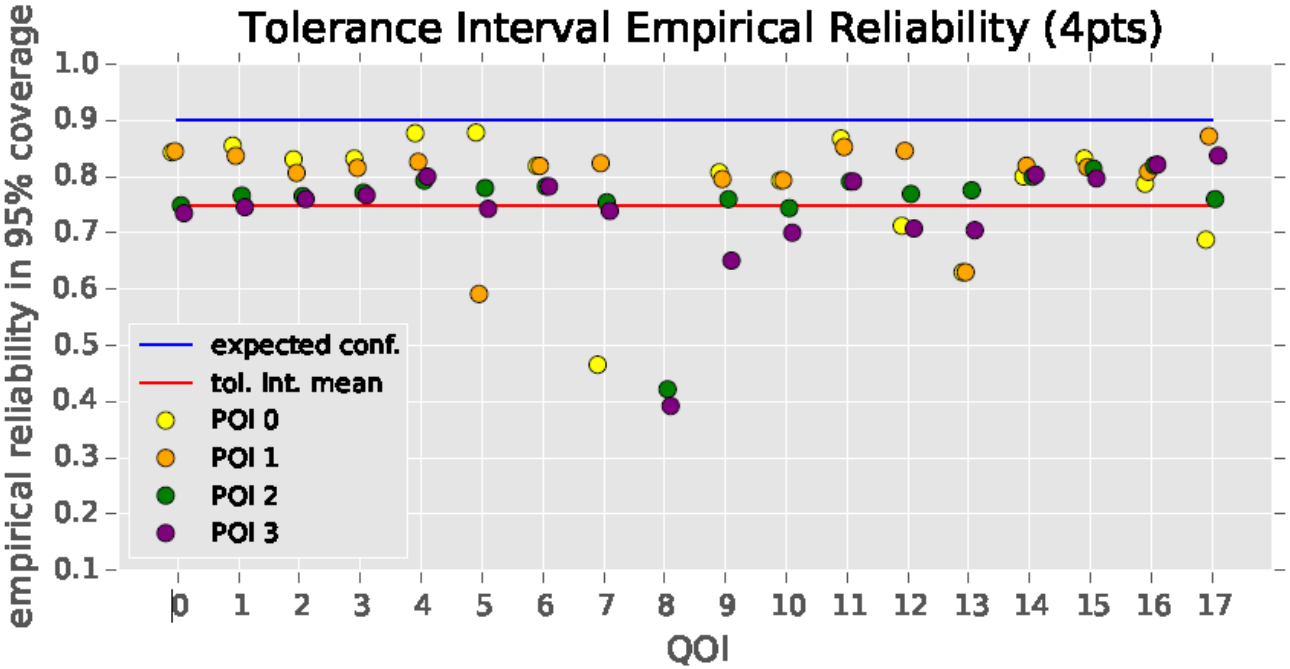
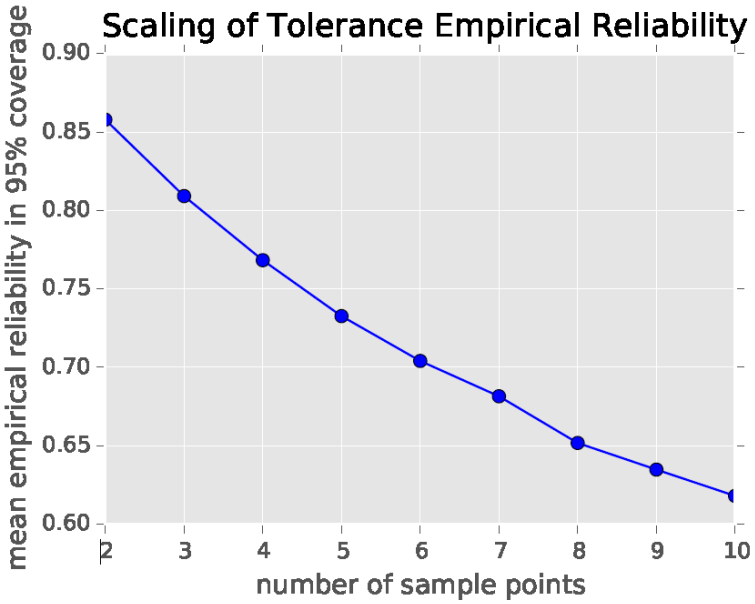


- Always less than the "advertised" 90% reliability or confidence that the TIs will successfully cover the 0.025 – 0.975 range of response
- 75% empirical average TI reliability for four σ-ϵ curves (averaged over ~70 highly nonlinear solid mechanics response quantities and >700,000 trials)
- 70% - 85% successful coverage rate for most QOIs

0.9/0.95 TI Empirical Reliability vs. # of σ-ϵ curves



- **TI reliability of successfully covering 0.025 – 0.975 range of response decreases significantly with # of σ-ϵ curves—but magnitudes of shortfall errors Δ_U and Δ_L decrease faster—more curves are better on average.**

- **>70% expected TI reliability in usual situation of ≤6 σ-ϵ curves**

Conclusions

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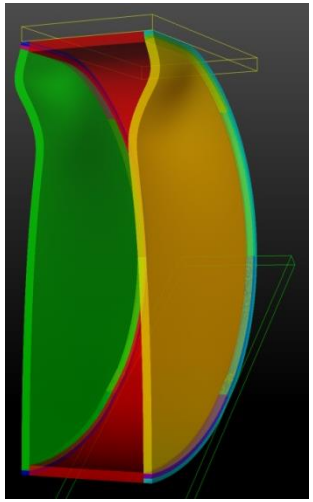
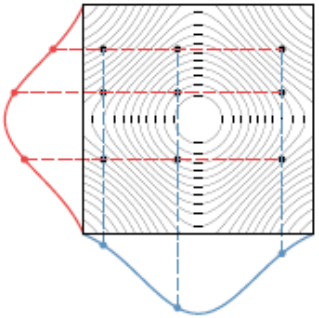
- **The proposed TI methodology is a simple way to manage risk of underestimating material behavior variability when only a small number of material characterization tests are available or affordable.**
- **The methodology is relatively affordable even with computationally expensive models.**
- **The Can-Crush study and results provide evidence that the TI methodology is reasonably reliable (>70% for ≤6 σ-ϵ curves) for providing conservative estimates of the true 0.025 – 0.975 range of response for a large variety of highly nonlinear solid mechanics QOI types, including:**
  - **Von Mises stresses**
  - **displacements and resisting load**
  - **material damage quantities EQPS, Tensile EQPS, and Tearing Parameter**

Next Steps: Assess Sparse-Data UQ Approaches for Multi-Material Solid Mechanics Problems, where only a few σ-ϵ curves exist for each material

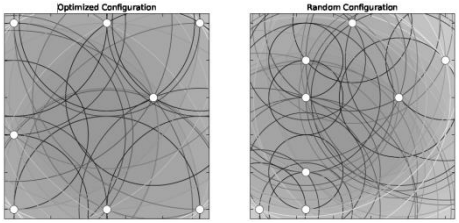
2D random-variable analogue of sparse random-function problem with all possible combinations of input realizations shown. Continuous-variable analogue problem helped determine a more efficient UQ approach than the all-combinations of stress-strain curves approach previously taken.

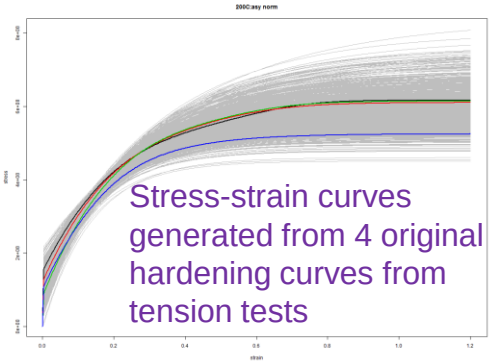
Can-crush simulation results with nominal mtl. hardening curves (half-symmetry Presto model)





Simulated Annealing for Optimal Sampling Design





1 of 18 output quantities to be used to assess performance of sparse-data UQ method applied to can-crush test problem