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SANDIA STATISTICAL SCIENCES

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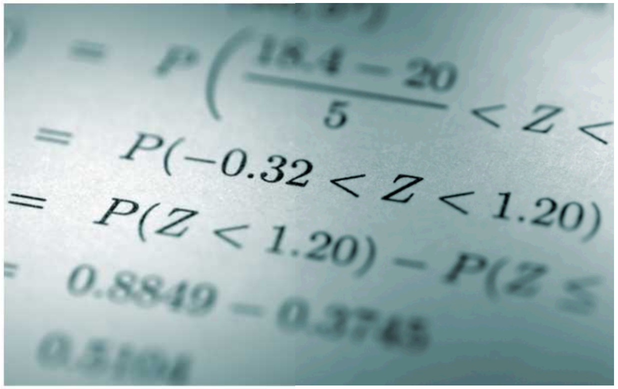

Design and Analysis of Margin Testing at Sandia

A Statistical Perspective

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Department of Statistical Sciences

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Sandia Statistical Sciences Mission

- Sandia Statistics Mission: Team with Sandia groups and projects to assist with statistical challenges
  - Promote optimization of Sandia product and process performance through application of statistical methods aimed at improving data collection, analysis, and communication of results
- Statistics is a foundational capability for a National Laboratory
  - There are many opportunities to advance the field of statistics, apply statistical methods more broadly in the engineering sciences, and team better to improve product quality
- The discipline of statistics has two main dependent focus areas:
  1. Making data make sense (planning and design of statistical studies)
  2. Making sense out of data (analysis of data and communication of results)
- Primary areas of statistical expertise provided by the group are:
  - Design of experiments, sampling and test plans, and sample size calculations
  - Statistical quality control
  - Statistical reliability and maintainability
  - Margin and uncertainty analyses
  - Measurement error, repeatability, and reproducibility plans and analysis
  - Bayesian Statistics
  - Probabilistic modeling and computer simulation
  - Spatial data analysis
  - Signal processing
  - Causal Inference
  - Statistical computing

Statistics Work at Sandia

- **Statistics is a foundational capability across all Sandia mission areas**
- **Customer interaction and teaming is critical to our success**
  - We work with our customers to:
    - Identify project objectives and requirements
    - Determine a technical approach based on the unique circumstances of the project
    - Perform the work based on a customized approach
    - Review the results and assist with communication
- Current Staff
  - 11 full-time statisticians (4 MS, 7 PhDs)
  - 2 year round MS level interns

7 Mission Areas

All Life Cycle Activities Benefit from Statistical Rigor

Concept/ Feasibility

- Critical Parameter Identification
- Initial Design Margin based on models
- Design Characterization
- Design of Experiments
- Mod/Sim optimization
- Statistical design of computer experiments

Development

- Design Margin based on test data
- Performance, Input, and Environmental Margin evaluation
- Design of Experiments
- Sample size calculations
- Sensitivity analyses
- Measurement uncertainty / tester R&R analyses
- Statistics support for reliability assurance

Qualification

- Design of test plans to demonstrate requirements are met
- Analysis of qualification data
- Analysis of early production process data
- Fusion of information to inform production readiness

Production

- Process Capability / Characterization
- Statistical Process Control planning & implementation
- Determination of production specifications
- Lot Acceptance test planning
- Production data reviews
- Statistical analysis for anomalies

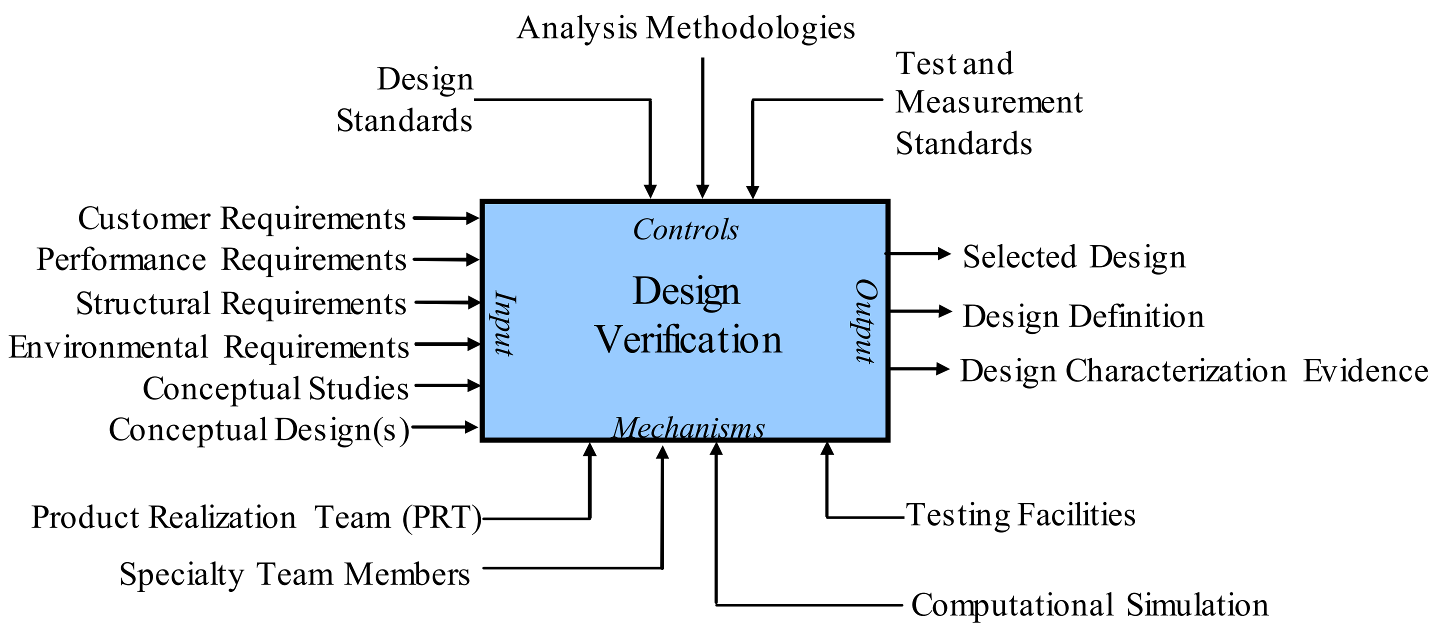
Surveillance

- Development of shelf life sample plans
- System and Component Data Reviews
- Aging trends
- Tester Correlation Studies
- Design of Experiments / Data Analysis needed for special investigations
- Statistical analysis for anomalies

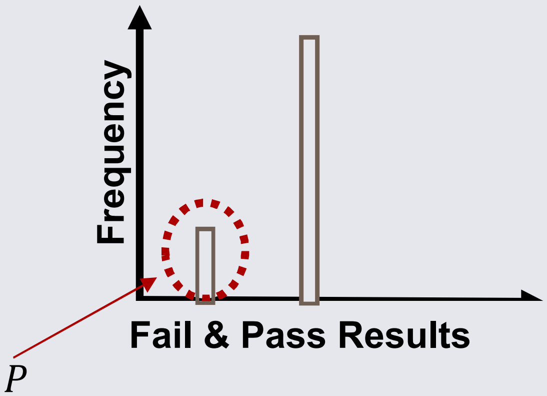
Multiple large scale development programs create a growing need for statistics support

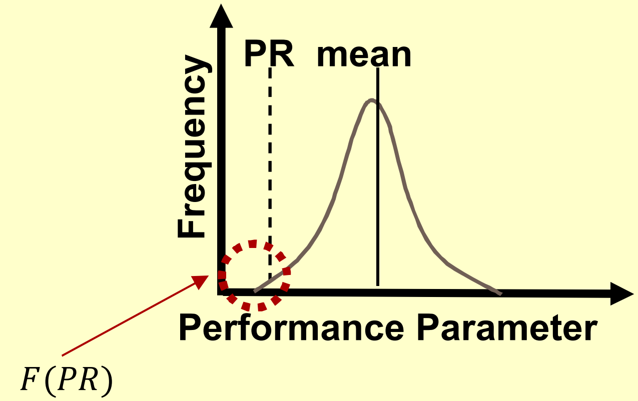
Objectives for Qualification

- Qualification is a planned set of evaluation activities to assure design intent and customer requirements are met
- Key design attributes that must be evaluated are
  - Robustness: The design is such that there is a demonstrated significant performance margin between the product requirements and the product performance
  - Reliable: The design provides for an acceptable probability that the item will perform a required function under stated conditions for a stated period of time



Component Unreliability Contributors





<u>Quality defect</u> – component did not meet design intent and is not capable of functioning properly in all design basis environments

Quality defect is a type of unreliability: Component did not function properly

Quality defects are often assembly errors, workmanship problems, mistakes, etc. caused by design or production problems

<u>Margin Insufficiency</u>: component did not have sufficient performance capability above a required level to compensate for uncertainty

Margin insufficiency is a type of unreliability: Component functioned but didn't meet requirements

A good design has adequate margin if the component works with expected production and environmental variations

Component Reliability Model

Component reliability

= Prob (no failure due to quality defects) * Prob (no failure due to low margin, given no quality defects)

= $\{1 - P\}$ * $\{1 - F(PR)\}$

Always in model

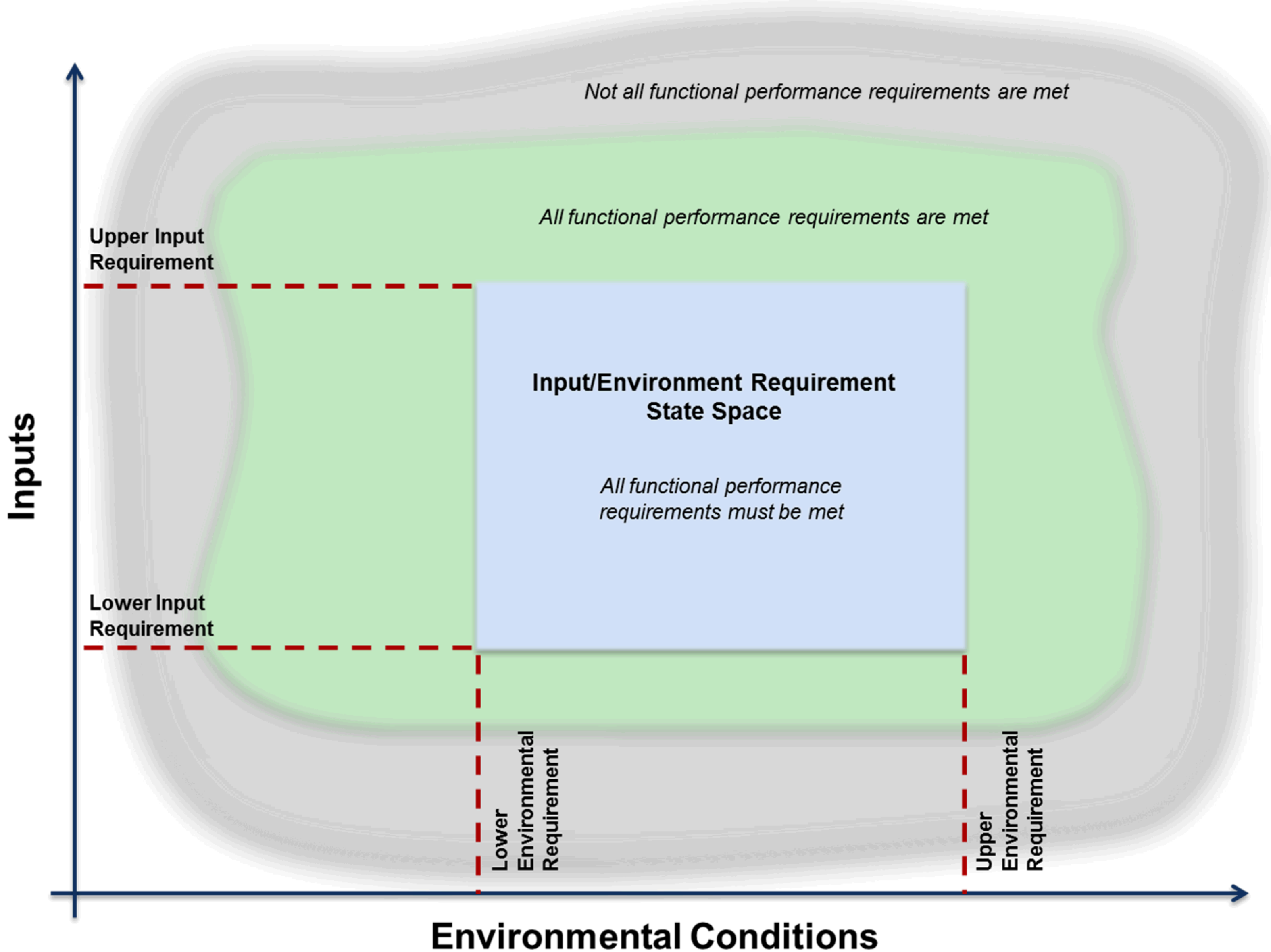
Included in model if needed

Margin studies identify and quantify margin insufficiency terms

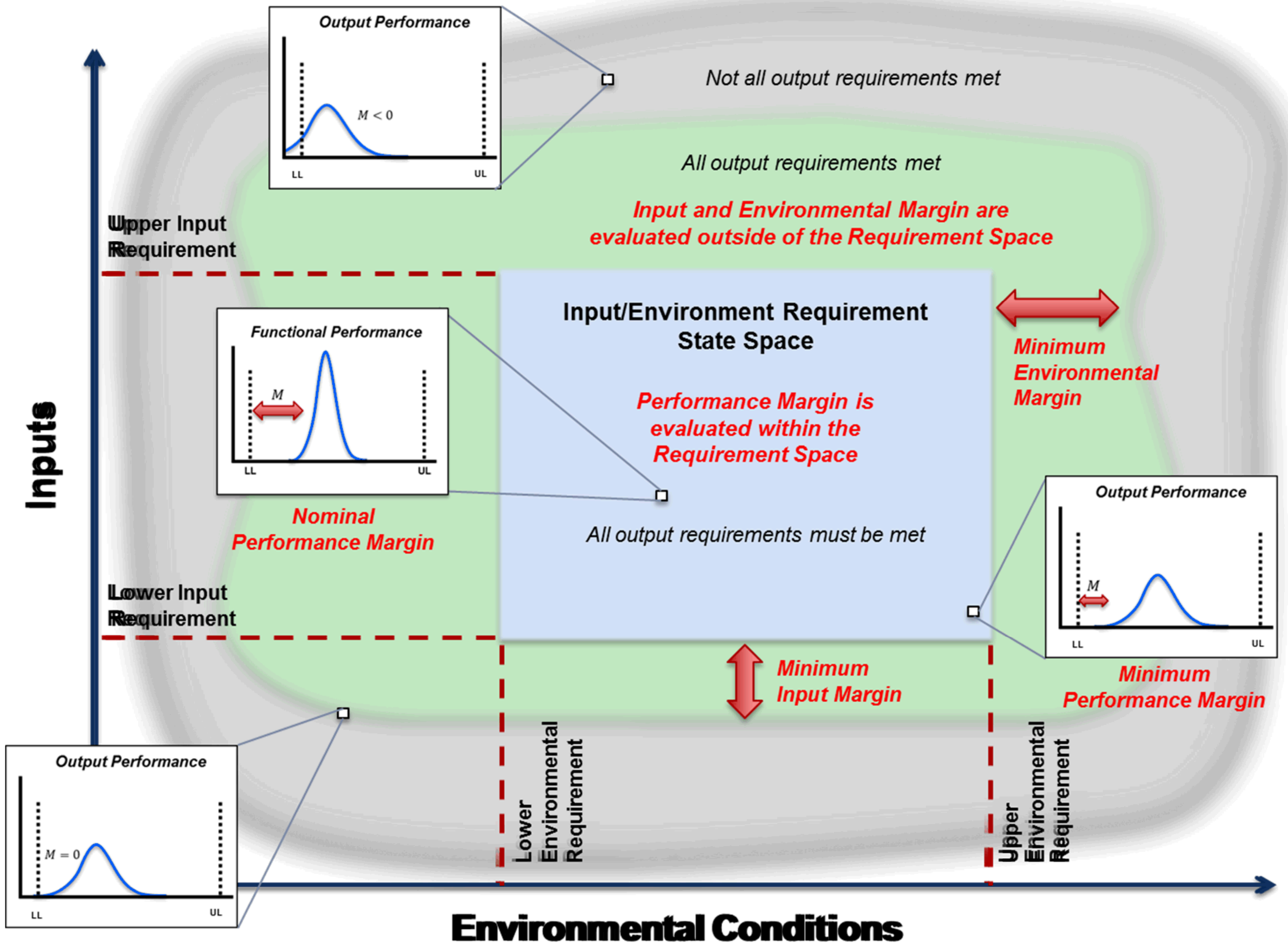
Benefits of Margin Analysis

- Improved understanding of product
  - Performance impacts due to margin insufficiency
  - What behavior is expected (baseline)
  - Performance margin, stability over time
  - *Motivates in-depth review of failure mechanisms*
  - *Motivates thinking about <u>what data may be needed</u>*
- Improved understanding of test programs
  - *Motivates in-depth review of tester, analysis, and monitoring points and their impact on data*
  - *Motivates thinking about <u>what data may be available</u>*
- Improved opportunity to detect defects
  - Margin analyses allow for detection of trends before they affect performance

Typical Requirements Space



Goals for Product Qualification



Challenges for Qualification

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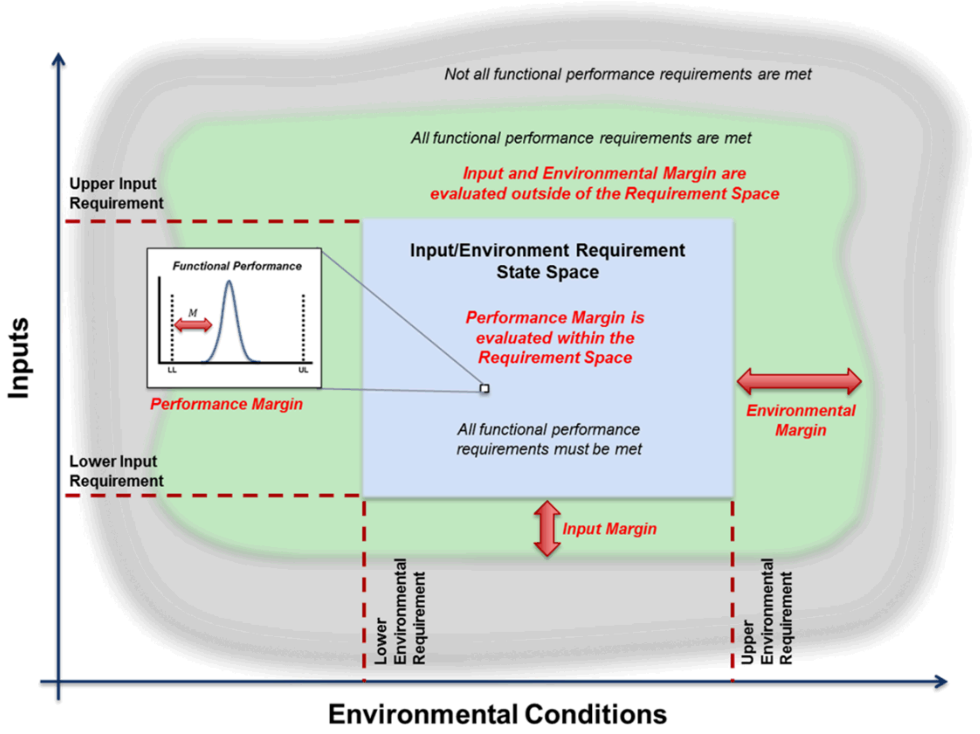
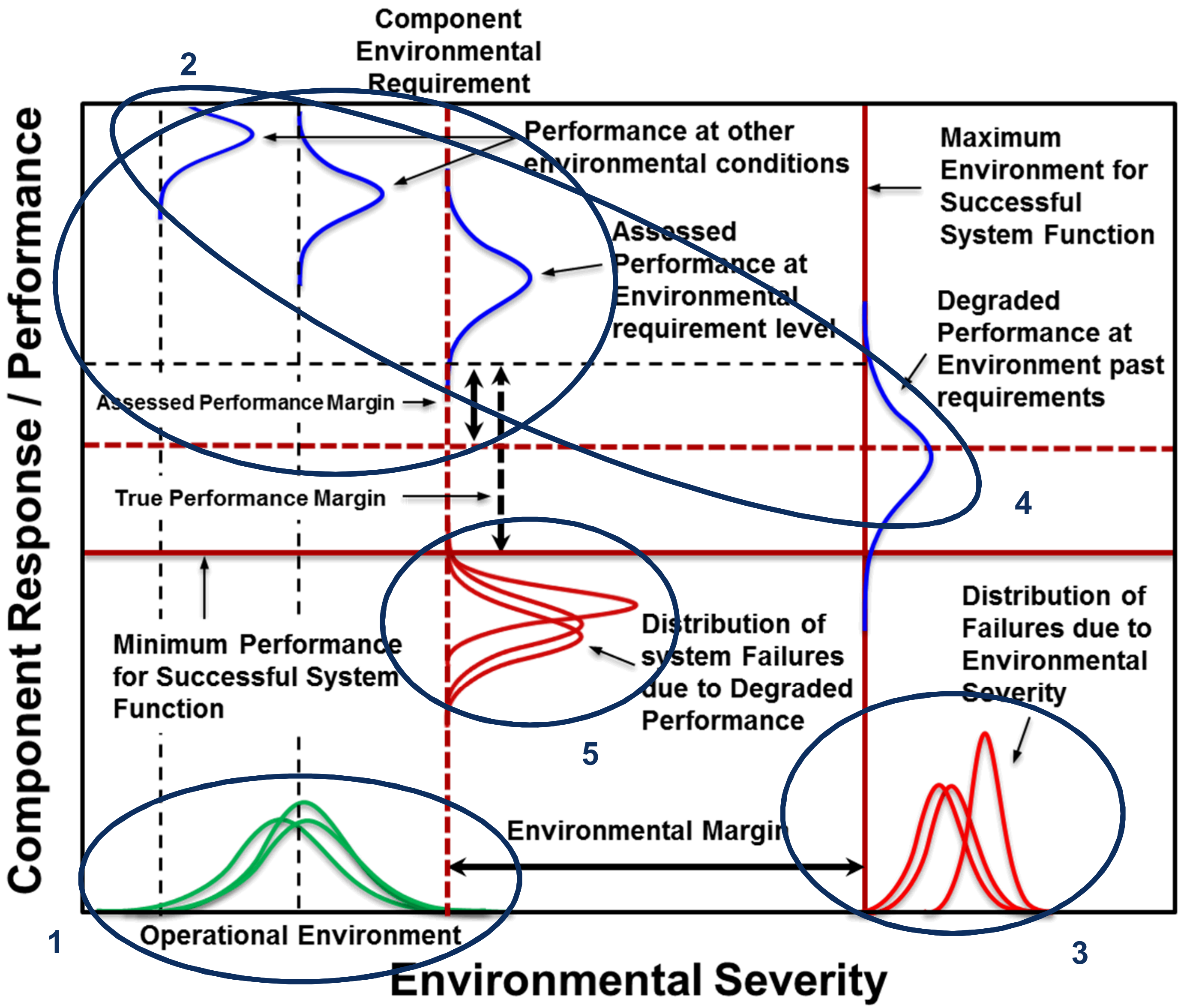
- Statistical Challenges:
  - How / where to test across this multi-dimensional space
    - Balance between testing inside and beyond the requirements space
  - How to evaluate performance at each test point in the space
    - Balance between binary (pass/fail) and continuous performance data
  - Inputs and Environmental severity may not be scalar



- Additional Considerations:
  - Worst case environments and inputs may be unknown
    - Computational models and engineering judgment is often leveraged to inform assumptions
  - Test facilities may not be capable of achieving all points in the requirements space (or too far beyond the requirement space)
    - Often results in censored data
  - Challenges are exacerbated when resources (both assets and test time) are limited

Evaluating Margin



Required

1. Evaluate Component Environmental Requirements
2. Evaluate Component Performance Margin
3. Evaluate Component Environmental Margin

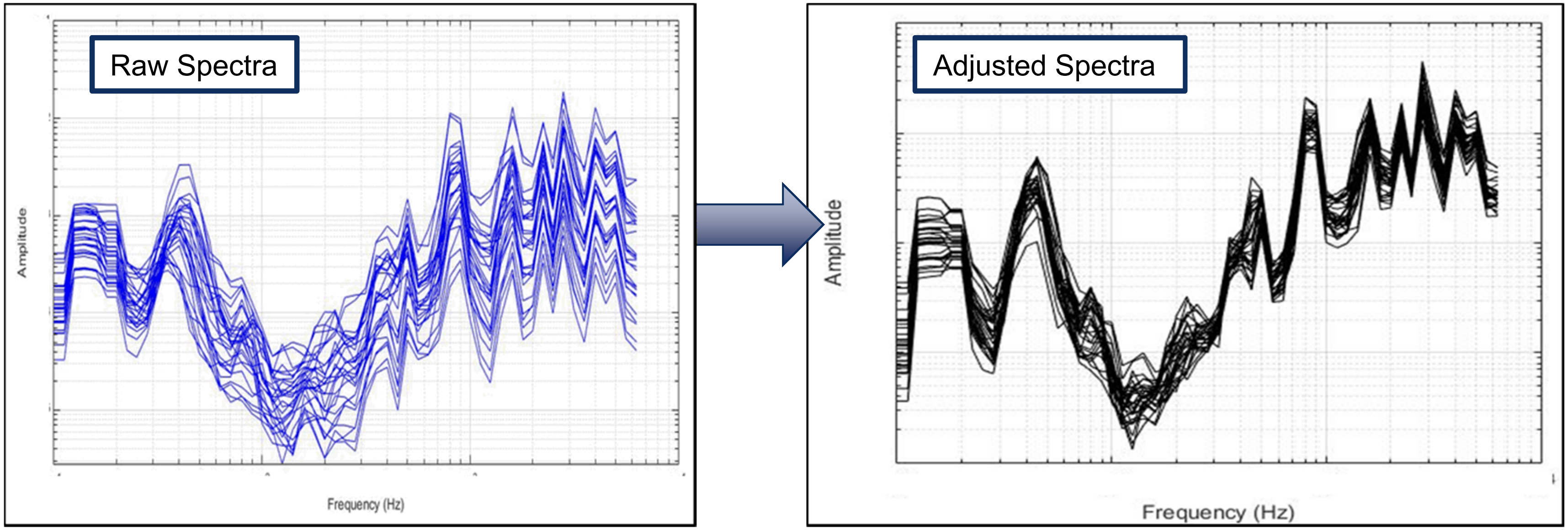
Component Requirement

Evaluated if Possible

4. Evaluate Component Performance as a function of Environments
5. Evaluate Component / System Performance Threshold Distribution

Characterization of System Environments

- A key part of system development and qualification is the characterization of the environments associated with system deployment
  - We have developed methods to construct tolerance bounds for the Acceleration Spectral Density (ASD), which is a measure of the vibration environment
  - There is a large predictable effect of dynamic pressure (*Q*), therefore the ensemble of ASDs are normalized to a target value of *Q* – which can then be used to construct an upper tolerance bound for the targeted conditions

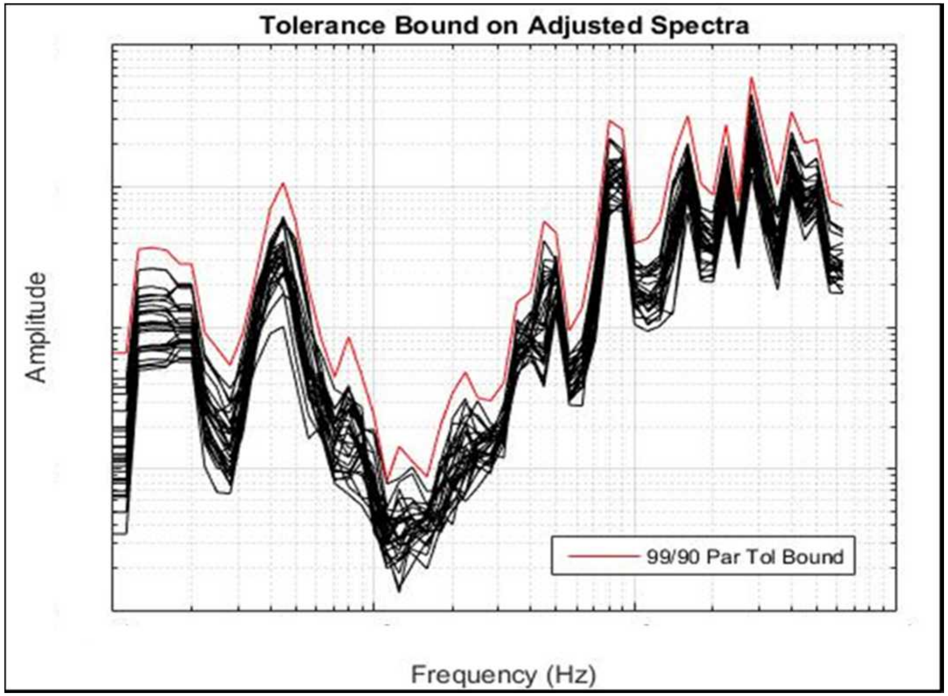


Parametric Bootstrap Approach

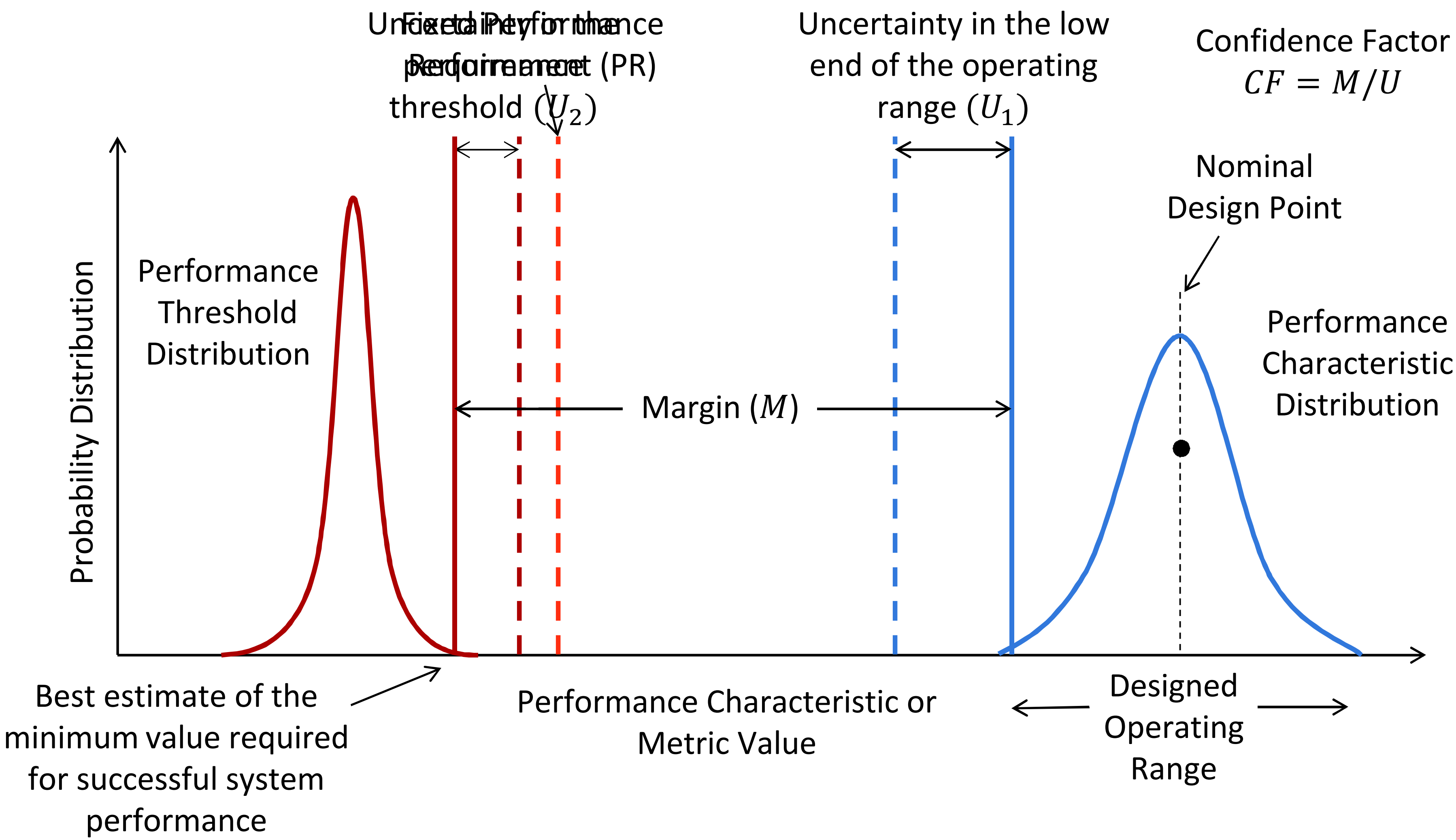
- The method used to derive the tolerance bound relies on a “parametric bootstrap” variant of the general bootstrap procedure
- It is assumed that an appropriate basis set has been determined such that *p* latent variables are sufficient to adequately represent the spectral variation
  - This method is based on a principal-components decomposition of an appropriate set of spectra where we project the log-transformed high-dimensional spectral data onto a smaller dimensional orthogonal space (defined by latent variables) which facilitates analysis.
- At each of *B* iterations, plausible values for the true (unknown) parameters of assumed normal distributions (σ_k and μ_k) are simulated
  - The (1-α) percentile for the i^{th} bootstrap iteration is

$$P_{boot}^{i}(j) = \bar{Y}_{boot}^{i}(j) + z_{1-\alpha} \cdot S_{boot}^{i}(j),$$

  - $\bar{Y}_{boot}^{i}$ and S_{boot}^{i} are functions of the simulated σ_k and μ_k
  - The γ-percentile of the values within $\{P_{boot}^{i}(j)\}_{i=1:B}$ is the γ-level upper confidence bound for the (1-α) percentile of the j^{th} spectral channel.

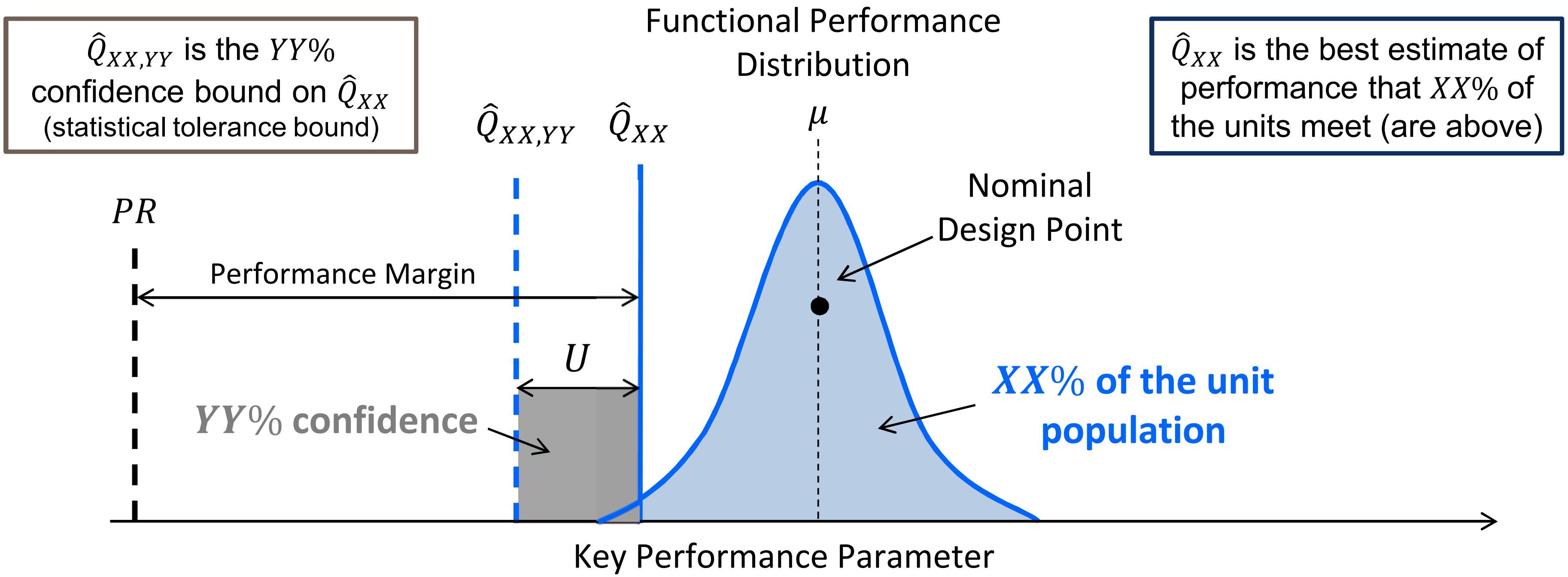


Conceptual Framework for Performance Margins



Statistical Framework for Performance Margin

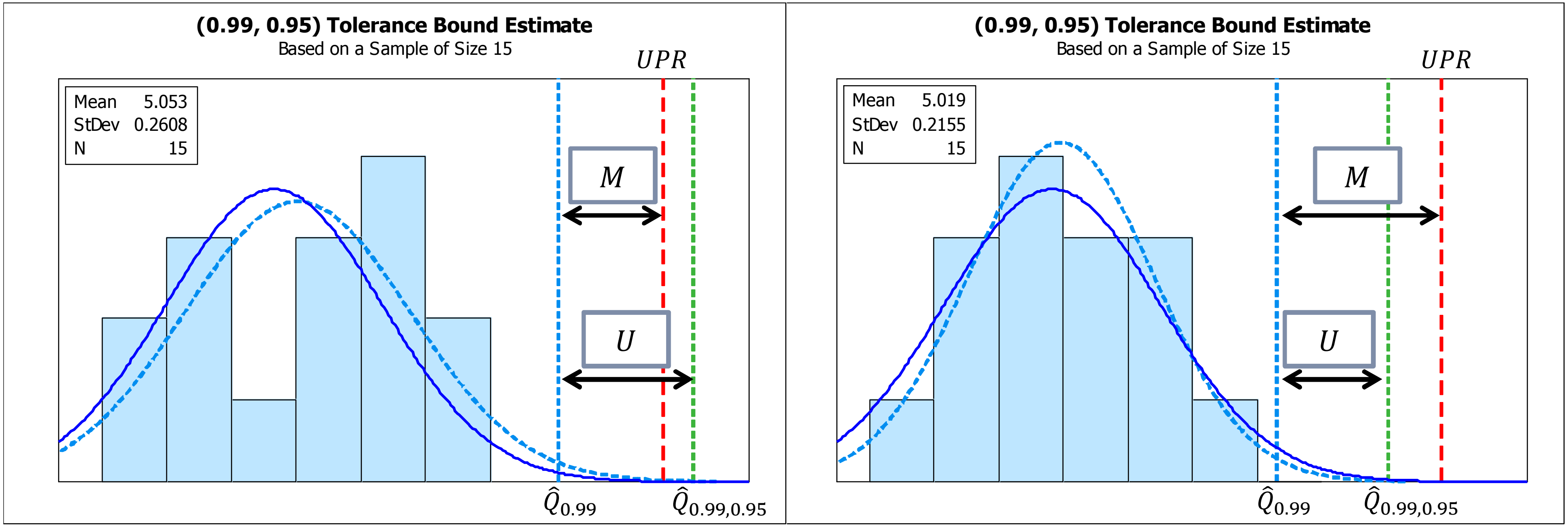
- Are we $\boldsymbol{YY}\%$ confident that **at-least** $\boldsymbol{XX}\%$ of the unit population will yield a response **greater than** the performance requirement $\boldsymbol{PR}$?



- Statistical Challenges:
  - Estimation of a parametric distribution requires unverifiable assumptions
    - Traditional goodness-of-fit tests are inadequate
  - Quantile estimates require extrapolation outside of observable data

Sample Size Considerations

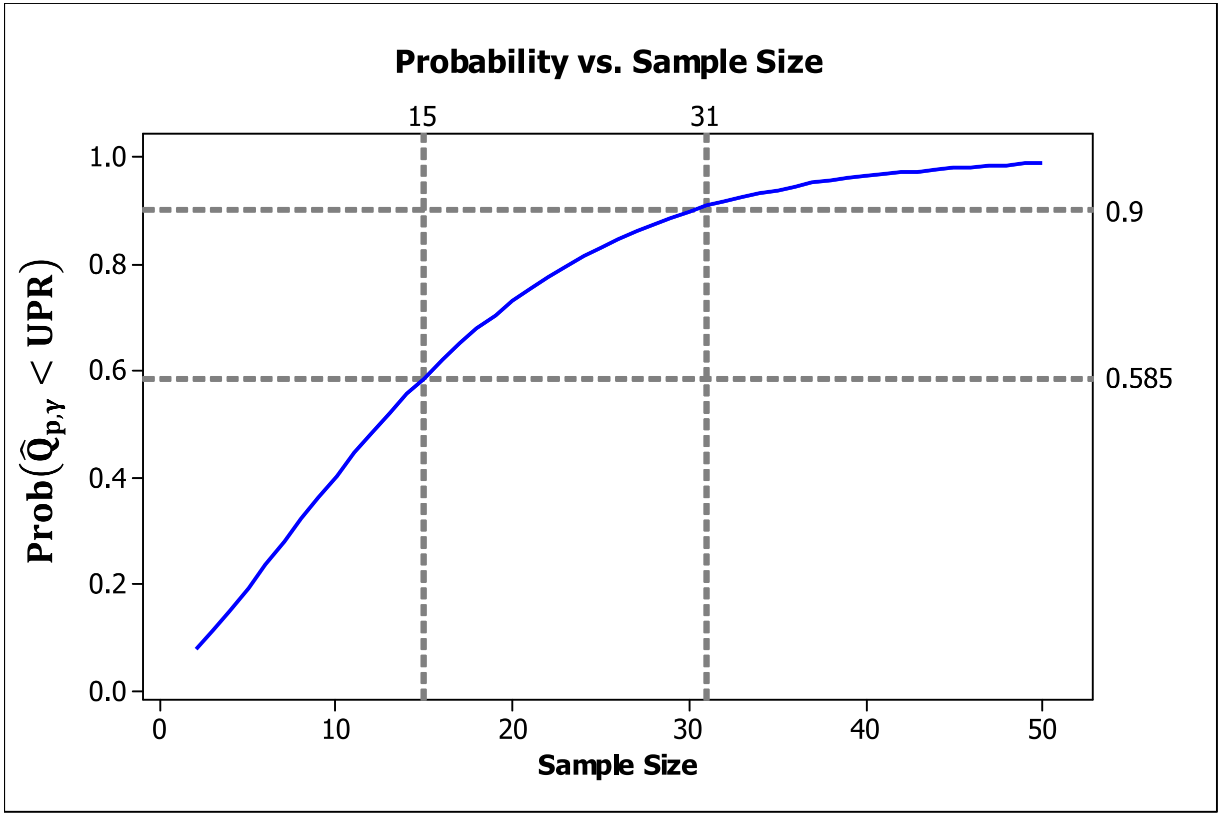
- How many samples do I need to **demonstrate** the performance characteristic has sufficient margin to requirements with a high probability, provided the true underlying distribution has positive margin?
  - Smaller sample sizes create more risk of not being able to make high confidence statements



Choose a Sample Size that Provides Acceptable Risk

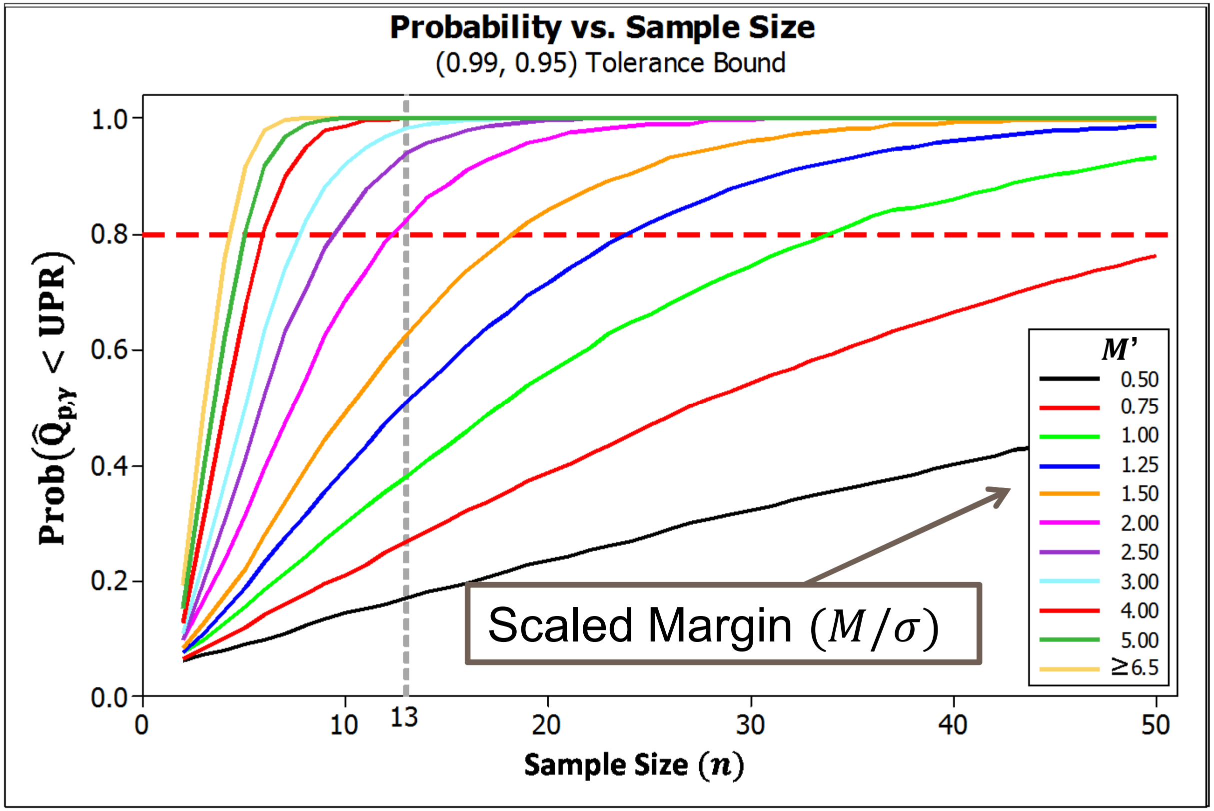
- "How many samples do I need to **demonstrate** the performance characteristic has sufficient margin to requirements <u>with a high probability</u>, provided the true underlying distribution has positive margin?"
  - Mathematically, we want $Prob\left(\hat{Q}_{p,\gamma} < UPR\right) \geq \beta$

Set to achieve a tolerable level of risk



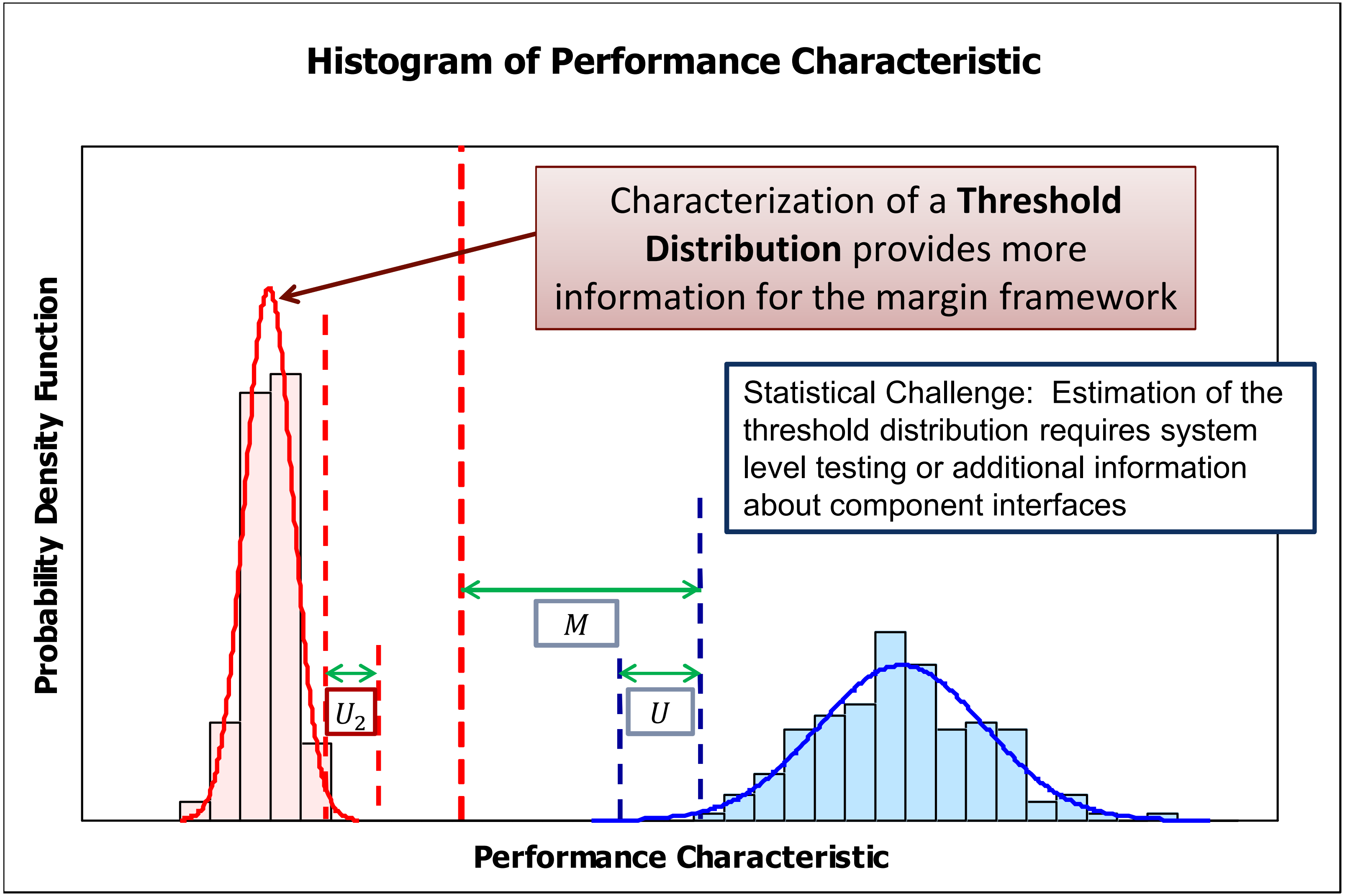
Generating Sample Size Curves

- Curves are generated via Monte Carlo Simulation
- Repeat steps below S times
  - Sampling Step: Draw a sample of size n from the distribution $f(x)$ that has a margin = M
  - Estimation Step: Estimate a (p, γ) upper tolerance bound $\hat{Q}_{p,\gamma}$
  - Comparison Step: Compare the estimated upper tolerance $\hat{Q}_{p,\gamma}$ to UPR and record if $\hat{Q}_{p,\gamma} < UPR$ (i.e. let $y_i = 1$ if $\hat{Q}_{p,\gamma} < UPR$ and $y_i = 0$ otherwise)
- Estimate the probability of $\hat{Q}_{p,\gamma}$ not exceeding UPR by
  - $Prob(\hat{Q}_{p,\gamma} < UPR) =$ (# of times $\hat{Q}_{p,\gamma} < UPR$)$/S = \sum_{i=1}^{S} y_i / S$
- **Recommend smallest sample size that achieves $\boldsymbol{Prob(\hat{Q}_{p,\gamma} < UPR) \geq 0.80}$**



$p = 0.99, \gamma = 0.95$			
M'	Minimum sample size for $Prob(\hat{Q}_{p,\gamma} < UPR) \geq 0.80$	M'	Minimum sample size for $Prob(\hat{Q}_{p,\gamma} < UPR) \geq 0.80$
0.5	110	2.5	10
0.75	55	3.0	8
1.0	34	3.5	7
1.25	24	4.0	6
1.5	19	5.0	5
2.0	13	≥6.5	4

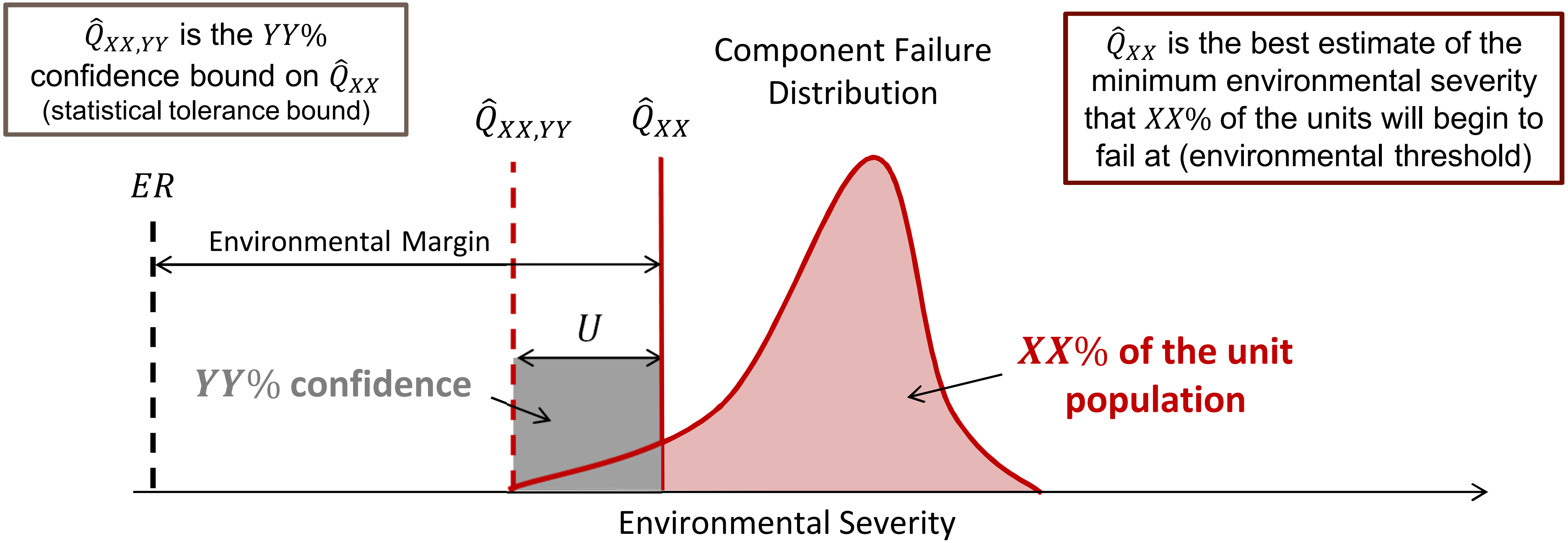
A More Informative Margin Framework



Graphical Depiction of a Margin Analysis with Overlapping Distributions

Statistical Framework: Environmental Analogue

- Are we $\boldsymbol{YY}\%$ confident that **at-least $\boldsymbol{XX}\%$** of the unit population failures will be in an environment **more severe than** the maximum required environment $\boldsymbol{ER}$?

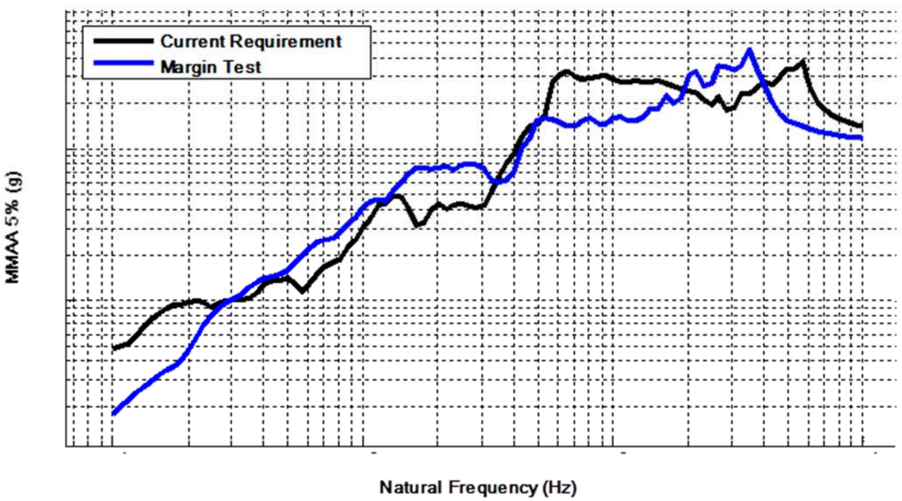


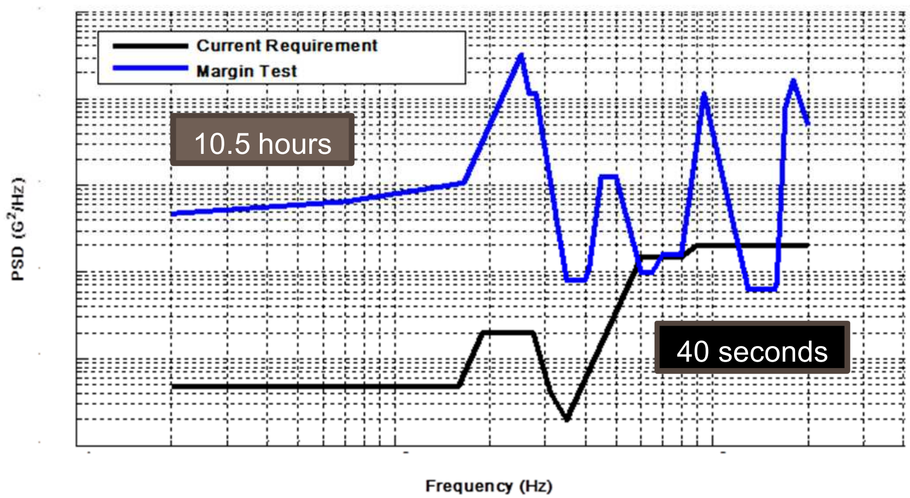
- Statistical Challenges:
  - Estimation of the component failure distribution with limited binary data
    - Highly robust components often lead to multiple censored observation
  - Environmental severity is often a functional variable

Need for a Scalar Severity Metric

- Many common tasks rely on assessment of environmental severity
  - Existing tools struggle to answer the question in a meaningful way.
  - New tools are needed to provide a rigorous answer

Which test is more severe?

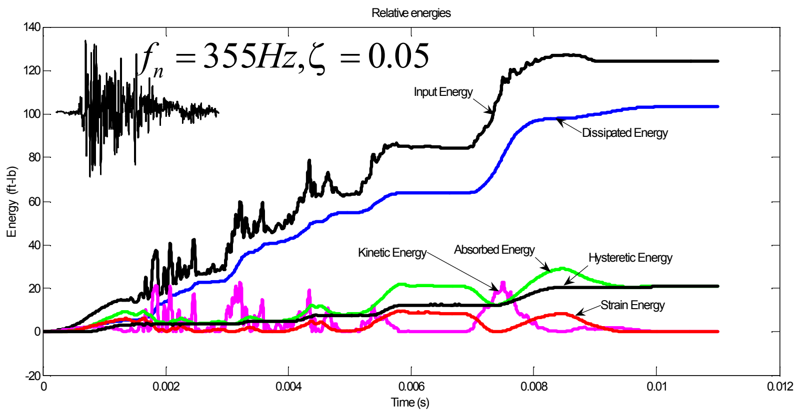




Energy Analysis – Mechanical Environments

- Most important characteristics of energy-based analysis:
  - Scalar margin estimates
  - Closely related to accepted material failure criteria
  - Accounts for multiple exposures and duration
  - Can be used to generate scalar estimates of margin for complex spectra
  - Can characterize failures of different types
    - Peak strain energy $\rightarrow$ first passage type failures
    - Total energy $\rightarrow$ fatigue type failures
    - Kinetic energy $\rightarrow$ electrical contact intermittency
    - Hysteretic energy $\rightarrow$ plastic failure

Energy metrics have quickly become an indispensable tool for environmental margin analyses



Total Input Energy

$$E_I^R = -\int \{\dot{z}\}^T [M]\{\ddot{x}_g\}dt$$

Dissipated Energy

$$E_D = \int \{\dot{z}\}^T [C]\{\dot{z}\}dt$$

Absorbed Energy

$$E_A = \int \{\dot{z}\}^T \{f_s\}dt$$

Kinetic Energy

$$E_K^R = \frac{1}{2}\{\dot{z}\}^T [M]\{\dot{z}\}$$

Strain Energy

$$E_\varepsilon = \int \{\dot{z}\}^T [K]\{z\}dt$$

Energy balance:

$$E_K + E_D + E_A = E_I$$

Summary

- Evaluating component margins is a key task for system qualification
  - Performance, Input, and Environmental margins all must be evaluated
  - There are many statistical challenges in assessing these margins

- The statistical sciences group at Sandia is leading the development and evaluation of novel statistical approaches to the design and analysis of margin testing

- References

1. J.T. Newcomer and K.E. Freeland, *A Margin Based Approach to Determining Sample Sizes via Tolerance Bounds,* SAND2013-8168, September 2013.
2. J.T. Newcomer, B.M. Rutherford, E.V. Thomas, R.L. Bierbaum, L.M. Hickman, J.W. Lane, S Fitchett, A Urbina, A.A. Robertson, and L.P. Swiler, *Handbook of Statistical Methodologies for QMU*, SAND2012-9603, November 2012.
3. J.T. Newcomer, *A New Approach to QMU for Physical Simulation Data*, SAND2012-7912, Sandia National Laboratories, Albuquerque, NM, September 2012.