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Reduced-order Modeling for Radiation Heat Transfer Applications

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Problem Definition

- Participating Media Radiation (PMR) is prohibitively computationally expensive due to highly nonlinear coupling with conduction/convection and the need to resolve dependence of the radiative intensity on direction (Ω).
- Nonlinear coupling is unavoidable but the cost of resolving the angular dependence of intensity may be significantly reduced.
- Successfully reducing this cost could allow the inclusion of additional physics while making UQ studies tractable for large problems without sacrificing model fidelity.

Governing Equations

- Radiative Transfer Equation — *5-dimensional PDE*

$$\vec{\Omega}\cdot\vec{\nabla}I(\vec{\Omega})+(\sigma_A+\sigma_S)I(\vec{\Omega})=\sigma_A I_b+\frac{\sigma_S}{4\pi}\int I(\vec{\Omega})d\vec{\Omega}$$

- Discrete Ordinates Approximation — *Up to several hundred coupled 3-dimensional PDEs*

$$\vec{\Omega}_i\cdot\vec{\nabla}I_i+(\sigma_A+\sigma_S)I_i=\sigma_A I_b+\frac{\sigma_S}{4\pi}\sum w_j I_j$$

$$I_i=\varepsilon I_{bw}+\frac{1-\varepsilon}{\pi}\sum_{\vec{n}\cdot\vec{\Omega}_j<0}w_j I_j|\vec{n}\cdot\vec{\Omega}_j|$$

- Discretized Model — *Hundreds or thousands of solutions of large linear systems (Full-Order Model, FOM) per time-step or nonlinear iteration*

$$\bar{\bar{K}}(\vec{\Omega}_i)\vec{I}(\vec{\Omega}_i)=\vec{S} \longleftarrow \text{FOM (N x N)}$$

Reduced Order Modeling

- Reduced order modeling offers to reduce the prohibitive cost of the discrete ordinates method by replacing a significant fraction of the linear system solutions with less expensive solutions to significantly smaller linear systems.
- Sample solution space and construct reduced basis through POD

$$\vec{\Omega}_1,\vec{\Omega}_2,\cdots,\vec{\Omega}_K\rightarrow\vec{I}_1,\vec{I}_2,\cdots,\vec{I}_K$$

$$\bar{\bar{M}}=[\vec{I}_1,\vec{I}_2,\cdots,\vec{I}_K]=\bar{\bar{U}}\bar{\bar{S}}\bar{\bar{V}}^T$$

$\bar{\bar{\phi}}$ is the primary modes of $\bar{\bar{M}}$ given by the first $k\leq K$ columns of $\bar{\bar{U}}$

- Approximate discretized intensity in low-dimensional space

$$\vec{I}(\vec{\Omega})\approx\bar{\bar{\phi}}\vec{x}$$

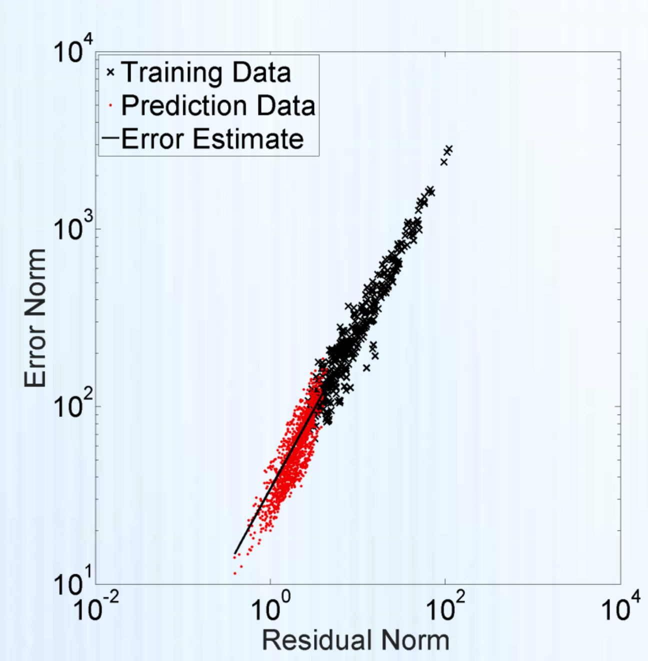
$$(\bar{\bar{K}}(\vec{\Omega})\bar{\bar{\phi}})\vec{x}=\vec{S}$$

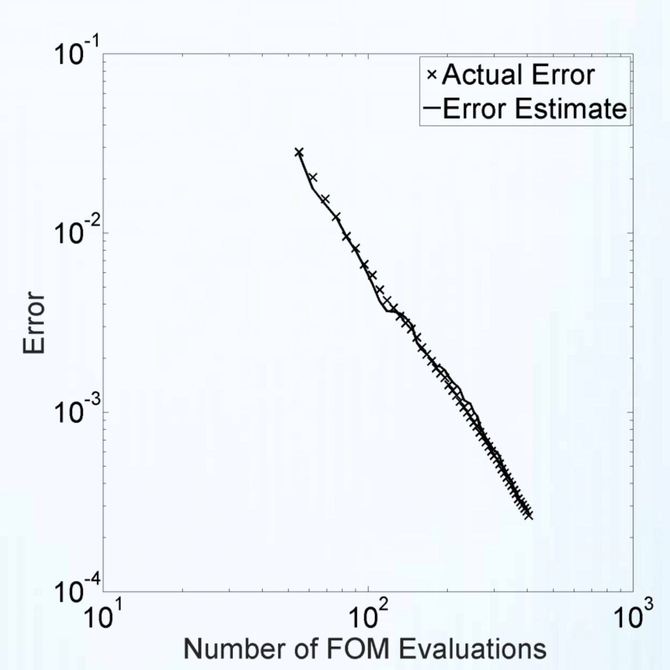
- Solve for $\vec{x}$ applying least-squares Petrov-Galerkin projection

$$(\bar{\bar{K}}(\vec{\Omega})\bar{\bar{\phi}})^T(\bar{\bar{K}}(\vec{\Omega})\bar{\bar{\phi}})\vec{x}=(\bar{\bar{K}}(\vec{\Omega})\bar{\bar{\phi}})^T\vec{S} \longleftarrow \text{ROM (k x k)}$$

Error Estimation

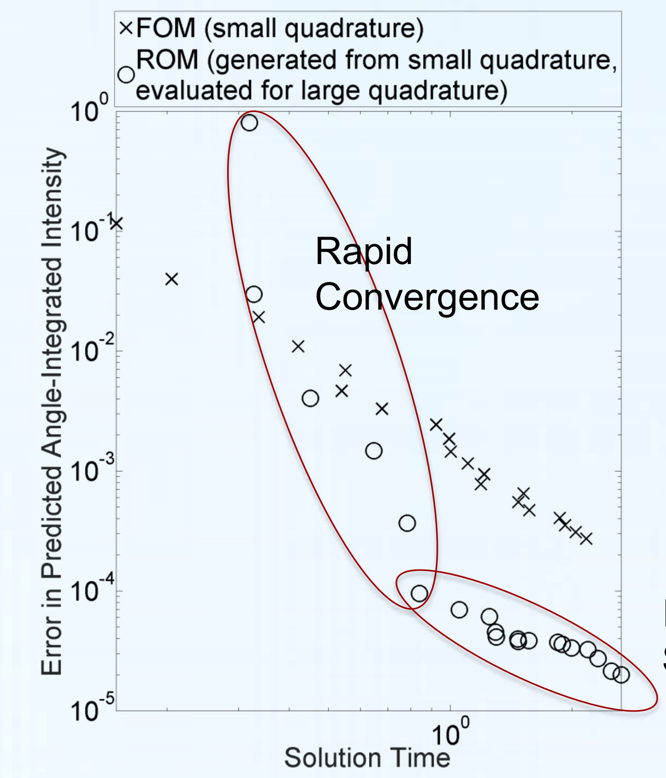
- Reduced-order model error surrogate (ROMES) model constructed as reduced basis is enriched through adaptive refinement

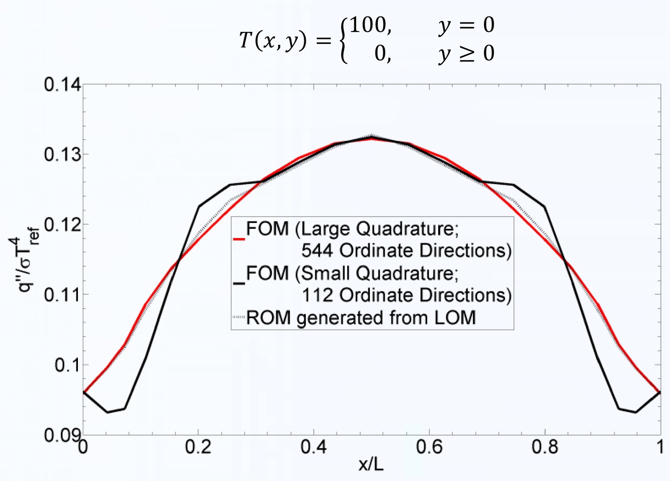


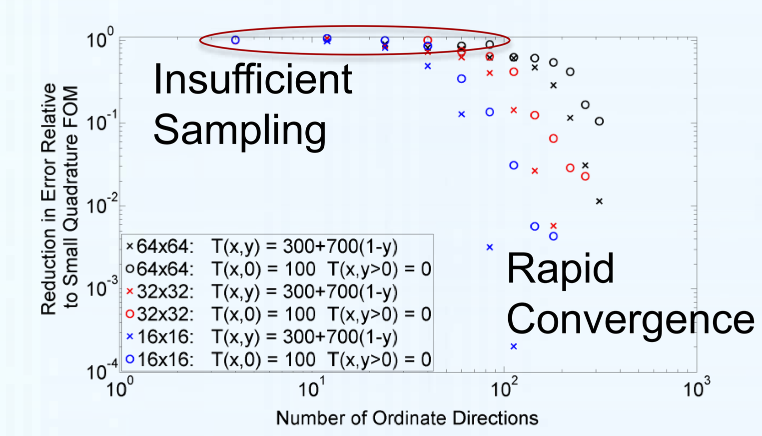


1D and 2D Results

- Enriching DOM quadrature with ROM solutions enhances accuracy
- Despite added cost of ROM solutions this is more efficient than increasing quadrature order
- Significant improvements in accuracy hold true for 2D
- Dramatic reduction in ray-effects
- Minimum number of snapshots required prior to rapid accuracy improvements

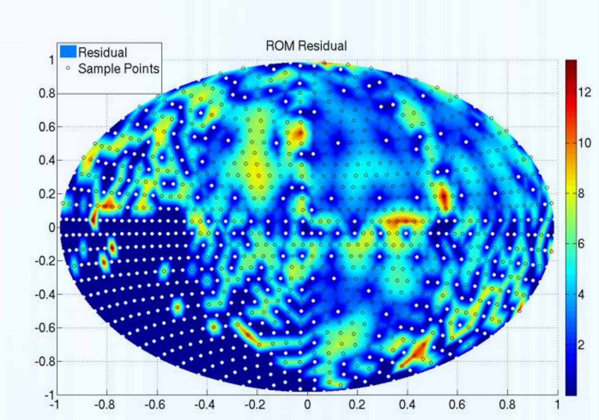


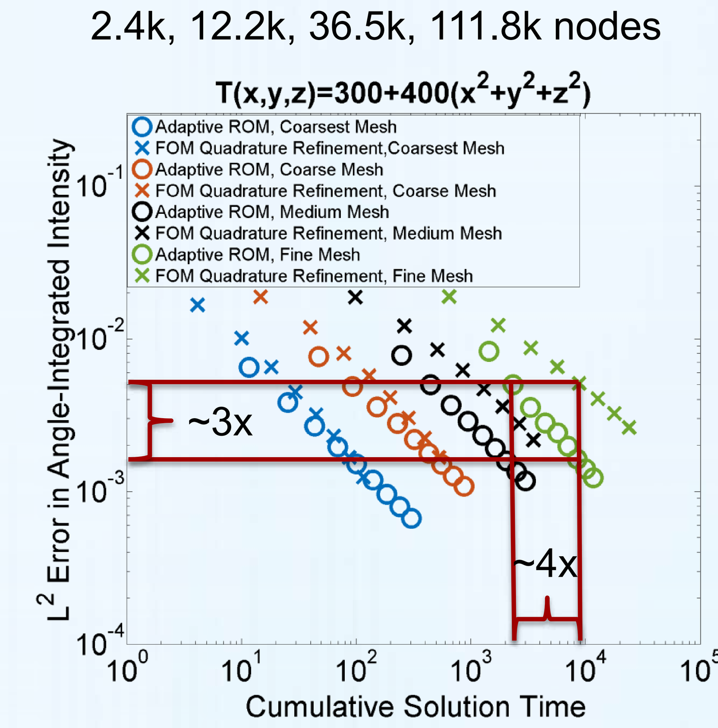




3D Results

- Enrich basis through greedy search to increase ROM solution accuracy
- No need to guess and check appropriate quadrature order
- Adaptive ROM benefits increase with





- Already 4x speedup with only 100k nodes
- Accurate error estimates provide stopping criteria

Summary / Future Work

- Proposed method significantly reduces computational cost of DOM in small test problems (3-4x)
- Improvement expected to scale with problem size (larger problem = greater speedup)
- Improvements hold for all tested source distributions
- Ongoing work to incorporate scattering
- Ongoing work to generalize error surrogate to provide a stochastic description of the error for use in UQ analyses
- Plan to make method available to SIERRA applications
- Publications

– J. Tencer, M. Larsen, K. Carlberg and R. Hogan, "Augmented Quadratures for the Discrete Ordinates Method Using Reduced Order Modeling Approaches," in Proceedings of the First Pacific Rim Thermal Engineering Conference, PRTEC, Hawaii's Big Island, USA, 2016.

– J. Tencer, K. Carlberg, M. Larsen and R. Hogan, "Reduced Order Modeling Applied to the Discrete Ordinates Method for Radiation Heat Transfer in Participating Media," in ASME 2016 Summer Heat Transfer Conference, HT2016, Washington, DC, USA, 2016.

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