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Structural Dynamics Lunchtime Series #1.3
The SMAC Modal Extraction Algorithm

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Introduction

- Motivation for developing SMAC to give confidence in presence of small nonlinearity – the power in a spatial filter
- Reliance on Complex Mode Indicator Function (CMIF) – a spatial filter
- Theory based on the spatial (modal) filter
- Your most important decision in the extraction process – the starting values of the roots to extract
- Convergence and uncertainty of roots
- Shape fitting least squares solution
- Checking your results with the CMIF/FRF resynthesis
- Randy believes that real mode extractions are almost always adequate for model validation purposes

Motivation for developing SMAC

- SMAC finds the frequency/damping based on a spatial filter instead of a matrix polynomial root finder
- I used the polynomial root finders for years and found the root stabilization diagram diverges for nonlinear response for some popular algorithms
- A spatial filter approach gives a fair estimate number of mode shapes, even if there is some nonlinearity

Modal Filter Theory in SMAC

- Roots are "found" by optimizing frequency and damping until a "match" is found

$$\overline{\Psi}^T \ddot{\overline{\mathbf{x}}} = \ddot{q}$$

$$\overline{\Psi}^T \overline{\mathbf{H}}(f) = H_q(f)$$

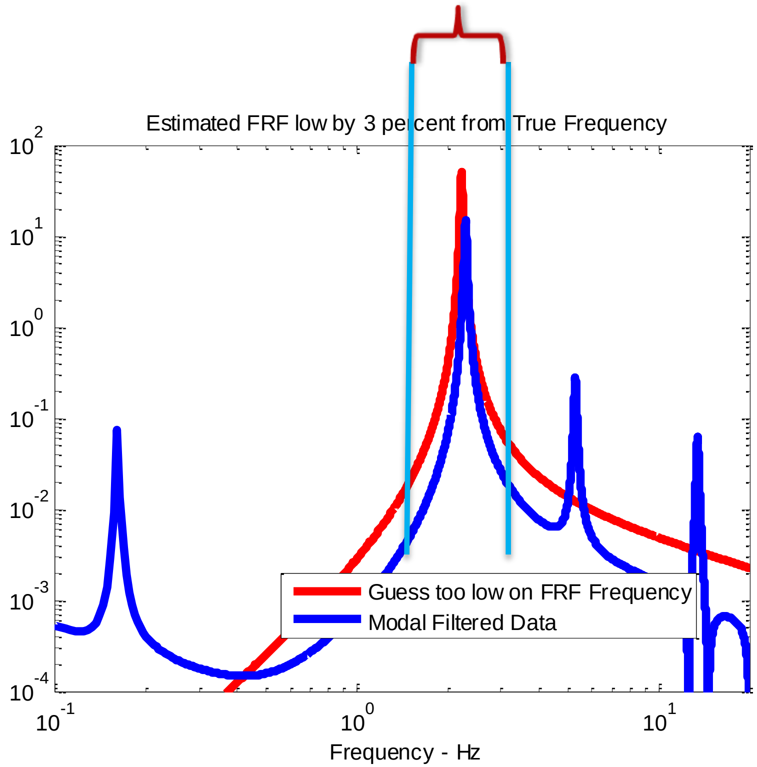
$$\overline{\mathbf{H}}^{.T}(f) * \overline{\Psi} = H_q^{.T}(f)$$

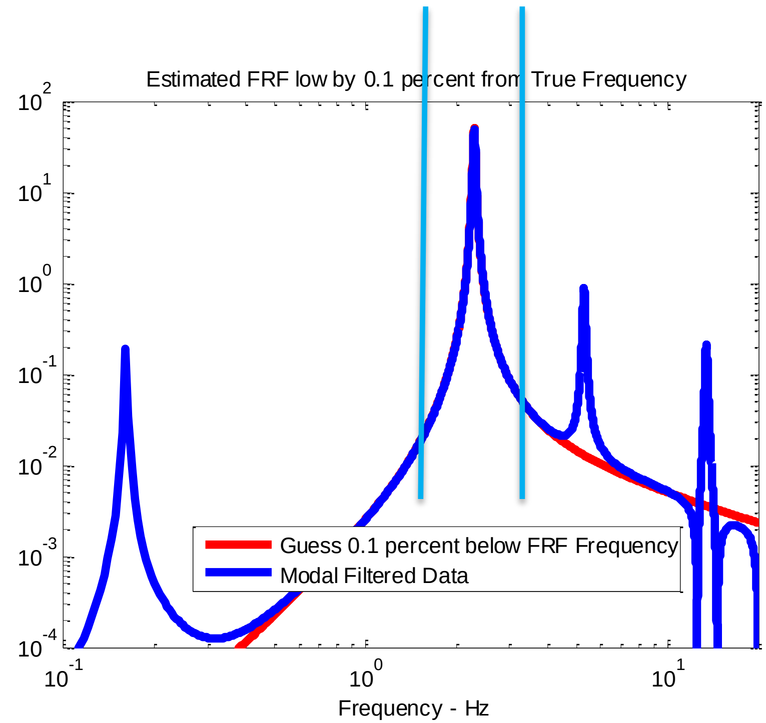
$$\overline{\Psi} = [\overline{\mathbf{H}}^{.T}]^+ \overline{H}_q^{.T}(f)$$

- Make guesses at frequency and damping of analytical H_q and find $\overline{\Psi}$
- Reconstruct H_q from $\overline{\Psi}$ and measured H and see how well they match

Modal Filter in SMAC modal algorithm Example of bad match and good match

- First plot shows match when guessed frequency is off by 3%
- 2nd plot shows match when guessed frequency is off by 0.1%
- Goodness of fit is calculated by correlation coefficient between the blue and red vectors in the frequency band near the guessed resonant frequency





Correlation Coefficient Plot and CMIF

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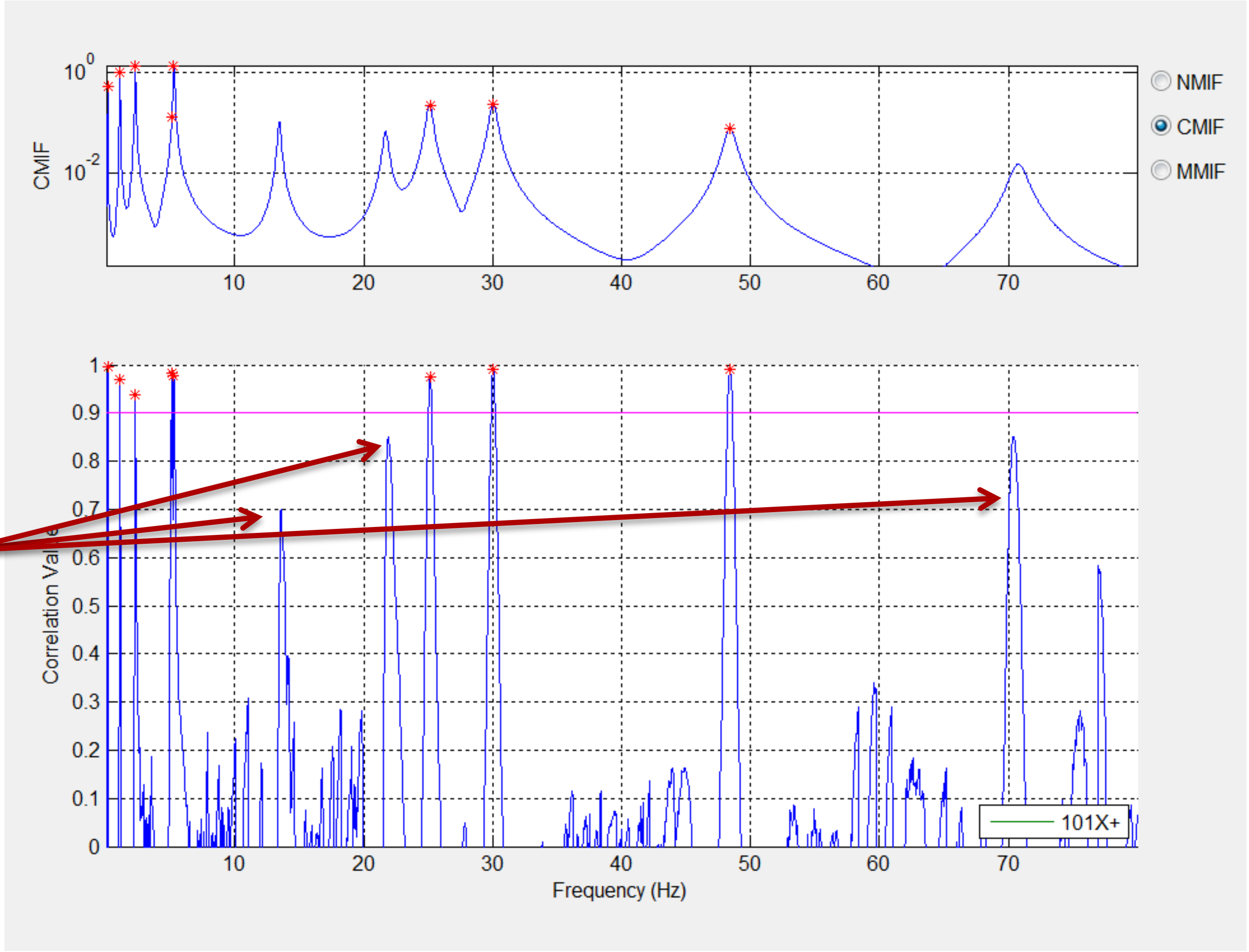
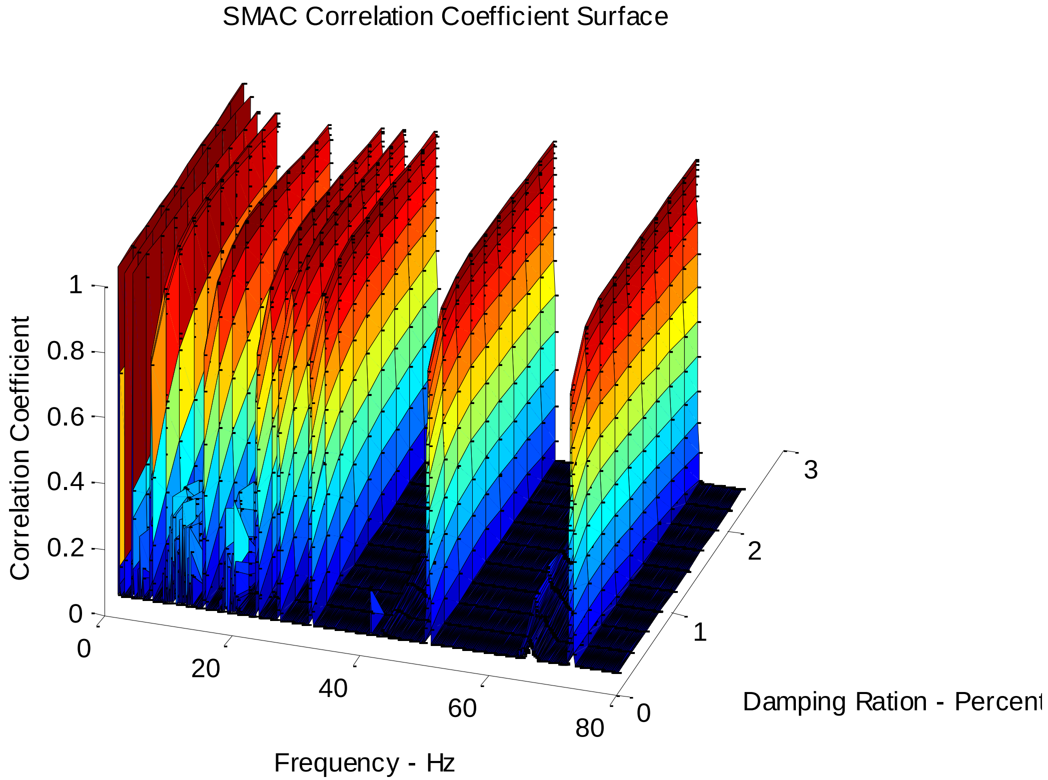
- Initial correlation coefficient plot with guess at damping slightly below the average
- SMAC automatically selects starting frequencies for any corr coef peak > .9

The modal filter approach cannot extract more modes than accelerometers. In this case there are 23 modes in the band and 16 accelerometers. These roots could not be extracted accurately.



Correlation Coefficient Surface Plot

- Correlation coefficient changes radically in frequency direction but slowly in damping direction
- Using starting root values from last plot, automatic optimization algorithm "climbs to the top of each hill" until damping and frequency do not change more than some tolerance. Final correlation coefficient is usually > .99
- Can also be done manually for closely spaced roots

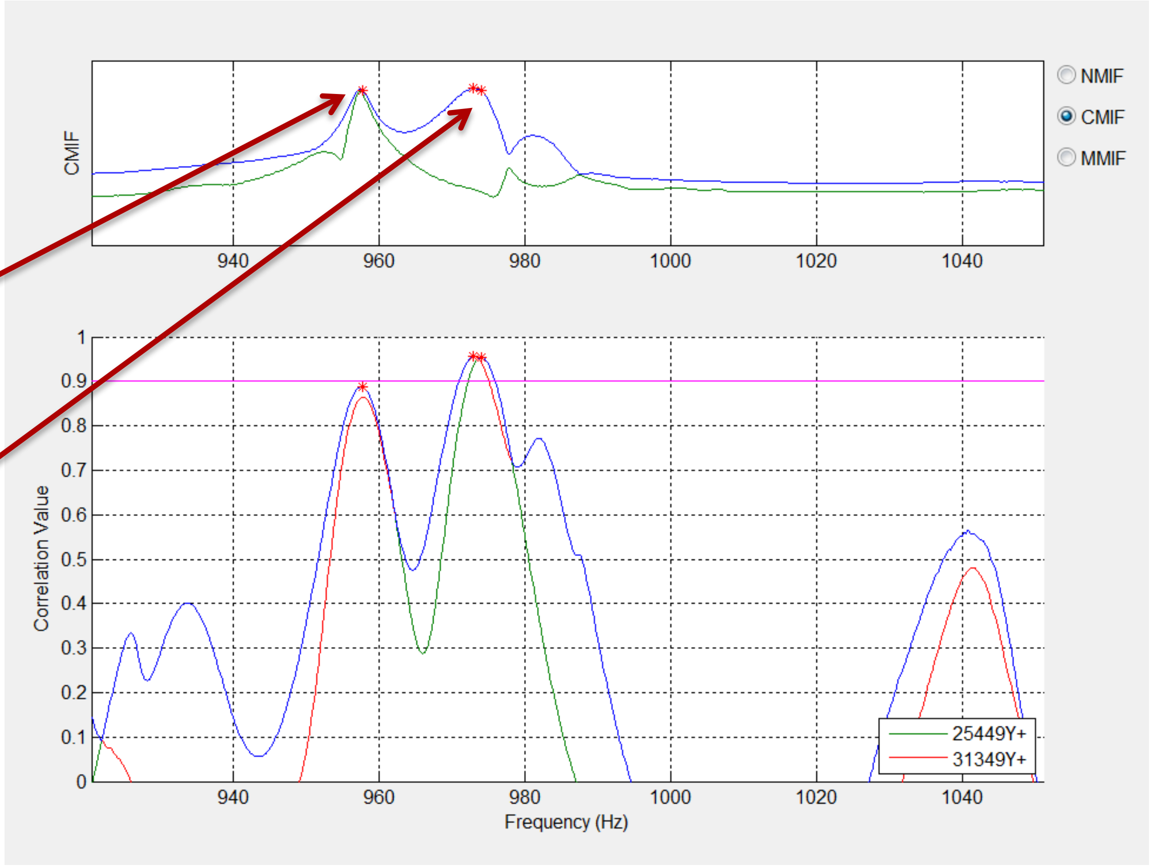


Your most important decision, starting values for the roots

- Put FRFs from all references in SMAC and look at Complex Mode Indicator Function to establish the number of roots
- CMIF is a spatial filter and tells how many different mode shapes at each frequency

Two shapes at this frequency

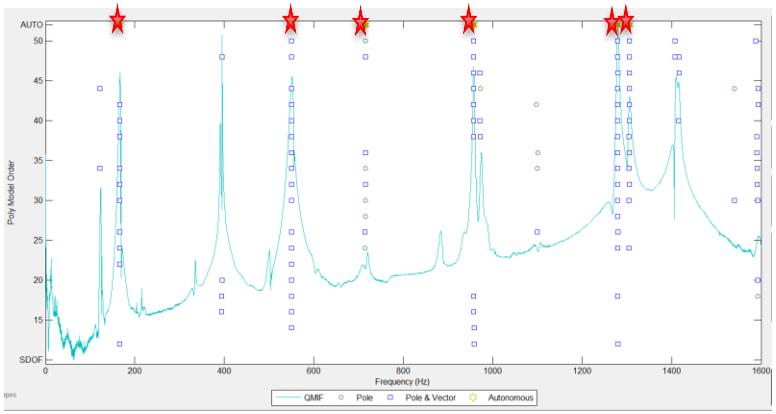
One shape at this frequency, but slight shift in frequency from one reference to another due to nonlinearity



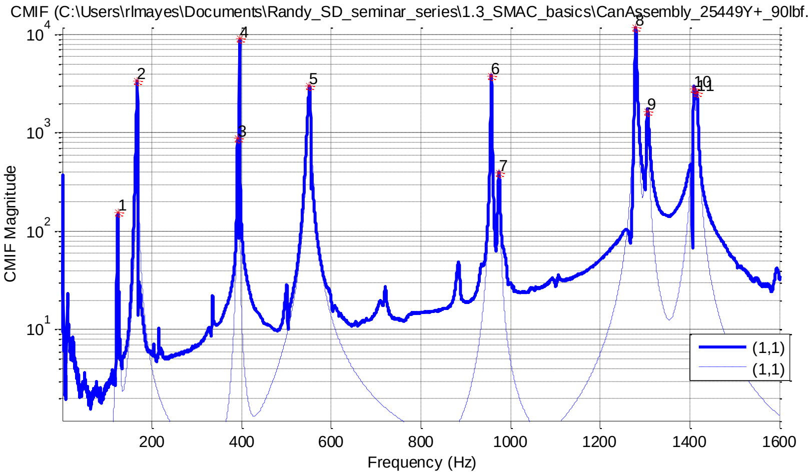
SMAC vs Best Commercial Root Finder on some mildly nonlinear data

- SMAC can converge on roots with automatic settings plus small amount of user interaction
- BCRF automated root extraction converged on 5 of 11 roots found by SMAC plus one weak root – See red stars
- I have previously seen other matrix polynomial root finders split one root into multiple roots for slightly nonlinear modes
- Besides bandwidth, BCRF has 18 parameters to set if you don't choose the defaults of fully automatic approach
- Besides bandwidth, SMAC has 6 parameters that can be adjusted from the defaults
- SMAC fit 8 modes automatically and user had to manually fit 3

BCRF Stability Diagram



SMAC CMIF Fit Plot



Uncertainty on modal frequency and damping

- Simmermacher analyzed uncertainty in modal frequency and damping with SMAC which showed (for a general cases) that frequency and damping estimates were unbiased and within about three times the convergence tolerance settings
  - Default frequency convergence percentage is 0.05percent
  - Default damping convergence percentage is 2 percent
  - These work well for modes with modal damping above 0.5 percent
  - If damping is less than 0.1 percent, I divide default tolerances by a factor of four
  - Exceptions to this are when the frequency delta is less than 1/20 of the frequency value (frequencies VERY low with respect to max frequency of the band). Then uncertainty goes up.

Shape fitting theory in SMAC

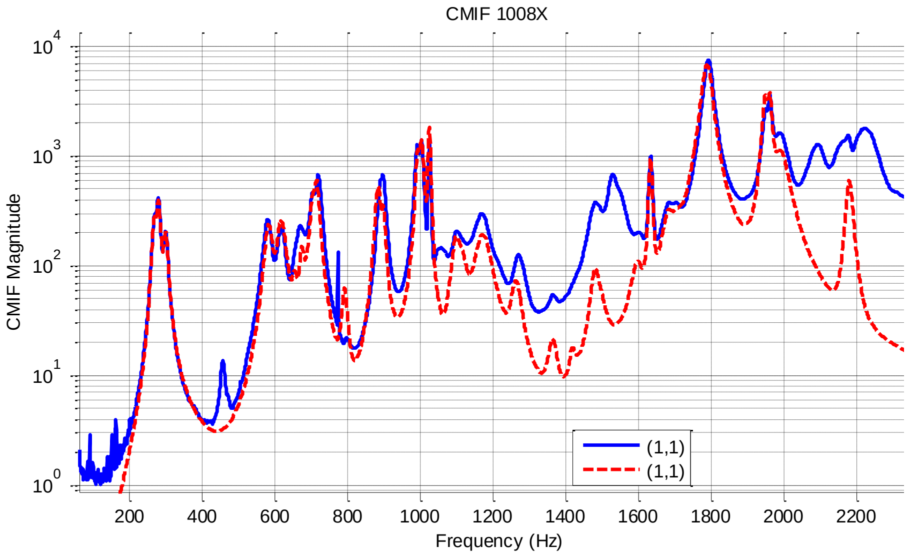
- Once the modal frequency and damping have been established, the FRFs from one reference are fit with least squares estimate of the residue (*A*)
  - The drive point is fit first – if this has great error, the entire shape will have a bias error
  - The cross points are then fit
- In mass normalized real mode fits, a frequency response kernel for a single mode number *r* is calculated at 4 frequency lines around each resonant frequency

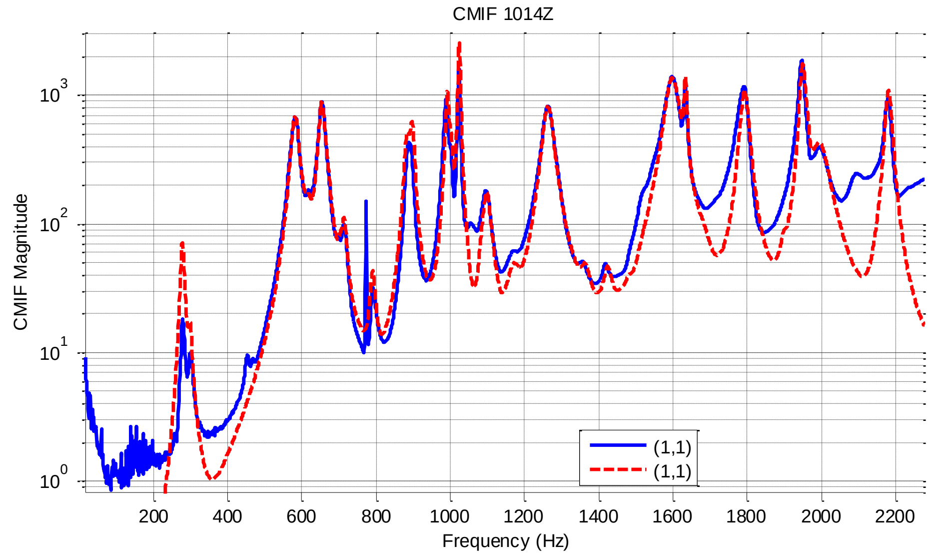
$$H^r(\omega_i) = \left\{ \frac{-\omega_i^2 \Phi_{dp}^r \Phi_{cp}^r}{\omega_r^2 - \omega_i^2 + j2\omega_i\omega_r} \right\} \qquad \ker^r(\omega_i) = \left\{ \frac{-\omega_i^2}{\omega_r^2 - \omega_i^2 + j2\omega_i\omega_r} \right\}$$

$$\mathrm{Imag}\begin{Bmatrix} H_m(\omega_1) \\ H_m(\omega_2) \\ \vdots \\ H_m(\omega_{4n}) \end{Bmatrix} = \mathrm{Imag}\begin{bmatrix} \ker^1(\omega_1) & \ker^2(\omega_1) & \cdots & \ker^n(\omega_1) \\ \ker^1(\omega_2) & \ker^2(\omega_2) & \cdots & \ker^n(\omega_2) \\ \vdots & \vdots & \ddots & \vdots \\ \ker^1(\omega_{4n}) & \ker^2(\omega_{4n}) & \cdots & \ker^n(\omega_{4n}) \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{Bmatrix}$$

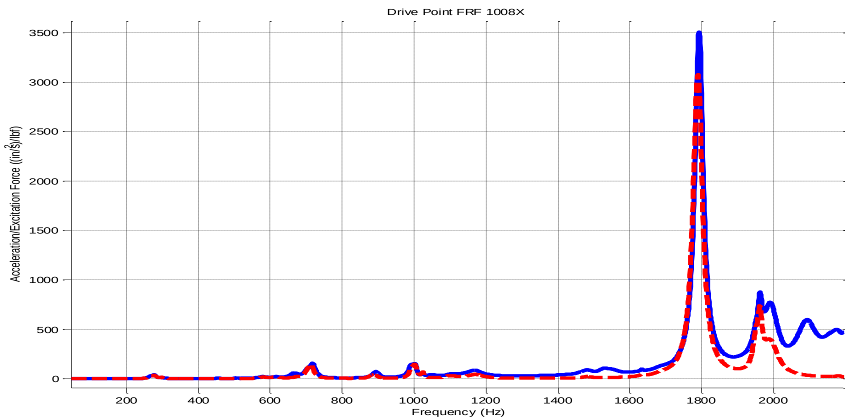
Check your results with CMIF and FRF Synthesis (dashed red line)

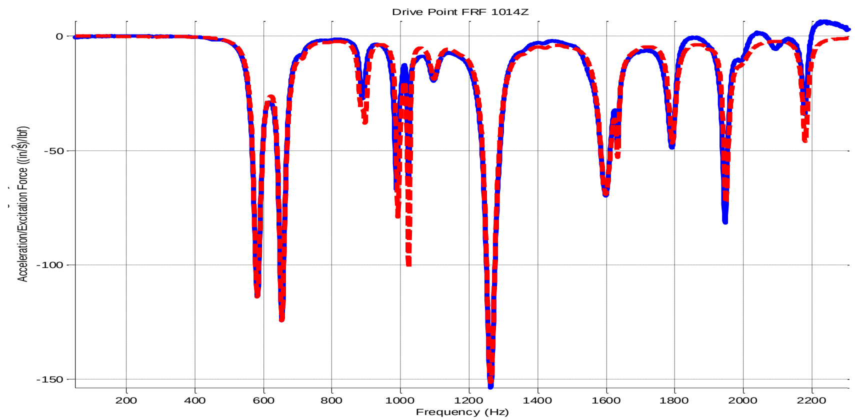
- CMIF of various references from same test give confidence on modal extraction





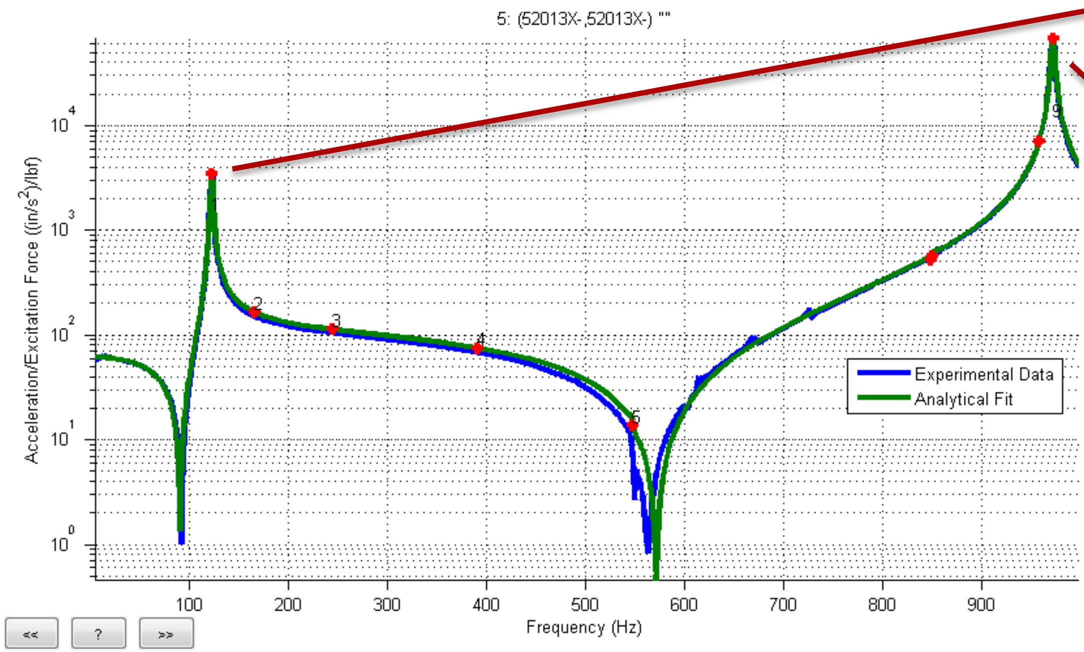
- Drive Point FRFs of various references also show confidence

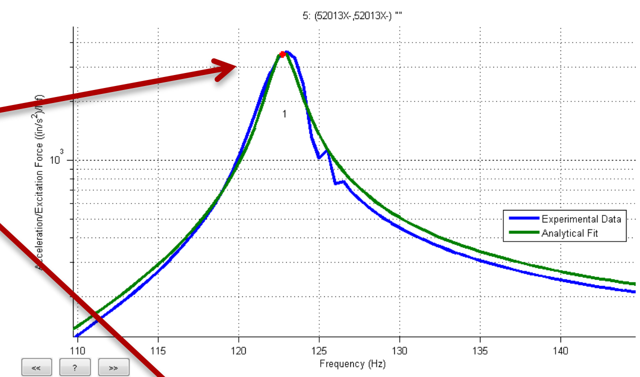


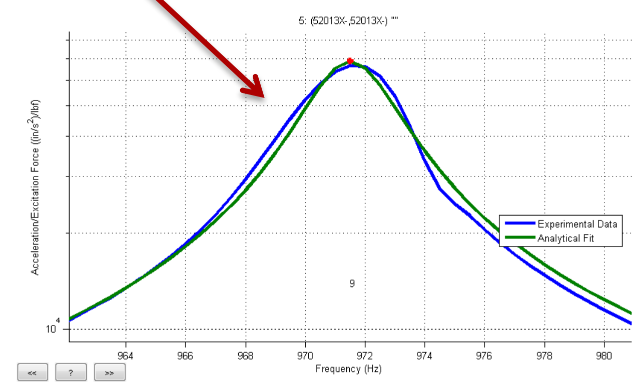


Focus on Real Mode Shapes

- FE Models only use real modes
- I believe that real modes are usually an adequate simulation of the response. Many modal experts are firm believers in complex modes, but I believe they are usually just fitting the residual effects of other modes, which they should have fit by adding residuals, not making the mode complex.
- Consider this real mode fit of nonlinear response







Conclusions – Randy Mayes is very biased toward using SMAC because:

- He spent years pulling his hair out trying to extract modes with matrix polynomial root finders that gave a lot of different answers depending on how one tweaked their input parameters and how many computational roots one requested.
- The modal filtering approach used by SMAC and CMIF helps one decide in advance how many modes to extract
- One does not have to pick through dozens of computational roots to find the true ones
- The uncertainty on frequency and damping is unbiased and quantified
- SMAC has nice checks with synthesis of CMIF and FRFs built in
- SMAC gives reasonable estimate of linear modal parameters for slightly nonlinear data (many of our systems are slightly nonlinear)
- For model validation and substructuring, Sandia tools generally require real mode estimates

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