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Partial Differential Equation-Constrained Optimization Framework for Viscoelastic Material Design

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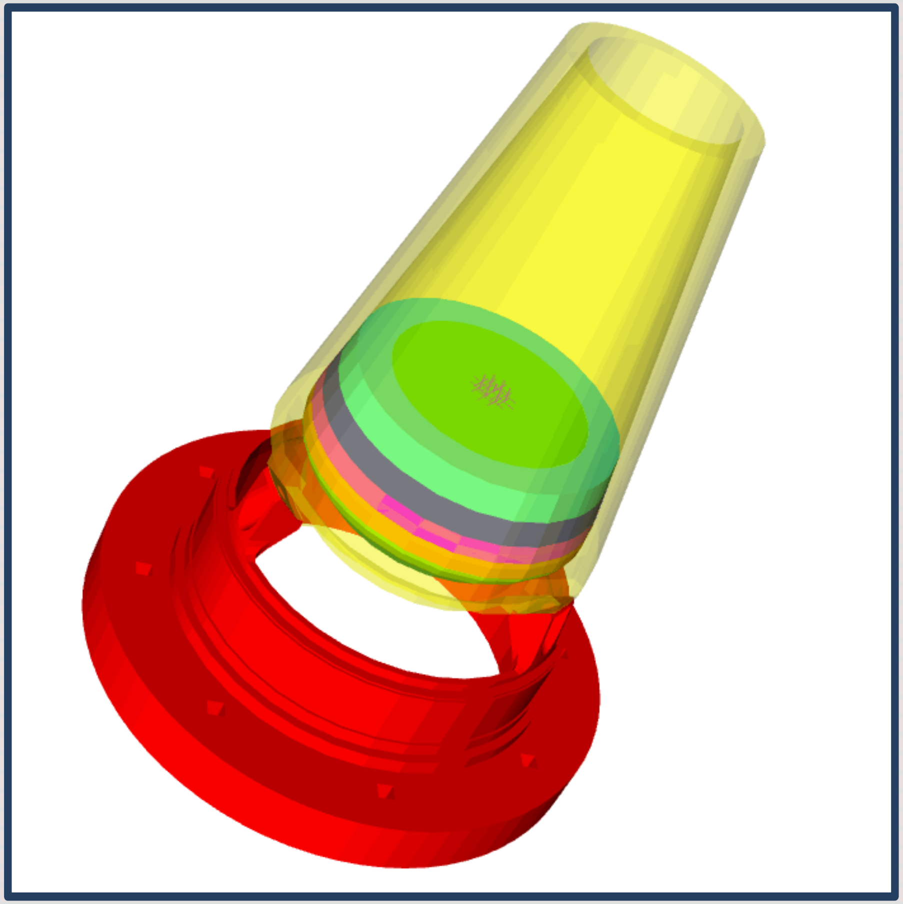
Outline

- Research Motivation
- PDE-Constrained Framework for Viscoelastic Material Inversion
  - Design Problem Statement
  - Coupled Acoustic-Structural Variational Boundary Value Problem Formulation
  - Numerical Optimization Formulation
- Numerical Implementation in Sierra SD + ROL
  - Example #1: Mechanical vibration reduction
  - Example #2: Acoustic cloaking with layered viscoelastic foam
- Questions?

Research Motivation

- Many engineering systems experience harsh vibration environments
  - Examples: Aerospace structures, aircraft structures, civilian structures
- Current engineering practice ‘ad hoc’ in the design of foam materials for damping
- Large scale PDE-constrained optimization can select materials that provide optimal vibration control



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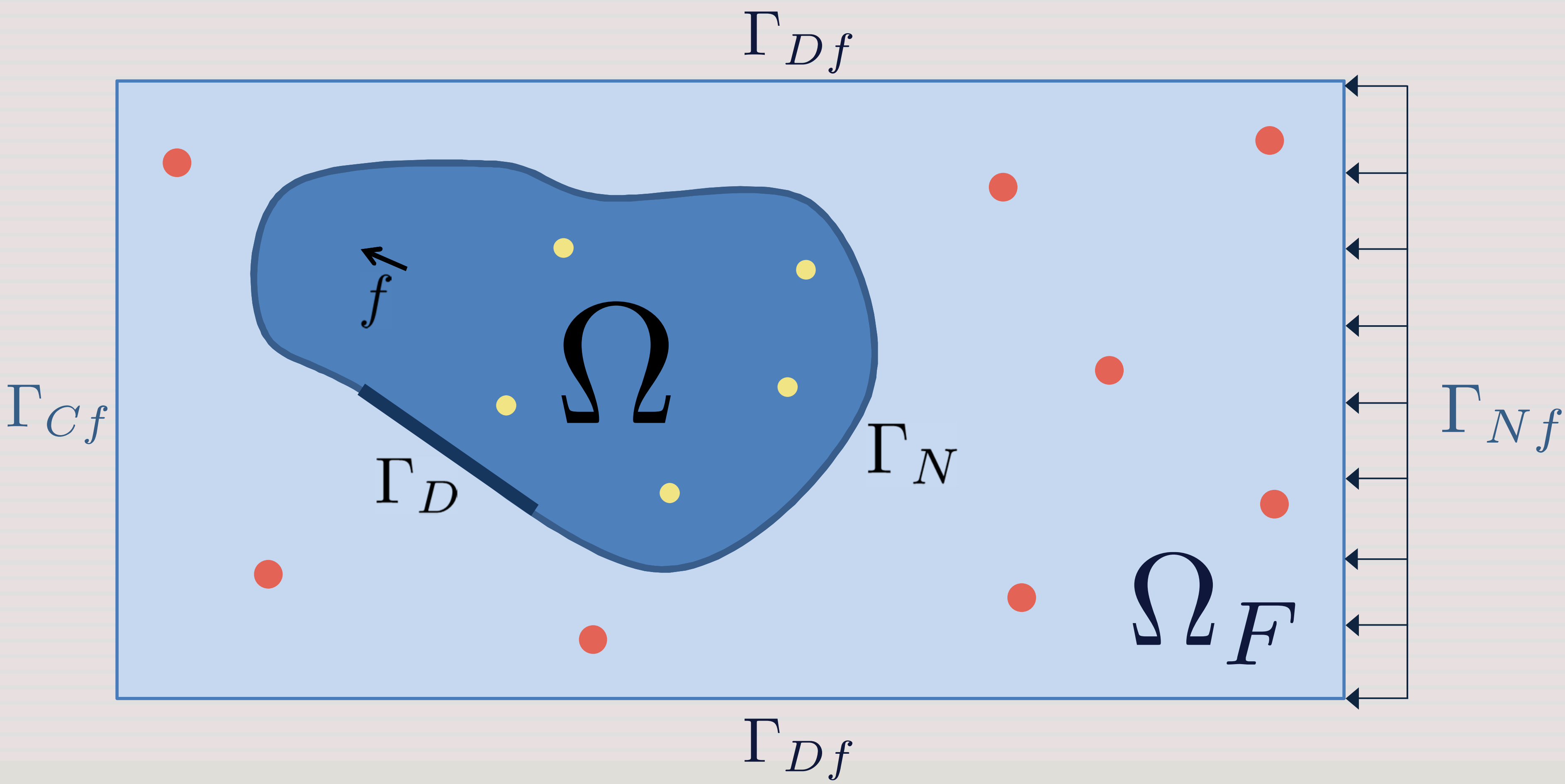
Inverse Problem as Design Problem

Objectives:

- Design material by solving a material-identification inverse problem
- Find optimal properties of viscoelastic materials to match structural acoustic response to desired behavior
  - Complex Bulk (b) and Shear (G) moduli and spring/dashpot constants are design variables
- Generate frequency-dependent design by solving Acoustic-Structural Interaction (ASI) system at multiple frequencies

Coupled ASI Domain



(● ●)= Measurement locations (Red: microphone, Yellow: accelerometer)

Governing Equations for Coupled ASI Problem

Coupled PDE's for ASI govern system behavior and provide constraints for optimization

Elastodynamics

$$\nabla \cdot \sigma = \rho \ddot{\mathbf{u}}, \ \text{in} \ \Omega \times (0,T)$$
$$\sigma \cdot \mathbf{n} = \mathbf{h}, \ \text{on} \ \Gamma_N \times [0,T]$$
$$\sigma = \mathbf{D} : \nabla \mathbf{u}, \ \text{in} \ \Omega \times [0,T]$$
$$\mathbf{u} = 0, \ \text{in} \ \Gamma_D \times [0,T]$$

$$\Omega \cup \partial\Omega = \bar{\Omega}$$
$$\Gamma_D \cap \Gamma_N = \emptyset$$
$$\Gamma_D \cup \Gamma_N = \partial\Omega$$

Acoustic Wave Equation

$$\nabla^2 \phi = \frac{1}{c^2}\ddot{\phi}, \ \text{in} \ \Omega_f \times (O,T)$$
$$\nabla \phi \cdot \mathbf{n}_f = -\rho_f \ddot{u}_n, \ \text{on} \ \Gamma_{Nf} \times [O,T]$$
$$\phi = 0, \ \text{on} \ \Gamma_{Df} \times [O,T]$$
$$\phi(0,T) = 0, \ \text{in} \ \Omega_f$$
$$\dot{\phi}(0,T) = 0, \ \text{in} \ \Omega_f$$

$$\Omega_f \cup \partial\Omega_f = \bar{\Omega}_f$$
$$\Gamma_{Df} \cap \Gamma_{Nf} = \emptyset$$
$$\Gamma_{Df} \cup \Gamma_{Nf} = \partial\Omega_f$$

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Coupled ASI Equations

- Fourier transform of time-domain equations for frequency domain/ steady-state analysis
- Finite Element discretization

$$\left(\begin{bmatrix}\mathbf{K}_s & 0 \\ 0 & \mathbf{K}_f/\rho_f\end{bmatrix} + i\omega\begin{bmatrix}\mathbf{C}_S & \mathbf{L} \\ \mathbf{L}^T & -\mathbf{C}_f/\rho_f\end{bmatrix} + \omega^2\begin{bmatrix}\mathbf{M}_s & 0 \\ 0 & -\mathbf{M}_f/\rho_f\end{bmatrix}\right)\begin{bmatrix}u \\ \phi\end{bmatrix} = \begin{bmatrix}f_s \\ f_f/\rho_f\end{bmatrix}$$

- $\{\mathbf{K}_s, \mathbf{C}_s, \mathbf{M}_s\}$ = Structural Stiffness, Damping, and Mass Matrices
- $\{\mathbf{K}_f, \mathbf{C}_f, \mathbf{M}_f\}$ = Fluid Stiffness, Damping, and Mass Matrices
- $\mathbf{L}$ = Coupling Matrix

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Design Variables

- Viscoelastic materials with frequency-dependent complex shear and bulk moduli

$$G(\omega) = G_R(\omega) + iG_I(\omega)$$
$$b(\omega) = b_R(\omega) + ib_I(\omega)$$

- Represented in structural stiffness matrices in finite element discretization

$$\mathbf{K}_s = \int_\Omega \mathbf{B}^T \mathbf{D} \mathbf{B} d\Omega$$
$$= \int_\Omega \mathbf{B}^T \left(G_R \mathbf{D} + b_R \mathbf{D}_R \right) \mathbf{B} d\Omega +$$
$$i \int_\Omega \mathbf{B}^T \left(G_I \mathbf{D} + b_I \mathbf{D}_R \right) \mathbf{B} d\Omega$$

$$\mathbf{p} = \{G_R, b_R, G_I, b_I\}$$

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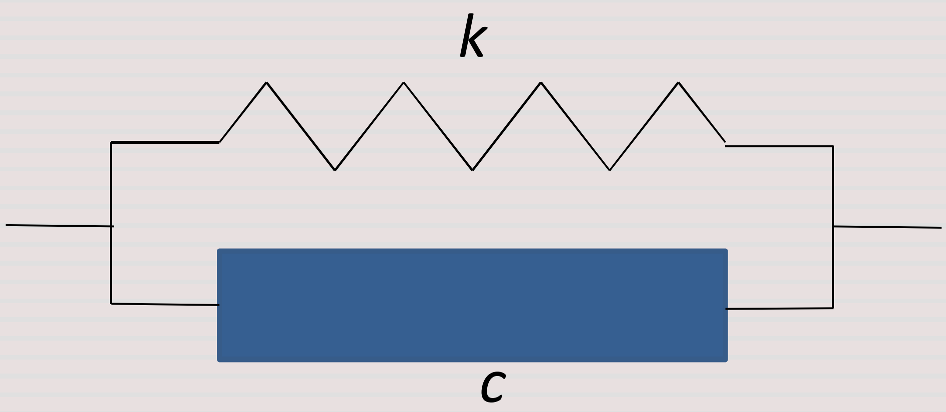
Design Variables: Spring & Dashpot

- Two-node spring and dashpots used for stiffness and damping in acoustic/structural system
- Spring and damping constants {***k, c***} serve as design variables



$$\mathbf{p} = \{k, c\}$$

Least-Squares Minimization Approach

Objective Function: Least-squares residual between computed and desired physical fields, with regularization term for design variable

$$\mathcal{J}(\mathbf{u},\mathbf{p}) = \frac{1}{2}\left(\hat{\mathbf{u}} - \mathbf{u}^h\right)^T [Q] \left(\hat{\mathbf{u}} - \mathbf{u}^h\right) + \mathcal{R}(\mathbf{p})$$

$$\hat{\mathbf{u}} = \text{Measured Data}, \in \mathbb{C}^{sd+ad}$$

$$\mathbf{u}^h = \text{Discrete Solution}, \in \mathbb{C}^{sd+ad}$$

$$[Q] = \text{Measurement Matrix}$$

$$\mathbf{p} = \text{Design Variable}, \in \mathbb{C}^{ndv}$$

$$\mathcal{R}(\mathbf{p}) = \text{Regularization Term}$$

$$sd = \text{Structural Degrees of Freedom}$$

$$ad = \text{Acoustic Degrees of Freedom}$$

$$ndv = \text{Number of Design Variables}$$

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Optimization Formulation

- Define the minimization problem:

$$\min_{\mathbf{u},\mathbf{p}\in\mathcal{U}\times\mathcal{P}} \mathcal{J}(\mathbf{u},\mathbf{p})$$

Objective Functional

$$\text{subject to } \mathbf{g}(\mathbf{u},\mathbf{p}) = 0$$

with PDE Constraint (e.g. Structural-Acoustic Helmholtz Equation)

- Reduced-Space Methods: Assume state variable $\mathbf{u}$ as function of design variable $\mathbf{p}$
- Gradient-based optimization implementation in Rapid Optimization Library/Sierra SD
  - Numerical optimization using Newton-Krylov methods with Trust-Region Search

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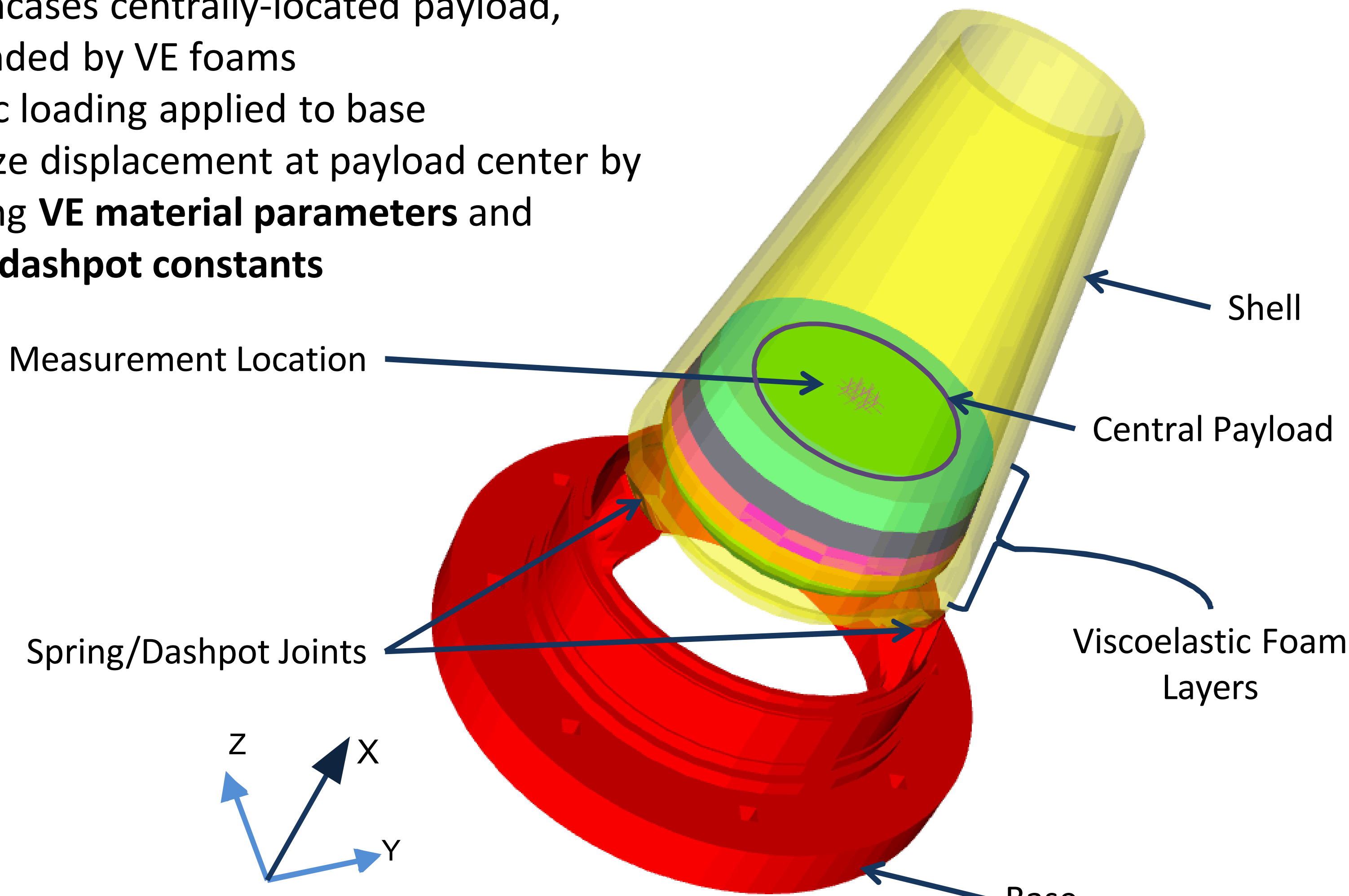
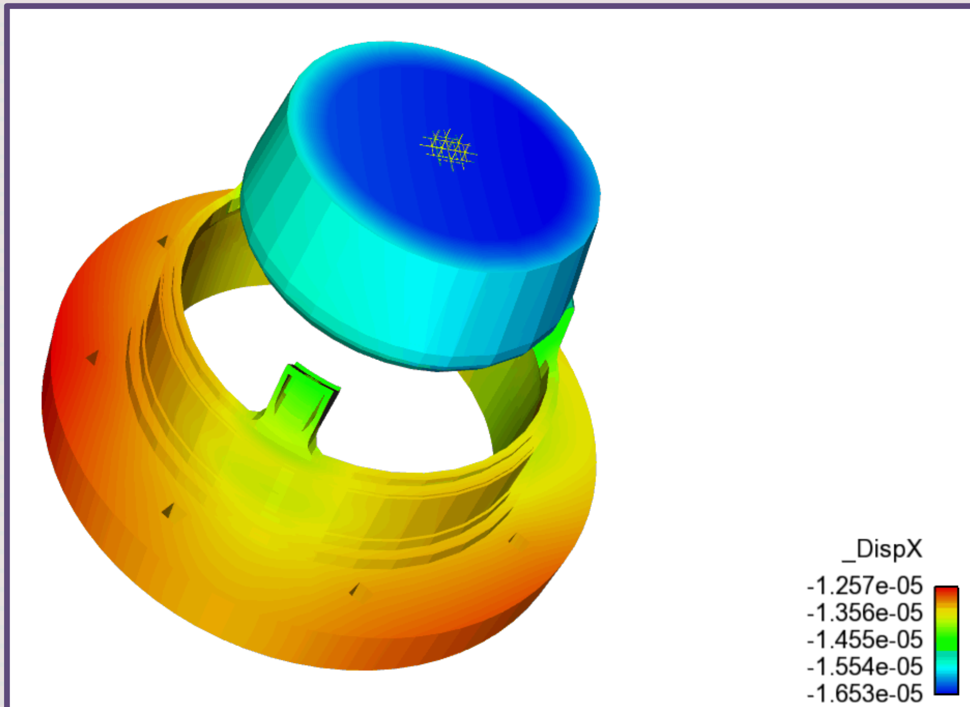
Case Study 1: *Mechanical Vibration Reduction*

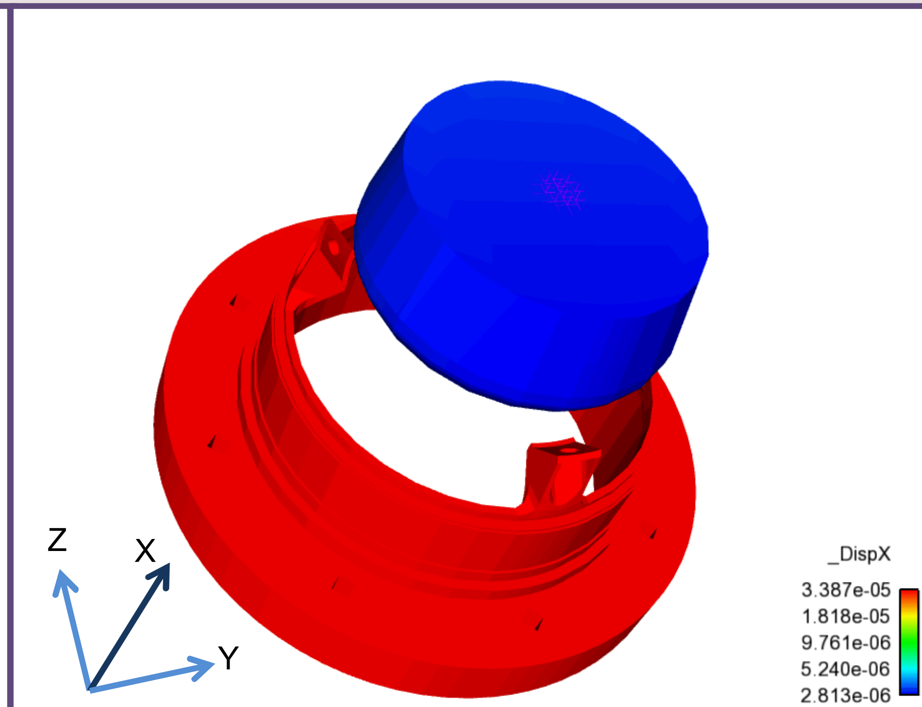
- Shell encases centrally-located payload, surrounded by VE foams
- Periodic loading applied to base
- Minimize displacement at payload center by adjusting **VE material parameters** and **spring/dashpot constants**



Case Study 1: *Mechanical Vibration Reduction*

- Displacement at measurement locations minimized (dependent on frequency)





Left: X-displacement in base and payload with initial material guesses, 440 Hz loading;
Right: X-displacement in design

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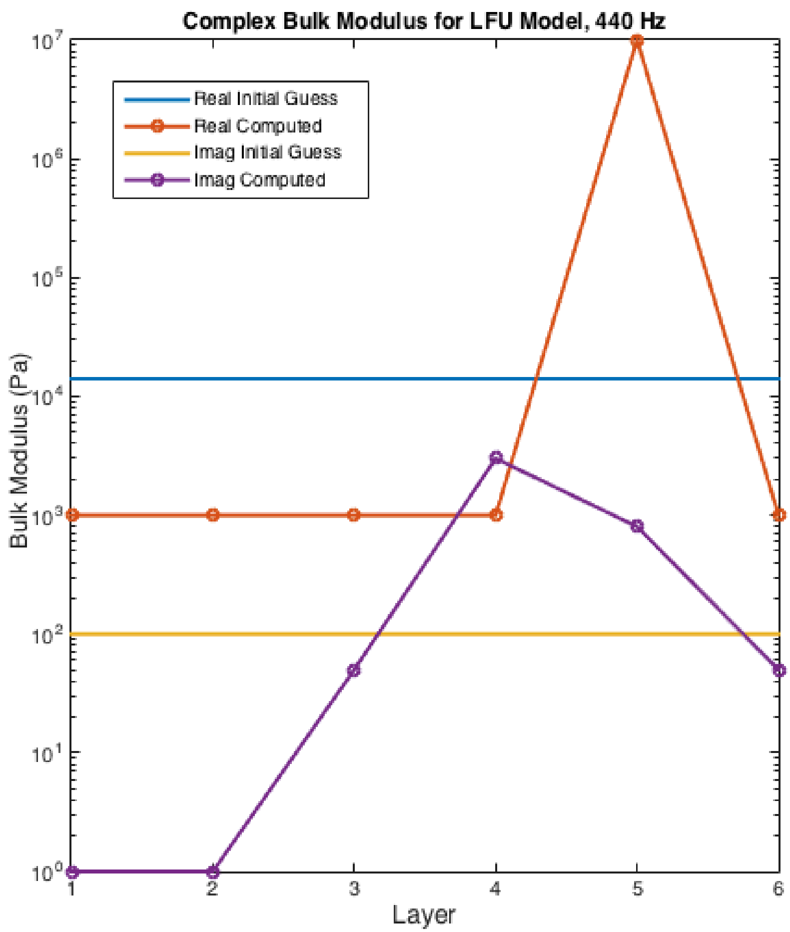
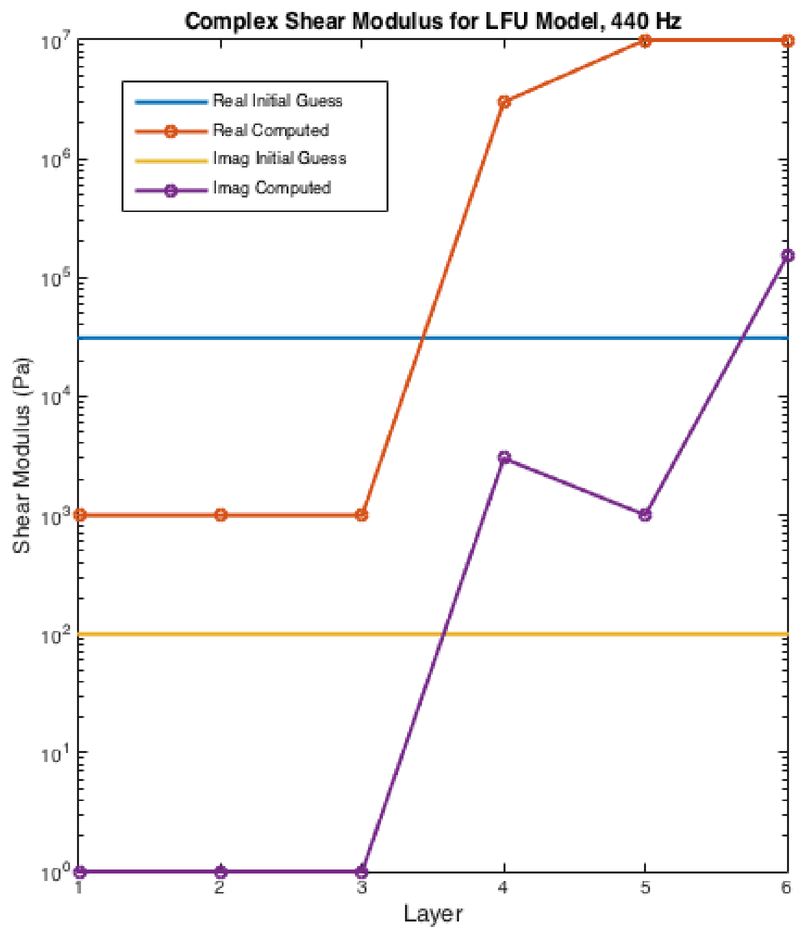
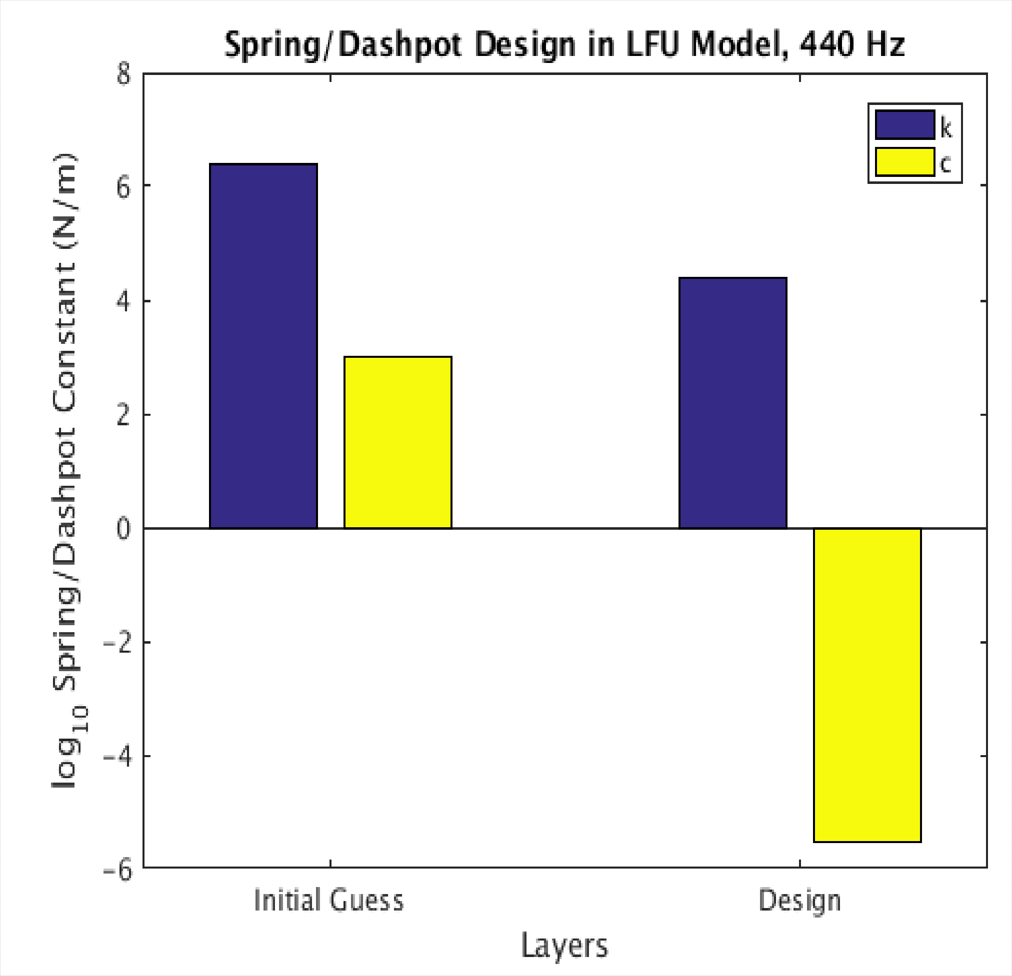
Case Study 1: *Mechanical Vibration Reduction*





Left: Real and imaginary Bulk moduli for 6 VE layers, compared with initial guess
Right: Real and imaginary Shear moduli for 6 VE layers, compared with initial guess

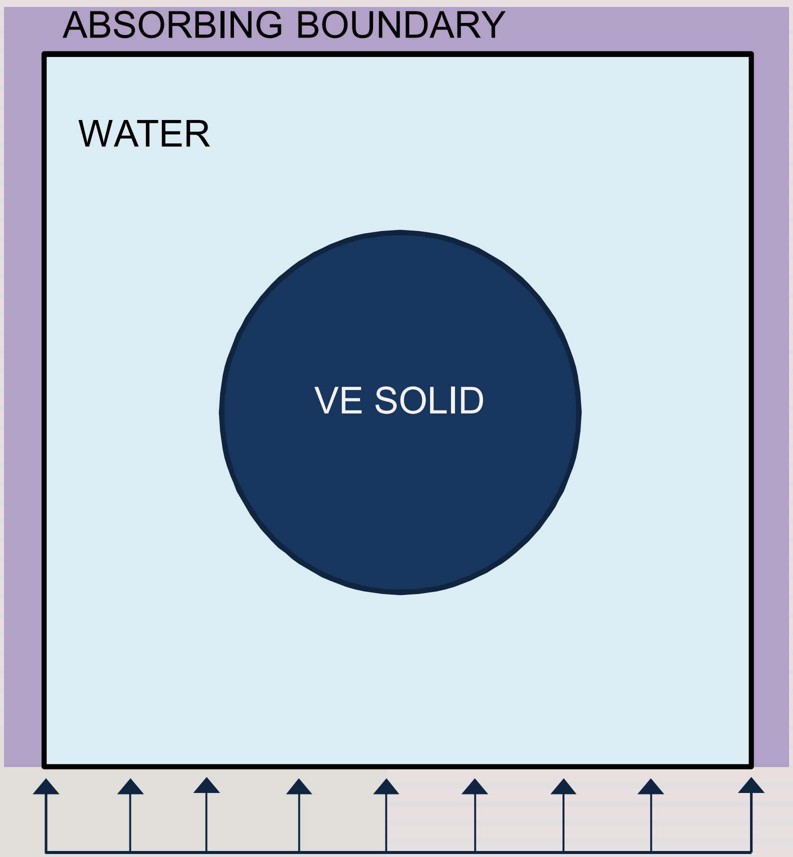
Case Study 1: *Mechanical Vibration Reduction*

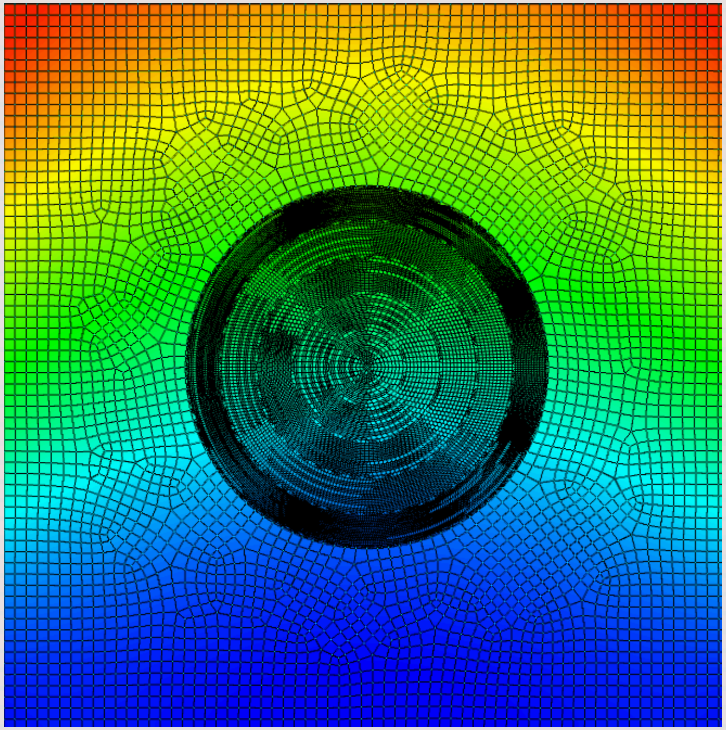


Right: Designed spring and damping constants, compared to initial guess

Case Study 2: *Acoustic Cloaking*

- 2-D fluid region with circular VE solid inclusion
- Inclusion consists of concentric rings w/ distinct material properties
- Periodic acoustic load applied to end
- Match forward problem pressure distribution by adjusting **VE material parameters**

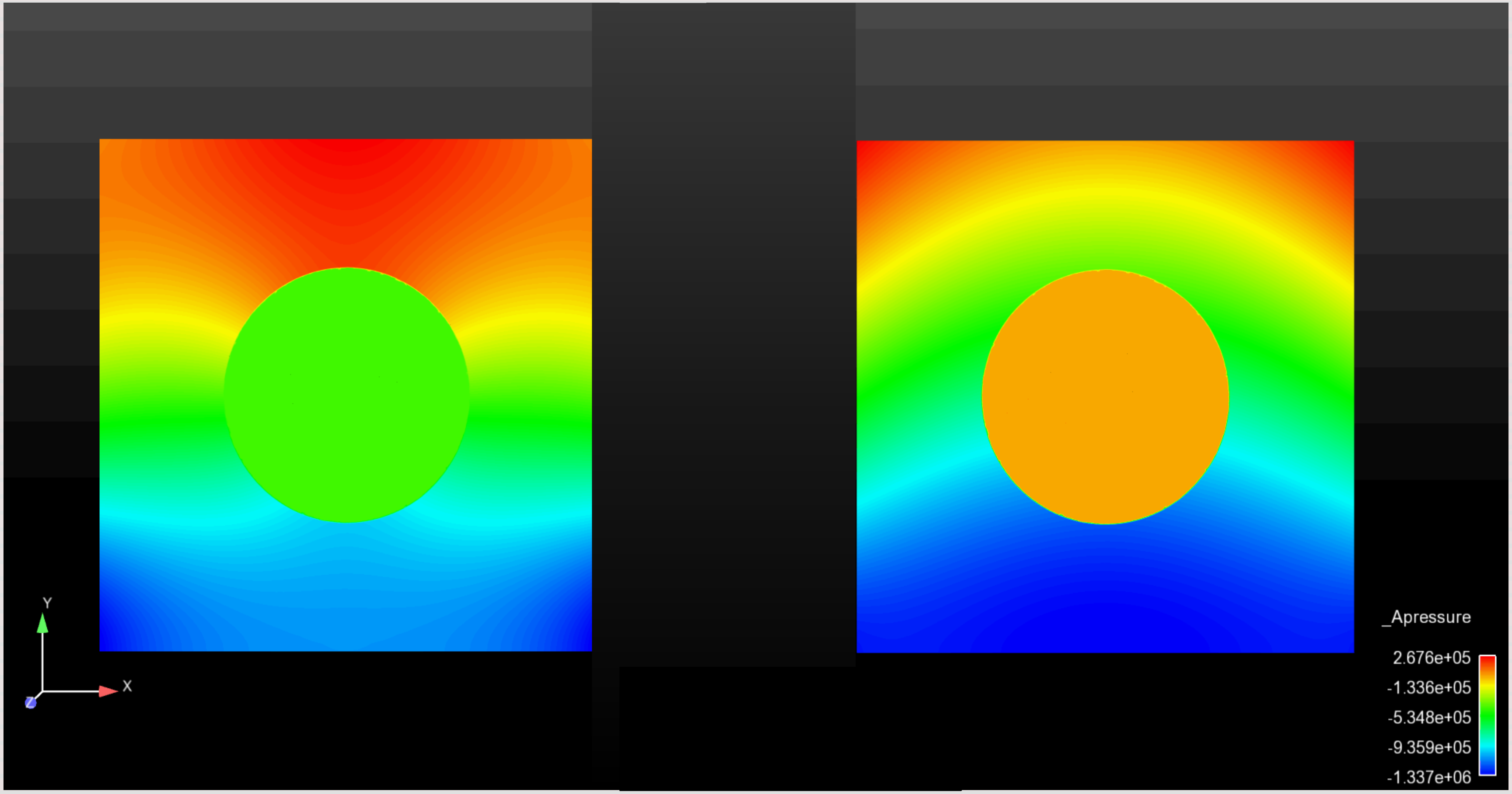




Left: Model Set up
Right: Forward problem pressure distribution (500 Hz loading) in model with 50 layers

Case Study 2: *Acoustic Cloaking*

- Optimized VE foams allow recovery of desired pressure distribution

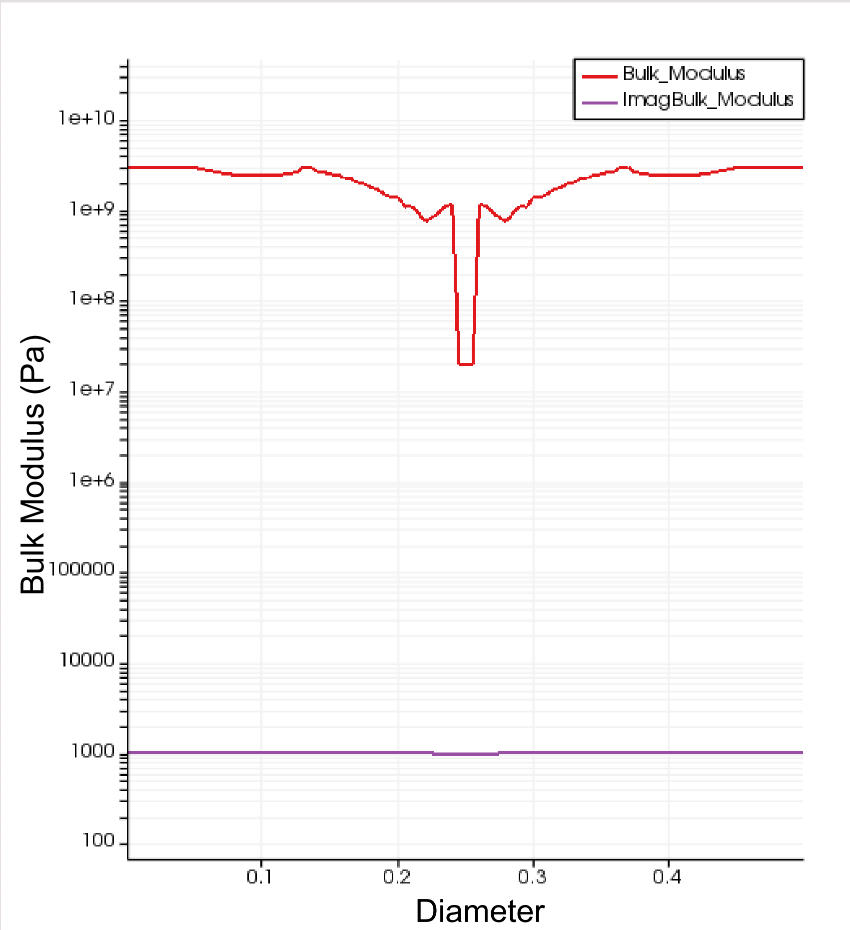


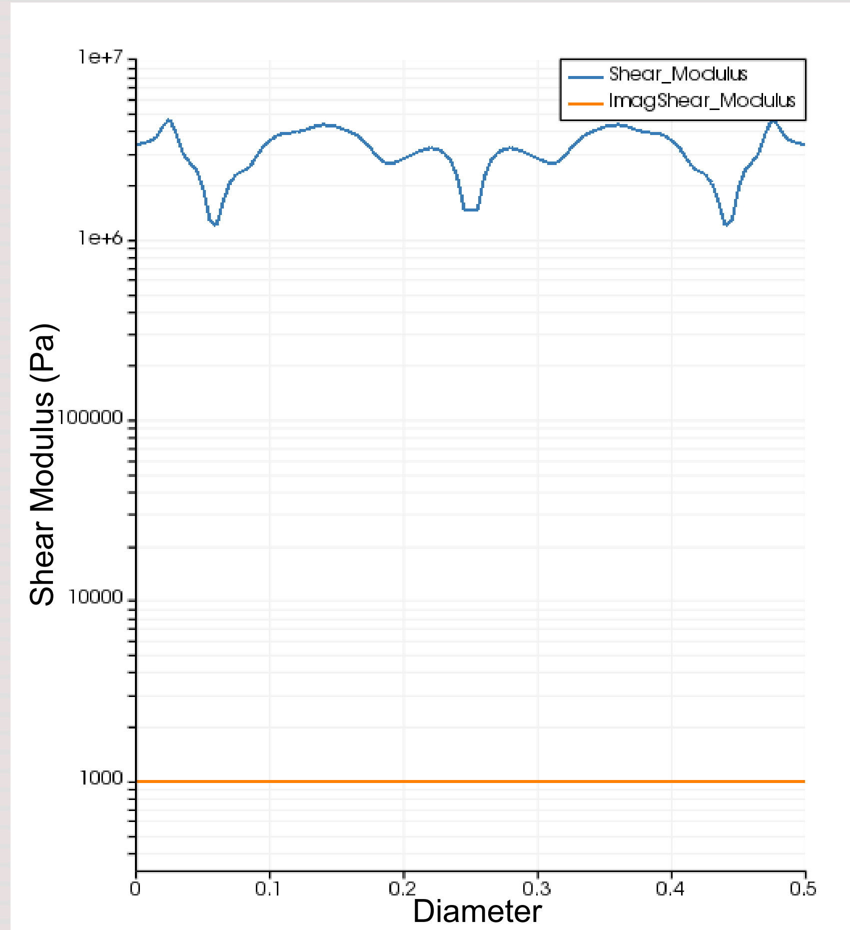
Left: Acoustic pressure distribution with initial material guess (500 Hz Loading)

Right: Pressure distribution after convergence to optimized design

Case Study 2: *Acoustic Cloaking*

Computed material parameters vary across disk diameter





Figures: Real & imaginary bulk moduli (left) and shear moduli (right) across inclusion diameter

Conclusions

- Abstract formulation for viscoelastic material design via numerical optimization
- Applications to mechanical and acoustic loading scenarios
- Frequency dependent material designs
  - Difficulties in computing solution for some frequencies (near resonance)
  - Sensitivity to initial guesses
- Directions for further development:
  - Improved objective (Modified Error in Constitutive Equations Method)
  - Heterogeneous viscoelastic materials
  - Metamaterials

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