Final Technical Report

Project: New Methods in Non-Perturbative QCD
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Supported personnel: Mithat Ünsal(PI), Alireza Behtash (graduate students).

1 New Methods in Non-Perturbative QCD

1.1 Infrared equivalence between compactified QCD and quantum magnets

In this work, we found a connection between QCD-like gauge theories and quantum antiferromagnets. It is known that in a spontaneously dimerized quantum antiferromagnet, spin-1/2 excitations (spinons) are confined in pairs by strings akin to those confining quarks in non-abelian gauge theories. The system has multiple degenerate ground states (vacua) and domain walls between regions of different vacua. For two vacua, we demonstrate that spinons on a domain wall are liberated, in a mechanism strikingly similar to domain-wall deconfinement of quarks in variants of quantum chromodynamics. This observation not only establishes a novel phenomenon in quantum magnetism, but also provides a new direct link between nuclear/particle physics and condensed-matter physics. The analogy opens doors to improving our understanding of particle confinement and deconfinement by computational and experimental studies in quantum magnetism.

1.2 Calculable chiral symmetry realizations in QCD

In *Chiral Lagrangian from Duality and Monopole Operators in Compactified QCD*, we showed that there exists a special compactification of QCD on $\mathbb{R}^3 \times S^1$ in which the theory has a domain where continuous chiral symmetry breaking is analytically calculable. We gave a microscopic derivation of the chiral lagrangian, the chiral condensate, and the Gell-Mann-Oakes-Renner relation. Abelian duality, monopole operators, and flavor-twisted boundary conditions, or a background flavor holonomy, play the main roles. The flavor twisting leads to the new effect of fractional jumping of fermion zero modes among monopole-instantons. Chiral symmetry breaking is induced by monopole-instanton operators, and the Nambu-Goldstone pions arise by color-flavor transmutation from gapless "dual photons". We also give a microscopic picture of the "constituent quark" masses. Our results are consistent with expectations from chiral perturbation theory at large S^1 and yield strong support for adiabatic continuity between the small and large circle regimes. This work provides the first framework in which chiral symmetry breaking happens at weak coupling where it is also calculable.

1.3 Hidden topological angles and Lefschetz thimbles

In *Hidden topological angles and Lefschetz thimbles*, and *"The curious incident of multi-instantons and the necessity of Lefschetz thimbles,"*, we found the existence of new topological angles (HTAs) in a large class of quantum field theories and quantum mechanical systems, including QCD. These new angles are distinct from topological theta-parameters in the lagrangian, and has crucial impact on observables such as mass gap, gluon condensates, and the realization of center symmetry.

In order to find these angles, we used some new mathematical techniques. In the calculation of the multi-instanton amplitudes, we used the idea of Lefschetz thimbles, (steepest descent or stationary phase method). The phase in the stationary phase is actually a topological invariant, characteristic of the saddle itself.

We examined this idea in QCD-like gauge theories, starting first with minimal supersymmetric Yang-Mills theory (the only QCD-like supersymmetric theory.) There, we showed that the microscopic mechanism for the vanishing of the gluon condensate is due to hidden topological angles. The same effect leads to an anomalously small condensate in a QCD-like $SU(N)$ gauge theory with fermions in the two-index representation, a natural generalization of $SU(3)$ QCD to large-N

Prior to our work, it was believed that the gluon condensate can only receive positive contributions (when the topological angle is set to zero). Our work has shown the existence of negative contributions emanating from HTAs.

This work is important in understanding general non-perturbative aspects of QCD, and other QFTs.

1.4 Toward Picard-Lefschetz Theory of Path Integrals

In *Toward Picard-Lefschetz Theory of Path Integrals, Complex Saddles and Resurgence*, and *Complexified path integrals, exact saddles and supersymmetry*, we proved that the proper semi-classical analysis of generic Euclidean path integrals necessarily requires complexification of the action and measure, and consideration of complex saddle solutions. This is a step that needs to be taken in order to perform a Lefschetz thimble decomposition even if the path integral is over real field configurations. Currently, there is building up evidence that this construction may be of practical utility in theories with sign problem, including finite density QCD.

Upon complexification, a surprising effect take place. There are complex saddle points which contribute to the path integral, and which have have a natural interpretation in terms of the Picard-Lefschetz theory. In quantum mechanical examples, we found new exact finite action non-BPS solutions to the holomorphic Newton equations (Euclidean equations of motions). This gives an immense generalization of the standard textbook discussion, and well-known instanton saddles. Some of the solutions are complex, multi-valued and even singular, and yet, one can demonstrate unambiguously that they must contribute to path integral. The multi-valuedness of the action is either related to the recently found hidden topological angles or to the resurgence theory and cancellation of ambiguities. (I will comment on resurgence below). I will argue that the present formalism is also the correct framework to find a more complete basis of saddles in general QFTs, and path integrals. These observations resolves some deep puzzles in the literature concerning the nature of the complex saddles.

1.5 Resurgence and mass gap in two dimensional sigma models

Two dimensional non-linear sigma models are asymptotically free theories, with a dynamically generated scale and a mass gap. In two related works, *"Decoding perturbation theory using resurgence: Stokes phenomena, new saddle points and Lefschetz thimbles"* and its follow-up *"Resurgence and Dynamics of O(N) and Grassmannian Σ Models"*, we studied the non-perturbative dynamics of these models, by using compactification with twisted boundary conditions on $\mathbb{R} \times S^1$, semi-classical techniques and resurgence theory.

Resurgence theory implies that the non-perturbative (NP) and perturbative (P) data in a QFT are quantitatively related, and that detailed information about non-perturbative saddle point field configurations of path integrals can be extracted from perturbation theory. Traditionally, only stable NP saddle points are considered in QFT, and homotopy group considerations are used to classify them. However, in many QFTs the relevant homotopy groups are trivial, and even when they are non-trivial they leave many NP saddle points undetected. Resurgence provides a refined classification of NP-saddles, going beyond conventional topological considerations. To demonstrate some of these ideas, we studies the $SU(N)$ principal chiral model, and $O(N)$ and Grassmannian Σ-Models. The use of resurgence and adiabatic continuity reveals the existence of new exact saddles. These saddles turn out to be the weak coupling realization of 't Hooft's renormalons. We provided ample evidence that the renormalon ambiguities are systematically cancelled in the semi-classical expansion.

These works strongly suggest that resurgence can be geometrized as a sum over Lefschetz thimbles, and the two, at least for semi-classical path integrals, are two sides of the same coin. It is expected that similar structures are omni-present, including QCD in 4d.

1.6 Infrared Renormalons versus Operator Product Expansions

In collaboration with Shifman and Dunne, I examined the connection between infrared (IR) renormalons and condensates in the operator product expansion for correlation functions to make predictions concerning the structure of singularities in the Borel plane for the perturbative series in quantum field theories. This work aims to build the resurgence technique in the strongly coupled regime of QCD-like theories. In this work, we showed that conventional bubble-chain method for detecting renormalon-induced factorial divergences in many QFTs is insufficient. So far, we were not able to improve bubble-chain techniques to make it work universally.

1.7 New Methods in QFT and QCD

In *New Methods in Non-perturbative QFT and QCD* prepared for Annual Reviews of Nuclear and Particle Science, we provide a conceptual introduction to new methods in non-perturbative QFT, and QCD. First, we describe large-N orbifold-orientifold equivalence, large-N volume independence with emphasis on QCD(adj) and double-trace deformation of Yang-Mills theory. These equivalences relate "difficult" theories to "easier" ones. Then, we describe semi-classical calculably on $\mathbb{R}^3 \times S^1$, and monopoles and magnetic and neutral bion saddles, brief description of resurgence in QFT, and calculable phase transitions on $\mathbb{R}^3 \times S^1$. These techniques provide a calculable regime of QFT and QCD continuously connected to more difficult strong coupling regime, and which provides insights that are justified based on first principles.

2 Publications, refereed (February 2015-January 2017)

1. G. Basar, G. V. Dunne and M. Unsal, "Quantum Geometry of Resurgent Perturbative/Nonperturbative Relations," JHEP **1705**, 087 (2017) doi:10.1007/JHEP05(2017)087

2. G. V. Dunne and M.Ünsal, *"Deconstructing zero: resurgence, supersymmetry and complex saddles,"* `arXiv:1609.05770 [hep-th]`, JHEP **1612**, 002 (2016)

3. A. Cherman, T. Schaefer and M. Ünsal, *"Chiral Lagrangian from Duality and Monopole Operators in Compactified QCD,"* Phys. Rev. Lett. **117**, no. 8, 081601 (2016) `arXiv:1604.06108 [hep-th]`

4. A. Behtash, G. V. Dunne, T. Schfer, T. Sulejmanpasic and M. Ünsal, "Complexified path integrals, exact saddles and supersymmetry," Phys. Rev. Lett. **116**, no. 1, 011601 (2016)

5. A. Behtash, G. V. Dunne, T. Schaefer, T. Sulejmanpasic and M. Unsal, "Toward Picard-Lefschetz Theory of Path Integrals, Complex Saddles and Resurgence," arXiv:1510.03435 [hep-th]. Annals of Mathematical Sciences and Applications, Volume 2, No. 1 (2017)

6. G. V. Dunne and M. Ünsal, "New Nonperturbative Methods in Quantum Field Theory: From Large-N Orbifold Equivalence to Bions and Resurgence," Ann. Rev. Nucl. Part. Sci. **66**, 245 (2016)

7. A. Behtash, E. Poppitz, T. Sulejmanpasic and M. Ünsal, "The curious incident of multi-instantons and the necessity of Lefschetz thimbles," JHEP **1511**, 175 (2015)

8. G. V. Dunne and M. Ünsal, *"Resurgence and Dynamics of O(N) and Grassmannian Sigma Models,"* JHEP **1509**, 199 (2015) [arXiv:1505.07803 [hep-th]].

9. G. V. Dunne, M. Shifman and M. Ünsal, *"IR Renormalons vs. Operator Product Expansion in Supersymmetric and Related Gauge Theories,"* Phys. Rev. Lett. **114**, no. 19, 191601 (2015) `arXiv:1502.06680 [hep-th]`.

10. A. Behtash, T. Sulejmanpasic, T. Schaefer and M. Ünsal, *"Hidden topological angles and Lefschetz thimbles,"* Phys. Rev. Lett. **115**, no. 4, 041601 (2015) `arXiv:1502.06624 [hep-th]`.

11. A. Cherman, P. Koroteev and M. Ünsal, *"Resurgence and Holomorphy: From Weak to Strong Coupling,"* J. Math. Phys. **56**, no. 5, 053505 (2015) [arXiv:1410.0388 [hep-th]].

12. A. Cherman, D. Dorigoni and M. Ünsal, *"Decoding perturbation theory using resurgence: Stokes phenomena, new saddle points and Lefschetz thimbles,"* JHEP **1510**, 056 (2015) [arXiv:1403.1277 [hep-th]].

3 Publications, submitted (February 2015-January 2017)

1. T. Sulejmanpasic, H. Shao, A. Sandvik and M. Unsal, *"Liberation of quarks and spinons on domain walls: analogies between gauge theories and quantum magnets,"* arXiv:1608.09011 [hep-th]. (submitted to PRL)

2. C. Kozçaz, T. Sulejmanpasic, Y. Tanizaki and M.Ünsal, *"Cheshire Cat resurgence, Self-resurgence and Quasi-Exact Solvable Systems,"* `arXiv:1609.06198 [hep-th]` (Submitted to Communications in Mathematical Physics)

3. G. V. Dunne and M. Ünsal, *"WKB and Resurgence in the Mathieu Equation,"* `arXiv:1603.04924 [math-ph]` (Conference proceeding, Resurgence, Physics and Numbers)

4. T. Sulejmanpasic and M.Ünsal, *"Aspects of Perturbation theory in Quantum Mechanics: The BenderWu Mathematica package,"* `arXiv:1608.08256 [hep-th]` (submitted to Journal of Physics A: Mathematical and Theoretical)

4 Publications, not refereed (February 2015-January 2016)

1. G. V. Dunne and M. Ünsal, *"What is QFT? Resurgent trans-series, Lefschetz thimbles, and new exact saddles,"* Proceedings, The 33rd International Symposium on Lattice Field Theory in press (2015), arXiv:1511.05977 [hep-lat].

5 Presentations (February 2015-January 2017)

1. *"Toward Picard-Lefschetz Theory of Path Integrals and the Physics of Complex Saddles"* Continous Advances in QCD, May 14, 2016, Minneapolis

2. *"Semi-classical Analysis of Gauge theories on $R^3 \times S^1$"* Recent Developments in Semiclassical Probes of Quantum Field Theories, March, UMass Amherst

3. *"Toward Picard-Lefschetz Theory of Path Integrals and the Physics of Complex Saddles"* Recent Developments in Semiclassical Probes of Quantum Field Theories, March, UMass Amherst

4. *"Toward Picard-Lefschetz Theory of Path Integrals, and Resurgence"* Monday Colloquia, Stanford Institute for Theoretical Physics, April 25, 2016, Palo Alto

5. M. Ünsal, *"Toward Picard-Lefschetz Theory of Path Integrals, and Resurgence"* Seminar, Harvard Physics Department, December 8, 2015, Boston

6. M. Ünsal, *" What is QFT? Resurgent trans-series, Lefschetz thimbles, and new exact saddles"* CMSA Colloquium, Harvard Mathematics department, September 9, Boston

7. M. Ünsal, *" What is QFT? Resurgent trans-series, Lefschetz thimbles, and new exact saddles"* Pleanary talk on 33rd International Symposium on Lattice Field Theory, July 2015, Lattice 2015, Kobe, Japan

8. M. Ünsal, *" Analytic continuation of path integrals and new exact saddles"* Instituto de Fisica Teorica (IFT) , 1-5 June 2015, Madrid, SPAIN

9. M. Ünsal, *"What is QFT? Resurgence, transseries, and Lefschetz thimbles"* Joint ICTP/SISSA Seminar, Trieste, May 2015

10. M. Ünsal, *"Resurgence and transseries in QFT: Towards a continuum definition"* Centro di Ricerca Matematica Ennio De Giorgi, Resurgence, Physics and Numbers Conference, May 18-22 2015, Pisa, Italy

11. M. Ünsal, *"What is QFT? Resurgence, transseries, and Lefschetz thimbles"* Stony Brook University, Resurgence and localization in string theory and quantum field theory workshop March 16- 20, 2015, NY

12. A. Behtash *"Lefschetz Thimbles and Quantum Mechanics"*, at Mathematical Physics Seminar, Harvard

6 Publications (published or submitted during 2016)

Investigator	publ	prep./sub.	conf.repts.	talks
Ünsal	10	4	0	10
Behtash	1	1	0	1
Total	11	5	0	11

7 Student Tracking Information

Name	Mentor	Entered	Joined	Ph.D.
Alireza Behtash	Ünsal	Sep 2014	Sep 2014	Fall 2018 (expected)