SOLID STATE LIGHTING LED LIFE PREDICTION USING EXTENDED KALMAN FILTER

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ABSTRACT:

Solid-state lighting (SSL) luminaires containing light emitting diodes (LEDs) have the potential of seeing excessive temperatures when being transported across country or being stored in non-climate controlled warehouses. They are also being used in outdoor applications in desert environments that see little or no humidity but will experience extremely high temperatures during the day. This makes it important to increase our understanding of what effects high temperature exposure for a prolonged period of time will have on the usability and survivability of these devices. The U.S. Department of Energy has made a long term commitment to advance the efficiency, understanding and development of solid-state lighting (SSL) and is making a strong push for the acceptance and use of SSL products to reduce overall energy consumption attributable to lighting.

Traditional light sources "burn out" at end-of-life. For an incandescent bulb, the lamp life is defined by B50 life. However, the LEDs have no filament to "burn". The LEDs continually degrade and the light output decreases eventually below useful levels causing failure. Presently, the TM-21 test standard is used to predict the L70 life of SSL Luminaires from LM-80 test data. The TM-21 model uses an Arrhenius Equation with an Activation Energy, Pre-decay factor and Decay Rates. Several failure mechanisms may be active in a luminaire at a single time causing lumen depreciation. The underlying TM-21 Arrhenius Model may not capture the failure physics in presence of multiple failure mechanisms. Correlation of lumen maintenance with underlying physics of degradation at system-level is needed.

In this paper, a Kalman Filter and Extended Kalman Filters have been used to develop a 70% Lumen Maintenance Life Prediction Model for a LEDs used in SSL luminaires. This model can be used to calculate acceleration factors, evaluate failure-probability and identify ALT methodologies for reducing test time. Ten-thousand hour LM-80 test data for various LEDs have been used for model development. System state has been described in state space form using the measurement of the feature vector, velocity of feature vector change and the acceleration of the feature vector change. System state at each future time has been computed based on the state space at preceding time step, system dynamics matrix, control vector, control matrix, measurement matrix, measured vector, process noise and measurement noise. The future state of the lumen depreciation has been estimated based on a second order Kalman Filter model and a Bayesian Framework. The measured state variable has been related to the underlying damage using physics-based models. Life prediction of L70 life for the LEDs used in SSL luminaires from KF and EKF based models have been compared with the TM-21 model predictions and experimental data.

INTRODUCTION:

The LEDs (Light Emitting Diodes) have been widely used since the last decade. Also, there is a tendency that LEDs would dominate the lighting market because of the LEDs' advantages in the light efficiency, energy saving, improved physical robustness and long operating hours. The industrial utilization of LEDs in some extreme environments requires the LEDs have a considerable long life. It is also required that those LEDs must withstand high temperature and high humidity environment without much lumen maintenance degradation after long term wearing. The most critical value for the failure of LEDs is 'L70' (the Lumen Maintenance reaches 70% of the original and pristine starts at 100%). There is extremely long-term degradation for the LEDs to reach its L70 that is normally over 30,000 hours for the Philips LUXEON Rebel lamps. The former methodology to quantify the LED's L70 is TM-21 (Technical Memorandum) wrote by the Philips, Osram, Nichia, Illumitex, GE and Cree. Here, we introduce another reliable method, the Extended Kalman Filtering, to quantify LED life.

The life of LEDs can be affected by many facts, including: the manufacturers, junction temperatures, humidity as well as the LED working current. Somehow, the LED working temperatures and currents are the most severe conditions that shorten the LED life scope. Theoretically, the higher junction temperature it is, the less lumen output will be. The working current would be around 350 mA to 1A for the high power white LEDs. In the Philips experiment, the test temperatures are ranged from 55 °C, 85°C, 105 °C and 120 °C, and the test currents are ranged from 350 mA, 500 mA, 700 mA and 1A. Under those conditions, we suppose the lumen degradation conforms to the Arrhenius Equation, which is exponentially decaying. Thus, with the general model assumption and current lumen maintenance data, we can derive the pseudo L70 life for the Philips high power LEDs, and it is necessary to introduce the EKF algorithm that is reliable method and was previous verified in the BGA prognostic health management system, to meet the requirements and accomplish dynamic life prediction goal.

The EKF algorithm has been previously used for monitoring and predicting the electronic system failures, which is a part from the Prognostic Health Management (PHM) of Electronic systems. [Lall 2004a-d, 2005a-b, 2006a-f, 2007a-c, 2008a-f]. The EKF could watch out for the tendency of resistance change and monitor it when crossing the failure threshold. Similarly, in this paper, we use EKF to catch the Lumen Maintenance degradation lines and to make an extrapolation for the future space to generate the Remaining Useful Life (RUL) of L70 threshold. The exponential function has been incorporated into the EKF to make exponential extrapolations. Those extrapolations provide us life estimations of LEDs that calculated from Newton Raphson method. Then, the L70 life could be viewed as following certain distributions, such as normal distribution, lognormal distribution and the weibull distribution. Once fitting the life data, we use the Maximum Likelihood Estimation (MLE) method to get the best fit for the life data. Then, we can get the cumulative probability function F(t) and obtain the reliability function R(t).

NOMENCLATURE

LED	Light Emitting Diodes
√(t)	Lumen Maintenance
L70	Lumen Maintenance at 70%
KF	Kalman Filter
EKF	Extended Kalman Filter
TM-21	Technical Memorandum-21
AES	Activated Energy of System
PHM	Prognostic Health Management
Xp	Filter Projection
Xe	Filter Estimation
H	Measurement Matrix
Q	Process Noise
F	System Dynamic Matrix
R	Measurement Noise
)(t)	Fundamental Matrix
M_k	the First Covariance Matrix
K	Kalman Gain
P_{k+1}	the Second Covariance Matrix
A	Pre-decay Factor
α	In-Situ temperature Coefficient 1
β	In-Situ temperature Coefficient 2
E_a	Activated Energy
L_o	Lumen Output
L_I	Initial Lumen Maintenance
C_1	Decay Constant 1
C_2	Decay Constant 2
T	Life Time
T_0	Initial Life Time
Ts	Sampling Time

LEDS' LUMEN MEASUREMENT SYSTEM:

The LEDs' Lumen Measurement System (**Figure 1**) contains parts: (1) Light Emitting Device. (2) Light Gathering System. (3) Light Transmitting System. (4) Light Analyzing System. The Light Emitting Device generates the light from the lamp, which made from our test LEDs. There are many kinds of LED lamps. Each LED lamp has its unique light generating system, which contains a LED driver and LED bulbs. The LED driver transfers the AC to the DC that is only allowed for LEDs. However, each driver, somehow, will produce the ripple current that would affect the quality and life of LEDs. The Light Gathering System collects all the light emitting from the LED, typically, the Light Gathering System includes the integrating sphere and cosine diffuser etc., and we use the integrating sphere to collect the light from LEDs. There is coating on the surface inside of Integrating Sphere, which makes the light to diffuse at all the inside can surface, which causes the light distribute evenly. Then the Light Transmitting System, including the cable optical fibers, can get the well-distributed light from the Integrating Sphere and transmit it into the Light Analyzing System. The Light Analyzing System mainly analyzes the Power of the light, the Luminous Flux, Luminance as well as the CCK that denotes the color shifting value of the light.

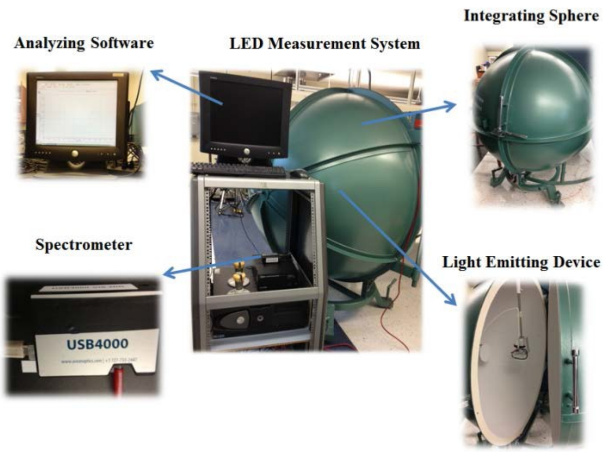


Figure 1 LEDs Measurement System

In general, we get the Lumen Maintenance data from the measurement system, which presents the power intensity of the light from LED bulb. The Lumen Maintenance says the percentage of the light power of LED compared to the pristine that is normalized and sets to be 100% at the beginning. For example, the test starts at the 100% lumen maintenance, and through many burning hours, the lumen maintenance drops to the 70% of its original. Therefore, it is the dropping percentage of lumen maintenance versus the time that we can use it to predict the pseudo life of LED throughout this system.

ACCELERATING TEST AND ACCELERATING MODEL:

The purpose of accelerated degradation is concerned with models and data analyses for degradation of product performance over time at overstress and design conditions. There are many advantages for the accelerated test. It can be analyzed in very early time. Also, it can estimate the time when the performance through the failure threshold. The extrapolation could let us explore the behaviors and consequences under the different stress level, and compare those results to generate much more accurate life prediction model.

There are many assumptions for the accelerating model. Those assumptions include: (1) Degradation is not reversible. (2) Single Degradation process is applied. (3) Degradation of specimen performance before the test beginning is negligible. (4) Performance is measured with negligible random error. We also apply the statistic model for the L70 life, which shows the typical performance around obvious value. Those distributions are very important for the high reliability management of LED systems.

In this paper, the exponential Arrhenius model is discussed, and the Arrhenius rate relationship is widely adopted for the temperature based accelerating experiments. For the Arrhenius rate relationship, in any temperature and exposure time, the distribution of performance μ is lognormal. Thus, $y=\log(\mu)$ is normal. The standard deviation $\int$ of log performance is a constant, which does not depend on the temperature and current.

PHILIPS DATASET:

The datasets were collected by the PHILIPS LUMILEDS, which published in documents DR05-1-LM80 and DR05-1-LM80. The test product is LUXEON LXM3-PW series LEDs, which is shown in the following **Figure 2** .

Figure 2 LED Test Product

Those documents contain the LEDs' lumen maintenance degradation and chromatic shifting, the very vital indication for the failure of LEDs. In order to estimate the pseudo life of LED, thus, we use the lumen maintenance degradation for extrapolating the L70 life (the Lumen Maintenance reaches 70% of the original and pristine starts at 100%), which is the standard criterion for industrial world to estimate the life of bulbs. In this dataset, there are many test conditions; each test condition includes 25 samples. In each sample, the lumen maintenance data was recorded from 0 to 9000 hours, the test matrix shows below Table 1:

	Current	T s	CCT
Test1	0.35A	55 ˚C	3000K
Test2	0.35A	85 ˚C	3000K
Test3	0.35A	105 ˚C	3000K
Test4	0.35A	120 ˚C	3000K
Test5	0.5A	55 ˚C	3000K
Test6	0.5A	85 ˚C	3000K
Test7	0.5A	105 ˚C	3000K
Test8	0.5A	120 ˚C	3000K
Test9	0.7A	55 ˚C	3000K
Test10	0.7A	85 ˚C	3000K
Test11	0.7A	105 ˚C	3000K
Test12	1A	55 ˚C	3000K
Test13	1A	85 ˚C	3000K
Test14	1A	105 ˚C	3000K

Table 1 Test Matrix

In the test matrix, there are 14 tests under 4 different currents and 4 different temperatures, and the test CCT is 3000K. In this paper, we use the data from the last test, which the current is 1 A; test temperature is 105 °C and CCT is 3000K. The dataset from the selected test would look like below Figure 1. Different color represents the lumen outputs at each time from 25 samples. There is a tendency of the degradation in the lumen maintenance (**Figure 4**), and we want to explore the degradation model in the degradation of lumen maintenance and to estimation the pseudo LEDs' L70 life. The general degradation on the lumen maintenance shows on the **Figure 4**. Throughout the 9000 hours experiment, the lumen maintenance decays from 100 % to the 92 %. However, this degradation may not look like the linear since there are multiple elements affect the degradation of lumen maintenance. The degradation would tend to be much slower in the future, which is called decelerating process. Generally, we use the exponential degradation function to present this process, and use the exponential family functions to predict the pseudo L70 life for the LEDs.

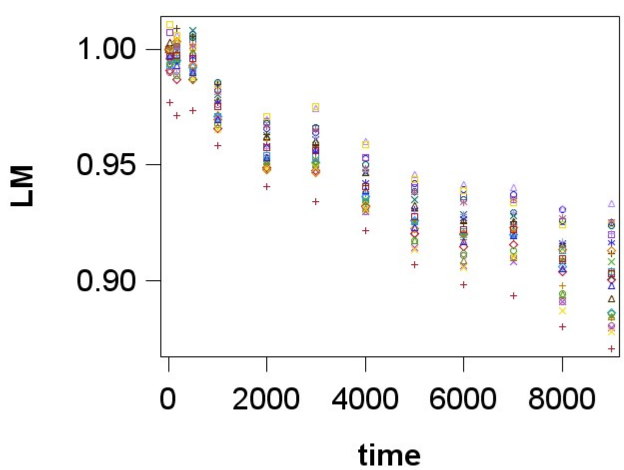
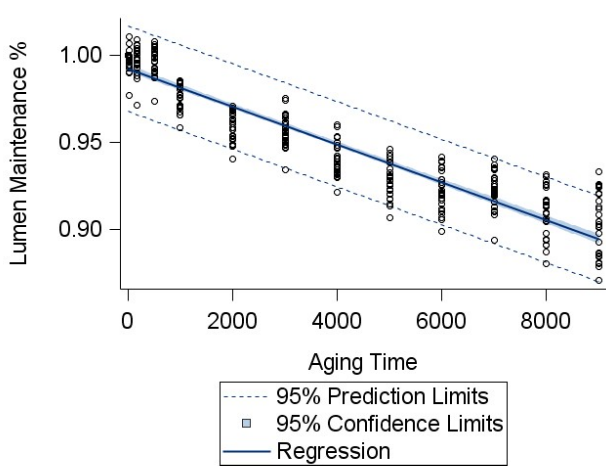


Figure 3 Test Lumen Degradation



Extended Kalman Filter Algorithm

System damage state estimation in the presence of measurement noise and process noise has been achieved using the Extended Kalman Filter (EKF). Previously, the Kalman Filter has been used in guidance and tracking applications [Kalman 1960, Zarchan 2000].

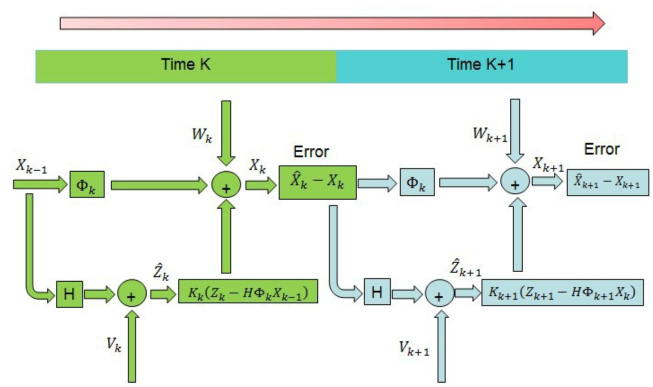


Figure 5 the Recursive Algorithm and Extended Kalman Filter

System state has been described in state space form using the measurement of the feature vector, velocity of feature vector change and the acceleration of the feature vector change. The equivalent Extended Kalman Filter equation for state space representation is in the presence of process noise and measurement noise is:

$$\dot{x} = Fx + w \quad (1)$$

$$\dot{x} = f(x) + w \quad (2)$$

Where the F is the system linear dynamic matrix; f(x) is non-linear dynamic matrix; the G is measurement matrix; u is measurement vector; and w is system white noise; the matrices are related to the nonlinear system and measurement equations according to

$$F = \frac{\partial f(x)}{\partial x}\bigg|_{x=\hat{x}} \quad (3)$$

$$H = \frac{\partial h(x)}{\partial x}\bigg|_{x=\hat{x}} \quad (4)$$

the process noise can be calculated by taking the expectation of white noise:

$$Q = E[ww^T] \quad (5)$$

Also, the EKF Equations require the measurement to be linearly related to the current state in the form of :

$$Z = H \oplus x + V \quad (6)$$

where the V is measurement noise. Similarly, the measurement noise matrix is derived from the measurement noise as following:

$$R = E[vv^T] \quad (7)$$

Figure 4 General Degradation Relationship of Lumen Maintenance

From the linear dynamic equations, we can clearly know that the world is linear as we supposed, and once we made this premise. All the problems can be simply solved through the matrix calculation. Therefore, we can get the first and second derivatives from the linear matrix calculation; the Equation 4-

1 is the model of the first order system while the Equation 4-2 is the second order system:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dddot{x} \end{bmatrix} = F \oplus \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} \quad (8)$$

The system dynamic matrix for the EKF is:

$$F_{EKF} = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial \dot{x}} & \frac{\partial \dot{x}}{\partial b} \\ \frac{\partial \ddot{x}}{\partial x} & \frac{\partial \ddot{x}}{\partial \dot{x}} & \frac{\partial \ddot{x}}{\partial b} \\ \frac{\partial \dot{b}}{\partial x} & \frac{\partial \dot{b}}{\partial \dot{x}} & \frac{\partial \dot{b}}{\partial b} \end{bmatrix} \quad (9)$$

We use this Jacobin Matrix to linearize the non-linear problem; therefore it can use the classical KF updates. This is for the second order system, and thus we can find the transfer function F to describe certain system, which it is the key to find fundamental matrix $\surd(t)$. In this paper for the EKF, we used the system model:

$$x = \langle \oplus e^{\circledR \oplus t} \quad (10)$$

The state vector is:

$$x_k = \begin{bmatrix} x \\ \dot{x} \\ \circledR \end{bmatrix} \quad (11)$$

This is an exponential function. The '$\langle$' and '$\circledR$' are two coefficients that are decided by different systems. The first derivation $\dot{x}$ and the second derivation $\ddot{x}$ are from:

$$x = \langle \oplus e^{\circledR \oplus t} \quad (12)$$

$$\dot{x} = \circledR \oplus x \quad (13)$$

$$\ddot{x} = \circledR \oplus \dot{x} = \circledR^2 \oplus x \quad (14)$$

Therefore the elements in system dynamic matrix will be calculated as:

$$F_{EKF} = \begin{bmatrix} \circledR & 1 & x \\ \circledR^2 & \circledR & 2\circledR x \\ 0 & 0 & 0 \end{bmatrix} \quad (15)$$

In the Kalman Filter, the Fundamental Matrix will be used directly to update the estimation from last time to the next.

Generally speaking, the process to find the ideal estimation can be expressed as following steps: First of all, we make the primary estimation, which should be approximate to the initiate value (the first data point) in the dataset, and secondly, we can find the first projection using the fundamental matrix $\surd(t)$ and simply calculate as:

$$\bar{x} = \surd(t)\hat{x} \quad (19)$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial \dot{x}} & \frac{\partial \dot{x}}{\partial b} \\ \frac{\partial \ddot{x}}{\partial x} & \frac{\partial \ddot{x}}{\partial \dot{x}} & \frac{\partial \ddot{x}}{\partial b} \\ \frac{\partial \dot{b}}{\partial x} & \frac{\partial \dot{b}}{\partial \dot{x}} & \frac{\partial \dot{b}}{\partial b} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ b \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix} \quad (20)$$

The next estimate could be obtained from the following equation:

$$\bar{x} = \hat{x} + K(Z - H \oplus \hat{x}) \quad (21)$$

K is Kalman Gain

H is measurement matrix

Z is measurement.

Each time we update the Kalman Gain and Covariance Matrix, which minimizes the errors and makes optimal calculation during each step. Thus, the Kalman Gain mainly conveys the information about how is our estimation close to the observation. The way to obtain Kalman Gain (K) is from three Riccati equations:

$$M_k = \surd P_k \surd_k^T + Q_K \quad (22)$$

$$K_k = M_k H_t (H M_k H_t + R_k)^{-1} \quad (23)$$

$$P_k = (I - K H) M_k \quad (24)$$

In the above equations, the M_k is the covariance matrix; the $\surd$ is the fundamental matrix; the $\surd^T$ is the transpose of that

0 ⌋

matrix; P_k is another covariance matrix that representing

Usually, the fundamental matrix $\surd(t)$ can be obtained from two ways: the first way, we can get it from Laplace Transform, simply as:

$$\surd(t) = \ell[(SI \,\square F)^{\square}] \quad \textbf{(16)}$$

Where the $\ell^{\square}$ is inverse of the Laplace Transform; However, the second way, known as the common way to find $\surd(t)$, derives from the Tylor Serise expansion:

$$\surd(t) = I + FT + \frac{(FT)^2}{2!} + \frac{(FT)^3}{3!} + \cdots \quad \textbf{(17)}$$

Normally, we only use the first two terms for representing the fundamental matrix $\surd(t)$, because the adding more terms cannot contribute much to the precision and filter convergence.

The Fundamental Matrix in the Extended Kalman Filter is:

$$\surd_{EKF}(T) \text{H} I + F_{EKF} T = \begin{bmatrix} 1+®T & T & xT \\ ®T & 1+®T & 2®\oplus xT \\ 0 & 0 & 1 \end{bmatrix} \quad \textbf{(18)}$$

error according to the time; the Q_k is the discrete process noise matrix, which is calculated from:

$$Q = \surd_s \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \textbf{(25)}$$

$$Q_k = \int_0^{Ts} \surd(\,|\,) \oplus Q \oplus \surd(\,|\,)^T d\,| \quad \textbf{(26)}$$

the H is the unit measurement matrix and H_t is transpose of it; the K is the Kalman Gain; R_k represents the measurement noise according to the different system. We notice that those three equations run like in the recursions: for the initial covariance error P_0, we can find variance matrix M_k that represents the current error in the first equation according to time. Then we use it in the second equation to find the Kalman Gain K, after that, we substitute the Estimate Kalman Gain K into the third equation to update last covariance error

P_k, thus we obtain the 'next' covariance error P_{k+1}. Therefore, as we go back to the first equation, we can obtain the updated M_k and updated Kalman Gain K. In the EKF, the Euler integration has been introduced to instead the performance of the KF's fundamental matrix, it can be found that:

$$\hat{x}_k = \bar{x}_k + \hat{\dot{x}}_{k□} T \quad (27)$$

$$\hat{\dot{x}}_k = \bar{\dot{x}}_k + \hat{\ddot{x}}_{k□} T \quad (28)$$

We call the equation above is the update equations, $\hat{x}_k$ represents the projection from the last time k ; $\hat{\dot{x}}_{k□}$ is the first derivative at time k-1; $\bar{x}_k$ is the estimation at time k, $\dot{x}_k$ is the first derivative at time k; T is the sample time.

$$\bar{x}_k = \hat{x}_k + K_1(Z □H \oplus \hat{x}_k) \quad (29)$$

$$\bar{\dot{x}}_k = \hat{\dot{x}}_k + K_2(Z □H \oplus \hat{x}_k) \quad (30)$$

The above equations are the basic Extended Kalman Filter equations, which are to find the estimation and its 'velocity'. Also, in those equations, it uses the same three Riccati Equations that expressed in the Kalman Filter to obtain the Kalman Gain K_1 and K_2.

Now, we can see that the Extended Kalman Filter actually have turned the non-linear problem into a linear one through integrating method. So at each step, the Extended Kalman Filter made a small integration, and if the integrate time is small enough, then the answer we get is becoming more precise. However, the difficulty within the Extended Kalman Filter is to find the dynamic non-linear model to describe the system, which always contains the unknown coefficients. Therefore, the better we know about the test system, for example, the theories and functions in the situation of LEDs failure, the better we can predict system model in the Extended Kalman Filter, therefore, the prediction of Remaining Useful Life (RUL) would be close to the real RUL

7. Extrapolate feature vector to threshold value:

$$LM = x_{e_{k+n}} \oplus ®_{k+n} + w_{k+n}$$

8. Report predicted RUL (and uncertainty);
9. Iterate to step 2 for next measurement (k = k +1);

Prognostic Degradation Models:

There are many prognostic models that could explain the degradation process for the lumen maintenance. Generally, we explore some simple constant rate models. The simplest degradation relationship for typical log performance μ(t) ,the mean of the population distribution of log performance at age t, is a simple linear function of product t:

$$\propto(t) = \langle □ ®' \oplus t \quad (31)$$

For a degradation accelerated by the temperature T, the Arrhenius Degradation Rate is:

$$®' = ® \oplus e^{(- \frac{©}{T})} \quad (32)$$

The β and γ are the coefficients for the typical product. With this rate parameter, the degradation model becomes:

$$\propto(t,T) = \langle □ t \oplus ® \oplus e^{(□\frac{©}{T})} \quad (33)$$

For a degradation accelerated by the current V, the Power Degradation Rate shows:

$$®' = ® V^{©} \quad (34)$$

With this rate parameter, the degradation model becomes:

$$\propto(\mathrm{t,V}) = \langle - \mathrm{t} \oplus ® V^{©} \quad (35)$$

For a degradation accelerated by the exponential decay rate, the decay stress rate could be expressed as:

$$®' = ® \oplus \mathrm{e}^{(© \oplus \mathrm{V})} \quad (36)$$

With this rate parameter, the degradation model becomes:

$$\propto(\mathrm{t,V}) = \langle - \mathrm{t} \oplus ® \oplus \mathrm{e}^{© V} \quad (37)$$

in our PHM.

Overall, since the μ taken from the natural logarithm, we can
simplify the models above by using the initial

Algorithm: Filtering and RUL prediction

1. Initiate x_0
2. Make the projections: $\hat{x}_k$

   $= \bar{x}_k + \hat{\dot{x}}_{k□} T + w \; \hat{\dot{x}}_k$

   $= \dot{x}_k + \hat{\ddot{x}}_{k□} T + w$
3. Calculate error covariance matrix before updata:

   $M_k = √_k P_k √_k^T + Q_K$
4. Calculate the Kalman Gain:

   $K = M_k \; H_t (H M_k H_t + R^k)^{□}$
5. Update the estimation with measurement:

   $\bar{x}_k = \hat{x}_k + K_1 (Z □ H ⊕ \hat{x}_k)$

   $\dot{x}_k = \hat{\dot{x}}_k + K_2 (Z □ H ⊕ \hat{x}_k)$
6. Calculate error covariance after measurement update:

   $P_k = (I □ K H) M_k$

degradation factor and overall degradation rate. We assumed the basic L70 extrapolation model is:

$$√ = 〈 ⊕ e^{(□ ® ⊕ t)} \quad (38)$$

In this model, the √is the Lumen Maintenance (%), and the 〈is the initial degradation factor, ®is the degradation rate. The model says the degradation line should be fitted within the exponential function. The coefficient 〈denotes the current lumen maintenance estimation, and the coefficient ® expresses the decay rate of the lumen maintenance. The EKF mainly incorporates this exponential model to make many extrapolations, which can calculate the time each extrapolation reach the threshold of the failure criterion.

LED failure criterion

Lumen Maintenance is described as the comparison between the time specified in the future and the brand new product. The Lumen Maintenance is 100% at the beginning. Lumen Maintenance often denotes as the L50 (50%) and L70 (70%). The customer decides a suitable lumen maintenance target for the LEDs. However, in the industrial world, the L70 (70%) is accepted as the LEDs failure criterion since the human eye cannot detect the change that light dropped 30% from 100%. Therefore, we apply the L70 for the failure threshold of LUXEON LEDs in this paper. Next, the EKF will extrapolate the degradation lines to reach the L70 threshold. Then, the Newton Raphson Method will calculate the Remaining Useful Life, which can be added with the current burning hours to generate the pseudo LEDs life.

Newton Raphson Method

When Kalman Filter made a prediction of the state vector from the data-set, and the remaining useful life could be estimated and calculated mathematically by solving the equation H(t) and find the time T-prediction:

$$H(t) = x_0 + \dot{x} \cdot t + \ddot{x} \cdot t^2 \,\square f(EoL) \quad (39)$$

$$T_{n+1} = T_n \,\square f(x) / f'(x) \quad (40)$$

In order to find the root of above equation, we introduce the Newton Rapson's Method:

T_{n+1} is the estimated root at time n;

T_n is the estimated root at time n+1;

$f'(x)$ is the derivation of target equation;

The Predicted RUL is known as the End of Life (EoL) minus the sampling time. So the algebra equations presents as following:

$$L70 = T_{predict} + T_{sample} \quad (41)$$

Extended Kalman Filter Prognostic Life:

As we mentioned before, the prediction model that EKF uses to make the extrapolations is the exponential degradation model. The L70 estimation model:

$$\surd = \langle \oplus \exp(\square ® \oplus t) \quad (42)$$

The following **Figure 6** shows the EKF estimations and data points; the blue dots present the EKF, and the red lines are the data points. We can see that in the first 9000 hours, the lumen maintenance drops to the 90%. Also, we need to eliminate the oscillation and unstable condition of the first 2000 hours estimation, therefore, we uses the 2000 to 9000 hours data to predict the LEDs L70 life. By tracking the current degradation lines, there is non-linear pattern in the degradation of lumen maintenance, which is close to the exponential decay. Therefore, the exponential function, we expressed above, is suitable for our lumen maintenance extrapolation and prediction.

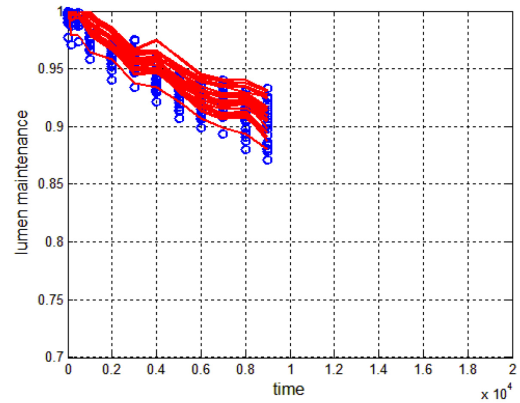


Figure 6 The LM Degradation Data and EKF Estimations

The following histogram (**Figure 7**) shows the EKF estimations of L70 life. Those estimations are derived by the extrapolations using the previous exponential decay function. There have 234 estimations, and in the Gaussian distribution, the mean value (EXP) is 43265hours, and the Standard Deviation (STD) is 2721 hours.

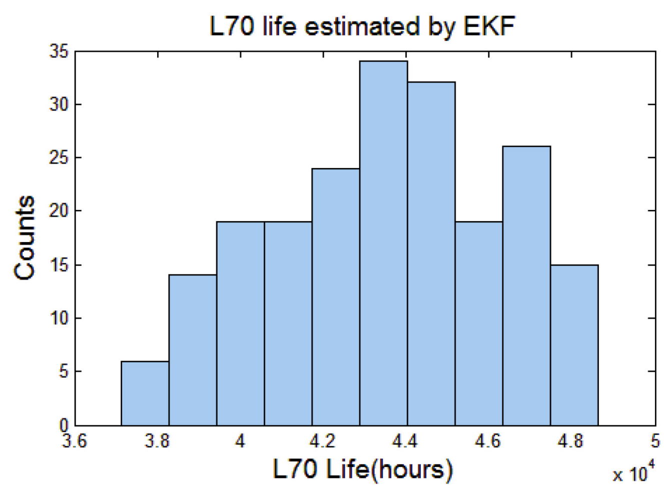


Figure 7 The Estimated L70 Distribution

PredictionTime(Hours)	2000	3000	4000	5000	6000	7000	8000	9000
Sample #1	40581.05	48328.18	47544.518	46551.295	47129.299	48270.638	48644.519	48559.844
Sample #2	37139	40432.47	39876.882	39347.764	39107.865	39196.167	39354.305	38714.029
Sample #3	40343.01	47538.42	46807.996	45535.511	45977.499	46647.778	47123.583	46404.442
Sample #4	39657.89	46284.88	45885.607	44782.444	44786.234	45349.321	45927.038	44758.924
Sample #5	40585.56	48300.43	47638.16	46404.623	46594.409	47545.836	48037.923	47655.785
Sample #6	39690.14	45783.38	44920.914	43643.97	43834.71	44540.419	44963.641	44632.181
Sample #7	38591.71	42711.06	43135.107	42152.169	42526.668	42950.825	43712.145	43018.365
Sample #8	39412.44	44066.42	43662.445	41924.066	42340.937	42370.93	43005.966	41590.11
Sample #9	40517.04	46577.43	45930.68	44449.577	44829.614	44663.529	45688.456	43591.552
Sample #10	39057.48	44754.35	44248.751	43139.174	43693.994	44301.315	45266.259	45015.984
Sample #11	40241.03	46648.05	47150.322	46068.148	46110.514	46865.028	47373.76	47202.99
Sample #12	40394.8	48186.27	48458.831	46896.343	46891.497	47686.635	47735.695	47328.932
Sample #13	39238.47	44293.97	44056.854	43169.081	43339.412	44045.515	44531.087	44456.596
Sample #14	38092.28	42628.4	42513.425	41555.958	41730.644	42195.451	42991.842	42573.269
Sample #15	40625.39	47365.91	46904.034	45474.656	45798.935	46692.401	46743.874	47073.545
Sample #16	40177.9	45655.5	45057.361	43611.085	43865.078	44436.749	45219.341	44959.22
Sample #17	38195.99	42081.4	41991.317	40732.745	40373.857	40521.137	41420.361	40380.246
Sample #18	38248.94	42035.84	42321.127	40858.333	40687.649	40750.655	41635.178	40706.824
Sample #19	38318.84	42761.51	42924.275	41706.098	41978.492	42552.248	43226.729	43296.668
Sample #20	40019.46	46748.61	46431.054	44988.31	45412.826	46137.468	46964.703	46895.891
Sample #21	38119.31	42046.75	42112.464	41245.807	41517.999	42363.504	43433.113	43314.785
Sample #22	38308.8	43353.43	43190.036	42078.166	41707.815	42153.047	42926.763	41664.043
Sample #23	38727.85	44681.68	45132.469	44099.904	43881.813	44875.139	45528.015	45032.258
Sample #24	37378.5	41196.96	41608.37	40431.859	39921.861	40046.069	41281.745	39564.175
Sample #25	38496.95	43634.35	44032.725	42717.433	42546.794	42962.622	43845.711	43079.093
Sample #26	39208.03	44646.46	44479.621	43307.067	43413.668	43941.511	44593.578	44016.175

Table 2 Pseudo L70 Life

The following pictures table 3 and table 4 show the estimated alpha and beta coefficients' histogram and distribution.

PredictionTime(Hours)	2000	3000	4000	5000	6000	7000	8000	9000
Sample #1	0.982421	0.961993	0.9635289	0.9527311	0.9420817	0.9393379	0.9372766	0.9305935
Sample #2	0.958453	0.937616	0.9342206	0.9216239	0.9069729	0.8986258	0.8934894	0.8805254
Sample #3	0.980731	0.959097	0.9607296	0.9467554	0.9349852	0.9288427	0.9280687	0.9160077
Sample #4	0.978229	0.958559	0.9599726	0.9466625	0.9323797	0.9260369	0.9256703	0.9088278
Sample #5	0.98504	0.965066	0.9656636	0.9531315	0.9402767	0.9362385	0.9349759	0.9254032
Sample #6	0.977715	0.95484	0.9527761	0.937872	0.9256752	0.9210307	0.9189774	0.9093089
Sample #7	0.971303	0.944714	0.9509839	0.9363986	0.9261648	0.9198272	0.920175	0.9063263
Sample #8	0.976254	0.950053	0.9505853	0.9301131	0.920365	0.9109351	0.9103754	0.8912402
Sample #9	0.984205	0.957153	0.9576653	0.9418604	0.9309728	0.9179594	0.921362	0.8979404
Sample #10	0.971556	0.948171	0.9497448	0.9352865	0.9254765	0.9201315	0.9231603	0.9141067
Sample #11	0.985516	0.966171	0.9740794	0.9602684	0.9459738	0.9415278	0.9402757	0.9314073
Sample #12	0.98515	0.967569	0.9748613	0.9589471	0.9439791	0.9390362	0.9339845	0.9241045
Sample #13	0.977406	0.955235	0.9557078	0.9423879	0.9306645	0.9267213	0.9244834	0.9160115
Sample #14	0.966263	0.945489	0.9471913	0.9322111	0.9203693	0.9148221	0.9156708	0.9042207
Sample #15	0.986009	0.964381	0.9659694	0.9501657	0.9385722	0.9352281	0.9296894	0.9258915
Sample #16	0.985242	0.959966	0.95811	0.9411829	0.9304608	0.9251273	0.9256756	0.915913
Sample #17	0.971411	0.94846	0.9479405	0.9299221	0.9144673	0.9066235	0.9081693	0.8912387
Sample #18	0.972011	0.948476	0.9512317	0.9312296	0.9172636	0.9087244	0.9100711	0.8940357
Sample #19	0.971297	0.951079	0.9535201	0.9359192	0.9256563	0.9214145	0.9205814	0.9129132
Sample #20	0.981816	0.963322	0.9651779	0.9485779	0.938266	0.9338277	0.9352703	0.9274054
Sample #21	0.969945	0.945034	0.9465229	0.9325028	0.9224563	0.9204115	0.9231716	0.9135207
Sample #22	0.967972	0.948656	0.9501901	0.9343075	0.9173836	0.9116235	0.9129614	0.8945405
Sample #23	0.970683	0.953756	0.9615592	0.9469918	0.9300877	0.9286157	0.9293612	0.9171219
Sample #24	0.96594	0.945612	0.9485419	0.930628	0.9137691	0.905822	0.9104594	0.8872595
Sample #25	0.970096	0.94996	0.956279	0.9388078	0.9232796	0.9171249	0.9194599	0.9052264
Sample #26	0.975707	0.954014	0.9560869	0.9406903	0.9278721	0.9222274	0.9220724	0.9096201

Table 3 Alpha Estimations

PredictionTime(Hours)	2000	3000	4000	5000	6000	7000	8000	9000
Sample #1	-0.87851	-0.70139	-0.733783	-0.74186	-0.722141	-0.712601	-0.718173	-0.719776
Sample #2	-0.89428	-0.78077	-0.804508	-0.8008	-0.782389	-0.775826	-0.778375	-0.772155
Sample #3	-0.87948	-0.70706	-0.739611	-0.744928	-0.724033	-0.71343	-0.720858	-0.719017
Sample #4	-0.88869	-0.72624	-0.754017	-0.758783	-0.739076	-0.729697	-0.736777	-0.730098
Sample #5	-0.88531	-0.70886	-0.737279	-0.745503	-0.726932	-0.717188	-0.722916	-0.722141
Sample #6	-0.88654	-0.72566	-0.753404	-0.756996	-0.738589	-0.73098	-0.736348	-0.734181
Sample #7	-0.89517	-0.75496	-0.782972	-0.78316	-0.766486	-0.759664	-0.7658	-0.759352
Sample #8	-0.88912	-0.74376	-0.771505	-0.769758	-0.753118	-0.744655	-0.750663	-0.741125
Sample #9	-0.88468	-0.71799	-0.747467	-0.752294	-0.734361	-0.719722	-0.729063	-0.719897
Sample #10	-0.88462	-0.72676	-0.758068	-0.759777	-0.740777	-0.733047	-0.742555	-0.740968
Sample #11	-0.89455	-0.73832	-0.765724	-0.769775	-0.750762	-0.743568	-0.749466	-0.747628
Sample #12	-0.89	-0.71638	-0.744992	-0.751272	-0.731261	-0.72204	-0.725744	-0.724635
Sample #13	-0.89644	-0.75284	-0.777325	-0.778999	-0.762782	-0.757373	-0.761419	-0.758529
Sample #14	-0.89314	-0.7586	-0.785235	-0.783672	-0.765994	-0.760466	-0.76754	-0.762491
Sample #15	-0.88694	-0.72219	-0.750633	-0.754932	-0.736903	-0.729888	-0.732426	-0.73457
Sample #16	-0.8953	-0.74039	-0.764497	-0.766767	-0.751615	-0.744859	-0.750802	-0.747628
Sample #17	-0.8953	-0.74039	-0.764497	-0.766767	-0.751615	-0.744859	-0.750802	-0.747628
Sample #18	-0.90565	-0.7782	-0.800283	-0.795981	-0.779282	-0.773204	-0.780262	-0.771649
Sample #19	-0.90188	-0.77089	-0.794055	-0.791282	-0.776638	-0.77303	-0.777606	-0.774304
Sample #20	-0.88987	-0.72987	-0.757068	-0.759931	-0.743294	-0.736408	-0.743635	-0.742324
Sample #21	-0.90301	-0.76867	-0.791643	-0.791242	-0.776957	-0.774076	-0.781006	-0.775834
Sample #22	-0.89268	-0.75326	-0.779743	-0.778694	-0.757384	-0.751419	-0.760486	-0.750764
Sample #23	-0.89011	-0.74212	-0.771837	-0.772918	-0.750224	-0.746175	-0.755215	-0.749773
Sample #24	-0.91022	-0.78737	-0.80792	-0.803738	-0.785622	-0.78001	-0.789829	-0.775605
Sample #25	-0.89409	-0.75143	-0.779286	-0.778236	-0.757527	-0.751234	-0.760777	-0.754435
Sample #26	-0.8925	-0.7434	-0.770186	-0.771486	-0.753237	-0.746346	-0.752983	-0.748074

Table 4 Beta Estimations (10^{-5})

One sample (**Figure 8**) shows the EKF' estimations and extrapolations, the blue line is the original data, the red line is estimations and the green line is extrapolations, which uses the exponential decay model. Obviously, there are two outliers for the estimation, and those two lines cannot be used for the true L70 distribution. The extrapolations tend to concentrate at 40,000 to 50,000 for L70 life.

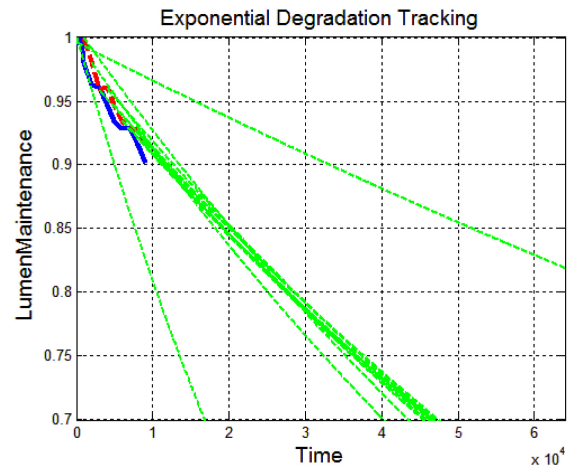


Figure 8 EKF L70 Extrapolations

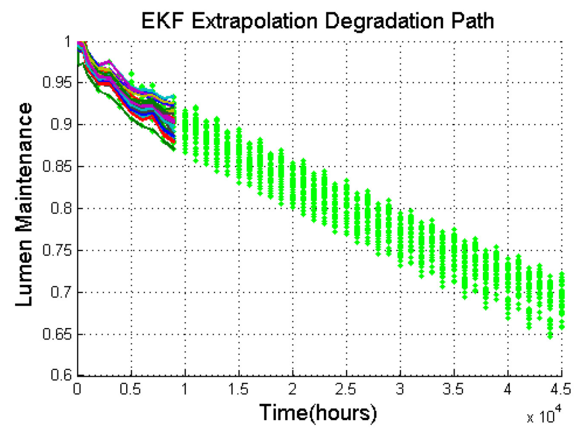


Figure 9 All Sample L70 Extrapolations and Degradation Path

It is the pseudo L70 life boxplot (**Figure 10**) that estimated through 1000 to 9000 hours, however, there is distinct two patterns in those estimations, before 2000 hours, the life is predicted shorter than that after 2000 hours. So we mainly apply the more recent estimations to form the life distribution. Also, in the boxplot, the variations are almost the same and the mean value can be derived from the fitting distributions, which is normal, lognormal and weibull distribution.

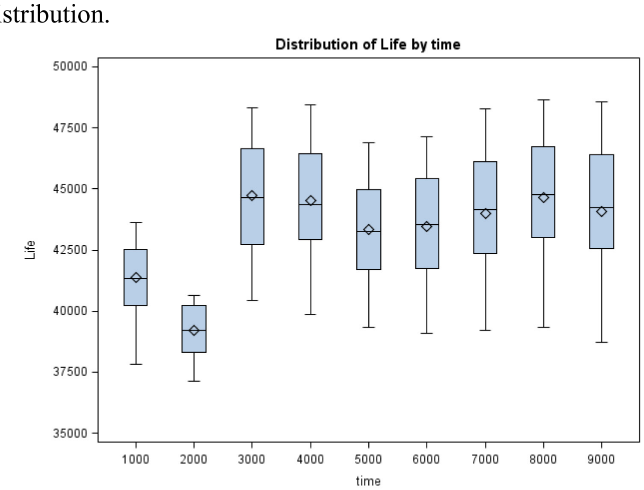


Figure 10 Distribution of Estimated L70 Life by Time

Prognostic Lumem Degradation at 16,000 hours using training EKF model:

The lumen degradation could be predicted at given time, for example it can predict the Lumen Maintenance at 16,000 hours. We consider normal distribution for the estimation of training decay rates. The following plots show the normal distributed beta factor and lumen estimation using previous 8,000 hours degradation information.

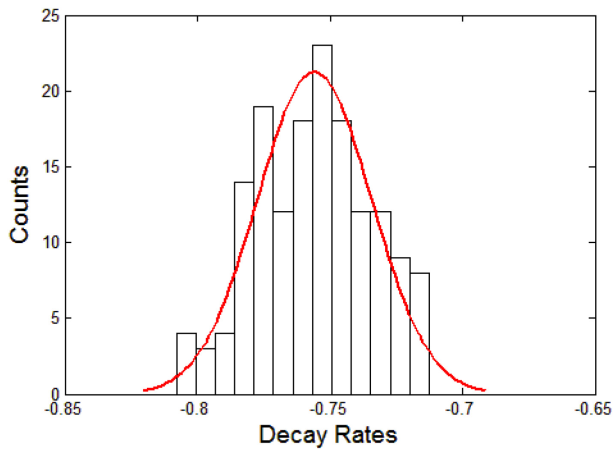


Figure 11 Fitted Distribution for Decay Rate

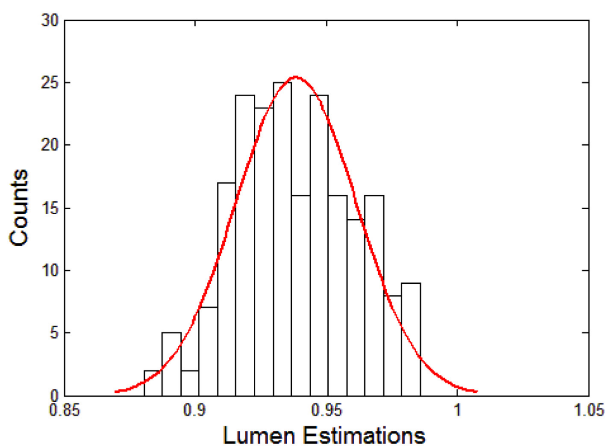


Figure 12 Fitted Distribution for Lumen Estimation

We use the basic exponential degradation function to model the degradation path from 8,000 hours to 16,000 hours.

In the plots, we can figure out that the lumen estimation and decay rate estimation are also normally distributed around its mean value with identified variance. The histogram plot showing below is the lumen estimation at 16,000 hours, the blue bar shows the counts of estimated lumen value, the red line is fitted distribution. The mean value of 16,000 hours lumen maintenance is 86.53%, and the mean value of 8000

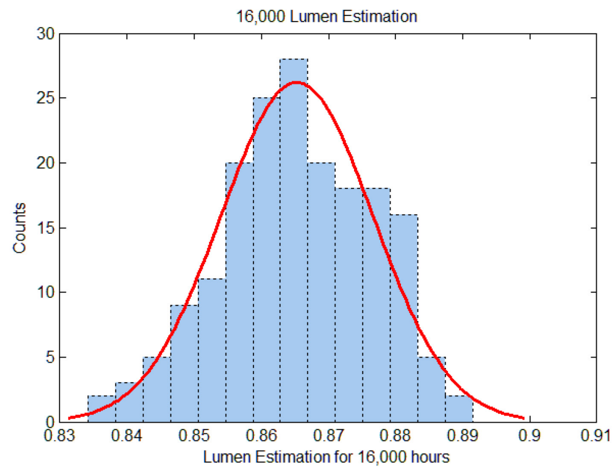


Figure 13 Lumen Estimation Summary for 16,000 hours

LED System Reliability Discussion

A cumulative distribution function F(t) represents the population fraction failing by age t. Any such continuous F(t) has the Mathematical properties: (1) It is a continuous function for all t. (2) $\lim_{t \to -\infty} F(t) = 0$ and $\lim_{t \to +\infty} F(t) = 1$.(3) $F(t) \leq F(t')$ for all $t < t2$. The range of t for most life distribution is from 0 to ∞, but some useful distributions have a range from -∞ to ∞. The reliability function R(t) for a life distribution is the probability of survival beyond age t, namely, the survivor or survivorship function can be represented as:

$$R(t) = 1 - F(t) \quad (43)$$

We can use the method that fitting the probability for the distribution of L70 life over the sample population. Here, we only use the normal distribution, lognormal distribution and Weibull distribution to fit the probability. The normal distribution has been used to describe the life of incandescent lamp filaments and of electrical insulations. It is also used as the distribution for product properties.

1. Normal Cumulative Distribution Function. The population fraction failing by age y is:

$$F(y) = \int_{-\infty}^{y} (2\pi\sigma^2)^{-1/2} \cdot e^{\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]} dx, -\infty < y < +\infty \quad (44)$$

2. Normal Probability Density. The probability density is:

$$f(y) = (2\pi\sigma^2)^{-1/2} \exp[-(y-\mu)^2/(2\sigma^2)], -\infty < y < +\infty \quad (45)$$

3. Normal Reliability Function. The population fraction surviving age t is:

$$R(t) = 1 - \Phi[(t - \mu)/\sigma] \quad (46)$$

hours is 92.21%. Therefore, the lumen maintenance degradation from 8000 hours to 16,000 hours is 5.68 %. The lumen maintenance variance at 8000 hours is 1.1196e-04, and the lumen maintenance variance at 16,000 hours is 1.2863e-04, so the variance increases 1.667e-5, which indicates the distribution shape is wider than our current's.

The lognormal distribution is widely used for life data, including metal fatigue, solid state components and electrical insulation. The lognormal and normal distributions are related; this fact is used to analyze lognormal data with methods for normal data.

1. Lognormal Cumulative Distribution. The population fraction failing by age t is:

$$F(t) = \surd\{[\log(t) - \propto]/ \int\}, \ t > 0 \qquad (47)$$

2. Lognormal Probability Density. For a lognormal distribution,

$$f(t)=\{0.4343/[(2\square)^{1/2}\,t\,ʃ]\}\oplus e^{\{-[\log(t)-\propto]^2/(2ʃ^2)\}},\ t>0 \quad (48)$$

3. Lognormal Reliability Function. The population fraction surviving age t is:

$$R(t)=1-\surd\{[\log(t)-\propto]/ʃ\}=\surd\{-[\log(t)-\propto]/ʃ\} \quad (49)$$

The weibull distribution is often used for product life, because it models either increasing or decreasing failure rates simply.

It is also used as the distribution for products properties such as strength (electrical or mechanical), elongation resistance, etc., in accelerated tests. It is used to describe the life of roller bearings, electronic components, ceramics, capacitors, and dielectrics in accelerated test.

1. Weibull Cumulative Distribution. The population fraction failing by age t is:

$$F(t)=1-e^{-(t/\langle)^{®}},\ t>0 \quad (50)$$

2. Weibull Probability Density. For a weibull distribution:

$$f(t)=(®/\langle^{®})t^{®\square}\oplus e^{\square(t/\langle)^{®}},\ t>0 \quad (51)$$

3. Weibull Reliability function. The population fraction surviving age t is:

$$R(t)=e^{-(t/\langle)^{®}},\ t>0 \quad (52)$$

The following table (**Table 5**) shows the fitting statistics for the normal distribution, lognormal distribution and weibull distribution, the lower value of Cramer-von Mises Criterion, the better fitting of distribution it is. Here, the normal distribution fitting shows lowest criterion value, which says best fitting.

Goodness-Fit Tests for Three Distributions		
Distributions	**Cramer-von Mises Criterion**	**P-Value**
Normal	0.0968	<0.005
Lognormal	0.0981	<0.005
Weibull	0.295	<0.010

Table 5

$$F(t)=\surd[(t-43265)/2720.9] \quad (53)$$

The reliability function, R(t), can thereby be written by:

$$R(t)=1-F(t)=1-\surd[(t-43265)/2720.9] \quad (54)$$

The L70 life follows the Lognormal Distribution with scale parameter $\backslash$, and shape parameter ʃ; the cumulative distribution can be written as:

$$F(t)=\surd\{[\log(t/\backslash)]/ʃ\}=\surd\{\log[(t/\backslash)^{1/ʃ}]\} \quad (55)$$

Therefore the cumulative distribution of LEDs' L70 life would be written as the following function:

$$F(t)=\surd\{[\log(t/10.7)]/0.06\} \quad (56)$$

This lognormal cumulative distribution function describes the population fraction failing by the age t. $\backslash$ is the mean time to failure (MTTF). $\backslash$ is in the same measurement units as t. In the LEDs' L70 distribution, we found that R(t) could be represented as:

$$R(t)=1\square\surd\{[\log(t/10.7)]/0.06\} \quad (57)$$

$$=\surd\{\square\log(t)\square 44356]/0.06\}$$

The L70 life follows the Weibull distribution with the shape parameter β (17.6) and scale parameter α (45000).

$$F(t)=1-e^{[-(t/45000)^{17.6}]},\ t>0 \quad (58)$$

The reliability function, R(t), can thereby be written by:

$$R(t)=1-F(t)=e^{[-(t/45000)^{17.6}]},\ t>0 \quad (59)$$

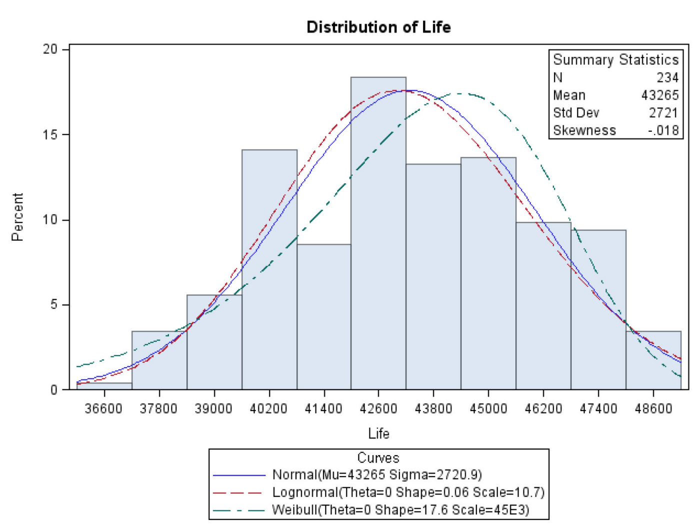


Figure 14 Distribution of L70 Life

As shown above, the L70 life (**Figure 14**) follows the normal distribution with expectation μ (43265) and variance $\int$ (2720.9).

The Normal Distribution, Lognormal Distribution and Weibull Distribution reliability function are shown below(**Figure 15**): the red line is the Log-normal reliability function of L70 life, the green line is the Normal reliability function of L70 life, and the blue one is Weibull reliability function of L70 life. Overall, the normal distribution could be the best fitting and description of distribution for LEDs L70 life. Therefore, we apply the normal distribution in this situation to describe the LEDs' reliability. The **Figure 16** describes the 95% confidence intervals (CI) of normal distribution. The parameters for the normal distribution fitting are estimated by the Maximum Likelihood Estimation.

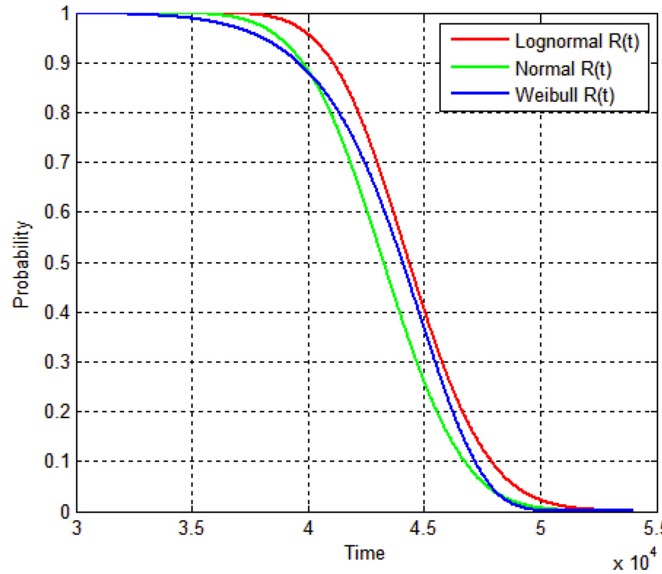


Figure 15 Three Reliability Functions based on three distributions

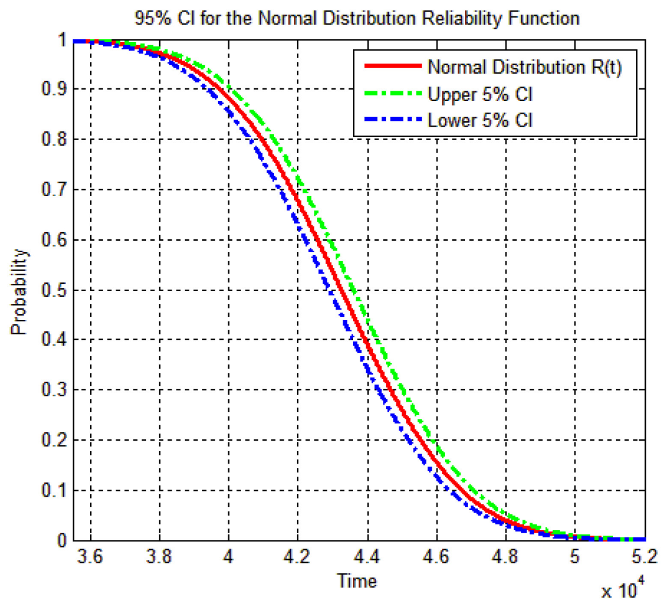


Figure 16 the 95% CI for the Normal Distribution

Conclusions:

In this paper, the Extended Kalman Filter algorithm has been introduced into the prognostic L70 life for the LEDs. The exponential model has been applied to make extrapolations, which could across the L70 criterion line. Therefore, we can calculate out the Pseudo L70 life according to the Status quo. Then, the Pseudo L70 life distributions have been generated using the fitting distribution method. The normal distribution, lognormal distribution and Weibull distribution are used into it. The Goodness-Fit Test (Cramer-von Mises Criterion) proves that the normal distribution is better than other two distributions. The normal distribution shows the mean time to failure (MTTF) is 43265 hours, the lognormal distribution's MTTF is 44356 hours, and the Weibull distribution has the longer MTTF, which shows 45000 hours. At last, the 95% confidence interval (CI) has been obtained. The estimated MTTF provide us with the information that the life for the LEDs is generally long term, the reliability tells us there is no LEDs would fail before 36,000 hours, so the reliability is '1'. When the time is at 43265, the reliability drops to the 50%, which means a half mount of LEDs would fail at this time. The maximum L70 life time is 50000, which all of LEDs' life will end before this time. So the LEDs life band will be from 36,000 to 50,000 hours. In the future work, we need to verify this degradation model by burning those LEDs to the failure threshold, then we can compare the Pseudo L70 life and Real L70 life to generate a better LEDs' prognostic model.

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