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Novel Maximum-based Timing Acquisition for Spread-Spectrum Communications

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***Abstract*—This paper proposes and analyzes a new packet detection and timing acquisition method for spread spectrum systems. The proposed method provides an enhancement over the typical thresholding techniques that have been proposed for direct sequence spread spectrum (DS-SS). The effective implementation of thresholding methods typically require accurate knowledge of the received signal-to-noise ratio (SNR), which is particularly difficult to estimate in spread spectrum systems. Instead, we propose a method which utilizes a consistency metric of the location of maximum samples at the output of a filter matched to the spread spectrum waveform to achieve acquisition, and does not require knowledge of the received SNR. Through theoretical study, we show that the proposed method offers a low probability of missed detection over a large range of SNR with a corresponding probability of false alarm far lower than other methods. Computer simulations that corroborate our theoretical results are also presented. Although our work here has been motivated by our previous study of a filter bank multicarrier spread-spectrum (FB-MC-SS) system, the proposed method is applicable to DS-SS systems as well.**

I. Introduction

Despite the ever increasing interest in OFDM-based techniques, such as LTE-A and emerging 5G waveforms, there still remains a need to improve upon spread spectrum techniques due to their unique solutions to numerous technologies. For instance, device-to-device (D2D) communication underlaying cellular networks has been considered, e.g., [1], [2]. Spread spectrum techniques offer tremendous advantages to this application due to their robustness and minimal interference characteristics. Recently, it has been shown that one spread spectrum system, namely filter bank multicarrier spread spectrum (FB-MC-SS), provides an attractive solution to both D2D communications and an underlay control channel (UCC) for dynamic spectrum access (DSA) [3], [4]. A full implementation of the FB-MC-SS system was recently reported in [4], providing credibility to its theoretical usefulness in these technologies. However, the timing acquisition methods that had previously been established for spread spectrum systems were determined to be insufficient for FB-MC-SS due to the wide variation in the signal-to-interference plus noise ratio (SINR) seen in these applications. A novel technique which allows for reliable synchronization with extremely low probabilities of false alarm was determined. This paper presents a detailed mathematical analysis of the method proposed in [4].

In spread spectrum communications, the transmitted signal spreads its data over a large amount of bandwidth while maintaining a low power spectral density (PSD). In order for the receiver to demodulate the spread signal, it must first synchronize the received signal with its spreading sequence. The initial synchronization is coarse (within a chip interval) and is known as acquisition, and subsequent tracking is performed to achieve fine-tuning. It is well known that the limiting factor in the performance of spread spectrum systems is the acquisition stage [5], [6], making it the most crucial stage to optimize. Most of the strategies proposed in the past have been tailored towards direct-sequence spread spectrum (DS-SS), in which a pseudo-noise (PN) sequence of a given length (chosen *a priori*) is used to spread its data. Minor modifications are made to account for problems unique to other classes of spread spectrum techniques, e.g., the presence of multiple acquisition phases and a larger search space in ultra-wideband (UWB) [7], but typically follow similar procedures.

Acquisition techniques can either employ (i) a programmable correlator to test the delays sequentially until synchronization is achieved (active method) or (ii) utilize a filter matched to the spreading code and subsequently determine where synchronization was achieved (passive method). It has been noted that passive methods are better suited for packetized communication [6], [8], [9]. Typical passive methods can be detailed as follows: (a) pass the received signal through a filter matched to the PN sequence; (b) take the power of the matched filter output to remove any carrier frequency or phase offset; (c) determine if any of the samples exceed some threshold; (d) pass any sample which exceeds the threshold to a verification stage to confirm that despreading has been achieved. Different passive methods change how the respective thresholds are determined and, accordingly, the verification step which follows in order to satisfy particular constraints on a spread spectrum system. In this paper, we argue that none of the previously proposed passive methods operate satisfactorily in a spread spectrum system which operates in a large dynamic range of signal-to-noise ratio (SNR), and that a novel method which focuses on local maximums instead of utilizing a threshold results in a considerable improvement in performance.

The least computationally expensive acquisition method is to choose a fixed threshold which gives reasonable results at some *a priori* minimum SNR, say $\mathrm{SNR}_{\min}$ [5], [6], [10]. In such systems, the probability of missed detection quickly increases if the SNR drops below $\mathrm{SNR}_{\min}$. On the other hand, it has been noted in [6] that if an automatic gain control (AGC) is utilized, the probability of false alarm increases as the SNR increases. Alternatively, it has been noted that thresholds may be chosen according to a constant false alarm rate (CFAR) [9], [11], [12]. This requires an accurate knowledge of the SNR at the receiver input. However, measurement of the SNR in a spread spectrum system is nearly impossible before despreading has been achieved [13], making this technique unreliable. Instead, adaptive thresholds can be utilized to increase the reliability of a threshold over unknown SNR values in real time. The most common adaptive thresholding methods use a moving average at the output of the power calculation to estimate the variance of the noise after the matched filter [8], [13], [14]. This average is then multiplied by some fixed parameter β to give the desired threshold. This allows the threshold to adapt with varying SNR without having to explicitly estimate it first. The parameter β is typically chosen to guarantee some performance metric at $\text{SNR}_{\min}$. This optimization decreases the system performance at more common operational SNR values which are much higher than $\text{SNR}_{\min}$. Additionally, the reported probabilities of false alarm associated with this method are quite high (around 10^{-3}) [8], [14]. This forces the verification stage to be more thorough and ultimately leads to a longer mean acquisition time (MAT).

Others have noted that maximum-based schemes perform better than threshold-based ones, but are more difficult to implement [5], [9]. Maximum-based methods reported in the literature require knowledge of a packet's arrival before searching for a maximum value, implying that some packet detection algorithm must be implemented before acquisition can be achieved. Packet detection can be particularly difficult in spread spectrum communications, and the additional hardware associated with the algorithm becomes costly. Once a packet is detected, the maximum-based scheme compares the amplitude of samples after the matched filter over some observation time, chosen *a priori*, and determines the location of the maximum value to be the correct timing. The false alarm rate must therefore account for false alarms in both the packet detection algorithm and in determining the maximum value. Note that in a high noise environment, there may be noise samples with similar magnitude to the pulses associated with the correct timing. In this scenario, this method is likely to fail.

Instead, we propose a maximum-based scheme which performs packet detection and acquisition simultaneously. This is done by utilizing a consistency metric of the location of maximum values across windows of samples instead of finding the location of a single maximum value across multiple windows. Not only will this limit the number of resources required in hardware compared to other maximum-based schemes but also will allow correct acquisition in high noise environments. Additionally, the probability of acquisition remains high over a large range of SNR values without having to estimate the SNR beforehand, similar to adaptive thresholding techniques. However, this method also provides a CFAR significantly lower than the adaptive thresholding methods. Our proposed scheme leads to a CFAR of 10^{-9} or lower, allowing the verification stage to be removed entirely and ultimately result in a much shorter MAT.

The rest of this paper is organized as follows. A brief introduction to the FB-MC-SS waveform and its main features is given in Section II. Equations related to the received signal during the preamble are discussed in Section III. An overview of the packet detection and timing acquisition algorithm is given in Section IV. The probability density function (PDF) of the signal as it moves through the proposed algorithm is given in Section V. Sections VI and VII use the derived PDF's to determine the probability of false alarm and missed detection, respectively. Section VIII considers the degradation of performance in the presence of subsample offsets, and proposes a modified method to combat this effect. The concluding remarks of this paper are given in Section IX.

II. FB-MC-SS OVERVIEW

In FB-MC-SS, the transmit signal is constructed according to the equation

$$x(t) = s(t) \star g(t) \tag{1}$$

where $\star$ denotes linear convolution,

$$s(t) = \sum_n s[n]\delta(t - nT) \tag{2}$$

is a train of impulses carrying the data symbols $s[n]$ with symbol interval T,

$$g(t) = \sum_{k=0}^{N-1} h_k(t) \tag{3}$$

is the transmit pulse-shape, N is the number of subcarriers,

$$h_k(t) = \gamma_k h(t) e^{j2\pi f_k t} \tag{4}$$

is the pulse-shape associated with the kth subcarrier, γ_k are a set of spreading gains (in the form of $\gamma_k = e^{j\theta_k}$), f_k are the subcarrier frequencies, and $h(t)$ is a square-root Nyquist pulse-shape. In [3], $h(t)$ is chosen to be a square-root raised-cosine pulse with a roll-off factor of 1.

III. RECEIVED PREAMBLE SEQUENCE

By assuming that, during the preamble, the transmit sequence $s[n]$ alternates between BPSK signals, (2) becomes

$$s(t) = \sum_n (-1)^n \delta(t - nT). \tag{5}$$

We observe that this choice of $s(t)$ is a periodic function with a period of $2T$. It thus can be expanded as

$$s(t) = \frac{1}{T} \sum_k e^{j\frac{(2k+1)\pi}{T}t}. \tag{6}$$

This indicates that $s(t)$ is a summation of infinite tones at frequencies $\{f = \frac{(2k+1)}{2T}, -\infty < k < \infty\}$. When $s(t)$ is passed through the pulse-shaping filter, those tones that are within the band of transmission will be retained and the rest will be removed. The receiver first demodulates the received signal to baseband, yielding $y(t)$, and passes it through a matched filter $g^*(-t)$. After straight-forward derivations, one finds the signal at the matched filter output to be

$$y'(t) = \frac{1}{2}\left(\sum_{k=-N}^{N-1} e^{j\frac{(2k+1)\pi}{T}t}\right) \star c(t) + v'(t) \tag{7}$$

where $c(t)$ is the channel impulse response and $v'(t)$ is the filtered channel noise.

Noting that the summation enclosed in parentheses of (7) is a truncated (equivalently, rectangular windowed) version of the summation in (6), (7) leads to

$$y'(t) = \left(\sum_k (-1)^k N \mathrm{sinc}\left(\frac{t-kT}{T/(2N)}\right)\right) \star c(t) + v'(t). \tag{8}$$

To have a better understanding of this result, consider the case where $c(t) = \delta(t)$, i.e., there is only a line of sight path with the gain of unity. In that case, (8) reduces to

$$y'(t) = \sum_k (-1)^k N \mathrm{sinc}\left(\frac{t-kT}{T/(2N)}\right) + v'(t). \tag{9}$$

We note that the sinc pulses have a main lobe width of T/N. For typical choices of N, this is a relatively narrow pulse. Accordingly, one may think of the sinc pulses as an approximation to a sequence of impulses and, hence, (9) resembles the preamble sequence (5). The gain factor N in front of the sinc pulses arises from the processing gain of the matched filter. Note that although perfect carrier synchronization has been assumed, a carrier frequency offset only slightly reduces the gain factor N for a large range of Δf_c [3].

IV. Packet Detection and Timing Acquisition Algorithm

To introduce the packet detection and timing acquisition methods, and present an analysis of their expected performance, we limit our presentation to the case where $c(t) = \delta(t)$. The developed results are trivially extendable to the case where the channel consists of a number of multipaths clustered in a time span that is a small fraction of one symbol period T [3]. Also, since the receiver is implemented based on samples of the received signal, we continue our discussion starting with the following sampled version of (9):

$$y'[n] = \sum_k (-1)^k N \mathrm{sinc}\left(\frac{n-kL}{L/(2N)}\right) + v'[n]. \tag{10}$$

Here, L denotes the number of samples within a symbol period of T seconds. According to (10), the transmit sequence of alternating $+1$ and -1 preamble symbols buried in background noise have been recovered and amplified with a gain of N.

Following the standard passive-search scheme, we take the power of the matched filter output as

$$z[n] = |y'[n]|^2. \tag{11}$$

The power calculation removes any carrier phase offset present in the signal, and limits the computational costs of the decision device by allowing faster hypothesis rejection [11]. We can see from (10) that the output of the power calculation consists of peaks every L samples, with an ideal amplitude of N^2 . The goal of the packet detection is to recognize the presence of these periodic peaks. The timing acquisition, then, takes the position of these samples as the correct timing phase for the rest of the processing steps at the receiver. The combined packet detection and timing acquisition algorithm can be detailed as follows:

Step 1: *Find Maximum Location* – take most recent L samples from $z[n]$, and find the index $\tau_0^{\max}$ of the maximum value

Step 2: *Store Maximum Location* – save $\tau_0^{\max}$ along with the $M-1$ most recent maximum indices in vector $\boldsymbol{\tau}$

Step 3: *Find Common Occurences* – determine if K out of M values in $\boldsymbol{\tau}$ share the same value

Step 4: *Detection* – if K indices share common value $\tau_C^{\max}$, exit algorithm with acquisition timing of $\tau_C^{\max}$; otherwise, repeat *Steps 1-3* with next L samples

This method offers a clear distinction from typical thresholding methods by maintaining the location of the maximum sample instead of its value. Rather than finding a threshold which offers an acceptable tradeoff between the probability of acquisition and false alarm in a large range of SNR, we rely on the fact that the maximum location should remain stationary over windows of L samples when the packet is present. However, in presence of a strong background noise, these peaks may not be easily observable. Therefore, we introduce flexibility into our algorithm by choosing $K < M$. To visualize this scenario, we introduce the matrix $\mathbf{Z}$, whose columns are built of L consecutive samples of $z[n]$ (i.e., the first column of $\mathbf{Z}$ contains samples $z[n]$ through $z[n-L+1]$, the second column contains $z[n-L]$ through $z[n-2L+1]$, and so on).

Fig. 1 presents a gray scaled example of matrix $\mathbf{Z}$, where $L = 32$ and the signal-to-noise ratio (SNR) at the receiver input is equal to -6 dB. In this example, the packet arrives at symbol number (column) 15 and a sequence of consistent strong samples can be seen following this symbol. However, there are also many elements of $\mathbf{Z}$ that are comparable with these observably strong samples. In the following sections, the probabilities associated with this strategy and the related parameters are derived. This allows us to make an informed decision of the selection of the parameters M and K that lead to low values of P_{fa} (probability of false alarm) and P_{md} (probability of missed detection).

V. Evolution of Probability Density Function

A. FB-MC-SS Signal Not Present

We begin by assuming that the received signal $y[n]$ is made up entirely of the additive noise signal $v[n]$ (i.e., no packet is present). If we assume that $v[n]$ is a complex white Gaussian signal with zero mean and variance σ_v^2, the probability density function (PDF) of the in-phase portion of the received signal is given by

$$f_Y^I(y) = \frac{1}{\sqrt{\pi\,\sigma_v^2}}\, \exp\left(\frac{-y^2}{\sigma_v^2}\right). \tag{12}$$

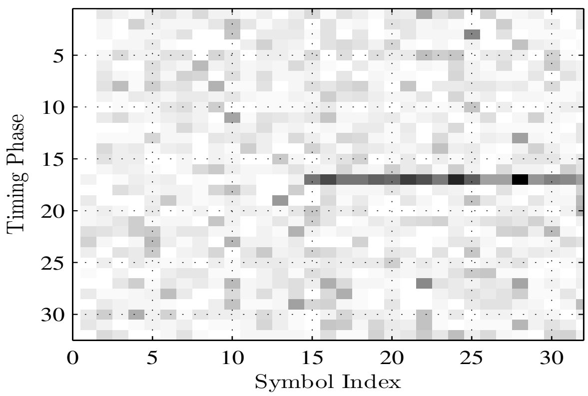

Fig. 1. Diagram of Timing Acquisition

Note that the variance used in (12) is actually $\sigma_v^2/2$ because we have assumed that the quadrature portion of the signal has an identical probability density function. The matched filter introduces coloring to the complex white Gaussian signal, but its distribution is still determined by a complex Gaussian variable. The matched filter introduces a gain of $\sqrt{N}$, changing its variance to $\sigma_{v'}^2 = N\sigma_v^2$ and giving the distribution

$$f_{Y'}^I(y') = \frac{1}{\sqrt{\pi\,\sigma_{v'}^2}} \exp\left(\frac{-y'^2}{\sigma_{v'}^2}\right). \tag{13}$$

The quadrature portion of $y'[n]$ also has the same distribution.

The distribution after the power calculation can be found by noting that the addition of k squared zero mean, complex Gaussian signals is determined by a generalized chi-squared distribution with k degrees of freedom for $x \geq 0$. Clearly, we have $k = 1$ (single complex Gaussian variable) with variance $\sigma_{v'}^2$, giving the desired PDF of $z[n]$ to be

$$f_Z(z) = \lambda e^{-\lambda z} u(z) \tag{14}$$

where $u(\cdot)$ is the unit step function, and the substitution $\lambda = 1/\sigma_{v'}^2$ has been made to give a standard exponential distribution.

B. FB-MC-SS Signal Present

Next, we assume that the received signal $y[n]$ is made up of the FB-MC-SS signal plus additive noise $v[n]$, defined to be complex white Gaussian with zero mean and variance σ_v^2. If we consider the signal after the matched filter (10), we see that the samples which correspond to incorrect timings lie outside the main lobe of the sinc function. Therefore, these samples can be considered zero mean with a very low variance and in the case of interest, where the signal is transmitted at or below the noise floor, these samples will be made up almost entirely of $v'[n]$. Therefore, this section will describe the PDF of the correct timing (center of main lobe) only, as we have described the PDF of all other timings in the previous section.

It is clear from (10) that the pulses associated with the correct timing, after the matched filter, have a gain of $\pm N$. Let the random variable Y'_+ represent the complex distribution associated with the pulses of gain $+N$. By including $v'[n]$, we see that the in-phase portion of Y'_+ is Gaussian with mean $+N$ and variance $\sigma_{v'}^2/2$, represented by

$$f_{Y'_+}^I(y') = \frac{1}{\sqrt{\pi\sigma_{v'}^2}} \exp\left(\frac{-(y'-N)^2}{\sigma_{v'}^2}\right). \tag{15}$$

A similar distribution is found for the negative pulses, where its mean has changed to $-N$. As the signal portion of $y'[n]$ is entirely real, the PDF associated with its quadrature components is Gaussian with zero mean and variance $\sigma_{v'}^2/2$, yielding

$$f_{Y'}^Q(y') = \frac{1}{\sqrt{\pi\,\sigma_{v'}^2}} \exp\left(\frac{-y'^2}{\sigma_{v'}^2}\right). \tag{16}$$

The power calculation multiplies Y' by its conjugate. If we consider Y'_+ only, this calculation implies the summation of two squared Gaussian distributions: the in-phase Gaussian with mean $+N$ and the quadrature Gaussian with mean 0. This is determined by a scaled noncentral chi-squared distribution, where the scaling is performed in order to have unit variance of the respective random variables. Note that the distribution of the Gaussian associated with the negative pulses, after the power calculation, will be identical to that of the positive pulses; therefore, the final distribution can be determined without this case. Note that the conjugate of the quadrature distribution is a time-reversed version of (16), an even function, making the quadrature distribution identical to (16). This is given by

$$Z = \left(Y_+'^{I}\right)^2 + \left(Y'^{Q}\right)^2. \tag{17}$$

To guarantee unit variance, we must divide each of the random variables by their respective standard deviations. With these new random variables, we can use the noncentral chi-squared distribution to get

$$\begin{aligned} Z' &= \left(\frac{Y_+'^{I}}{\sqrt{\sigma_{v'}^2/2}}\right)^2 + \left(\frac{Y'^{Q}}{\sqrt{\sigma_{v'}^2/2}}\right)^2 \\ &= \frac{2}{\sigma_{v'}^2}\left(\left(Y_+'^{I}\right)^2 + \left(Y'^{Q}\right)^2\right). \end{aligned} \tag{18}$$

Therefore, we can find the PDF of Z', and scale its distribution by $\sigma_{v'}^2/2$ to attain the desired PDF of Z.

To solve (18), we note that the degrees of freedom is $k = 2$ and the noncentrality parameter is $\lambda = 2\,(N/\sigma_{v'})^2$, giving the distribution of

$$f_{Z'}(z') = \sum_{i=0}^{\infty} \frac{e^{-(N/\sigma_{v'})^2}(N/\sigma_{v'})^{2i}}{i!} f_{C_{2+2i}}(z') \tag{19}$$

where f_{C_q} is the chi-squared distribution with q degrees of freedom. The final modification needed to obtain the desired distribution of Z is to apply the scaling factor $\sigma_{v'}^2/2$ by

$$\begin{aligned} f_Z(z) &= \frac{2}{\sigma_{v'}^2} f_{Z'}\left(\frac{2z}{\sigma_{v'}^2}\right) \\ &= \frac{2}{\sigma_{v'}^2} \sum_{i=0}^{\infty} \frac{e^{-(N/\sigma_{v'})^2}(N/\sigma_{v'})^{2i}}{i!} f_{C_{2+2i}}(2z/\sigma_{v'}^2). \end{aligned} \tag{20}$$

VI. Probability of False Alarm

A false alarm occurs when there is no incoming packet, but the packet detection algorithm perceives a packet has arrived. Under this condition, for a given column of $\mathbf{Z}$, each of the L timings are equally likely to be the maximum value, as they all belong to the distribution given in (14). It is worth noting that the magnitude response of the matched filter is not white which implies that some correlation exists between samples after the power calculation. However, the elements across each row of $\mathbf{Z}$ make up an L-fold decimated version of $z[n]$, which spreads out the magnitude response to produce a flat spectrum. Therefore, the elements across rows of $\mathbf{Z}$ are a set of independent random variables. With these observations, the probability of false alarm, after adding a new column to $\mathbf{Z}$, narrows down to the following: given a set of M independent integer random variables (the number of columns to be analyzed), uniformly distributed in the interval 0 to $L-1$, what is the likelihood that at least K of them are equal. This is similar to the famous 'Birthday Problem' probability [15], which determines how likely it is that at least $K = 2$ of M people share the same birthday. This probability becomes more difficult to find for $K > 2$, but it can be closely approximated by a Poisson distribution [15]:

$$P_{\text{fa}} = 1 - e^{-\binom{M}{K}L^{-(K-1)}} \tag{21}$$

where $\binom{M}{K}$ is the binomial coefficient.

Fig. 2 presents plots of P_{fa} for choices of $M = 8$, 12, and 16 in a range of K. Also, to confirm the accuracy of (21), we have included the results of simulations for $K \leq 5$. Higher values of K become infeasible to determine in simulation as their probabilities are too small to find within a reasonable amount of time.

As seen, simulations match theoretical formula (21). Very low probability of false alarm can be guaranteed, with moderate values of M and an informed selection of the corresponding value for K. Rather than maintaining a typical value of P_{fa} around 10^{-3} or 10^{-4} [8], [12], [14], we argue that a P_{fa} around 10^{-9} gives reliable acquisition rates and allows one to remove the verification block from the synchronization process entirely. Ultimately, this will lead to a decreased MAT.

VII. Probability of Missed Detection

A missed detection occurs whenever a preamble passes through the packet detection algorithm without a successful detection being made. Here, we develop mathematical formulations that allow us to evaluate the probability of missed detection for different choices of parameters M and K, and as the SNR varies.

In order for a preamble to pass through without a detection, an incorrect timing ($L-1$ choices) must be greater than the correct timing for $M-K+1$ or more columns within every M-column window of the preamble. Let the samples associated with the correct timing belong to the random variable Z_x, with distribution (20), and the incorrect timing samples belong to the random variable Z_v, with distribution (14).

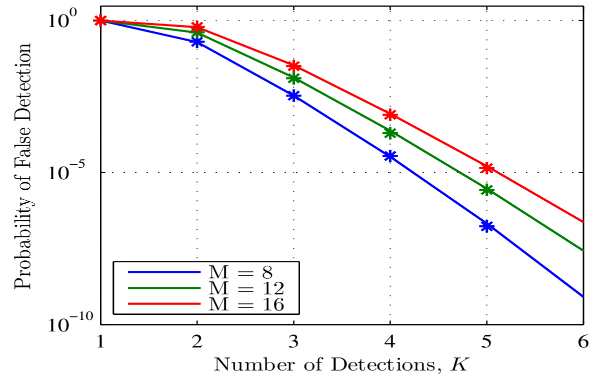

Fig. 2. Probability of false alarm: Theoretical (line) and Simulated (*)

If we consider a single column of $\mathbf{Z}$, the probability of missed detection at this column may be expressed as

$$P_{\text{md}}^{1} = P\left(z_v^{\text{m}} > z_x\right) \tag{22}$$

where z_v^{m} is the maximum of the $L-1$ z_v terms in the considered column. If we assume that the z_v terms are a set of independent exponential random variables, the distribution of z_v^{m} can be found with ease. First, the cumulative distribution factor (CDF) of each of the $L-1$ exponential random variables, given by

$$F_Z(z) = \left(1 - e^{-\lambda z}\right)u(z) \tag{23}$$

should be multiplied together, giving the CDF of z_v^{m}. The corresponding PDF is then found by taking the derivative of the CDF, by

$$\begin{aligned} f_{Z_v^{\text{m}}}(z_v^{\text{m}}) &= \frac{d}{dz_v^{\text{m}}}\left(F_Z(z_v^{\text{m}})^{L-1}\right) \\ &= (L-1)(1-e^{-\lambda z_v^{\text{m}}})^{L-2}\lambda e^{-\lambda z_v^{\text{m}}}u(z_v^{\text{m}}). \end{aligned} \tag{24}$$

Next, we define $z_d = z_v^{\text{m}} - z_x$ and note that

$$f_{Z_d}(z_d) = f_{Z_v^{\text{m}}}(z_d) \star f_{Z_x}(-z_d) \tag{25}$$

where $\star$ denotes linear convolution. Analytical evaluation of this convolution, given the form of the functions $f_{Z_x}(z_x)$ and $f_{Z_v^{\text{m}}}(z_v^{\text{m}})$, is not trivial. Here, we choose to obtain $f_{Z_d}(z_d)$ numerically. Once $f_{Z_d}(z_d)$ is obtained, P_{md}^{1} is evaluated as

$$\begin{aligned} P_{\text{md}}^{1} &= P(z_d > 0) \\ &= \int_0^{\infty} f_{Z_d}(z_d)dz_d. \end{aligned} \tag{26}$$

We note that (26) gives the probability of a missed detection in a single column of $\mathbf{Z}$ (i.e., probability that an incorrect timing has a higher amplitude than the correct timing). The packet detection algorithm uses a sliding window of M columns across W_{pa} columns of $\mathbf{Z}$, where W_{pa} is the length of the preamble, and indicates a detection if K or more columns in any window are associated with a single timing. This indicates that in order for a missed detection to occur, P_{md}^{1} must occur $M-K+1$ or more times in every window

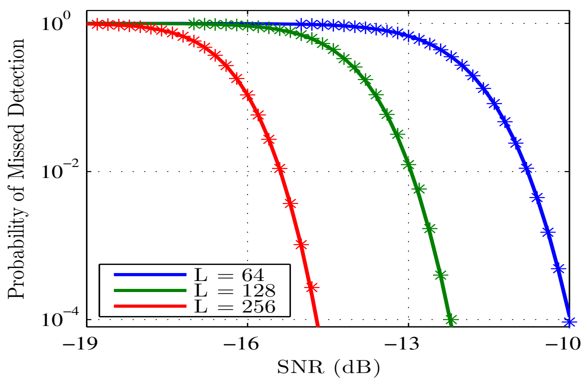

Fig. 3. Probability of Missed Detection: Theoretical (line) and Simulated (*)

across the W_{pa} preamble columns of $\mathbf{Z}$. The probability of such an event can be evaluated by taking the following steps.

We use a Markov chain process [16] with 2^M states describing the sign of z_d in the latest M columns of $\mathbf{Z}$. The transitional matrix, $\mathbf{\Pi}$, of this process can then be constructed using P_{md}^1. The Markov chain is started with a state that assumes z_d has been positive in the past columns of $\mathbf{Z}$, i.e., we are near the start of a preamble. Assuming that this is the first state of the Markov chain, the initial value of the state probability vector in the process will be the vector $\mathbf{p}_0 = [1, 0, 0, \cdots]^{\mathrm{T}}$. As the Markov chain process proceeds (equivalently, the preamble samples are processed), the state probability vector of the process evolves as

$$\mathbf{p}_n = \mathbf{\Pi}\mathbf{p}_{n-1} \tag{27}$$

where $n = 1, 2, \cdots$ is the time index.

The probability of missed detection at time index n is obtained by adding the elements of $\mathbf{p}_n$ that correspond to missed detection states. For a preamble with length W_{pa}, the probability of missed detection is calculated when n has reached a fraction of W_{pa}. The rest of the preamble is reserved for channel estimation.

Fig. 3 presents a set of plots of P_{md} as a function of SNR, for parameters $M = 8$, $K = 6$, and a few choices of L and N. The segment of preamble that is considered for packet detection is equal to 88. These parameters were chosen to mimic the implementation that has been presented in [4]. This specification allows the FB-MC-SS system to wait extended periods of time for a packet to arrive without getting false alarms, due to $P_{\mathrm{fa}} \approx 10^{-9}$, while guaranteeing a high likelihood of correct synchronization when packets arrive, due to $P_{\mathrm{md}} < 10^{-4}$. However, these parameters can be easily adjusted to operate in the scenarios which other proposed acquisition strategies assume, e.g., $P_{\mathrm{fa}} = 10^{-3}$, which will further decrease P_{md}.

Besides missed detection of a packet, the content of a packet may be unrecoverable when a wrong timing phase is picked. Nevertheless, in the presence of high-powered sinc pulses associated with the symbols of a packet, such events are very unlikely and their probability of occurrence is much lower than P_{md}. Hence, their consideration has very minor impact to our conclusions and, on that basis, they are not considered in this paper.

VIII. Subsample Offset

In hardware, the sampling done on the receiver using the analog-to-digital converter (ADC) introduces some offset that is a fraction of one sample due to the uncertainty of the propagation delay. In DS-SS, these subsample offsets can cause dramatic SNR degradation, consequently decreasing the acquisition performance [17]. This forces the acquisition methods to oversample the received signal, which increases complexity and does not solve the problem entirely. Therefore, additional processing algorithms have been proposed to further alleviate this problem. A parabolic interpolation algorithm is used to construct the correct timing samples in [11], but it has a downside of increased system complexity and power consumption. Alternatively, [17] takes the power of the summation of adjacent samples as an additional decision variable to be used with an independent threshold. Similarly, [12] introduces another threshold (lower than the conventional threshold) that must be jointly passed by adjacent samples as an additional decision variable.

Degradation is also seen in FB-MC-SS in the presence of a subsampling offset, although oversampling is not required and the degradation is not as severe. Consider L samples of the power calculation $z[n]$ in an ideal channel with no noise present. With ideal sampling, the sample associated with the correct timing will be at the center of the sinc pulse with amplitude N^2, and the remaining samples will be centered around zero. If we assume that T_s is the time between samples, and there exists a subsample offset $0 < |\epsilon| < T_s/4$, the sample associated with the correct timing will move away from the center of the sinc pulse and decrease in magnitude. Clearly, this will result in a degradation in the timing acquisition performance in terms of SNR. Even more concerning, however, is the case where a subsample offset $T_s/4 < |\epsilon| < T_s/2$ exists. Not only will the magnitude of the sample associated with the correct timing decrease further, but a neighboring point (dependent on the sign of ϵ) will approach the same magnitude. In this case, the proposed algorithm is equally likely to choose either as maximum (depending on the additive noise), and the K out of M requirement may be difficult to satisfy.

To alleviate this problem, we propose the following modification to the original acquisition algorithm. When the maximum value of the current L samples is found (*Step 1*), its neighboring points ($\tau_0^{\max} \pm 1$) are also determined. All three values are then shifted into $\boldsymbol{\tau}$, and its three oldest elements are shifted out (*Step 2*). This modification will force $\boldsymbol{\tau}$ to hold $3M$ indices, and the new condition of K out of $3M$ elements in $\boldsymbol{\tau}$ being equal will allow one of the samples associated with the main lobe of the sinc to pass. Note that if the passed sample happens to be a neighbor of the 'ideal' timing, FB-MC-SS will work with negligible degradation, whereas DS-SS can see massive degradation if this is the case. This modification significantly reduces the probability of missed detection in the

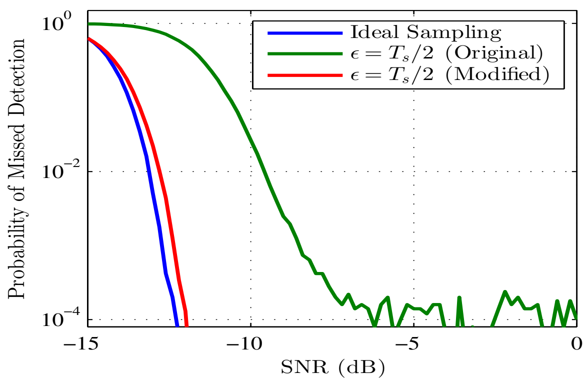

Fig. 4. Probability of Missed Detection Using Neighbor Technique

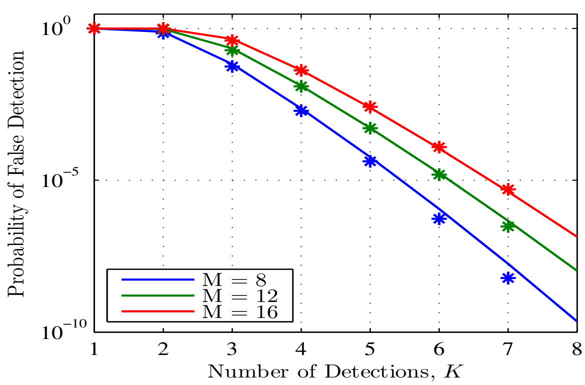

Fig. 5. Probability of false alarm Using Neighbor Technique: Theoretical (line) and Simulated (*)

presence of subsample offset, as shown from simulations in Fig 4. However, it will also increase the probability of false alarm.

At first glance, one may think that the probability of false alarm can be found by replacing M in (21) with $3M$. At closer inspection, however, the additional $2M$ samples are not uniformly distributed, and have a direct correlation with the M maximum timings. A closed form solution has not yet been determined, but a close approximation was found using simulations, indicating that M should be replaced by $2.5M$. This approximation is shown in Fig 5.

IX. Conclusion

In this paper, we proposed a timing acquisition strategy for the filter bank multicarrier spread-spectrum (FB-MC-SS) system that utilizes a maximum-based scheme instead of the conventional thresholding techniques. The novelty of the proposed method lies in its consistency metric, which allows the utilization of maximum values without relying on a separate packet detection algorithm. This allows the method to work in a large dynamic range of SNR without explicitly estimating it or determining a proper threshold to utilize, making it ideal for protocols that experience large changes in SNR continuously, e.g., the underlay control channel (UCC) for dynamic spectrum access (DSA), or a communication channel in underlaying cellular networks.

The probability of false alarm and missed detection of the proposed algorithm were determined and indicate how well the algorithm will perform in a complex AWGN channel. Subsample offsets present in hardware due to the analog-to-digital converter (ADC) were also considered, and a small modification to the algorithm was given to combat this effect. In addition to the analyses, simulations were presented to confirm the validity of the probabilities found.

References

[1] L. Wei, R. Q. Hu, Y. Qian, and G. Wu, "Enable device-to-device communications underlaying cellular networks: challenges and research aspects," *IEEE Communications Magazine*, vol. 52, no. 6, pp. 90–96, June 2014.

[2] ——, "Energy efficiency and spectrum efficiency of multihop device-to-device communications underlaying cellular networks," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 1, pp. 367–380, Jan 2016.

[3] D. L. Wasden, H. Moradi, and B. Farhang-Boroujeny, "Design and implementation of an underlay control channel for cognitive radios," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 10, pp. 1875–1889, 2012.

[4] T. Haddadin, S. A. Laraway, A. Majid, T. Sibbett, D. L. Wasden, B. F. Lo, L. Landon, D. Couch, H. Moradi, and B. Farhang-Boroujeny, "An underlay communication channel for 5G cognitive mesh networks: packet design, implementation, analysis, and experimental results," *IEEE International Conference on Communications Workshops: CoCoNet8*, 2016.

[5] U. Madhow and M. B. Pursley, "Acquisition in direct-sequence spread-spectrum communication networks: an asymptotic analysis," *IEEE Transactions on Information Theory*, 1993.

[6] D. L. Noneaker, A. R. Raghavan, and C. W. Baum, "The effect of automatic gain control on serial, matched-filter acquisition in direct-sequence packet radio communications," *IEEE Transactions on Vehicular Technology*, vol. 50, no. 4, pp. 1140–1150, 2001.

[7] S. R. Aedudodla, S. Vijayakumaran, and T. F. Wong, "Timing acquisition in ultra-wideband communication systems," *IEEE Transactions on Vehicular Technology*, vol. 54, no. 5, pp. 1570–1583, September 2005.

[8] R. D. Gaudenzi, "Signal recognition and signature code acquisition in CDMA mobile packet communications," *IEEE Transactions on Vehicular Technology*, 1998.

[9] J. H. J. Iinatti, "On the threshold setting principles in code acquisition of DS-SS signals," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 1, 2000.

[10] P. Driessen and Z. Shi, "A new rapid acquisition scheme for burst mode DS spread spectrum packet radio," *IEEE Military Communications Conference*, 1991.

[11] F. Benedetto and G. Giunta, "A self-synchronizing method for asynchronous code acquisition in band-limited spread spectrum communications," *IEEE Transactions on Communications*, vol. 57, no. 8, 2009.

[12] F. Benedetto, G. Giunta, and E. Guzzon, "Reducing mean acquisition time in code synchronization for wireless communications," *IEEE International Symposium on Personal Indoor and Mobile Radio Communications*, vol. 24, 2013.

[13] S. Yeom, Y. Jung, and S. Lee, "An adaptive threshold technique for fast PN code acquisition in DS-SS systems," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 6, 2011.

[14] A. Swaminathan and D. L. Noneaker, "A technique to improve the performance of serial, matched-filter acquisition in direct-sequence spread-spectrum packet radio communications," *IEEE Journal on Selected Areas in Communications*, 2005.

[15] A. DasGupta, "The matching, birthday and the strong birthday problem: a contemporary review," *Journal of Statistical Planning and Inference*, vol. 130, no. 1, pp. 377–389, 2005.

[16] R. Yates and D. Goodman, *Probability and Stochastic Processes. A Friendly introduction for Electrical and Computer Engineering*. John Wiley & Sons, Inc, 1999.

[17] S. Ding, Y. Zhengrong, and X. Liang, "An improved code acquisition scheme for band-limited DSSS systems with sampling offset," *IEEE Communications Letters*, vol. 16, no. 8, 2012.