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The Gaussian Laser Angular Distribution in HYDRA's 3D Laser Ray Trace Package

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The Gaussian Laser Angular Distribution in HYDRA's 3D Laser Ray Trace Package (LLNL-TR-728858)

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In this note, the angular distribution of rays launched by the 3D LZR ray trace package is derived for Gaussian beams (`npower==2`) with `bm_model`=±3. Beams with `bm_model`=+3 have a nearly flat distribution, and beams with `bm_model`=-3 have a nearly linear distribution when the spot size is large compared to the wavelength.

To model a laser beam in the 3D laser ray trace package, HYDRA chooses points on the lens — based on whether rays have equal power or power proportional to solid angle — and then samples points in the focal plane toward which the rays propagate. For the analytic beam models — `bm_model` ±3 and ±4 — the probability density function sampled is the parametrized super Gaussian.

Note that two options are available for the distribution of the rays within the bundle and the allocation of the laser power among the rays when using the analytical models 3 and 4. As already alluded to, these are controlled by the sign of the parameter `bm_model`. The sine weighting model (`bm_model` > 0) corresponds to equal power from each element of the surface of a spherical lens. This results in high power assigned to the rays originating from the outer portion of the lens. The equal power (`bm_model` < 0) option corresponds to equal power from each element of the surface of a flat lens, which also results in equal power in each ray.

Consider a round Gaussian beam with `spotx`=`spoty`=σ and `npower`=2 that comes from a lens with a radius defined in terms of the focal length f and beam angle a_2 as $r_{lens} = f\tan(a_2)$. The probability that a ray at a radius r_0 on the lens is launched at an angle between ϕ and $\phi+\,\mathrm{d}\phi$ with respect to the beam axis is

$$p(\phi)\,\mathrm{d}\phi \propto \exp\left(-\left[\frac{r_0 + f\tan\phi}{\sigma}\right]^2\right)\,\mathrm{d}\phi$$

where f is the lens focal length.

When `bm_model` is positive, rays are spaced linearly on the lens. The total probability of a ray being launched between ϕ and $\phi+\,\mathrm{d}\phi$ is then simply the normalized integral over the lens radius, $r\in[0, f\tan(a_2)]$

$$f_+(\phi) = f_{+0}\left[\frac{\displaystyle\int_0^{f\tan(a_2)} \exp\left(-\left[\frac{r + f\tan\phi}{\sigma}\right]^2\right)\,\mathrm{d}r}{\displaystyle\int_0^{f\tan(a_2)} \mathrm{d}r}\right],$$

where f_{+0} sets the normalization of this distribution. Below f_{+0} is defined such that the integral of the distribution is 1.

When `bm_model` is negative and each ray has the same power, rays are distributed following a square root law. Using this fact, the negative beam model angular distribution function may be computed as

$$f_-(\phi) = f_{-0}\left[\frac{\displaystyle\int_0^1 \exp\left(-\left[\frac{f\tan(a_2)\sqrt{\nu} + f\tan\phi}{\sigma}\right]^2\right)\,\mathrm{d}\nu}{\displaystyle\int_0^1 \mathrm{d}\nu}\right],$$

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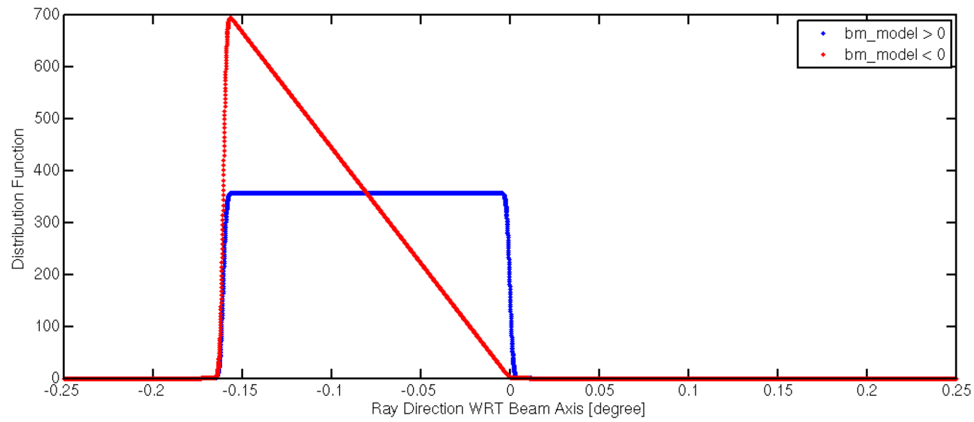

Figure 1: The total angular distributions of rays for `bm_model` = +3 (blue) and `bm_model`=-3 (red). In both cases, the spot sizes `spotx`=`spoty`=64 μm, the focal length $f = 177$ cm, and the F number is 178.4 giving a beam angle $a_2 = 0.00280268461943753$ radians.

where f_{-0} again sets the normalization defined below such that the integral of the distribution is 1.

These integrals may be carried out in closed form in terms of well known functions to give

$$\begin{aligned} f_+(\phi) &= f_{+0}\left(\frac{\sigma\sqrt{\pi}}{2f\tan(a_2)}\right)\left[\mathrm{erf}\left(\frac{f}{\sigma}[\tan\phi+\tan(a_2)]\right)-\mathrm{erf}\left(\frac{f}{\sigma}\tan\phi\right)\right] \\ f_-(\phi) &= f_{-0}\left\{\sqrt{\pi}\left(\frac{f\tan\phi}{\sigma}\right)\left[\mathrm{erf}\left(\frac{f}{\sigma}\tan\phi\right)-\mathrm{erf}\left(\frac{f}{\sigma}[\tan\phi+\tan(a_2)]\right)\right]\right. \\ &\quad \left.+\exp\left(-\left[\frac{f\tan\phi}{\sigma}\right]^2\right)-\exp\left(-\left[\frac{f}{\sigma}(\tan\phi+\tan(a_2))\right]^2\right)\right\}. \end{aligned}$$

These functions are plotted for a loosely focused beam in Figure 1. The equal solid angle model f_+ has a nearly flat angular distribution, while the equal power model has a nearly linear — actually, a $\tan\phi$ — angular distribution.

The normalization factors f_{+0} and f_{-0} are not, in general, expressible in terms of standard functions. In many laser configurations, the normalization factors may be well approximated as,

$$\begin{aligned} f_{+0} &= \frac{f}{\sigma\sqrt{\pi}}\left[\frac{\tan(a_2)}{\arctan\left(\tan(a_2)+\frac{3\sigma}{f}\right)}\right] \xrightarrow{3\sigma\ll f} \frac{f}{\sigma\sqrt{\pi}}\frac{\tan(a_2)}{a_2} \xrightarrow{|a_2|\ll 1} \frac{f}{\sigma\sqrt{\pi}} \\ f_{-0} &= \frac{\sigma}{\sqrt{\pi}f\arctan^2\left(\tan(a_2)+\frac{3\sigma}{f}\right)} \xrightarrow{3\sigma\ll f} \frac{\sigma}{fa_2^2\sqrt{\pi}}, \end{aligned}$$

where $f_+(\phi)$ and $f_-(\phi)$ have been approximated as flat and linear distributions, respectively, for the purposes of computing this normalization. For many sets of reasonable parameters, this is a very good approximation in part because these simple models capture the essence of the majority of the distribution and also in part because of compensating errors made by making these assumptions.

Finally, using the approximate results for f_{+0} and f_{-0}, the round Gaussian `bm_model`=$\pm$3 angular distributions of rays may be expressed as

$$\begin{aligned} f_+(\phi) &= \left(\frac{1}{2a_2}\right)\left[\mathrm{erf}\left(\frac{f}{\sigma}[\tan\phi+\tan(a_2)]\right)-\mathrm{erf}\left(\frac{f}{\sigma}\tan\phi\right)\right] \\ f_+(\phi) &= -\left(\frac{\tan\phi}{a_2^2}\right)\left[\mathrm{erf}\left(\frac{f}{\sigma}[\tan\phi+\tan(a_2)]\right)-\mathrm{erf}\left(\frac{f}{\sigma}\tan\phi\right)\right] \\ &\quad +\left[\frac{\sigma}{fa_2^2\sqrt{\pi}}\right]\left\{\exp\left(-\left[\frac{f\tan\phi}{\sigma}\right]^2\right)-\exp\left(-\left[\frac{f(\tan\phi+\tan(a_2))}{\sigma}\right]^2\right)\right\} \end{aligned}$$

where σ=`spotx`=`spoty`, f=`flen`, and a_2=`a2` as specified by the user on the `superg3d` card.