LA-UR-17-23695

Approved for public release; distribution is unlimited.

Title: Predictive Fusion of Geophysical Waveforms using Fisher's Method, under the Alternative Hypothesis

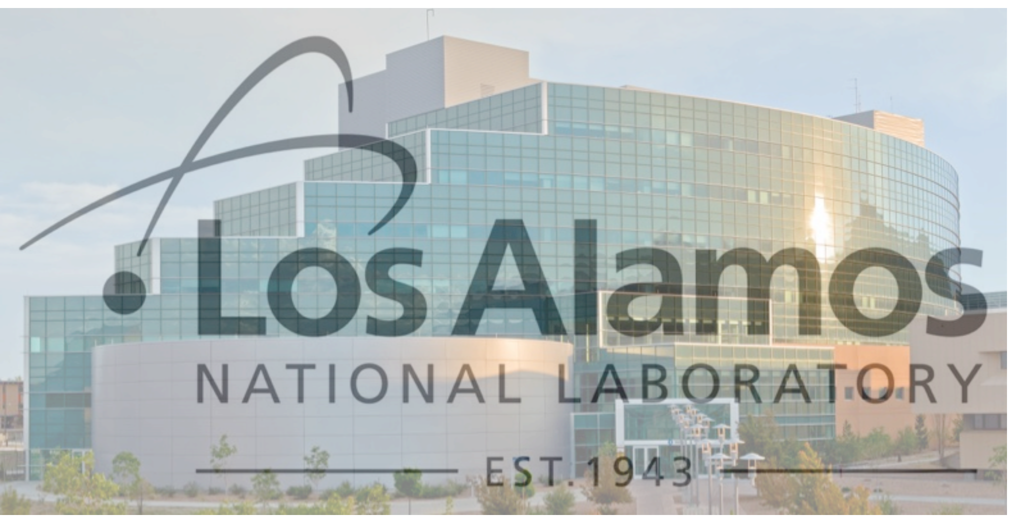
Author(s): Carmichael, Joshua Daniel
Nemzek, Robert James
Webster, Jeremy David

Intended for: Report

Issued: 2017-05-05

Predictive Fusion of Geophysical Waveforms using Fisher's Method, under the Alternative Hypothesis

Seismic, Acoustic and Radio Emissions from Near Surface Explosions



Joshua D. Carmichael
Bob Nemzek
Jeremy Webster

12-February-2017

Research Outline (1/5)

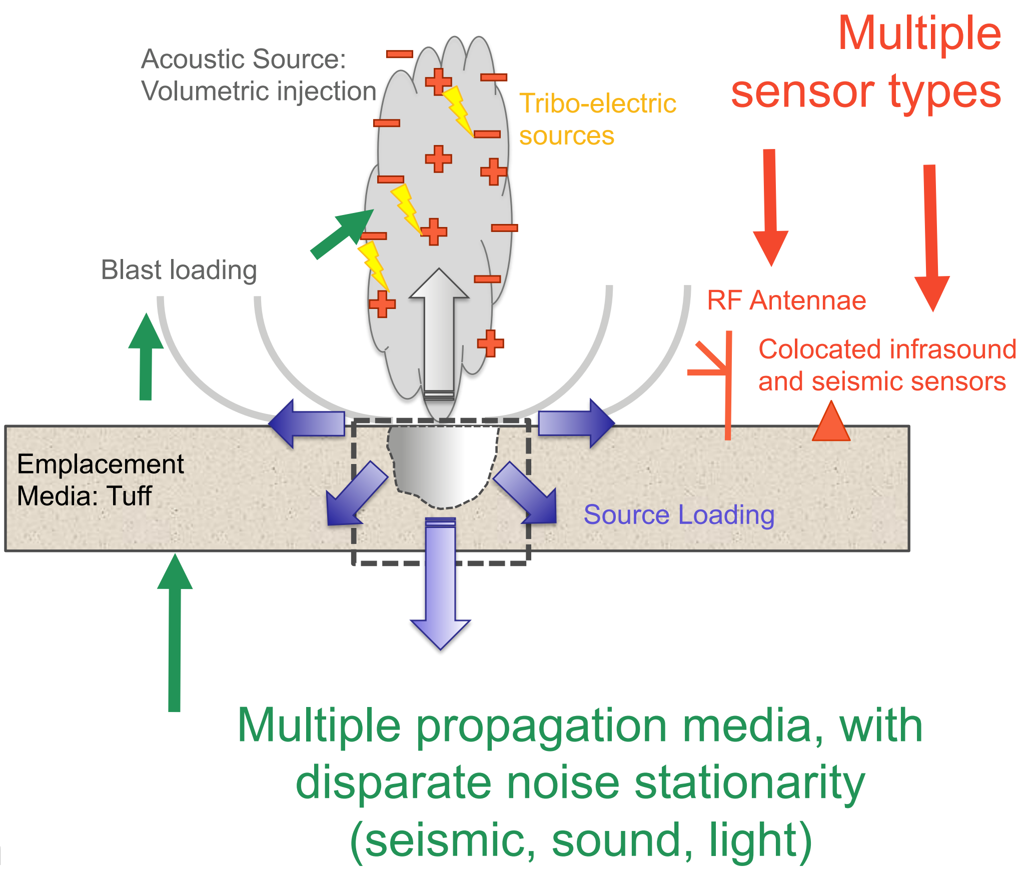
General Problem

How do we combine different signatures from an event or source together, in a **defensible** way?

Challenges

- Near surface explosions produce multiple signals that include **radio**, **acoustic**, and **seismic emissions**
- **However,** each signature (acoustic, seismic, radio emission) can exploit different **detection statistics**.
- Each detection statistic might give *marginal* evidence of an explosive source
- **Objective:** build a digital detector that continuously combines detection statistics recording explosions to screen sources of interest from "null" sources

Near-Surface Explosion Scenario



Research Outline (2/5)

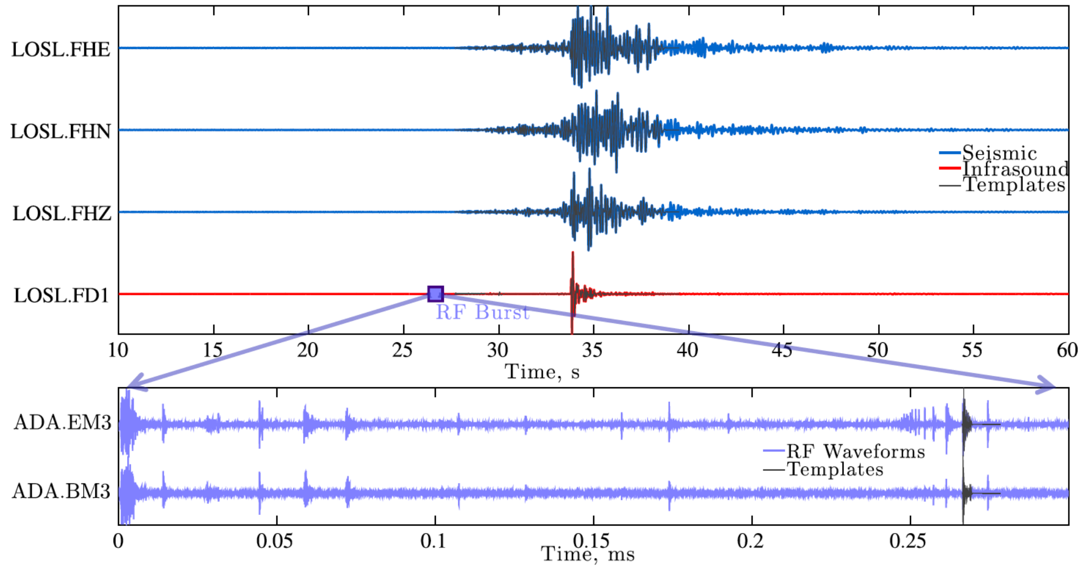
General Problem

How do we combine different signatures from an event or source together, in a **defensible** way?

Challenges

- Near surface explosions produce multiple signals that include **radio**, **acoustic**, and **seismic emissions**
- **However,** each signature (acoustic, seismic, radio emission) can exploit different **detection statistics**.
- Each detection statistic might give *marginal* evidence of an explosive source
- **Objective:** build a digital detector that continuously combines detection statistics recording explosions to screen sources of interest from "null" sources

Near-Surface Explosion Data



Seismic, acoustic and radio emission waveforms recording the same above ground explosion at the Los Alamos Testing Range.

Research Outline (3/5)

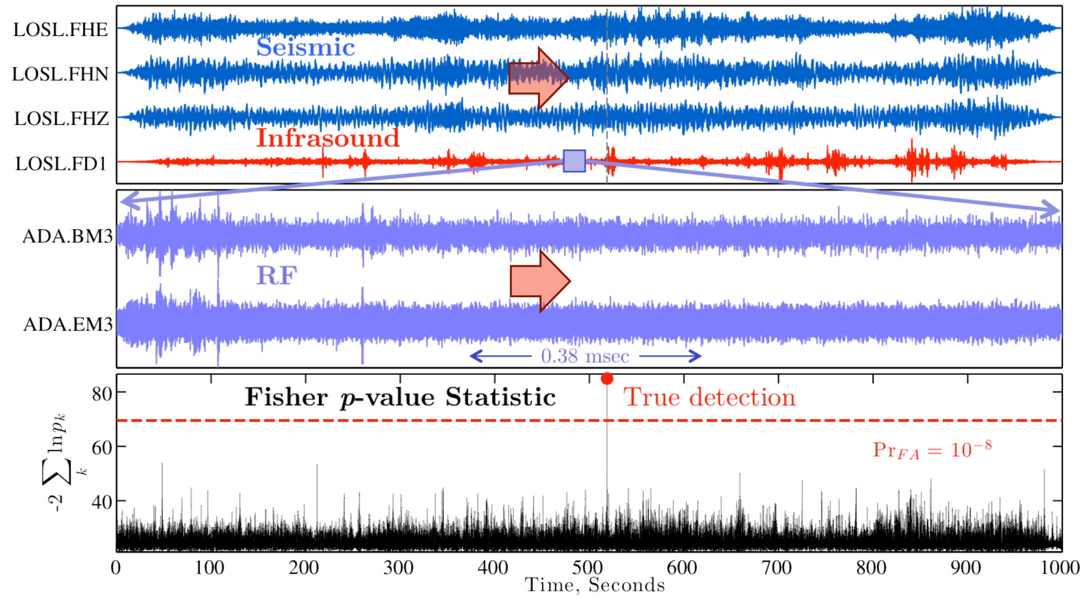
General Problem

How do we combine different signatures from an event or source together, in a **defensible** way?

Challenges

- Near surface explosions produce multiple signals that include **radio**, **acoustic**, and **seismic emissions**
- **However,** each signature (acoustic, seismic, radio emission) can exploit different **detection statistics**.
- Each detection statistic might give *marginal* evidence of an explosive source
- **Objective:** build a digital detector that continuously combines detection statistics recording explosions to screen sources of interest from ”null” sources

Explosion Data Statistics



Seismic, acoustic and radio emission each give weak individual evidence of an explosion when used alone

Research Outline (4/5)

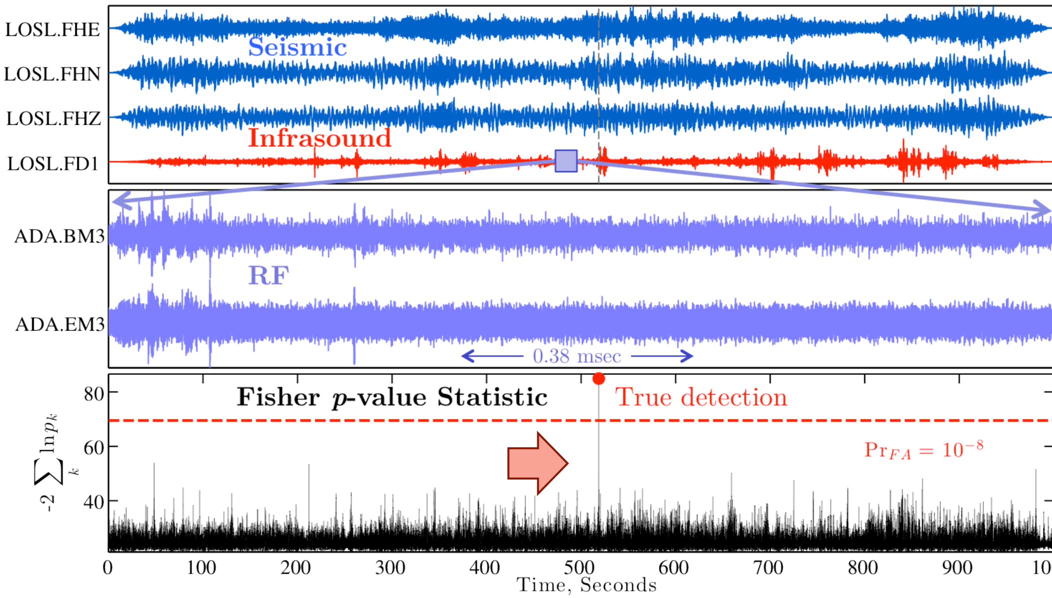
General Problem

How do we combine different signatures from an event or source together, in a **defensible** way?

Challenges

- Near surface explosions produce multiple signals that include **radio**, **acoustic**, and **seismic emissions**
- **However,** each signature (acoustic, seismic, radio emission) can exploit different **detection statistics**.
- Each detection statistic might give *marginal* evidence of an explosive source
- **Objective:** build a digital detector that continuously combines detection statistics recording explosions to screen sources of interest from "null" sources

Explosion Data Statistics



Seismic, acoustic and radio emission each give strong evidence of an explosion when combined together into a single statistic

Research Outline (5/5)

General Problem

How do we combine different signatures from an event or source together, in a **defensible** way?

Challenges

- Near surface explosions produce multiple signals that include **radio**, **acoustic**, and **seismic emissions**
- **However,** each signature (acoustic, seismic, radio emission) can exploit different **detection statistics**.
- Each detection statistic might give *marginal* evidence of an explosive source
- **Objective:** build a digital detector that continuously combines detection statistics recording explosions to screen sources of interest from ”null” sources

Fun Metaphor

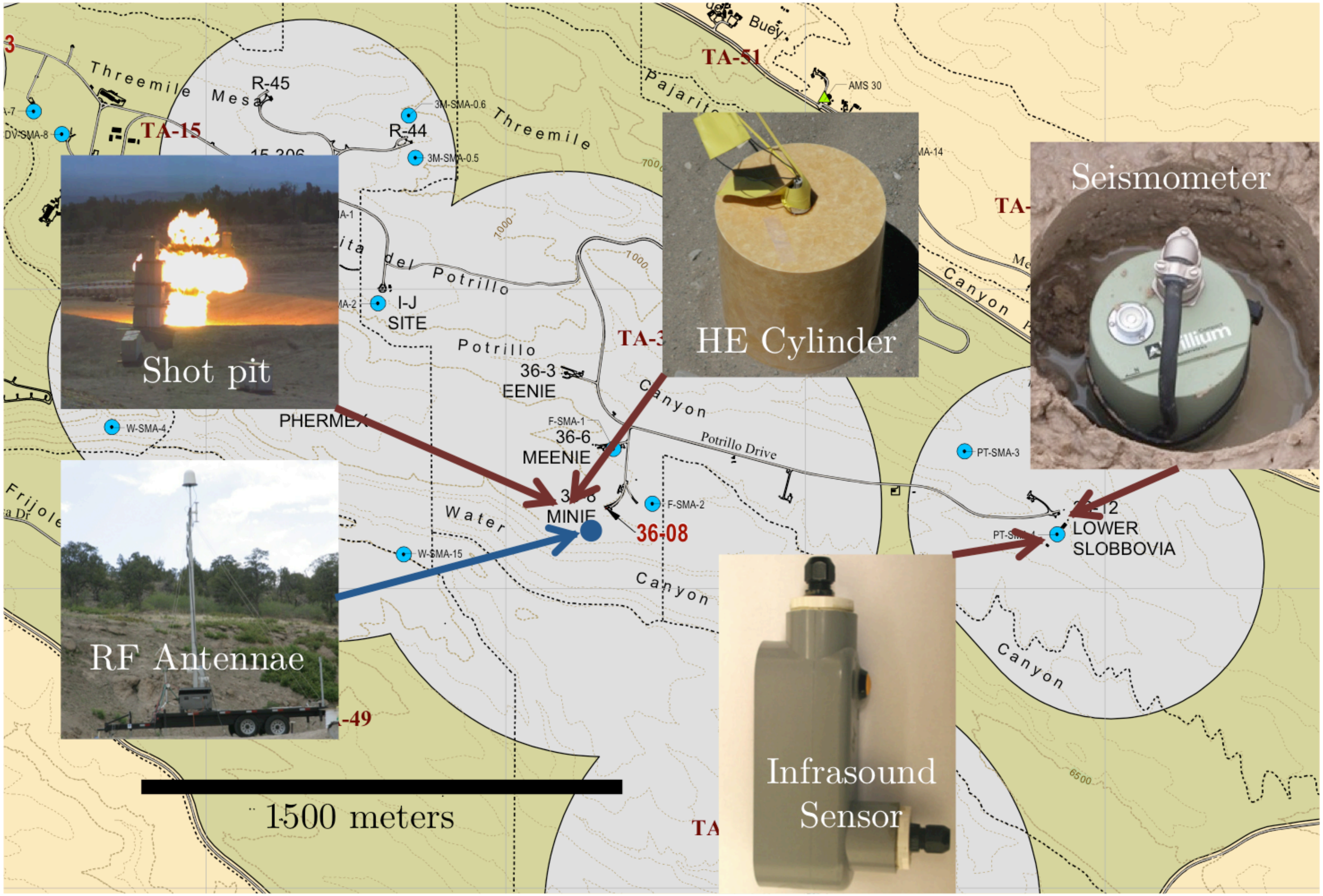
Individual pieces of evidence are insufficient for conviction of a crime. Does the *sum* of evidence make a case?

Cumulative Evidence of Crime

LANL Minie Parametric Experiments (2013): Bare COMP-B Explosions

Seismic, Acoustic and Radio Emission Records

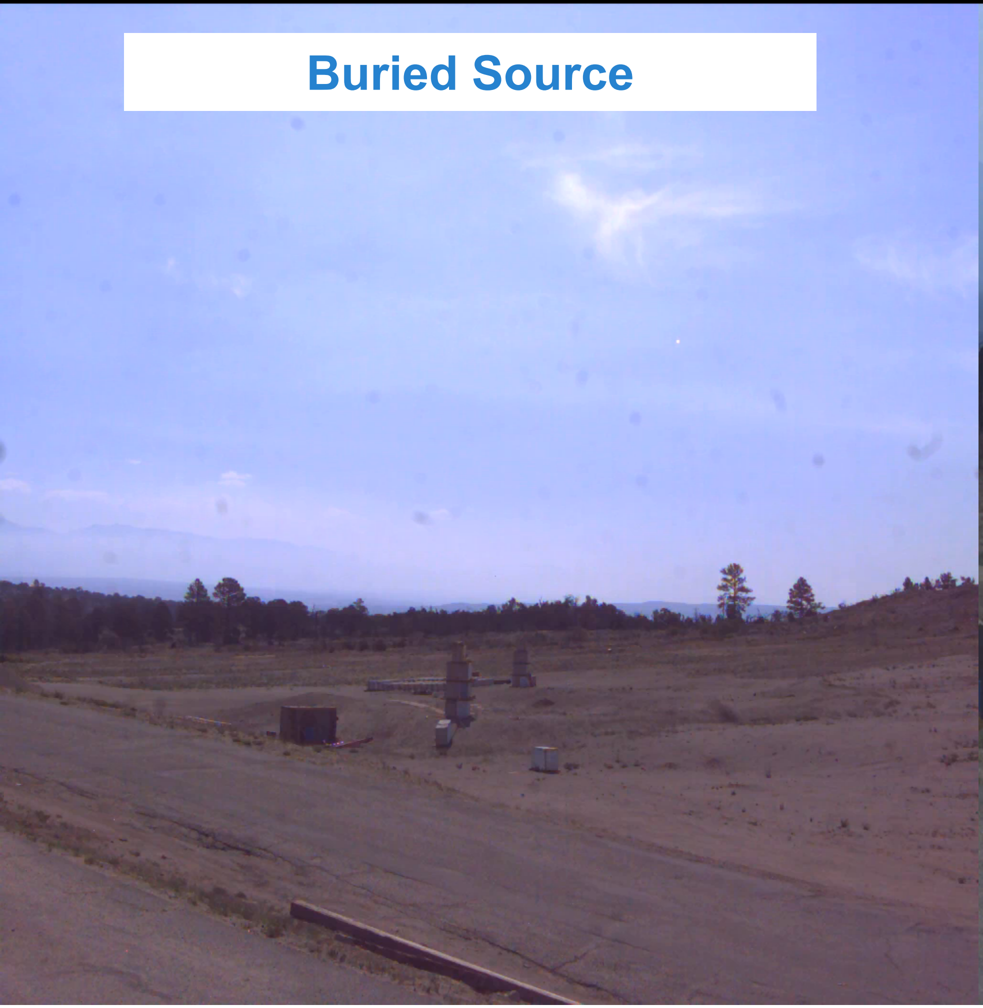
Minie Experiments: 70 Shots



Near Surface Explosions at LANL

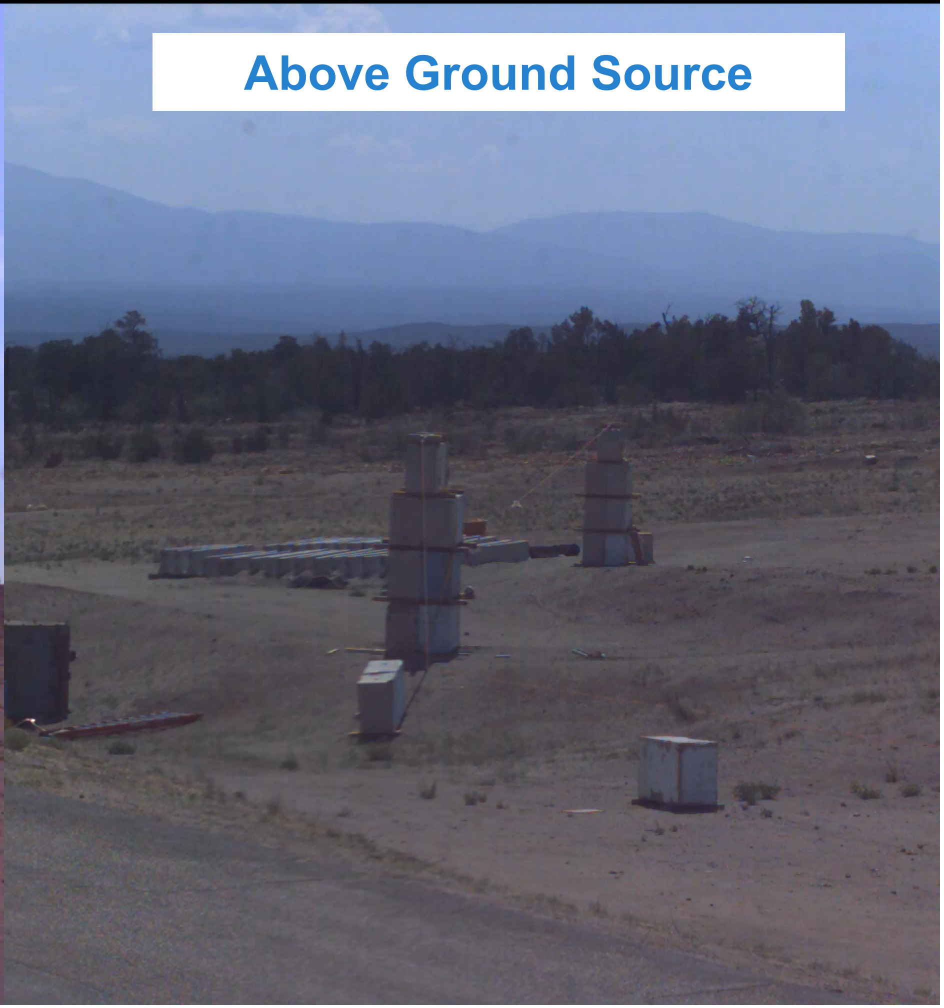
Did this happen?

Buried Source



Did this happen?

Above Ground Source



Quantitative Theory for Fusing Detection Statistics

Fisher's Combined Probability Test

Binary Hypothesis Testing/Detection (1/4)

Accumulating Evidence for an Explosive Source

Generalized Likelihood Ratio Detector

$$\frac{\max_{\boldsymbol{\theta}_1}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_1\right);\mathcal{H}_1\right)\}}{\max_{\boldsymbol{\theta}_0}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_0\right);\mathcal{H}_0\right)\}} = s_k\left(\boldsymbol{x}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

Detection Probability with $s_k(x)$

$$\Pr_D = \int_{\eta}^{\infty} f_S\left(\bar{s}_k;\mathcal{H}_1\right) d\bar{s}_k$$

Binary Hypothesis Testing/Detection (2/4)

Accumulating Evidence for an Explosive Source

Generalized Likelihood Ratio Detector

$$\frac{\max_{\boldsymbol{\theta}_1}\{f_{\mathbf{X}}(\boldsymbol{x}_k(\boldsymbol{\theta}_1);\mathcal{H}_1)\}}{\max_{\boldsymbol{\theta}_0}\{f_{\mathbf{X}}(\boldsymbol{x}_k(\boldsymbol{\theta}_0);\mathcal{H}_0)\}} = s_k(\boldsymbol{x}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

Detection Probability with $s_k(x)$

$$\Pr_D = \int_{\eta}^{\infty} f_S(\bar{s}_k;\mathcal{H}_1)\, d\bar{s}_k$$

Binary Hypothesis Testing/Detection (4/4)

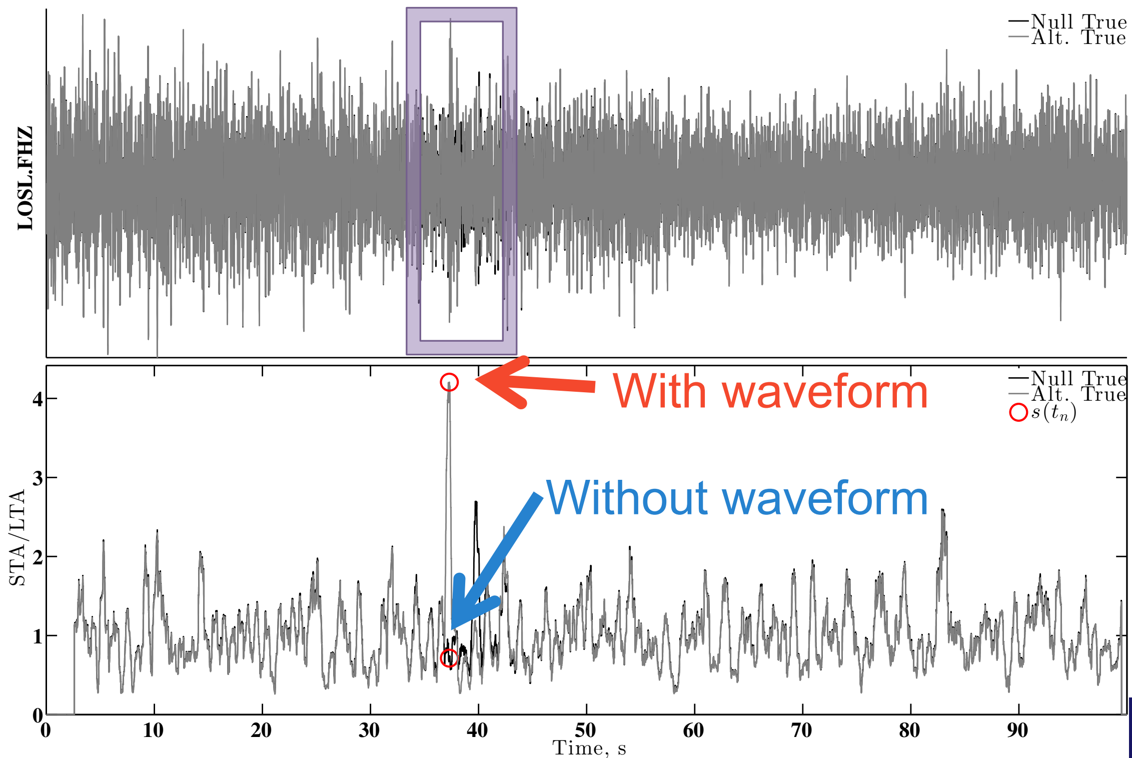
Accumulating Evidence for an Explosive Source

Generalized Likelihood Ratio Detector

$$\frac{\max_{\boldsymbol{\theta}_1}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_1\right);\mathcal{H}_1\right)\}}{\max_{\boldsymbol{\theta}_0}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_0\right);\mathcal{H}_0\right)\}} = s_k\left(\boldsymbol{x}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

Detection Probability with $s_k(x)$

$$\Pr_D = \int_{\eta}^{\infty} f_S\left(\bar{s}_k;\mathcal{H}_1\right) d\bar{s}_k$$



Binary Hypothesis Testing/Detection (3/4)

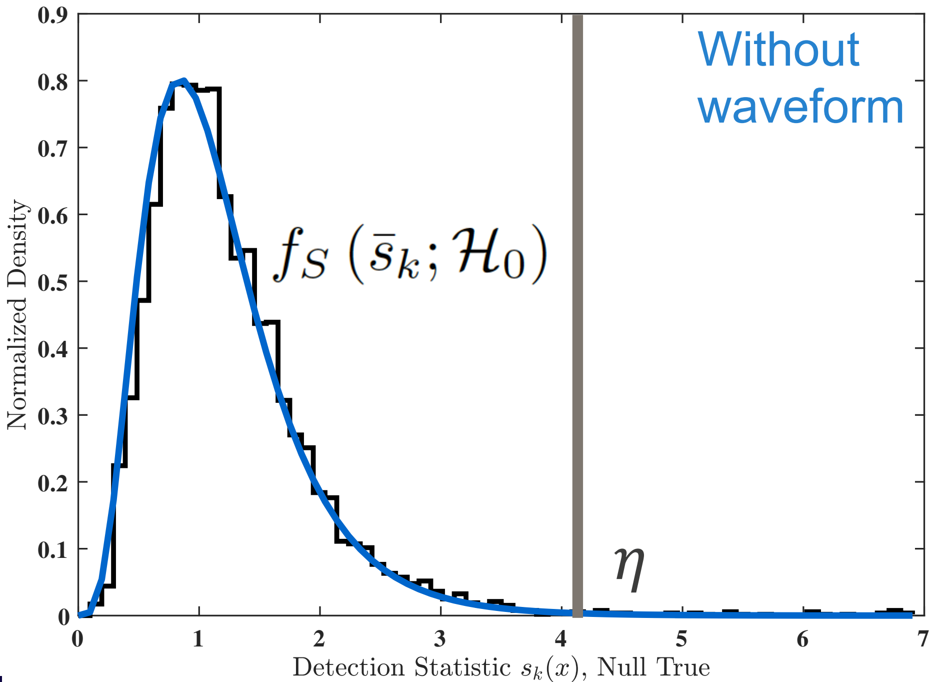
Accumulating Evidence for an Explosive Source

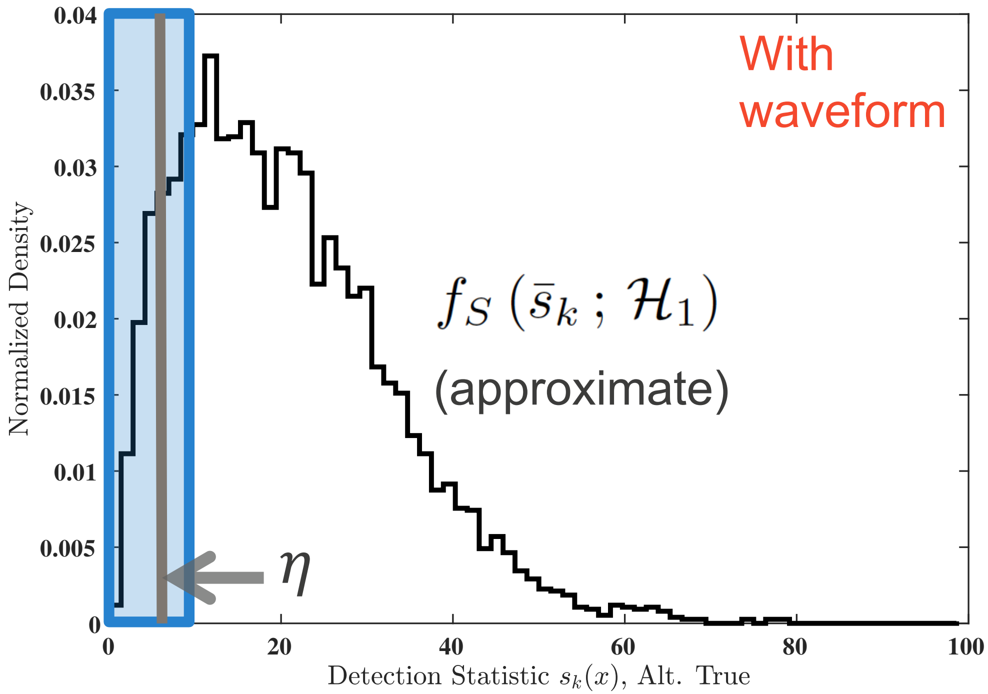
Generalized Likelihood Ratio Detector

$$\frac{\max_{\boldsymbol{\theta}_1}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_1\right);\mathcal{H}_1\right)\}}{\max_{\boldsymbol{\theta}_0}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_0\right);\mathcal{H}_0\right)\}} = s_k\left(\boldsymbol{x}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

Detection Probability with $s_k(x)$

$$\Pr_D = \int_{\eta}^{\infty} f_S\left(\bar{s}_k\,;\mathcal{H}_1\right) d\bar{s}_k$$





Null-Rejection/Detection with *p*-values

Accumulating Evidence for an Explosive Source

Generalized Likelihood Ratio Detector

$$\frac{\max\limits_{\boldsymbol{\theta}_1}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_1\right);\mathcal{H}_1\right)\}}{\max\limits_{\boldsymbol{\theta}_0}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_0\right);\mathcal{H}_0\right)\}} = s_k\left(\boldsymbol{x}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

p-value computed from $s_k(x)$ under $\mathcal{H}_0$ is uniformly distributed

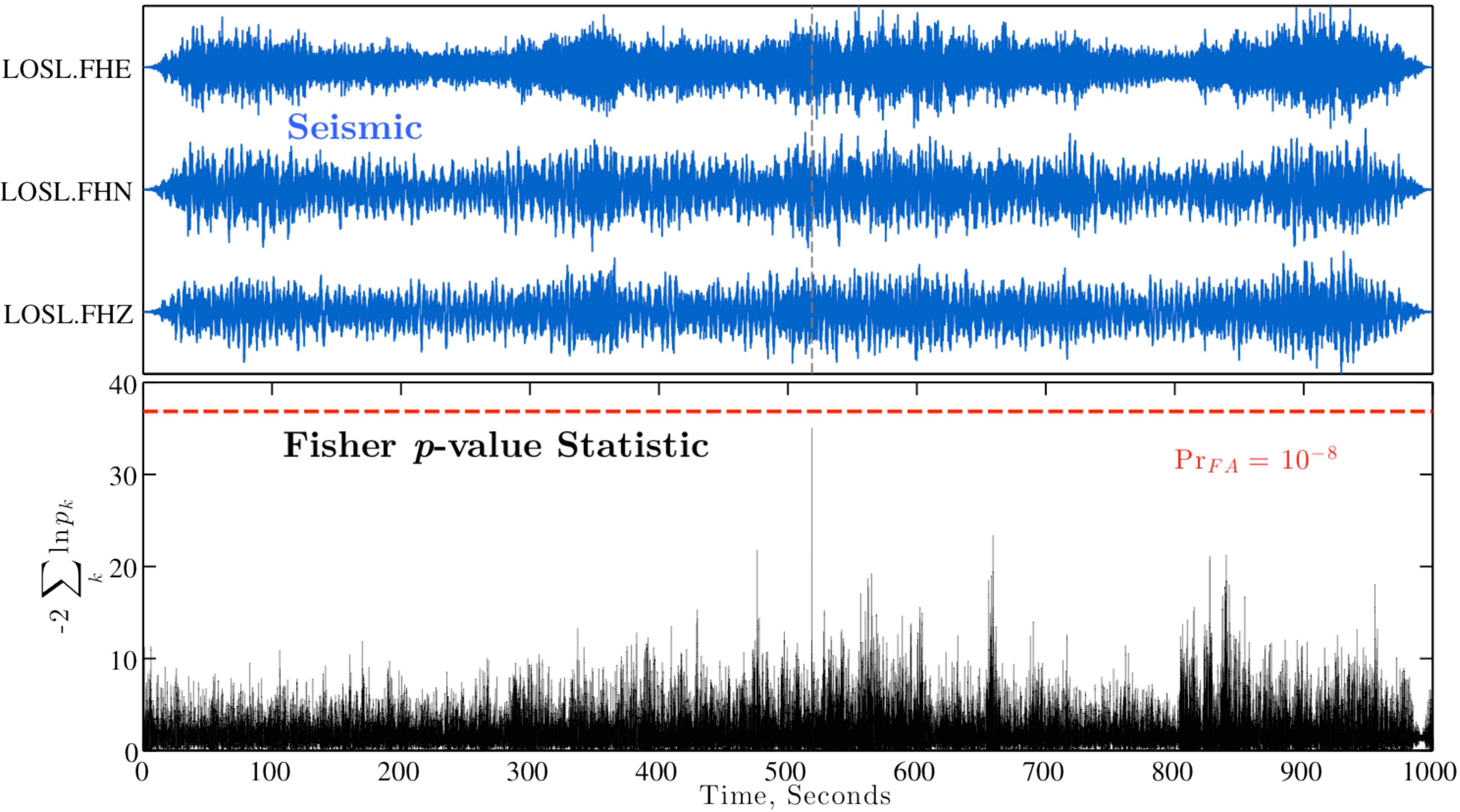
$$p_k\left(t_n;\mathcal{H}_0\right) \triangleq \int_{s_k\left(t_n;\mathcal{H}_0\right)}^{\infty} f_S\left(\bar{s}_k;\mathcal{H}_0\right) d\bar{s}_k \sim \mathcal{U}(0,1)$$

Summation of log-transformed *p*-values is therefore χ^2-distributed

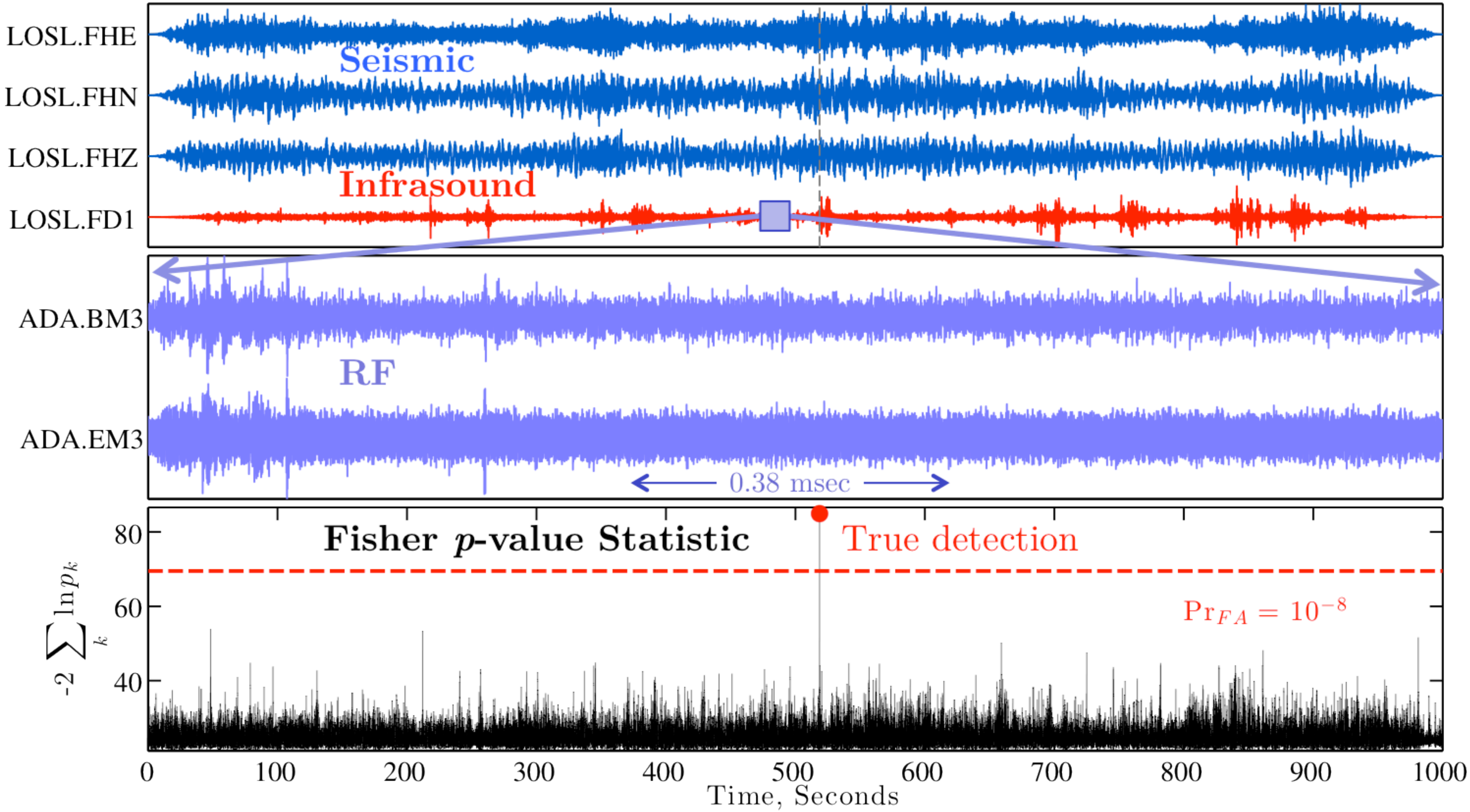
$$Z_{M\cdot P}\left(t_n;\mathcal{H}_0\right) \triangleq -2\sum_{k=1}^{M\cdot P} \ln\left[p_k\left(t_n;\mathcal{H}_0\right)\right] \sim \chi^2_{2M\cdot P}$$

Result invariant to distributional form of the screening statistic $s_k(x)$. This is ***why* Fisher's Method is useful**.

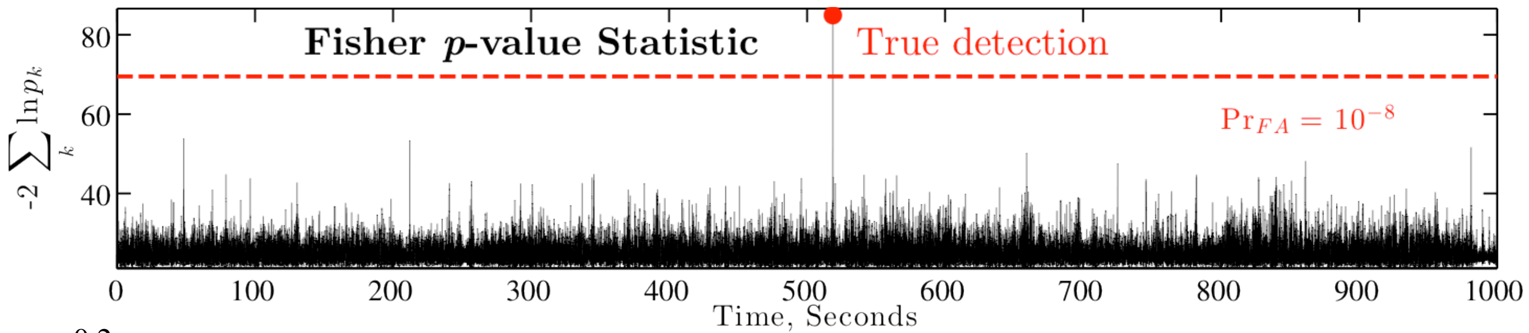
"Fusing" 1 Explosion-Signature *p*-value

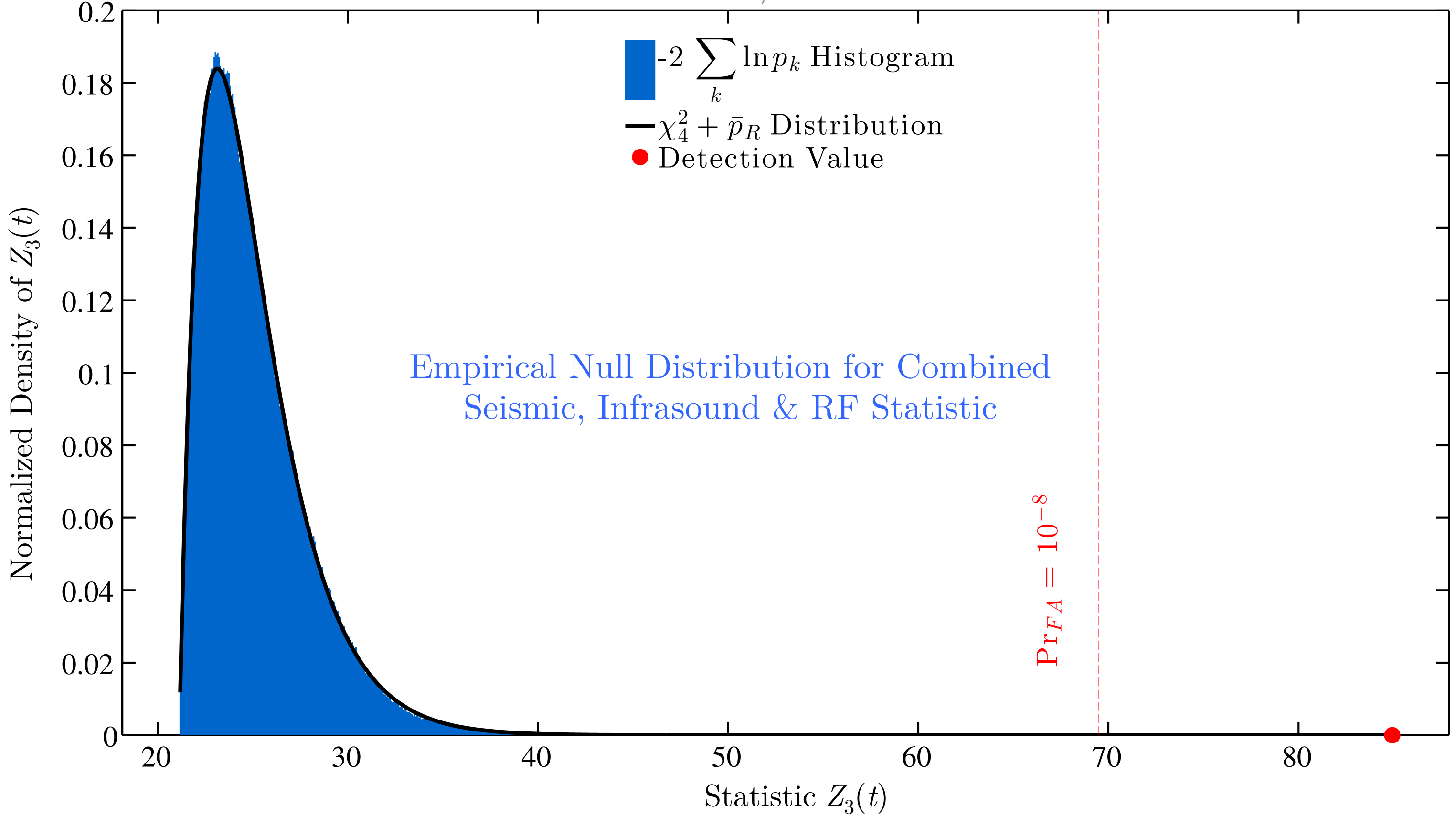


Fusing 3 Explosion-Signature *p*-values



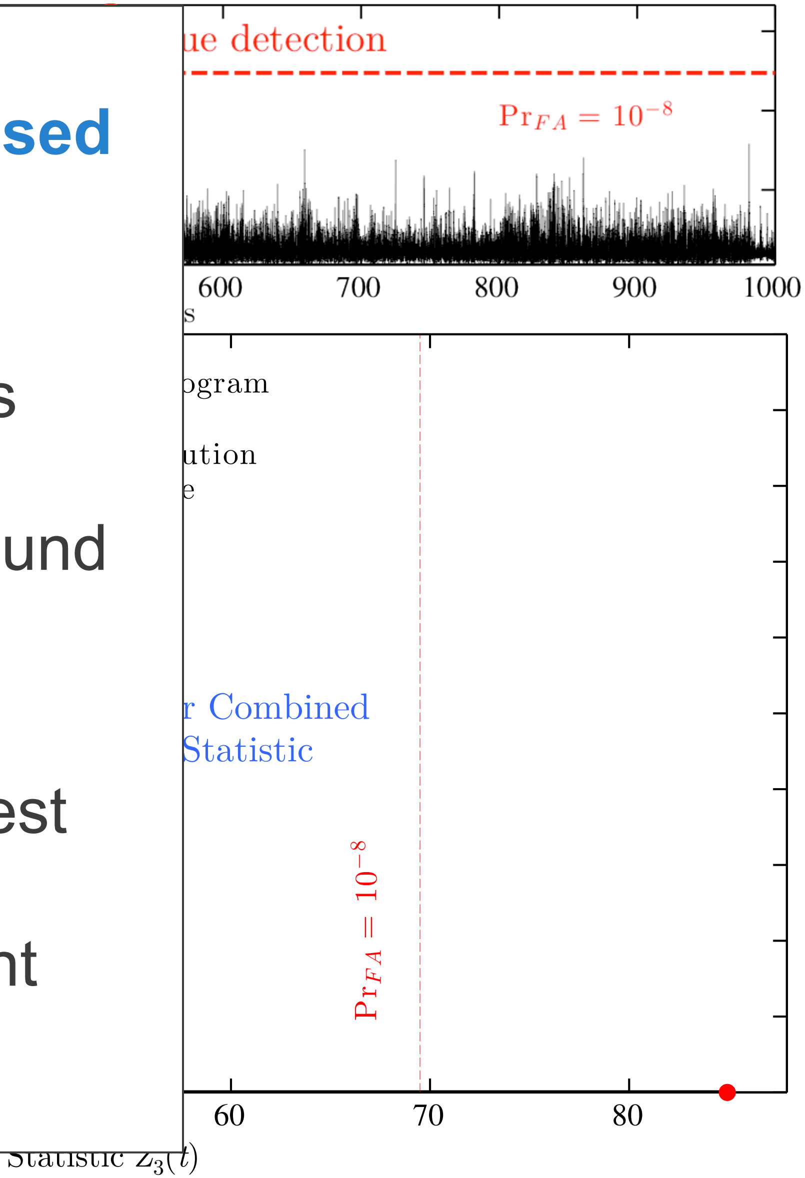
Fused Signatures Reject Null (1/2)



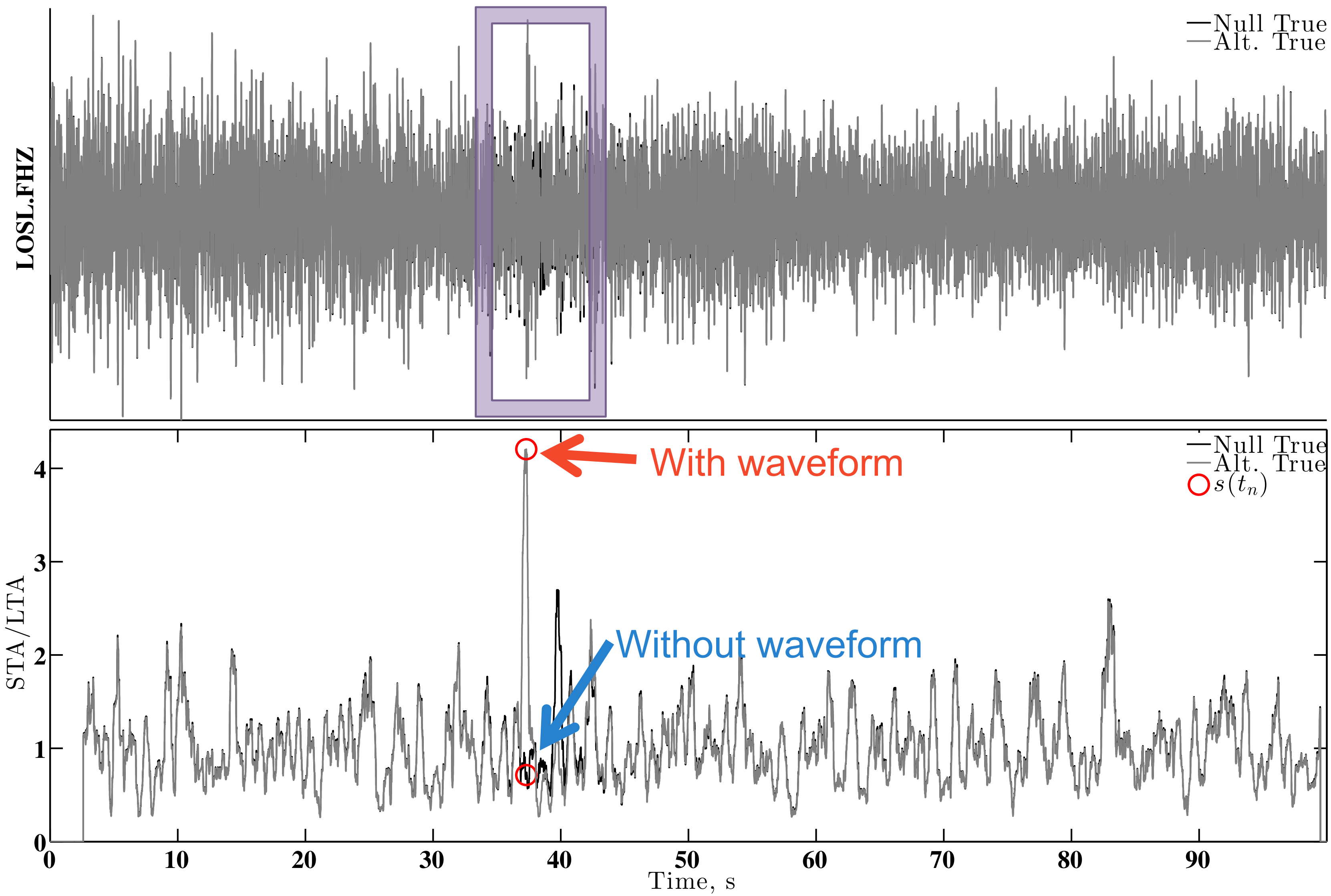


Fused Signatures Reject Null (2/2)

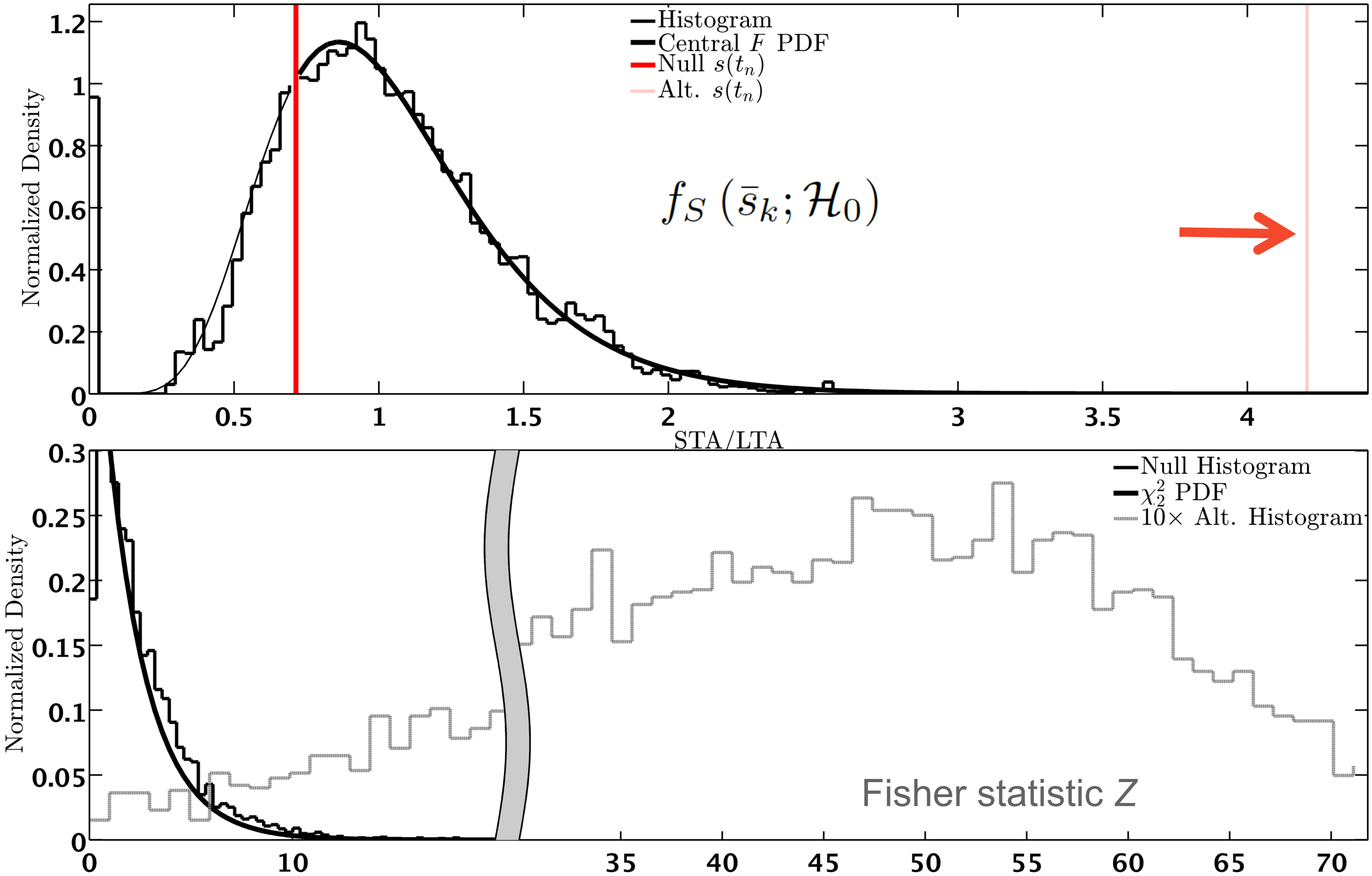
- What is the quantitative significance of “large” **fused *p*-values?**
- How sensitive is Fisher’s Method under the **null hypothesis** if a background signal **is** present?
- Can we apply Fisher’s test under the alternative hypothesis with a present signal?



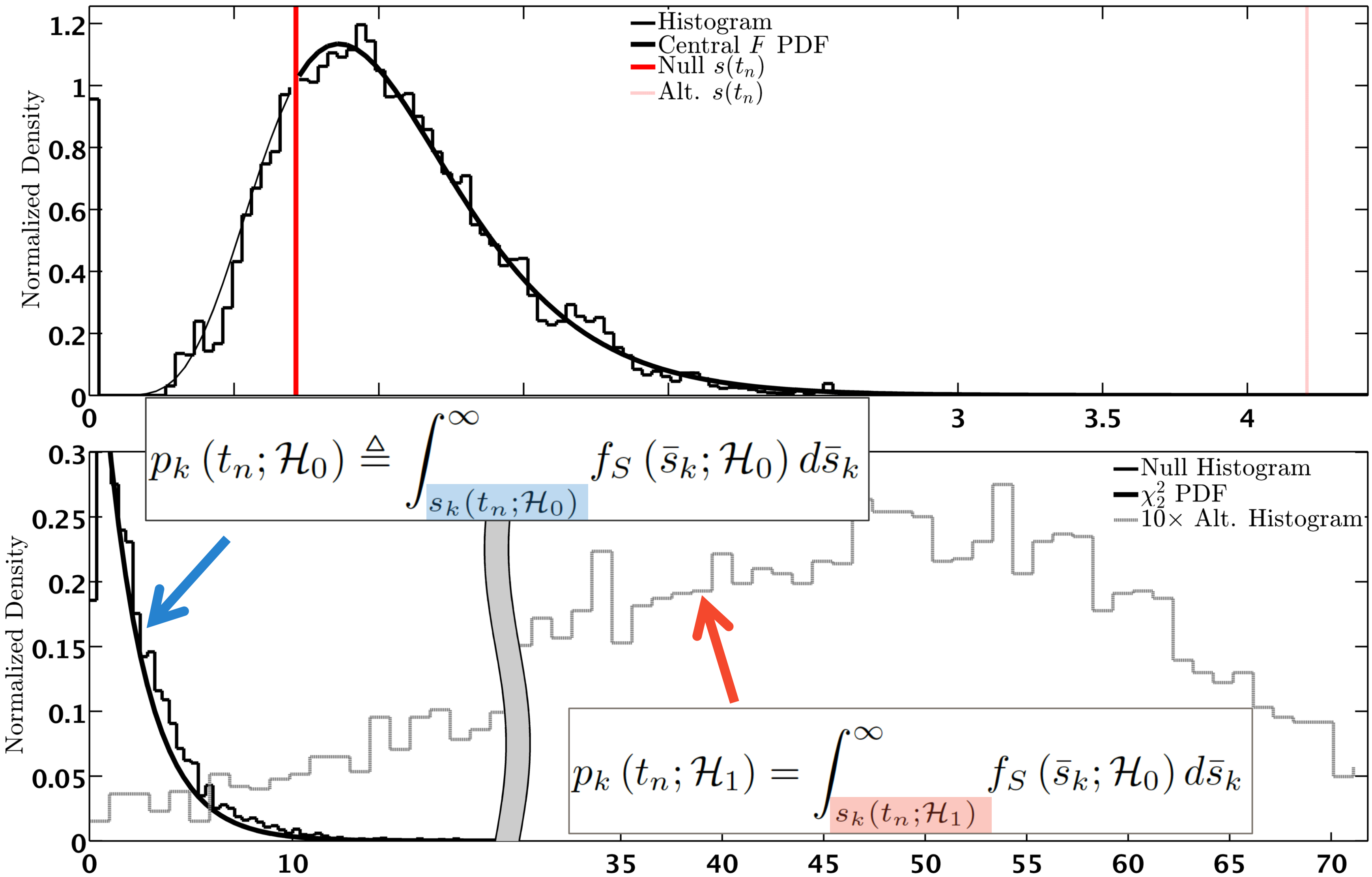
What if Alternative Hypothesis is True?



P-values & Alt. Hypothesis True (1/8)



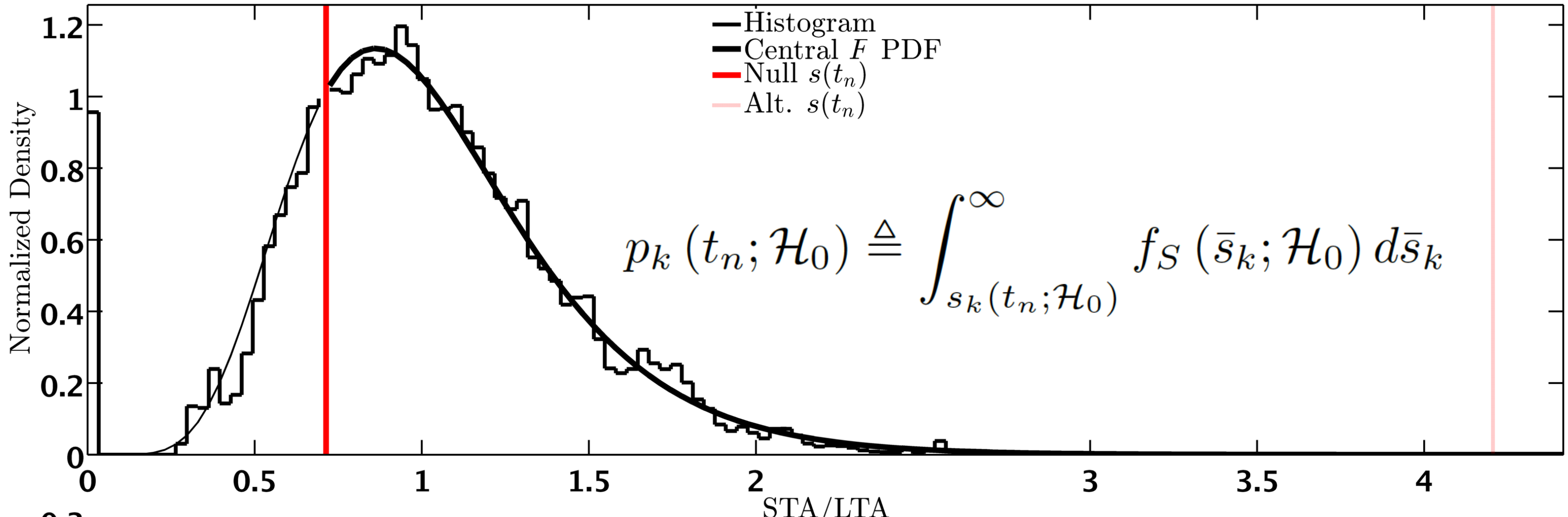
P-values & Alt. Hypothesis True (2/8)

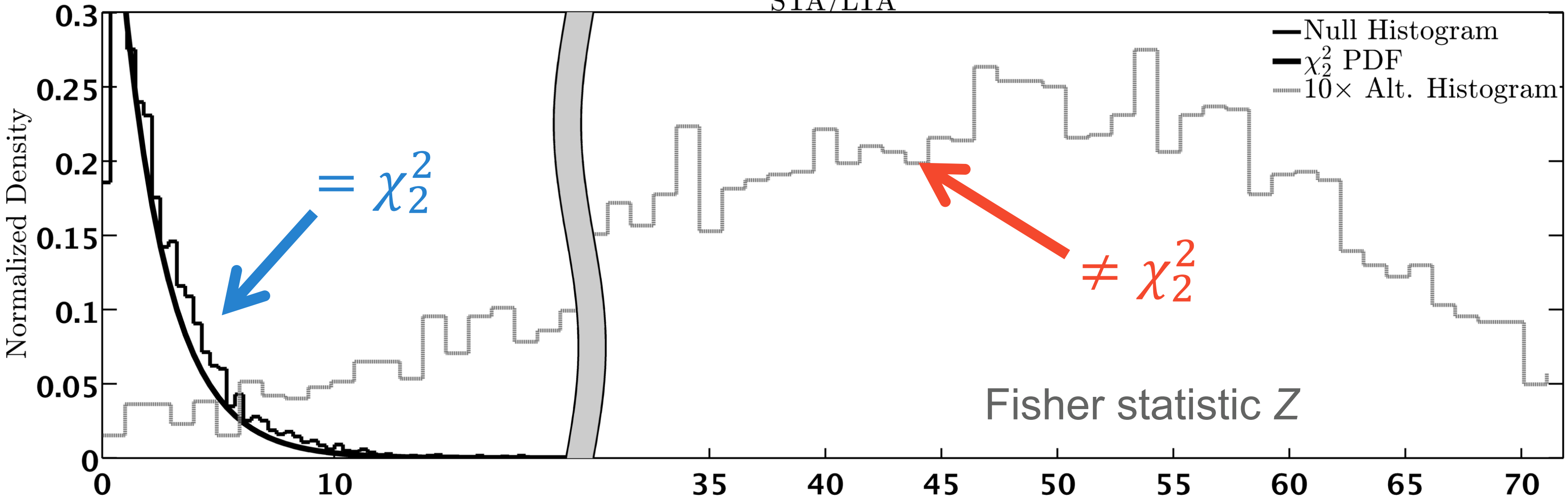


$$p_k\left(t_n;\mathcal{H}_0\right) \triangleq \int_{s_k(t_n;\mathcal{H}_0)}^{\infty} f_S\left(\bar{s}_k;\mathcal{H}_0\right) d\bar{s}_k$$

$$p_k\left(t_n;\mathcal{H}_1\right) = \int_{s_k(t_n;\mathcal{H}_1)}^{\infty} f_S\left(\bar{s}_k;\mathcal{H}_0\right) d\bar{s}_k$$

P-values & Alt. Hypothesis True (3/8)





P-values & Alt. Hypothesis True (4/8)

- Histogram for Z under $\mathcal{H}_0$ agrees well with predicted χ^2 PDF
- Histogram for Z under **weak** $\mathcal{H}_1$ show no analogous agreement
- Example shows weak signal provides a large distributional disagreement for Z under the competing hypotheses.
- While small p-value measured under $\mathcal{H}_1$ suggests that hypothesis tests should reject $\mathcal{H}_0$, we lacked **predictively capability to quantify screening power**
- Quantification requires the PDF for Z under $\mathcal{H}_1$

P-values & Alt. Hypothesis True (5/8)

Accumulating Evidence for an Explosive Source

Generalized Likelihood Ratio Detector

$$\frac{\max_{\boldsymbol{\theta}_1}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_1\right);\mathcal{H}_1\right)\}}{\max_{\boldsymbol{\theta}_0}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_0\right);\mathcal{H}_0\right)\}} = s_k\left(\boldsymbol{x}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

p-value computed from $s_k(x)$ under $\mathcal{H}_1$ is NOT uniformly distributed

$$p_k\left(t_n;\mathcal{H}_1\right) = \int_{s_k\left(t_n;\mathcal{H}_1\right)}^{\infty} f_S\left(\bar{s}_k;\mathcal{H}_0\right) d\bar{s}_k \nsim \mathcal{U}(0,1)$$

Summation of log-transformed *p*-values is NOT χ^2-distributed

$$Z_{M\cdot P}\left(t_n;\mathcal{H}_1\right) \triangleq -2\sum_{k=1}^{M\cdot P} \ln\left[p_k\left(t_n;\mathcal{H}_1\right)\right] \nsim \chi^2_{2M\cdot P}$$

Result now depends on distributional form of the screening statistic $s_k(x)$. **Is Fisher's Method still useful?**

P-values & Alt. Hypothesis True (6/8)

Accumulating Evidence for an Explosive Source

Generalized Likelihood Ratio Detector

$$\frac{\max_{\boldsymbol{\theta}_1}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_1\right); \mathcal{H}_1\right)\}}{\max_{\boldsymbol{\theta}_0}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_0\right); \mathcal{H}_0\right)\}} = s_k\left(\boldsymbol{x}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

PDF for $s_k(x)$ under $\mathcal{H}_1$

$$f_S\left(\bar{s}_k; \mathcal{H}_1\right)$$

$$Q_p \triangleq F_S^{-1}\left(1 - p|\mathcal{H}_1; \mathcal{H}_0\right)$$

p-value for $s_k(x)$ under $\mathcal{H}_1$

$$p_k\left(t_n; \mathcal{H}_1\right) = \int_{s_k(t_n;\mathcal{H}_1)}^{\infty} f_S\left(\bar{s}_k; \mathcal{H}_0\right) d\bar{s}_k$$

PDF for *p*-value for $s_k(x)$ under $\mathcal{H}_1$

$$f_{P|\mathcal{H}_1}\left(p|\mathcal{H}_1\right) = \frac{f_S\left(Q_p; \mathcal{H}_1\right)}{f_S\left(Q_p; \mathcal{H}_0\right)}$$

PDF for *Z* under $\mathcal{H}_1$ for signature *k*

$$f_Z^{(k)}\left(z_k; \mathcal{H}_1\right) = \frac{1}{2}\exp\left\{-\frac{1}{2}z_k\right\} \cdot f_{P|\mathcal{H}_1}\left(\exp\left\{-\frac{1}{2}z_k\right\}\right)$$

P-values & Alt. Hypothesis True (7/8)

Accumulating Evidence for an Explosive Source

Generalized Likelihood Ratio Detector

$$\frac{\max_{\boldsymbol{\theta}_1}\{f_{\mathbf{X}}(\boldsymbol{x}_k(\boldsymbol{\theta}_1);\mathcal{H}_1)\}}{\max_{\boldsymbol{\theta}_0}\{f_{\mathbf{X}}(\boldsymbol{x}_k(\boldsymbol{\theta}_0);\mathcal{H}_0)\}} = s_k(\boldsymbol{x}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

PDF for $s_k(x)$ under $\mathcal{H}_1$

$$f_S(\bar{s}_k;\mathcal{H}_1)$$

$$Q_p \triangleq F_S^{-1}(1-p|\mathcal{H}_1;\mathcal{H}_0)$$

p-value for $s_k(x)$ under $\mathcal{H}_1$

$$p_k(t_n;\mathcal{H}_1) = \int_{s_k(t_n;\mathcal{H}_1)}^{\infty} f_S(\bar{s}_k;\mathcal{H}_0)\, d\bar{s}_k$$

PDF for *p*-value for $s_k(x)$ under $\mathcal{H}_1$

$$f_{P|\mathcal{H}_1}(p|\mathcal{H}_1) = \frac{f_S(Q_p;\mathcal{H}_1)}{f_S(Q_p;\mathcal{H}_0)}$$

PDF for *Z* under $\mathcal{H}_1$ for signature *k*

$$f_Z^{(k)}(z_k;\mathcal{H}_1) = \frac{1}{2}\exp\left\{-\frac{1}{2}z_k\right\}\cdot f_{P|\mathcal{H}_1}\left(\exp\left\{-\frac{1}{2}z_k\right\}\right)$$

P-values & Alt. Hypothesis True (8/8)

Accumulating Evidence for an Explosive Source

Generalized Likelihood Ratio Detector

$$\frac{\max\limits_{\boldsymbol{\theta}_1}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_1\right);\mathcal{H}_1\right)\}}{\max\limits_{\boldsymbol{\theta}_0}\{f_{\mathbf{X}}\left(\boldsymbol{x}_k\left(\boldsymbol{\theta}_0\right);\mathcal{H}_0\right)\}} = s_k\left(\boldsymbol{x}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

PDF for $s_k(x)$ under $\mathcal{H}_1$

$$f_S\left(\bar{s}_k;\mathcal{H}_1\right)$$

$$Q_p \triangleq F_S^{-1}\left(1-p|\mathcal{H}_1;\mathcal{H}_0\right)$$

p-value for $s_k(x)$ under $\mathcal{H}_1$

$$p_k\left(t_n;\mathcal{H}_1\right) = \int_{s_k(t_n;\mathcal{H}_1)}^{\infty} f_S\left(\bar{s}_k;\mathcal{H}_0\right) d\bar{s}_k$$

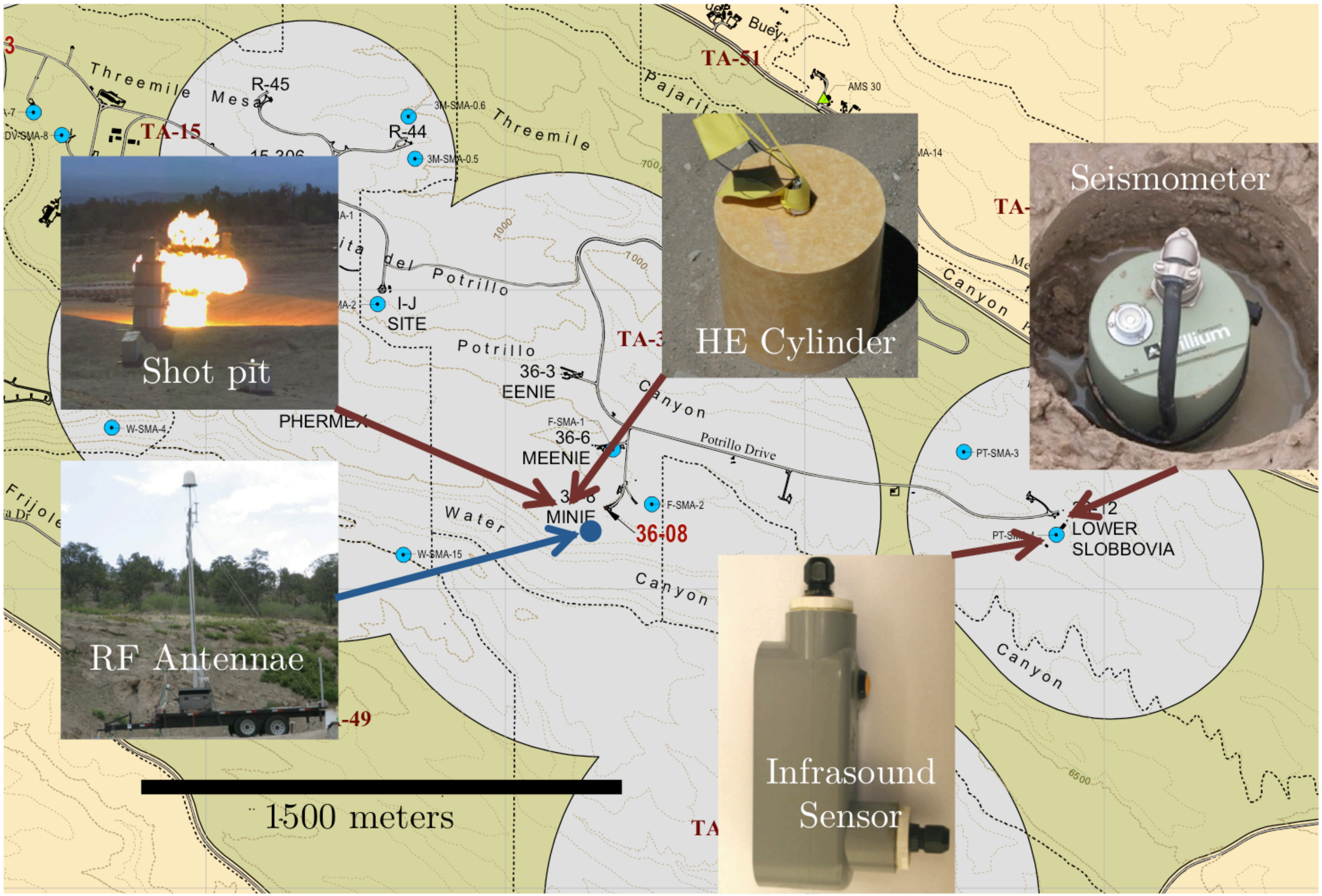
PDF for *p*-value for $s_k(x)$ under $\mathcal{H}_1$

$$f_{P|\mathcal{H}_1}\left(p|\mathcal{H}_1\right) = \frac{f_S\left(Q_p;\mathcal{H}_1\right)}{f_S\left(Q_p;\mathcal{H}_0\right)}$$

PDF for Z under $\mathcal{H}_1$ for fused signatures with characteristic method

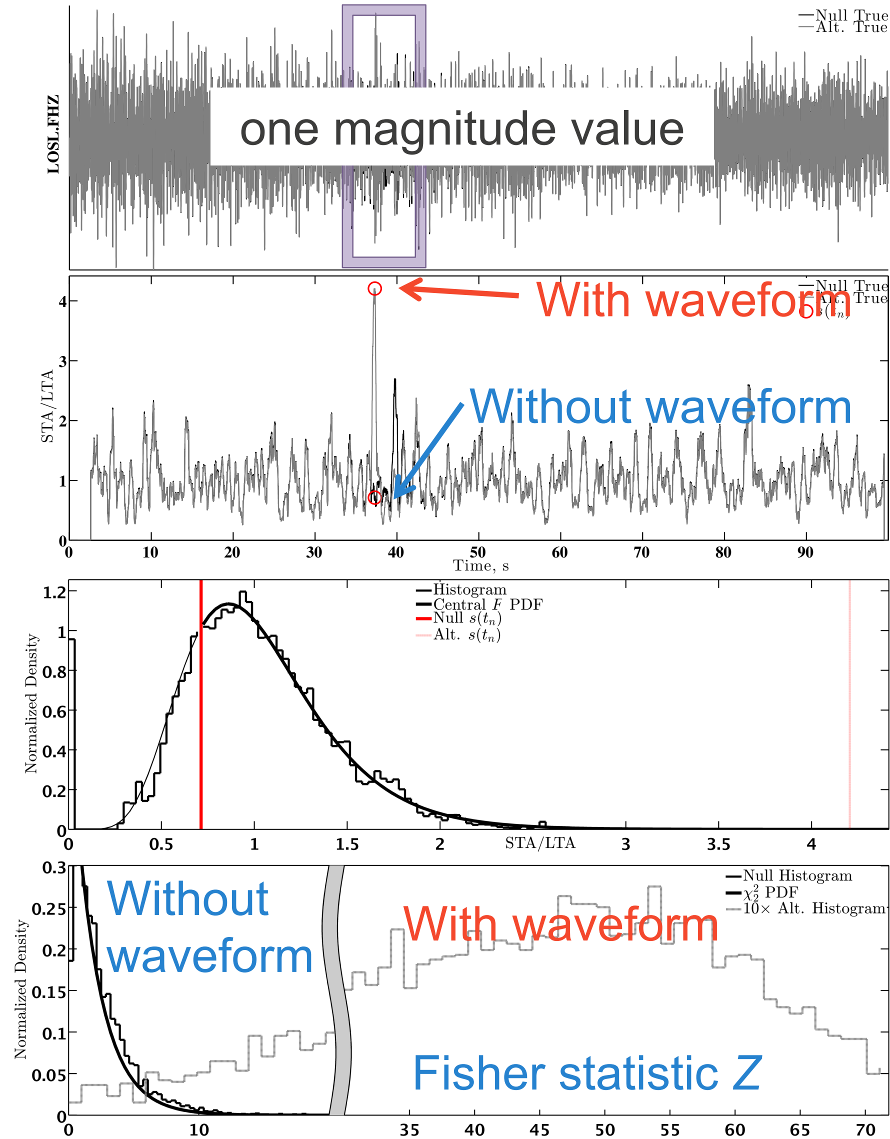
$$f_{Z_{M\cdot P}}\left(z_{M\cdot P};\mathcal{H}_1\right) = \mathcal{F}^{-1}\left\{\prod_{k=1}^{M\cdot P}\mathcal{F}\left\{f_Z^{(k)}\left(z_k;\mathcal{H}_1\right)\right\}\right\}$$

Data Collection Reminder

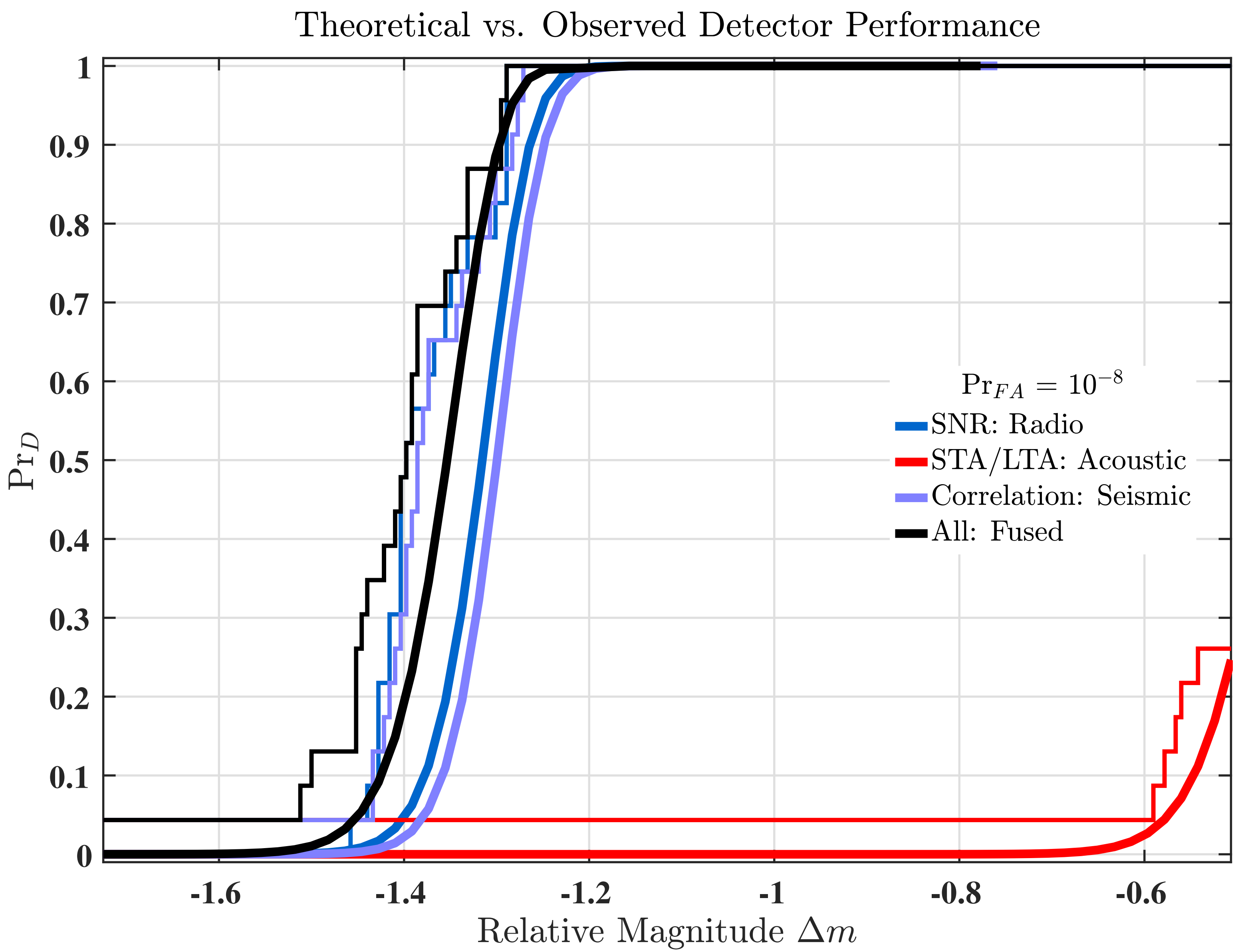


Semi-Empirical Test: p-values

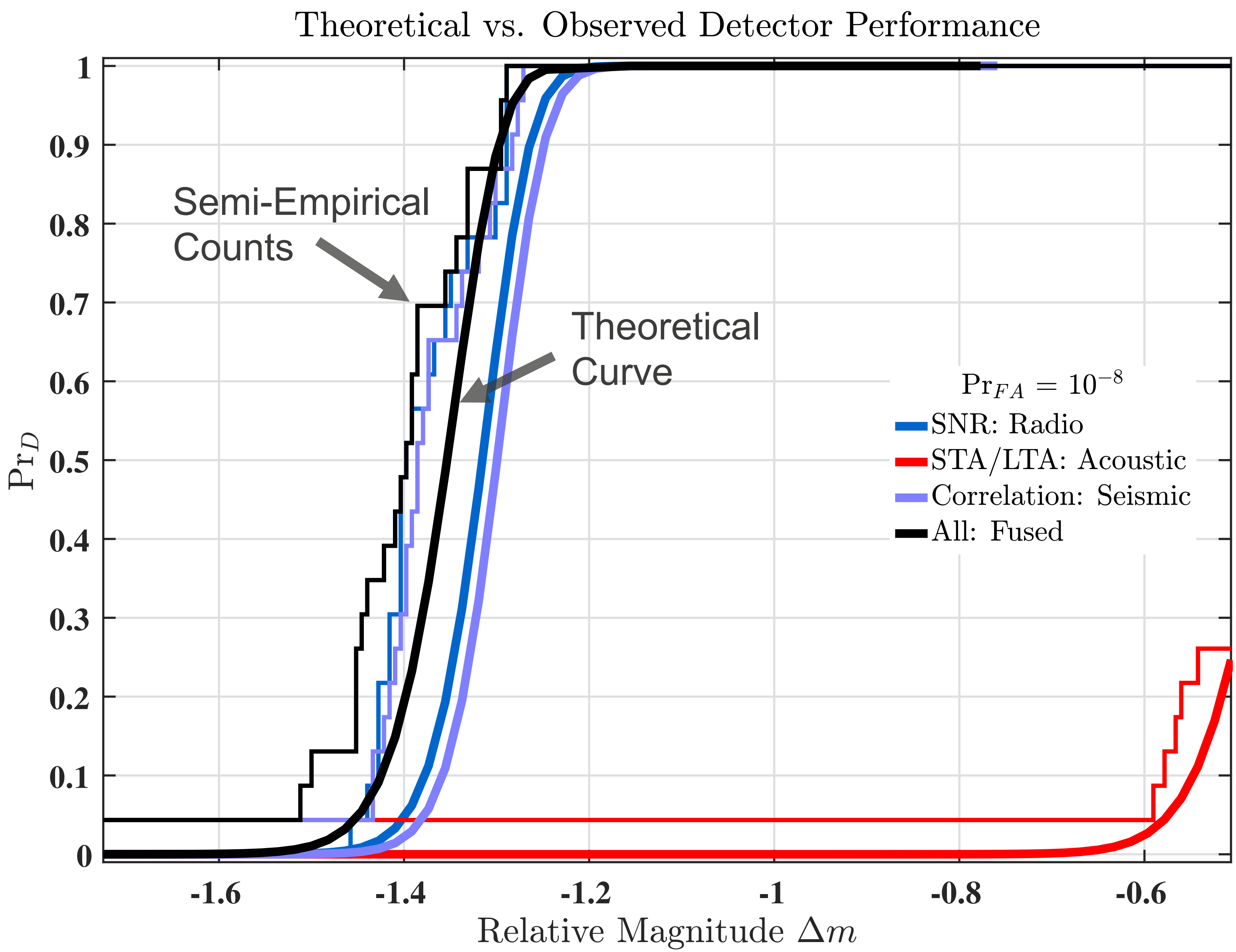
- Bury scaled waveforms sampled over a **relative magnitude grid** & recorded in real noise thousands of times
- Process data with an **signature-specific** detectors. Measure detection statistics when waveform is present vs. absent
- Bin detection statistic, and compare compute p-values in two cases.
  - **Signal Absent** $\mathcal{H}_0$
  - **Signal Present** $\mathcal{H}_1$
- Form Fisher statistics in both cases, and compare vs. magnitude



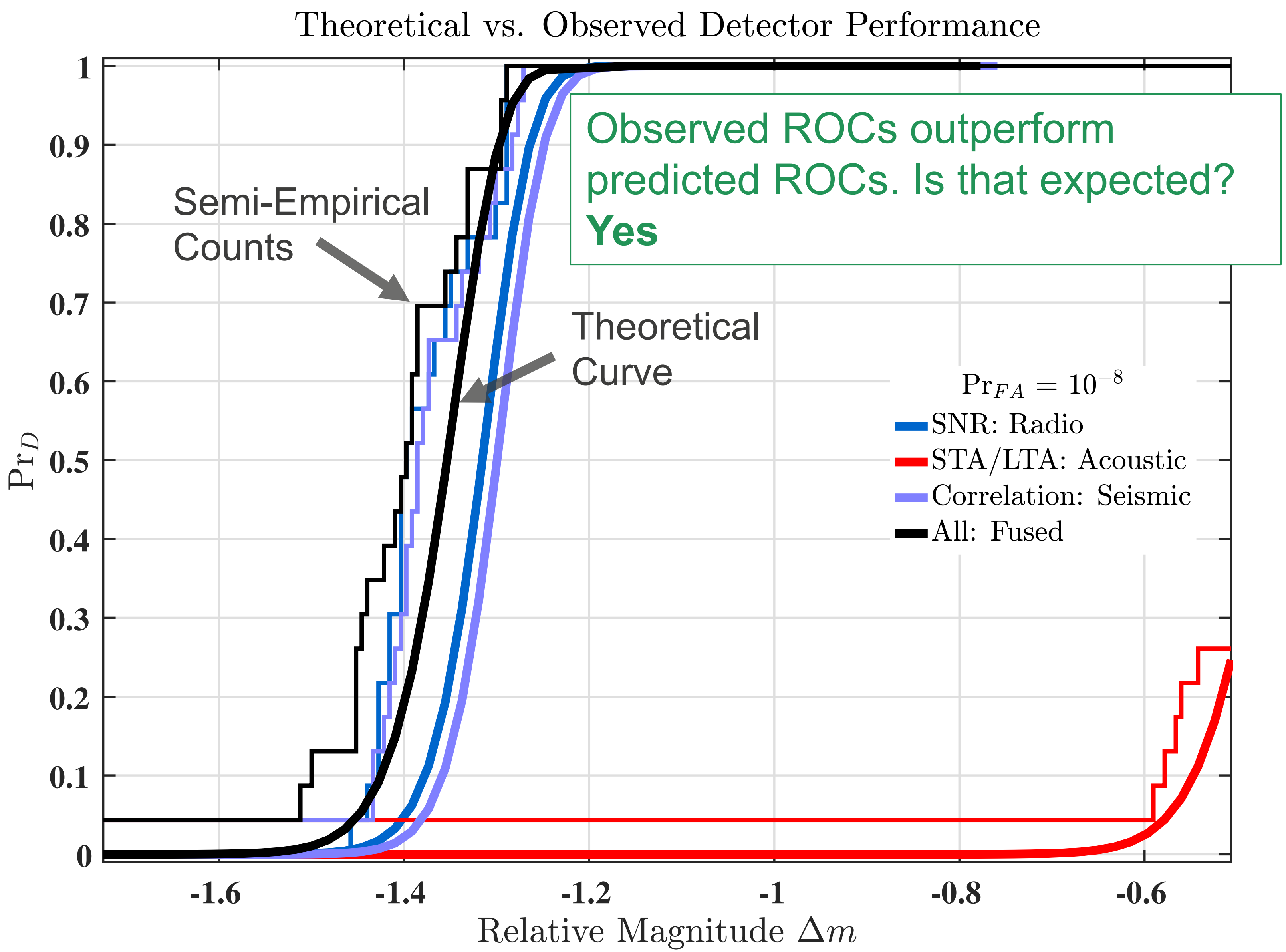
Fisher's Test at a Constant False Alarm Rate (1/3)



Fisher's Test at a Constant False Alarm Rate (2/3)

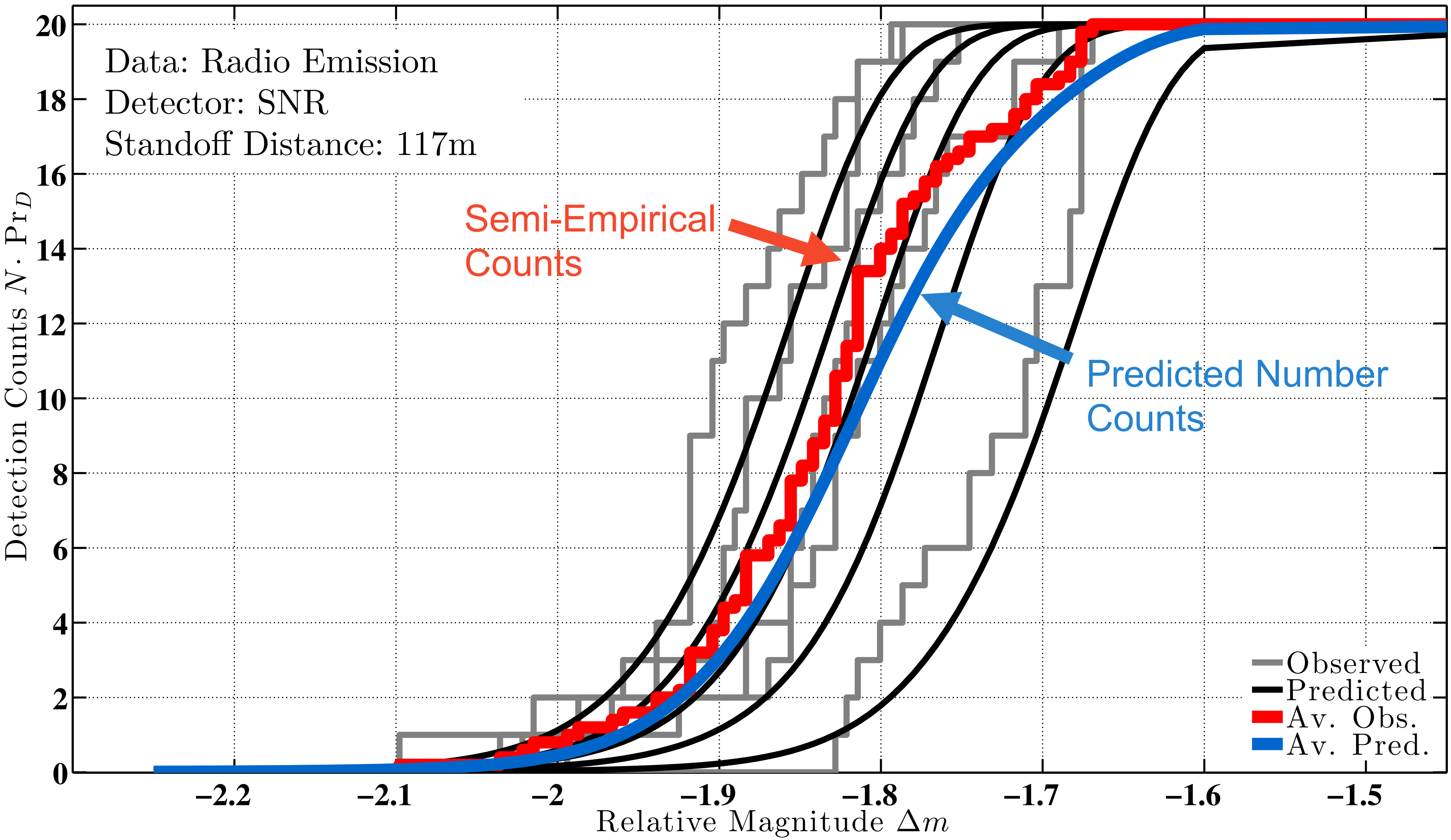


Fisher's Test at a Constant False Alarm Rate (3/3)



SNR Detector ROC Curves (1/2)

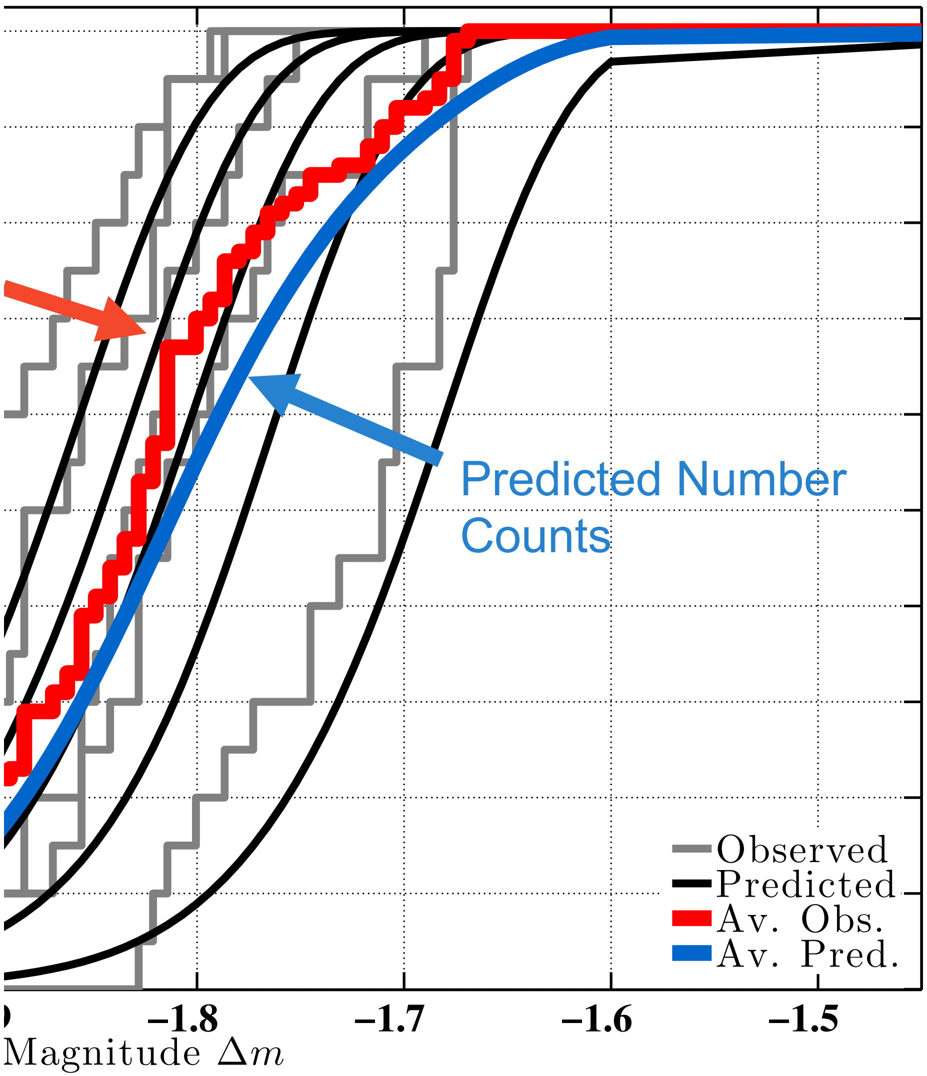
Semi-Empirical vs. Predicted ROC Curves



SNR Detector ROC Curves (2/2)

Semi-Empirical vs. Predicted ROC Curves

- Observed detection capability **appears** to outperform predicted capability
  - Compare red stair plot vs. smooth plot
- Observed capability includes more **detection opportunities** per waveform
  - Detector accepts event declarations over a multiple point duration sliding window.
- **Explicitly**: if the SNR statistic exceeds it's concurrent threshold η at least twice over a intervals greater than $\Delta t = 0.1\mu\text{sec}$ over the duration of the infused waveform segment, the true detector declares an event.
- The **predicted performance** only considers single detection opportunities over each *N*-point processing window.



Research Conclusions (1/4)

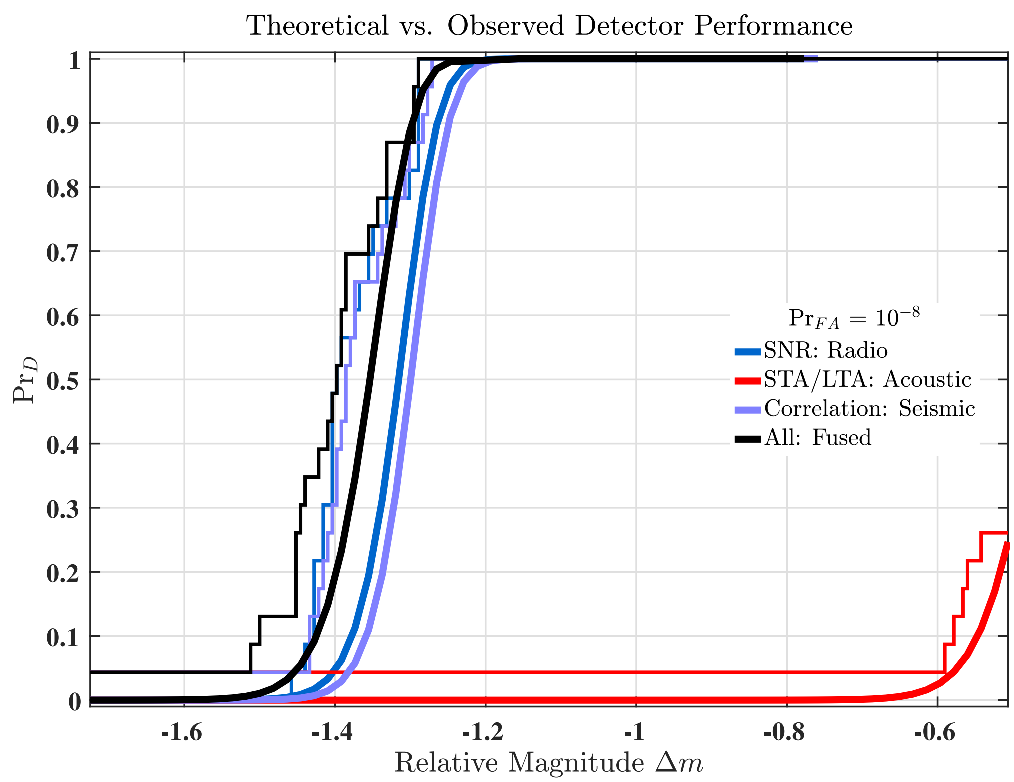
General Problem

How do we combine different signatures from an event or source together, in a **defensible** way?

Synthesis

- Fisher's Method is useful for rejecting the null hypothesis since fused statistics are **invariant to detectors**
- **However,** Fisher's statistics *Z* is sensitive to errors under null
- Fisher's method also lacked a **predictive capability** if signal present
- **Result:** we derived a PDF for Fisher's statistic *Z* under the **alternative hypothesis** and developed **ROC** curves over magnitude to provide Fisher's Test with a ***predictive capability.***

Near-Surface Explosion Scenario



Research Conclusions (2/4)

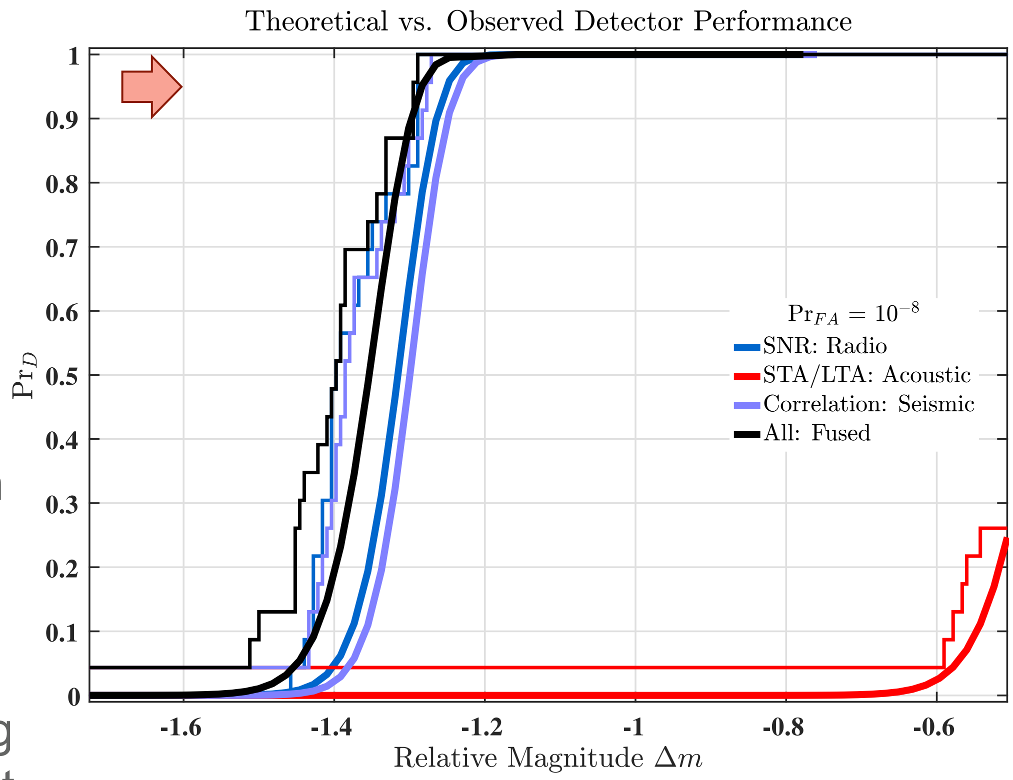
General Problem

How do we combine different signatures from an event or source together, in a **defensible** way?

Remaining Implications

- Determine how deploying signature-specific sensors increases screening/detection capability **predictively**
- Determine at what magnitude *N*-fused signatures provides a desired detection probability for fixed false alarm rate.
- **Example:** *N*-acoustic sensors that exploit power detectors provide a given detection capability. Can supplementing these receivers with a seismometer that exploits a correlation detector increase the detection capability by Δm magnitude units?

Near-Surface Explosion Scenario



Research Conclusions (3/4)

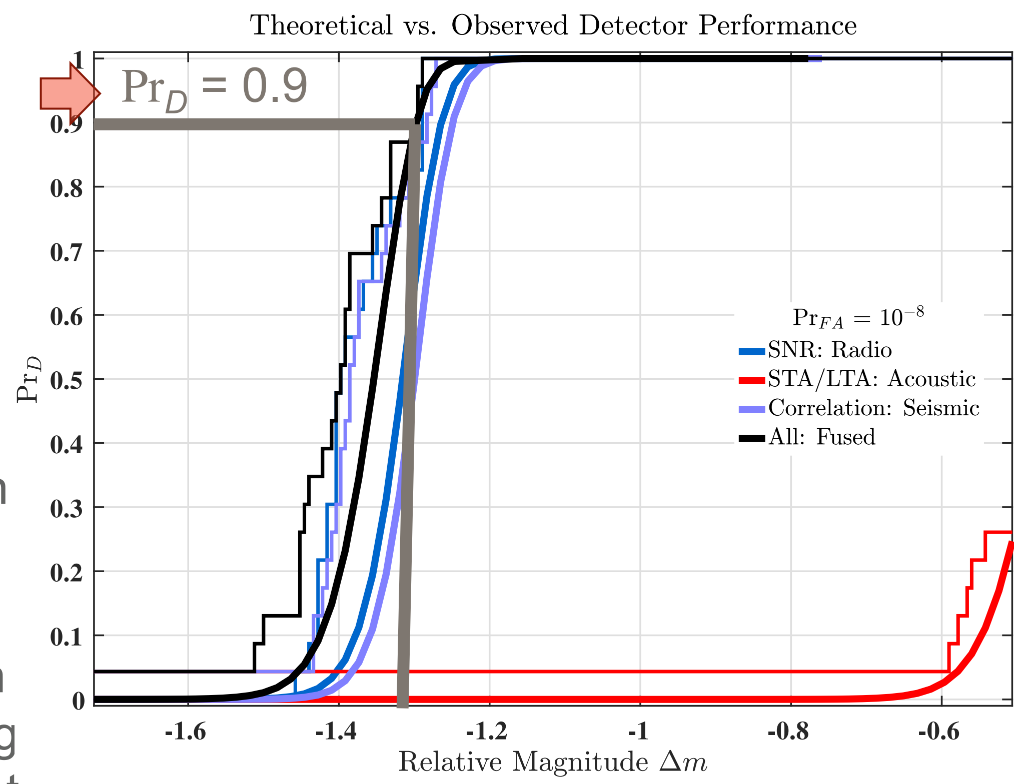
General Problem

How do we combine different signatures from an event or source together, in a **defensible** way?

Remaining Implications

- Determine how deploying signature-specific sensors increases screening/detection capability **predictively**
- Determine at what magnitude *N*-fused signatures provides a desired detection probability for fixed false alarm rate.
- **Example:** *N*-acoustic sensors that exploit power detectors provide a given detection capability. Can supplementing these receivers with a seismometer that exploits a correlation detector increase the detection capability by Δm magnitude units?

Near-Surface Explosion Scenario



Supplementary slides provide examples that quantify that in very synthetic tests

Research Conclusions (4/4)

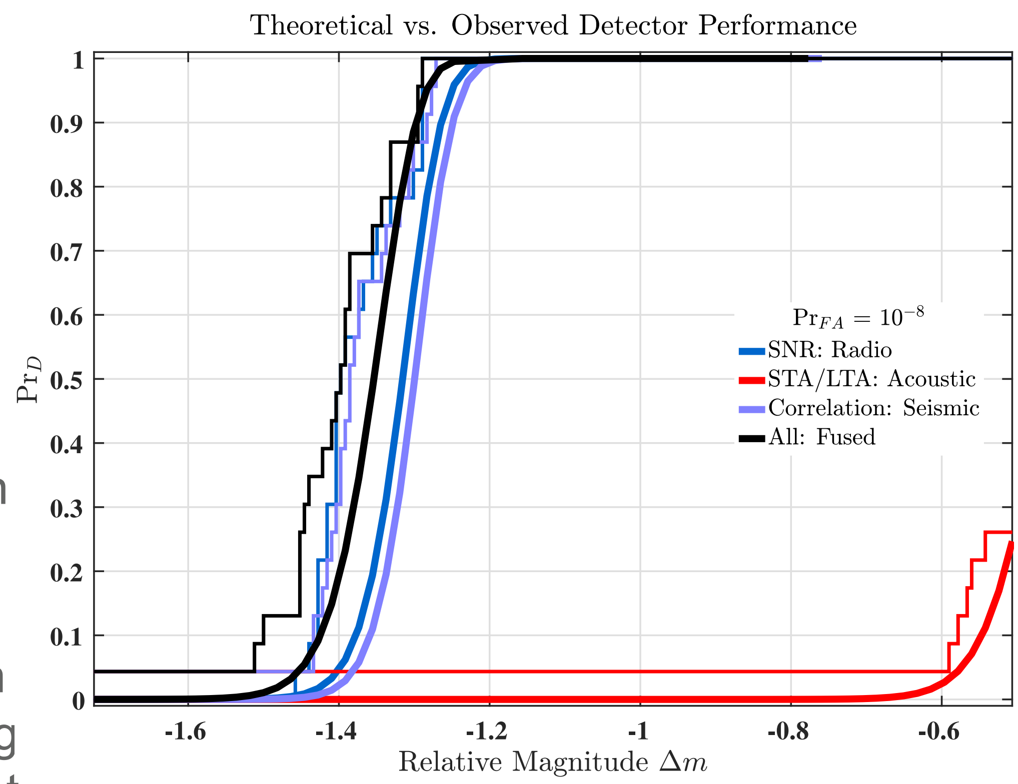
General Problem

How do we combine different signatures from an event or source together, in a **defensible** way?

Remaining Implications

- Determine how deploying signature-specific sensors increases screening/detection capability **predictively**
- Determine at what magnitude *N*-fused signatures provides a desired detection probability for fixed false alarm rate.
- **Example:** *N*-acoustic sensors that exploit power detectors provide a given detection capability. Can supplementing these receivers with a seismometer that exploits a correlation detector increase the detection capability by Δm magnitude units?

Near-Surface Explosion Scenario

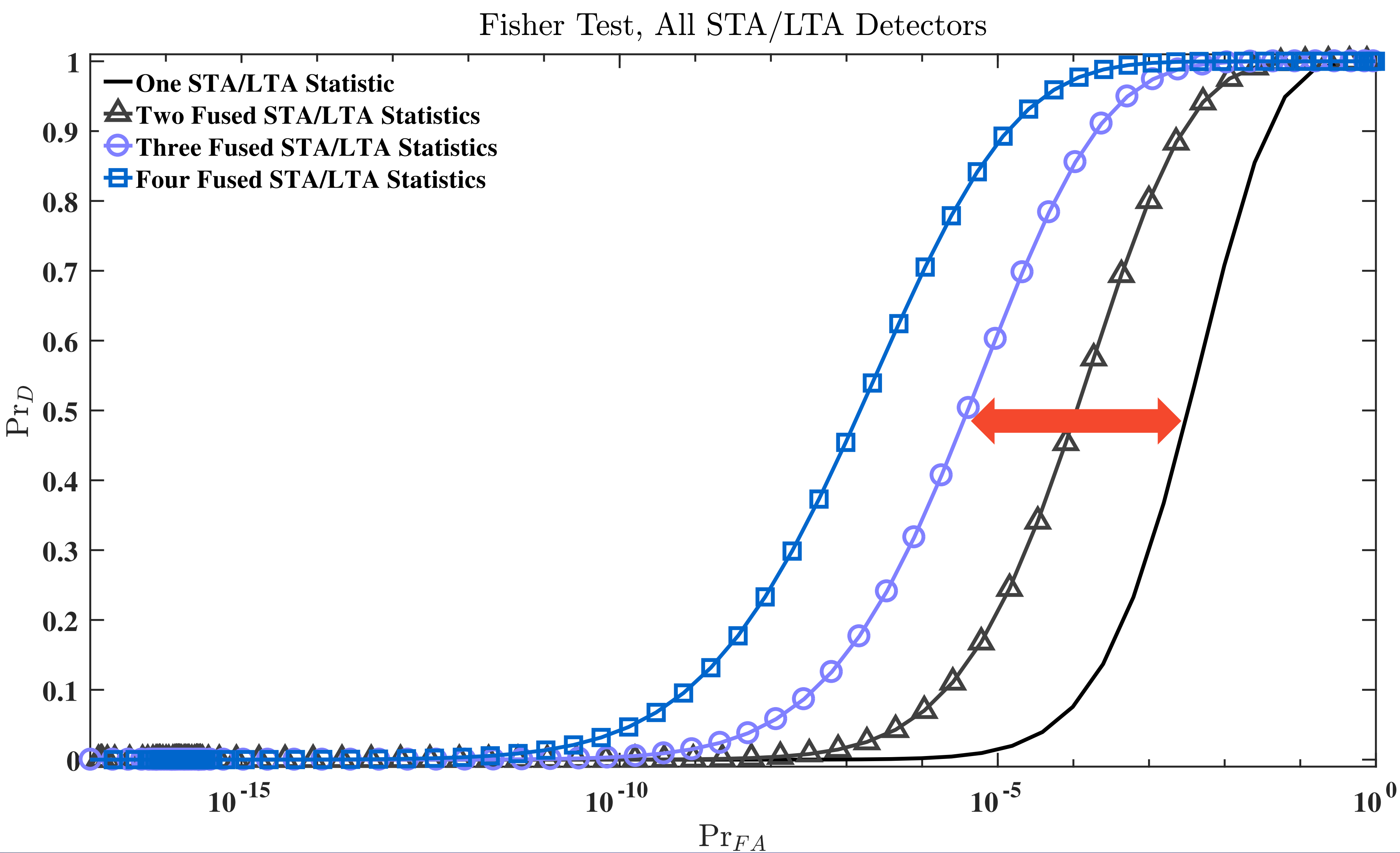


Supplementary slides provide examples that quantify that in very synthetic tests

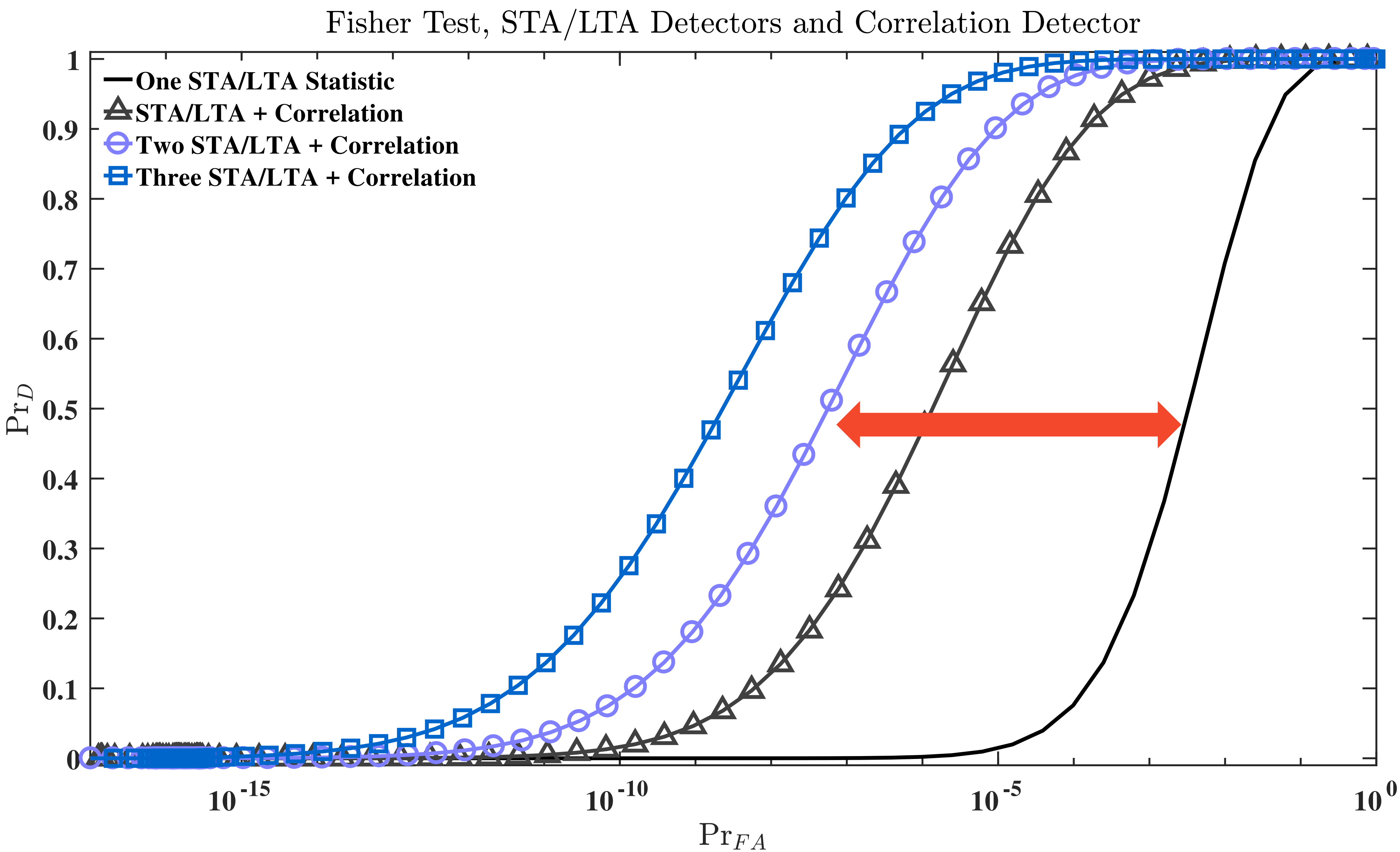
Synthetic Tests for Combining Detection Statistics

Fisher’s Combined Probability Test

Synthetic Test Fusing *F* Statistics



Synthetic Test Fusing *F* and Correlation Statistics



Synthetic Test Fusing *F*, Correlation, and SNR Statistics