“Meson Spectroscopy from QCD”

funded as DE-SC0006765

Project Results

Research Highlights 2012-2016

This document reports on progress to date in DE-SC0006765.

Highlights of the research include

- the determination of the form of the **lowest energy gluonic excitation** within QCD and the spectrum of hybrid hadrons which follows
- the first calculation of the spectrum of **hybrid baryons** within a first-principles approach to QCD
- a detailed mapping out of the **phase-shift of elastic $\pi\pi$ scattering** featuring the **ρ resonance** at two values of the light quark mass within lattice QCD
- the first (and to date, only) determinations of **coupled-channel meson-meson scattering** within first-principles QCD
- the first (and to date, only) determinations of the **radiative coupling of a resonant state**, the ρ appearing in **$\pi\gamma \to \pi\pi$**
- the first (and to date, only) determination of the properties of the broad **σ resonance in elastic $\pi\pi$ scattering** within QCD without unjustified approximations

Research products have appeared in various forms:

- Eighteen papers in peer-reviewed journals including two PRLs and a Nature review
- Thirty-nine presentations by Dudek (mostly invited)
- Lattice conference plenary talks by Dudek and group members Briceno and Wilson
- Reference to our research results in the NSAC Long-Range Plan document

Research Progress

An understanding of the hadron spectrum as it arises from the quarks and gluons of QCD is one of the main goals of Dudek's research. Particular emphasis falls upon investigating the possible existence of *hybrid mesons*, states in which quarks are partnered with an excitation of the gluonic field. Such states might be expected within a strongly coupled theory of quarks and self-interacting gluons, but no unambiguous experimental observation has yet been forthcoming. Determining the properties of such states, such as their mass, decay width and branching fractions to various final states, within a first principles approach to QCD, is an immediate target.

Over the course of this grant award, a number of items of major progress have bought this target closer.

Using a large basis of interpolating fields of quark bilinear structure, $\bar{\psi}\Gamma D \ldots D\psi$, correlation matrices have been computed which have then been 'diagonalized' to find the spectrum of excited mesons. An example is presented in Ref [1] where we determined the spectrum of excited isospin=0 mesons for several values of the light quark mass and several lattice volumes, following up on our early work presented in Ref [2]. By considering the overlap of various states with operators having hidden-light, $\bar{u}u + \bar{d}d$, and hidden-strange, $\bar{s}s$, composition, we also determined the light-strange mixing of the states.

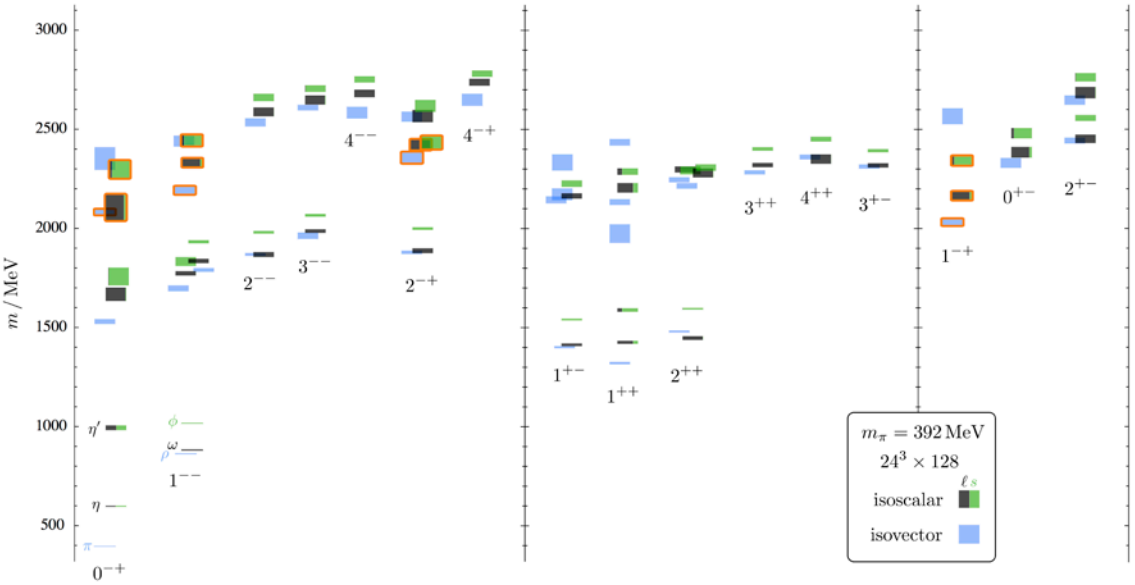

Figure 1: Spectrum of isoscalar and isovector mesons across J^{PC} determined from lattice QCD. Hidden light, hidden strange mixing indicated by black/green composition of box. Lightest supermultiplet of hybrid mesons shown in orange.

The determined spectrum resembles in gross structure much of the experimental spectrum, but there are clear signs for hybrid mesons with exotic and non-exotic quantum numbers. The particular distribution of such states, and the form of the operators found to best interpolate them from the vacuum, suggests a simple phenomenology [3] for the lightest gluonic excitation in QCD: a chromomagnetic configuration associated with an energy increase of ~1.3 GeV.

The same phenomenology describes a set of states observed in Ref [4] where we determined the light baryon spectrum, suggesting that there are also hybrid baryons. This paper inspired a PAC-approved search for such states in Hall-B at Jefferson Lab.

In work done in collaboration with researchers at Trinity College, Dublin, we determined the excited state spectrum of charmonium [5], and again found a

spectrum of hybrid mesons, which may be candidates to explain some of the recently experimentally observed X,Y,Z states.

These studies suggest a rich spectrum of excited meson and baryon states, including those of hybrid character, but they are not capable of resolving their resonant nature, the fact that they decay into lighter hadrons. This physics can be studied within lattice QCD by making use of a finite-volume formalism which relates the discrete spectrum of states in a periodic cube (defined by the lattice boundary) to the infinite-volume scattering amplitudes of QCD.

We have focused much of our research effort into such studies over the past few years, initially determining *elastic* scattering amplitudes in unprecedented detail and latterly performing the first (and, to-date, only) extractions of *coupled-channel* scattering amplitudes.

In Ref [6], by computing spectra in several volumes and a number of moving frames, we were able to determine the S-wave and D-wave $\pi\pi$ isospin=2 phase-shifts over the entire elastic scattering region. We found, as one would expect, a non-resonant weakly repulsive effect, smaller in D-wave than S-wave.

Computing using a large basis of $q\bar{q}$*-like* operators and $\pi\pi$*-like* operators, we were able to determine the isospin=1 spectrum and the P-wave scattering phase-shift which we mapped out in great detail, observing a clear ρ resonance. Computing at two values of the light quark mass, we observed the expected decrease in the ρ mass and increase in the decay as the light quark mass is reduced [7, 8]. Work by group postdocs Briceno and Wilson and their collaborator Bolton [9] in unitarized chiral effective theory showed that our results extrapolate to ρ properties at the physical light quark mass that are in excellent agreement with experiment.

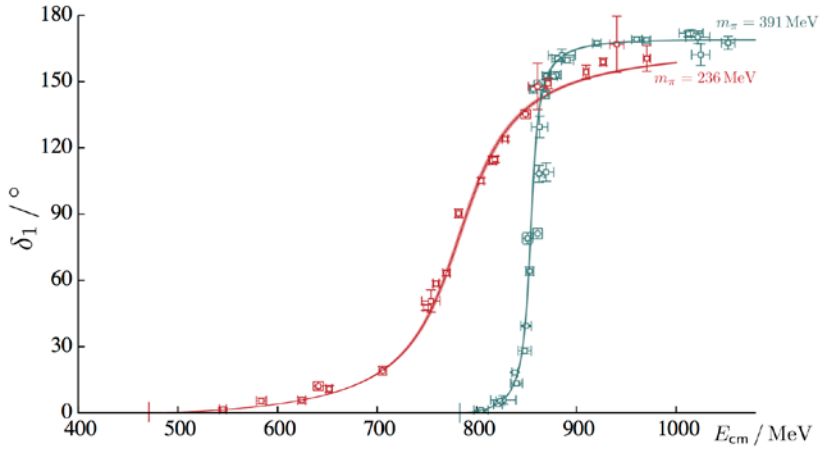

Figure 2: Elastic $\pi\pi$ P-wave phase-shift extracted from lattice QCD at two light quark masses. The resonant behavior due to the ρ is clearly visible. The ρ is observed to decrease in mass and increase in decay width as the quark mass is reduced.

Most recently we have released a preprint [10] describing the application of these techniques to the isospin=0 channel, which has always been considered the most challenging, owing to the need to compute diagrams in which the quarks all annihilate, leading to a completely disconnected diagram. Using the distillation technology we were able to compute such diagrams with high statistical precision. From the resulting finite volume spectra we determined the $\pi\pi$ S-wave phase-shift, observing an interesting transition between a $\pi\pi$ bound-state at higher quark mass to a broad resonance, analogous to the experimental σ meson, at lighter quark mass.

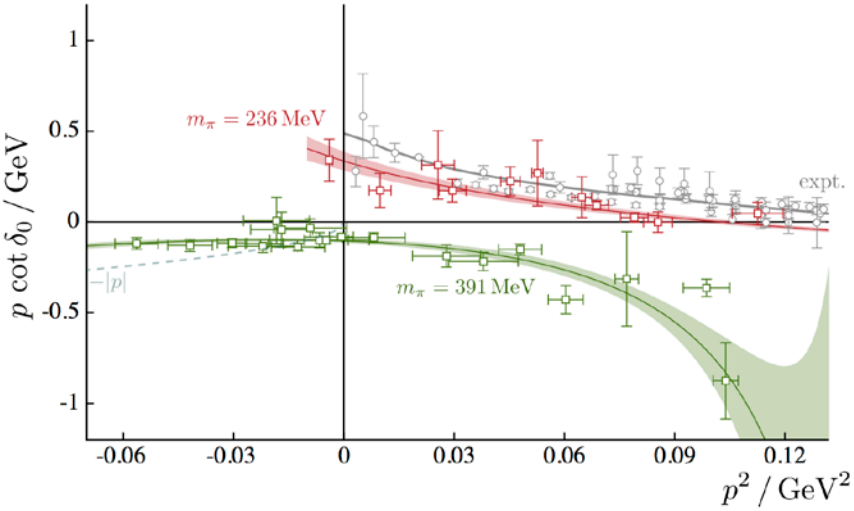

Figure 3: Elastic $\pi\pi$ S-wave phase-shift extracted from lattice QCD at two light quark masses. A clear qualitative change in behavior as the quark mass is reduced is observed, which we find to be due to the σ meson evolving from a bound-state to a broad resonance.

Since most excited resonances decay to more than one final state, it was necessary to extend the finite-volume formalism to handle coupled-channel scattering. This has been done by several group using different methods, all of which lead to the same formalism. We presented one derivation in Ref [11], and therein also illustrated how the formalism could be practically applied in a calculation by building a toy model whose finite-volume spectrum we could interpret.

Subsequently, we performed the first explicit calculation within lattice QCD of the coupled system πK, ηK where we determined the t-matrix in S, P and D-waves. The results were presented in two papers [12, 13]. While this was a considerable achievement, and gave us some good insight into properties of excited K^* resonances, it turned out that the ηK channel is largely decoupled, so the full variety of possibilities within coupled-channel scattering was not manifested in this calculation. Our next calculation investigated a system where the two channels prove to be very strongly coupled, that of $\pi\eta$, $K\bar{K}$. Here we found a t-matrix which features a very strong *cusp-like* behavior at the $K\bar{K}$ threshold, coupled with a rapid turn-on of amplitudes leading to $K\bar{K}$ final states. By analytically continuing the

amplitudes which parameterize the energy dependence of the scattering we were able to find a single nearby resonant pole on the fourth Riemann sheet, close to the $K\overline{K}$ threshold. This pole appears to be the analogue of the experimental $a_0(980)$ at the higher quark mass used in the calculation.

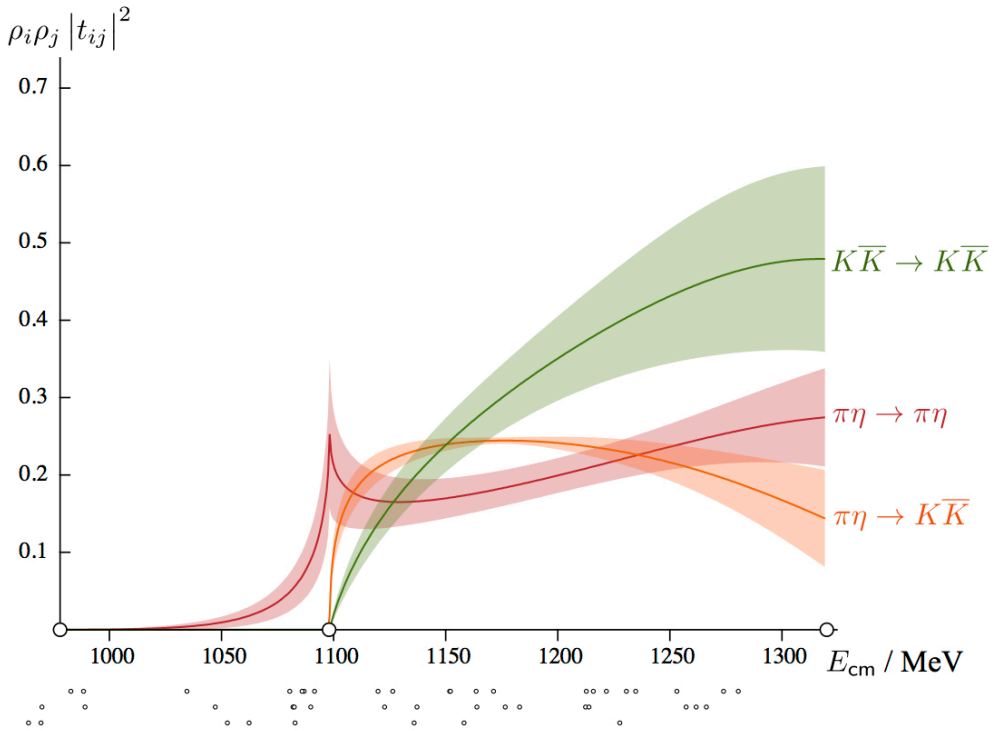

Figure 4: Coupled channel $\pi\eta$, $K\overline{K}$ scattering matrix extracted from lattice QCD with m_π = 391 MeV. The rapid change at $K\overline{K}$ threshold is traced to the presence of an a_0 resonance pole.

Jefferson Lab's GlueX and CLAS12 experiments will use photoproduction as a means to create new meson and baryon states. As such it makes sense to study within lattice QCD the coupling of states to photon currents. The PhD work of Christian Shultz investigated this using the distillation correlation construction technology and operators optimized to overlap onto particular states in the excited spectrum. We found that we could determine transition form-factors between various excitations with π and ρ quantum numbers [14].

In order to determine the coupling to external currents of *resonant* (decaying) states, it is again necessary to consider a finite-volume formalism. This formalism was derived by Raul Briceno [15], and he lead our first explicit lattice QCD calculation, determining the $\pi\gamma^* \to \pi\pi$ P-wave amplitude as a function of photon virtuality and $E_{\pi\pi}$ – the expected resonant behavior due to the ρ was clearly visible and analytic continuation to the ρ -pole lead to a rigorous determination of the $\rho \to \pi\gamma^*$ amplitude. This work was written up in two papers [16, 17].

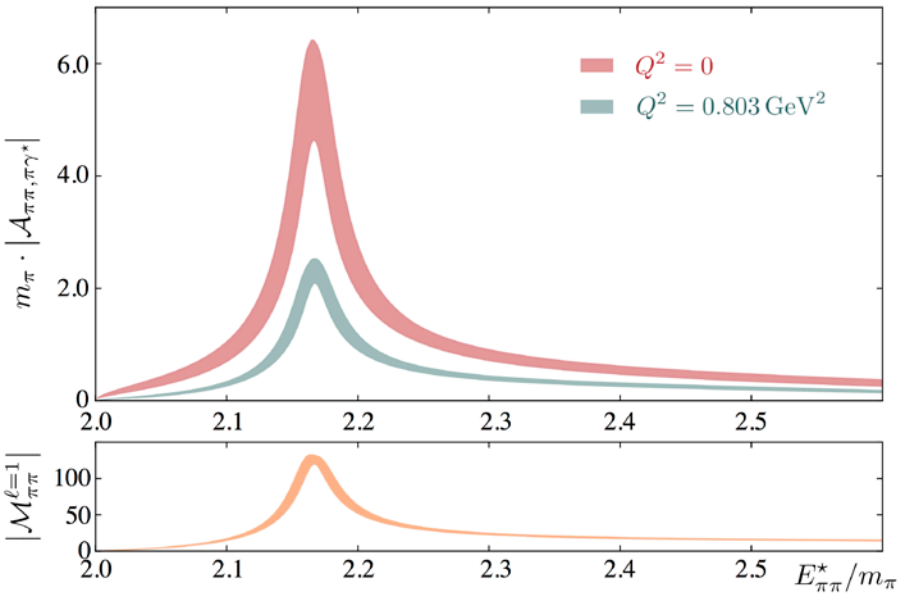

Figure 5: The amplitude $\pi\gamma^*\to\pi\pi$ in *P*-wave extracted from lattice QCD with m_π = 391 MeV at two values of the photon virtuality, Q^2. The presence of a ρ resonance is clearly visible. The elastic $\pi\pi\to\pi\pi$ amplitude is shown below for comparison.

A number of other projects have not yet reached the publication stage. One involved the development of a basis of *tetraquark* operators constructed as the product of a diquark operator and an antidiquark operator, including the full set of possible color-spin-flavor constructions for the diquark. These operators are being used in lattice QCD calculations by Gavin Cheung, a PhD student of Christopher Thomas in DAMTP, Cambridge. He has found that the inclusion of such operators in the double charm channel, in addition to *DD*-like operators, does not change the spectrum of states which resembles that of weakly interacting meson-meson pairs with no sign of tetraquark resonances. He reported on this in a parallel session talk at Lattice 2016. Extension to other flavor channels in ongoing.

Another effort of Dudek's which has not yet reached the publication stage involves the development of a codebase implementing the finite-volume formalism which deals with particles of arbitrary spin, as outlined in Briceno's paper, Ref [18]. A working 'beta' version of the code is being tested currently.

Dudek advocated for experimental and theoretical study of hadron spectroscopy in two documents written for a more general audience. In "*Physics Opportunities with the 12 GeV upgrade at Jefferson Lab*", Dudek co-authored a chapter on meson spectrum studies with GlueX collaboration chairman Curtis Meyer. More recently Dudek coauthored, with GlueX/BES III experimentalists Matt Shepherd and Ryan Mitchell, a review published in Nature entitled "*Searching for the rules that govern hadron construction*".

At ODU Dudek advised two graduate students, Diego Franco and Christian Shultz. Diego Franco chose to leave the PhD program in December 2014 in order to pursue opportunities outside physics. Christian Shultz successfully defended his PhD in March 2015. His dissertation work on radiative transitions computed within lattice QCD is reported on in PRD [14] and he has subsequently collaborated on two papers considering the radiative coupling of the resonant ρ meson [16, 17]. Dr. Shultz is now working in quantitative finance in Boston, MA.

The postdoc support provided by the grant has proven invaluable in moving forward the program of hadron spectroscopy in lattice QCD. By hiring him as a postdoc, Dudek brought Dr. David Wilson, who previously worked on Dyson-Schwinger equations, into the lattice QCD field, and he has matured into an expert in the area of finite-volume studies of coupled-channel scattering. Reflecting this, he was this summer invited to present a prestigious topical plenary at the Lattice conference. He is currently a postdoc at DAMTP in Cambridge and will soon move to a postdoc position at Trinity College, Dublin.

The grant also supported Dr. Raul Briceno for one year. Raul has lead much of the development of the formalism needed for finite-volume studies, and was invited to give a Lattice conference plenary in 2014. The grant has supported him as he has become increasingly involved in numerical lattice QCD calculations. He is currently employed by Jefferson Lab in the Theory Center as the Nathan Isgur Distinguished Postdoctoral Fellow.

Research Metrics (during grant period)

Dudek Refereed Journal Publications

1. *"Searching for the rules that govern hadron construction"*
   M.R. Shepherd, J.J. Dudek, and R.E. Mitchell
   Nature 534 7608 487-493 (2016) [3 cite]

2. *"The $\pi\pi \to \pi\gamma^*$ amplitude and the resonant $\rho \to \pi\gamma^*$ transition from lattice QCD"*
   R.A. Briceno, J.J. Dudek, R.G. Edwards, C.J. Shultz, C.E. Thomas, and D.J. Wilson
   Phys. Rev. D93 114508 (2016) [13 cites]

3. *"An a_0 resonance in strongly coupled $\pi\eta$, KK scattering from lattice QCD"*
   R.A. Briceno, J.J. Dudek, R.G. Edwards, and D.J. Wilson
   Phys. Rev. D93 094506 (2016) [32 cites]

4. *"The resonant $\pi\gamma \to \pi\pi$ amplitude from Quantum Chromodynamics"*
   R.A. Briceno, J.J. Dudek, R.G. Edwards, C.J. Shultz, C.E. Thomas and D.J. Wilson
   Phys. Rev. Lett. 115 242001 (2015) [26 cites]

5. *"Coupled $\pi\pi$, KK scattering in P-wave and the ρ resonance from lattice QCD"*
   D.J. Wilson, R.A. Briceno, J.J. Dudek, R.G. Edwards, and C.E. Thomas
   Phys. Rev. D92 094502 (2015) [66 cites]

6. *"Excited meson radiative transitions from lattice QCD using variationally optimized operators"*
   C.J. Shultz, J.J. Dudek, and R.G. Edwards
   Phys. Rev. D91 114501 (2015) [18 cites]

7. *"Resonances in coupled πK, ηK scattering from lattice QCD"*
   D.J. Wilson, J.J. Dudek, R.G. Edwards, and C.E. Thomas
   Phys.Rev. D91 054008 (2015) [59 cites]

8. *"Resonances in coupled πK, ηK scattering from quantum chromodynamics"*
   J.J. Dudek, D.J. Wilson, R.G. Edwards, and C.E. Thomas
   Phys.Rev.Lett. 113 182001 (2014) [91 cites]

9. "*Toward the excited isoscalar meson spectrum from lattice QCD"*
   J.J. Dudek, R.G. Edwards, Peng Guo and C.E. Thomas
   Phys.Rev. D88 094505 (2013) [63 cites]

10. *"Coupled channel scattering on a torus"*
    Peng Guo, J.J. Dudek, R.G. Edwards and A.P. Szczepaniak
    Phys.Rev. D88 014501 (2013) [66 cites]

11. *"Energy dependence of the ρ resonance in $\pi\pi$ elastic scattering from lattice QCD"*
    J.J. Dudek, R.G. Edwards & C.E. Thomas
    Phys.Rev. D87 034505 (2013) [126 cites]

12. *"Physics Opportunities with the 12 GeV upgrade at Jefferson Lab"*

J.J. Dudek *et al*
Eur. Phys. J. A48 187 (2012) [139 cites]

13. "*S and D-wave phase shifts in isospin-2 ππ scattering from lattice QCD*"
J.J. Dudek, R.G. Edwards & C.E. Thomas
Phys.Rev. D86 034031 (2012) [86 cites]

14. *"Excited and exotic charmonium spectroscopy from lattice QCD"*
L. Liu, G. Moir, M.J. Peardon, S.M. Ryan, C.E. Thomas, P. Vilaseca,
J.J. Dudek, R.G. Edwards, B. Joo & D.G. Richards
JHEP 07 (2012) 126 [156 cites]

15. *"Hybrid baryons in QCD"*
J.J. Dudek and R.G. Edwards
Phys.Rev. D85 054016 (2012) [66 cites]

16. *"Helicity operators for mesons in flight on the lattice"*
C.E. Thomas, R.G. Edwards, and J.J. Dudek
Phys.Rev. D85 014507 (2012) [47 cites]

17. *"The lightest hybrid meson supermultiplet in QCD"*
J.J. Dudek
Phys.Rev. D84 074012 (2011) [80 cites]

18. *"Excited state baryon spectroscopy from lattice QCD"*
R.G. Edwards, J.J. Dudek, D.G. Richards and S.J. Wallace
Phys.Rev. D84 074508 (2011) [259 cites]

Dudek Presentations (39)

1. *"An a_0 resonance in strongly coupled πη, KK scattering from lattice QCD"*
Lattice 2016
Southampton, UK, July 2016

2. *"Hadrons from lattice QCD"* [invited]
Lepton-Nucleus scattering XIV
Elba, Italy, June 2016

3. *"Thinking inside the box – hadron resonances from lattice QCD"* [invited]
A new era for hadron-particle physics
Jefferson Lab, June 2016

4. *"The elusive excited glue of QCD"* [invited]
Physics Department Colloquium
College of William and Mary, Jan 2016

5. *"Hadron spectroscopy and QCD"* [invited]
Theory Center Seminar
Jefferson Lab, Jan 2016

6. *"Hadron scattering and resonances from QCD"* [invited]
"Modern Exotic Hadrons" workshop
INT, Seattle, WA, Nov 2015

7. *"Hadron spectroscopy and resonances from QCD"* [invited]
XVI Conference on Hadron Spectroscopy
Newport News, VA, Sept 2015

8. *"Hadron resonances from QCD (?)"* [invited lecture series]
2015 International Summer School on Reaction Theory
Bloomington, IN, June 2015

9. *"Hadron resonances from QCD"* [invited]
21st International Conference on Few-Body Problems in Physics
Chicago, IL, May 2015

10. *"The excited hadron spectrum from lattice QCD"* [invited]
Bound states in QCD and beyond
St. Goar, Germany, Mar 2015

11. *"Coupled $\pi K/\eta K$ scattering from QCD"* [invited]
Theory Center Seminar,
Jefferson Lab, October 2014

12. *"New hadrons from lattice QCD"* [invited]
4th Joint Meeting of the Nuclear Physics Divisions of the APS and the Phys. Soc. of Japan, Hawaii, October 2014

13. *"Hadron Spectroscopy from lattice QCD"* [invited]
Advances and perspectives in computational nuclear physics,
Hawaii, October 2014

14. *"Overview of lattice spectroscopy and the connection to experiments"* [invited]
USQCD All-hands meeting
Jefferson Lab, April 2014

15. *"The elusive excited glue of QCD"* [invited]
Physics Department Colloquium
Indiana University, Bloomington, IN, Feb 2014

16. *"Lattice QCD update"* [invited]
GlueX Collaboration Meeting
Jefferson Lab, Feb 2014

17. *"Hadron resonances from QCD ?"* [invited]
Physics Department Nuclear Seminar
Florida State University, Tallahassee, FL, Nov 2013

18. *"The elusive excited glue of QCD"* [invited]

Physics Department Colloquium
Florida State University, Tallahassee, FL, Nov 2013

19. *"Understanding the hadron spectrum from QCD"* [invited]
Physics Department and INFN Seminar
Genova University, Genova, Italy, Oct 2013

20. *"Gluonic excitations in the hadronic spectrum"* [invited]
MENU 2013
Rome, Italy, Sept 2013

21. *"Hadron resonances from QCD?"* [invited]
Hall-B/Physics Analysis Center Meeting
Jefferson Lab, Sept 2013

22. *"Understanding the hadron spectrum in QCD"*
ATHOS 2013
Kloster Seeon, Germany, May 2013

23. *"Hadron scattering amplitudes from lattice QCD"*
ATHOS 2013
Kloster Seeon, Germany, May 2013

24. *"Gluonic excitations in QCD"* [invited]
LNS Colloquium
MIT, Cambridge, MA, Apr 2013

25. *"Advances in meson spectroscopy"* [invited]
7th LNF mini-workshop series: JLab at 12 GeV
INFN, Frascati, Italy, Dec 2012

26. *"Meson spectroscopy and GlueX"* [invited]
Presentation to NSAC subcommittee
Rockville, MD, Sept 2012

27. *"The elusive excited glue of Quantum Chromodynamics"* [invited]
Physics Department Colloquium
Old Dominion University, Norfolk, VA, Aug 2012

28. *"Meson spectra from lattice QCD"* [invited]
Seventh International Workshop on Chiral Dynamics
Jefferson Lab, Aug 2012

29. *"The hadron spectrum from lattice QCD"* [invited]
Fourth Workshop on Hadron Physics in China and Opportunities in US
KITPC, Beijing, China, July 2012

30. *"The hadron spectrum from lattice QCD"* [invited]
From nucleon structure to nuclear structure and compact astrophysical objects
KITPC, Beijing, China, July 2012

31. *"ππ scattering"* [invited]
4th International workshop on Lattice Hadron Physics
Adelaide, Australia, July 2012

32. *"Spectroscopy Overview"* [invited]
Lattice 2012
Cairns, Australia, June 2012

33. *"Lattice QCD and the hadron spectrum"* [invited lecture series]
HUGS
Jefferson Lab, June 2012

34. *"Lattice QCD and the hadron spectrum"* [invited lecture series]
Jefferson Lab Advanced Study Institute
Williamsburg, VA, June 2012

35. *"Gluonic excitations in QCD"* [invited]
Physics Division Colloquium
Argonne National Lab, IL, Mar 2012

36. *"The role of gluonic excitations in the hadron spectrum – a view from lattice QCD"* [invited]
Workshop on confinement physics
Jefferson Lab, Mar 2012

37. *"Gluonic excitations in QCD"* [invited]
Physics department seminar
U. Maryland, College Park, MD, Feb 2012

38. *"Gluonic hadrons"* [invited]
Physics seminar
LBNL, Berkeley, CA, Oct 2011

39. *"Hybrid Baryons"* [invited]
GlueX Collaboration Meeting
Jefferson Lab, Oct 2011

Dudek also declined a large number of invitations to speak (typically 5-10 invitations per year), including a Lattice conference plenary, in order to retain sufficient time to devote to research and teaching.

Student & Postdoc Publications and Presentations in grant period

"Relativistic, model-independent, multichannel 2→2 transition amplitudes in a finite volume"
R.A. Briceno and M.T. Hansen
Phys. Rev. D94 013008 (2016) [12 cites]

"Multichannel 0→2 and 1→2 transition amplitudes for arbitrary spin particles in a finite volume"

R.A. Briceno and M.T. Hansen
Phys. Rev. D92 074509 (2015) [20 cites]

"Connecting physical resonant amplitudes and lattice QCD"
D.R. Bolton, R.A. Briceno and **D.J. Wilson**
Phys. Lett. B757 50 (2016) [10 cites]

"Studies of Nucleon Resonance Structure in Exclusive Meson Electroproduction"
I.G. Aznauryan et al (incl. **D.J. Wilson**)
Int. J. Mod. Phys. E22 1330015 (2013) [84 cites]

"Electric dipole moment of the ρ meson"
M. Pitschmann et al (incl. **D.J. Wilson**)
Phys. Rev. C87 045207 (2013) [15 cites]

"Spectrum of hadrons with strangeness"
Chen Chen et al (incl. **D.J. Wilson**)
Few Body Syst. 53 293 (2012) [51 cites]

David Wilson gave 17 talks while supported by the grant, Raul Briceno gave 10 talks and Christian Shultz gave 9 talks.

Bibliography and references cited

[1] "*Toward the excited isoscalar meson spectrum from lattice QCD",* **Phys.Rev. D88 094505 (2013)**
J.J. Dudek, R.G. Edwards, Peng Guo and C.E. Thomas

[2] "*Isoscalar meson spectroscopy from lattice QCD",* **Phys.Rev. D83 111502 (2011)**
J.J. Dudek, R.G. Edwards, B. Joo, M.J. Peardon, D.G. Richards and C.E. Thomas

[3] "*The lightest hybrid meson supermultiplet in QCD",* **Phys.Rev. D84 074012 (2011)**
J.J. Dudek

[4] *"Hybrid baryons in QCD",* **Phys.Rev. D85 054016 (2012)**
J.J. Dudek and R.G. Edwards

[5] *"Excited and exotic charmonium spectroscopy from lattice QCD",* **JHEP 07 (2012) 126**
L. Liu, G. Moir, M.J. Peardon, S.M. Ryan, C.E. Thomas, P. Vilaseca,
J.J. Dudek, R.G. Edwards, B. Joo & D.G. Richards

[6] "*S and D-wave phase shifts in isospin-2 ππ scattering from lattice QCD*", **Phys.Rev. D86 034031 (2012)**
J.J. Dudek, R.G. Edwards & C.E. Thomas

[7] *"Energy dependence of the ρ resonance in ππ elastic scattering from lattice QCD",* **Phys.Rev. D87 034505 (2013)**
J.J. Dudek, R.G. Edwards & C.E. Thomas

[8] *"Coupled ππ, KK scattering in P-wave and the ρ resonance from lattice QCD",* **Phys. Rev. D92 094502 (2015)**
D.J. Wilson, R.A. Briceno, J.J. Dudek, R.G. Edwards, and C.E. Thomas

[9] *"Connecting physical resonant amplitudes and lattice QCD",* **Phys. Lett. B757 50 (2016)**
D.R. Bolton, R.A. Briceno and **D.J. Wilson**

[10] *"Isoscalar ππ scattering and the σ meson resonance from QCD",* **arXiv:1607.05900 [hep-ph]**
R.A. Briceno, J.J. Dudek, R.G. Edwards, and D.J. Wilson

[11] *"Coupled channel scattering on a torus",* **Phys.Rev. D88 014501 (2013)**
Peng Guo, J.J. Dudek, R.G. Edwards and A.P. Szczepaniak

[12] *"Resonances in coupled πK, ηK scattering from quantum chromodynamics",* **Phys.Rev.Lett. 113 182001 (2014)**
J.J. Dudek, D.J. Wilson, R.G. Edwards, and C.E. Thomas

[13] *"Resonances in coupled πK, ηK scattering from lattice QCD",* **Phys.Rev. D91 054008 (2015)**
D.J. Wilson, J.J. Dudek, R.G. Edwards, and C.E. Thomas

[14] *"Excited meson radiative transitions from lattice QCD using variationally optimized operators",* **Phys. Rev. D91 114501 (2015)**
C.J. Shultz, J.J. Dudek, and R.G. Edwards

[15] *"Multichannel 1→2 transition amplitudes in a finite-volume*", **Phys. Rev. D91 034501 (2015)**
R.A. Briceno, M.T. Hansen and A. Walker-Loud

[16] *"The resonant πγ→ππ amplitude from Qunatum Chromodynamics",* **Phys. Rev. Lett. 115 242001 (2015)**
R.A. Briceno, J.J. Dudek, R.G. Edwards, C.J. Shultz, C.E. Thomas and D.J. Wilson

[17] *"The ππ→πγ* amplitude and the resonant ρ→πγ* transition from lattice QCD",* **Phys. Rev. D93 114508 (2016)**
R.A. Briceno, J.J. Dudek, R.G. Edwards, C.J. Shultz, C.E. Thomas and D.J. Wilson

[18] *"Two-particle multichannel systems in a finite-volume with arbitrary spin",* **Phys. Rev. D89 074507 (2014)**
R.A. Briceno

[19] *"From controversy to precision on the sigma meson: a review on the status of the non-ordinary $f_0(500)$ resonance"*, **arXiv:1510.00653 [hep-ph]**
J.R. Pelaez