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Energy Changes in Transforming Solids

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During the period covered by this report, significant advances in several diverse areas have been achieved as outlined below under separate headings:

1. Damage Mechanics
2. Functionally Gradient Materials with Defects
3. Problems in Heterogenization
4. Conservation Laws with Application to Fracure and Defect Mechanics

Damage Mechanics

In collaboration with the late Professor J. Kestin of Brown University we succeeded during the last few years and with the sponsorship, in part, of the Department of Energy, to develop a rudimentary model of damage based on thermodynamic considerations. This initial theory leads to results which are in encouraging agreement with the few experimental data which have been published so far.

In developing the initial elements of this theory we obviously based ourselves on the "classical" or "conventional" thermodynamics, as expounded by J. Kestin in a series of recent papers and who was one of the very few thermodynamicists interested in applications to solid mechanics.

It is the study of irreversible processes which leads thermodynamicists to rather divergent views. We adopted the following position: A distinction has to be made between intensive parameters which appear in physical space and those which describe states of constrained equilibrium in the Gibbsian phase (or state) space. The latter consists of a set of extensive variables, namely the internal energy, the external deformation variables and the internal deformation variables, which, by contrast to the intensive ones, can be measured (in principle) in equilibrium as well as in nonequilibrium. For a given system under study, the introduction of these different variables is based on physical insight, experimental findings, intuition, etc.

Since temperature and entropy can be defined (or introduced) only for reversible processes it is imperative to introduce the so-called principle of local state (or the method of local equilibrium). This principle is applied by associating with every nonequilibrium state an accompanying equilibrium state of equal values of internal energy, as well as external and internal deformation variables. It is then asserted that the temperature and entropy in physical space (irreversible processes) can be approximated by their values in the Gibbsian phase space (reversible processes) by standard, classical methods. A continuous sequence of accompanying equilibrium states may be called an accompanying reversible process and it is conceived as an adiabatic projection of the continuous sequence of nonequilibrium states which constitute the irreversible process. This allows to express the classical Gibbs equation in rate form and to derive explicit expressions for the rate of entropy production by eliminating the rate of internal energy between it and the energy balance equation.

The essential part of this methodology consists in the formulation of the Gibbs equation for the accompanying process in phase space. This is obtained from the knowledge of the physics of the situation and leads to the identification of the internal deformation variables (which can be observed and measured, but not controlled) and the imagined virtual (i.e. reversible) work done against them by the associated affinities.

Based on the methodology briefly summarized above we were successful in obtaining, for one-dimensional systems and for isothermal processes, a basic theory of damage in brittle solids. It was found that the response of the elastic bar depends not only on the loading or straining processes, but also on some global geometric parameters, as well as on the temperature. These conclusions seem to agree with the very few experimental results. A summary of this work was published in [1-2].

Functionally Gradient Materials with Defects

Functionally gradient materials (FGM) were first produced in Japan for the primary purpose of developing thermal protection for a future space plane which would have to withstand severe aerdynamic heating. Since that time it was found that FGM, whose material properties vary continuously in a particular direction, would have useful applications in a variety of industries. This writer, together with his associates, has already been involved in some aspects concerning stress analysis of FGM (e.g., crack mechanics in smoothly nonhomogeneous materials as well as composites with nonhomogeneous fibers) and has been invited to present a paper [3] at the 3rd International Symposium on Structural and Functional Gradient Materials held at the Swiss Federal Institute of technology in Lausanne, Switzerland, in October 1994.

The micromechanical analysis is effected by employing the newly proposed methodology of heterogenization, whereby the solution to a heterogeneous problem is obtained by performing a transformation on the solution to a corresponding homogeneous problem.

We proceeded to write down the solution for a multilayered fiber perfectly bonded to an infinite matrix, which is subjected to arbitrary loading. The solution is then expressed in terms of the solution of the corresponding homogeneous problem i.e., when the fiber is absent and the matrix material still subjected to the same loading (singularities) occupies the whole space. We achieve this by exploiting a connection between the solution to the heterogeneous problem and a group structure on the set $(-1, 1)$ of real numbers x such that $-1 < x < 1$.

We then considered the case where the shear modulus is a continuous function of r throughout an annular region. The problem is formulated in a manner that leads to a Riccati differential equation, the solution of which can be obtained by considering the limit of our multilayered fiber solution.

Finally, the results are illustrated by considering the problem of reducing stress concentration around holes.

Problems in Heterogenization

It is recalled that in elastostatics this term has been introduced by the writer and his colleagues and refers to the passage from a homogeneous region subjected to some loads (singularities), which represents a simple problem, to the same region with inclusions (or holes) and subjected to the same loads. This novel approach turned out to extremely fruitful in treating in a most effective manner a variety of problems as follows:

To assess the influence of boundary conditions on the stress distribution around a single circular inclusion in plane elastostatics, by contrast to previous work for a bonded inclusion, an inclusion with a slipping interface was considered. This work, [4], may be summarized as follows:

It is shown that the solution, in plane elastostatics, for an infinite domain subjected to arbitrary loading and into which a circular inclusion of a different elastic material has been *inserted (heterogeneous problem) may be obtained, in the case of a slipping interface, from the solution of the corresponding homogeneous problem (i.e., when there is no inclusion) merely by a single quadrature and simple algebraic manipulations. This novel procedure of heterogenization is illustrated by several specific examples.*

A summary of the work on heterogenization at that time was also presented at the Eighth Symposium on Energy Engineering Sciences and published in the Proceedings [5]. An extension of the heterogenization procedure from pure elastostatics to piezoelectricity was accomplished in [6] and that work can be summarized as follows:

It was recently shown that the solution, in plane elastostatics, for an infinite domain with a bonded circular inclusion (heterogeneous problem), may be obtained from the solution of the corresponding homogeneous problem, merely by substitution into a simple algebraic expression, (heterogenization). This relation is universal in the sense of being independent of the loading considered.

In the present work the heterogenization procedure is extended to piezoelectric materials and worked out for antiplane deformation. Both the matrix and the bonded inclusion are taken to possess the symmetry of a hexagonal crystal in the 6mm class. The system is subjected to mechanical and electric sources, which produce only out-of-plane displacements and in-plane electric field, but are otherwise arbitrary.

The solution is obtained as a simple transformation, based on involution, applied to the solution of the corresponding homogeneous problem (i.e. the problem of the matrix material occupying the full space and subjected to the same sources). Several special cases are discussed and specific examples illustrate the general methodology.

Some earlier work on bonded inclusions, both with circular and straight boundaries was presented and published during the period of this contract [7], and was summarized as:

It is shown that the solution, in plane elastostatics, for an infinite domain with a bonded circular inclusion (heterogeneous problem), may be obtained from the solution of the corresponding homogeneous problem merely by substitution into a simple algebraic expression (heterogenization). This relation is universal in the sense of being independent of the loading considered. The case of two half-planes occupied by two dissimilar materials and bonded along a straight boundary is obtained as a limiting case.

The passage from a single circular inclusion to two inclusions was carried out for harmonic problems (e.g. anti-plane strain) in [8] with the following summary:

In this paper, we derive the solution for two circular cylindrical elastic inclusions perfectly bonded to an elastic matrix of infinite extent, under anti-plane deformation. The two inclusions have different radii and possess different elastic properties. The matrix is subjected to arbitrary loading. The solution is obtained, via iterations of Möbius transformations, as a rapidly convergent series with an explicit general term involving the complex potential of the corresponding homogeneous problem, i.e., when the inclusions are absent and the matrix material occupies the entire space and is subjected to the same loading. This procedure has been termed "heterogenization."

The technique used can be applied to problems governed by Laplace's equation.

Finally some remarks are included concerning the relation of our solution to the theory of discontinuous groups and automorphic functions and possible generalizations to multiple inclusions.

This same problem was somewhat reformulated in [9] with the important finding that it was possible to obtain several exact results. The abstract of this work reads as follows:

The heterogenization technique, recently developed by the authors, is applied to the problem, in antiplane elastostatics, of two circular inclusions of arbitrary radii and of different shear moduli, and perfectly bonded to a matrix, of infinite extent, subjected to arbitrary loading. The solution is formulated in a manner which leads to some exact results. Universal formulae are derived for the stress field at the point of contact between two elastic inclusions. It is also discovered that the difference in the displacement field, at the limit points of the Apollonius family of circles to which the boundaries of the inclusions belong, is the same for the heterogeneous problem as for the corresponding homogeneous one. This discovery leads to a universal formula for the average stress between two circular holes or rigid inclusions. Moreover, the asymptotic behavior of the stress field at the closest points of two circular holes or rigid inclusions approaching each other is also studied and given by universal formulae, i.e., formulae which are independent of the loading being considered.

Some years ago this writer was asked by IUTAM (International Union of Theoretical and Applied Mechanics) and CISM (International Center for Mechanical Sciences) to organize a summer school at Udine (Italy), the seat of CISM. He chose the subject

"Modeling of Defects and Fracture Mechanics" and this Second International Summer School on Mechanics was held September 2-6, 1991. The Proceedings were published by Springer-Verlag (Wien-New York) as "Courses and Lectures No. 331" in 1993. As a part of the contributions by this writer these proceedings contain an article on the application of the heterogenization methodology to the analysis of elastic bodies with defects [10] whose summary is as follows:

Numerous defects in materials may be characterized as cavities or inclusions within the framework of linear elasticity. Recently, the author and co-workers have developed a general procedure termed "Heterogenization," which permits an efficient analysis of elastic bodies with circular cavities or inclusions in terms of correspondingly loaded homogeneous bodies without such defects. One of the features of this novel methodology is that the expressions derived are completely independent of the loading. The present contribution summarizes the essentials of this methodology, which is based on a certain involution correspondence, and considers specific applications to cavities, elastic inclusions with different boundary conditions, as well as to coupled fields such as piezoelectricity and thermoelasticity.

At an International Conference on Micromechanics of Concrete and Cementitious Composites held at the Swiss Federal Institute of Technology in Lausanne (Switzerland) this writer was invited to present the opening general lecture on the foundations and current trends in micromechanics [11] where the heterogenization procedure was also mentioned, as noted in the abstract:

In this introductory lecture the goals and the modes of activity in micromechanics will be briefly described, placing emphasis on current trends and mentioning some gaps in our knowledge.

The most developed area of analytical micromechanics, namely that which is based on a continuum theory of elasticity, will then be discussed in more specific terms. This area deals with a mathematical description of defects in materials, such as dislocations, voids, inclusions, inhomogeneities, cracks, etc., and attempts to provide predictive tools to cover a wide range of material behavior, such as plasticity, creep, fracture, fatigue, damage, phase transformations, including the influence of residual stresses, texture and thermal effects, among others. Polycrystalline and composite materials are frequently of special concern.

It will be pointed out that, remarkably, the work of one man in micromechanics has *led to the naming of two different tensors in his memory. It is the late J. D. Eshelby who made in the nineteen fifties the now classical contributions to the stress distributions in and around inclusions and inhomogeneities, as well as to the concept of forces acting on defects, such as voids or cracks. In each of these two areas he found it expedient and necessary to introduce a new tensor, in the former of fourth rank, in the latter of second rank. These two tensors will be derived and their usefulness in micromechanics emphasized. Some current work which involves both Eshelby tensors will be described and applications to modeling of damage on the basis of thermodynamics with internal variables will be mentioned. A recently developed methodology to deal with inclusions, termed "Heterogenization" and based on the Kelvin transformation, will also be discussed.*

Returning to piezoelectricity, two piezoelectric fibers embedded in an intelligent material were considered in [12] where, again, some exact results were established as described in the following summary:

In this paper we are concerned with the problem of two circular piezoelectric fibers, of different radii and distinct material properties, perfectly bonded to a host intelligent material, of infinite extent. The matrix material may be piezoelectric or nonpiezoelectric but, together with fibers' materials, it possesses the symmetry of a hexagonal crystal in the 6mm class. The system is subjected to electromechanical loading (singularities) which produce out-of-plane displacement and in-plane electric fields, but are otherwise arbitrary.

Within the framework of the procedure of heterogenization, recently developed by the authors, the solution is sought as a transformation applied to the solution of the corresponding homogeneous problem (i.e., the problem of the host material occupying the full space and subjected to the same sources). The solution is formulated in a manner which leads to some exact results. Universal formulae are derived for the electromechanical field at the point of contact of two piezoelectric fibers. Some quantities which are invariant under the transformation, i.e., quantities which take the same values in the heterogeneous as in the corresponding homogeneous problems, are also discovered. The ramifications of this discovery are investigated. Moreover, the asymptotic behavior of the electromechanical field at the closest points of two plated circular holes or rigid conductors, in an intelligent matrix material, approaching each other is also studied and given by universal formulae, i.e., formulae which are independent of the electromechanical sources. The interaction of *the fibers with host-material microdefects, such as dislocations, electric line charges and microvoids, is scrutinized. The possibility of manipulating the electrical potential to reduce the high stress level is also discussed.*

A step from homogeneous inclusions to layered ones was undertaken in [13] as summarized below:

In this paper we consider, within the framework of the linear theory of elasticity, the problem of circularly cylindrical and plane layered media under antiplane deformations. The layers are, in the first instance, coaxial cylinders of annular cross-sections with arbitrary radii and different shear moduli. The number of layers is arbitrary and the system is subjected to arbitrary loading (singularities). The solution is derived by applying the heterogenization technique recently developed by the authors. Our formulation reduces the problem to solving linear functional equations and leads naturally to a group structure on the set t of real numbers such that $-1 < t < 1$. This allows us to write down the solution explicitly in terms of the solution of a corresponding homogeneous problem subjected to the same loading. In the course of these developments, it is discovered that certain types of inclusions do not disturb a uniform longitudinal shear. That these inclusions, which may be termed "stealth," are important in design and hole reinforcements is pointed out. By considering a limiting case of the aforementioned governing equations, the solution of plane layered media can be obtained. Alternatively, our formulation leads, in the case of plane layered media, to linear functional equations of the finite difference type which can be solved by several standard techniques.

Further aspects of heterogenization were considered in [14], giving emphasis to elastically embedded inclusions which could simulate a coating around fibers in a composite material, with the following summary:

In this paper, we briefly review some of the recent developments in the methodology of heterogenization. A connection between a group structure on the set $(-1, 1)$ of real numbers t such that $-1 < t < 1$, and the elastostatics of a multilayered fiber perfectly bonded to an infinite matrix is pointed out. Also, universal formulae, pertaining to the solution of two circular elastic inclusions perfectly bonded to a matrix, of infinite extent, which is subjected to arbitrary loading, are discussed. As a novel illustration of the heterogenization procedure, we study here the case where the inclusions are elastically (i.e., "imperfectly") embedded *in the matrix. Several cases are presented and discussed.*

It is to be stressed that the heterogenization methodology, by contrast to other procedures to treat heterogeneous bodies, proved extremely fruitful and can be exploited further in numerous novel settings.

More recently, in one study this methodology was applied to the problem, in plane elastostatics, of two circular inclusions of arbitrary radii and of different elastic moduli, and perfectly bonded to a matrix, of infinite extent, which is subjected to arbitrary loading. The solution was formulated in a manner which leads to governing functional differential equations, i.e., equations were then solved by employing novel techniques. Several illustrative examples are being worked out. Particular attention is devoted to the limiting, but important, cases of two rigid inclusions or circular holes. Moreover, the asymptotic behavior of the stress field at the closest points of these two defects as they approach each other is being investigated.

Some of the results were presented by invitation at the recent National Congress of Applied Mechanics and a paper is being prepared for journal publication.

In another study the effect of a dislocation in a fiber-reinforced composite was investigated and presented by invitation at the ASME Annual Meeting in November 1994. A paper summarizing these findings is being completed and will be also submitted for journal publication.

Conservation Laws with Application to Fracture and Defect Mechanics

Our work in the area of conservation laws during the life of the subject contract has successfully proceeded in several different directions.

It has to be realized that the principal basis for constructing conservation laws, i.e. divergence-free expressions, was E. Noether's first theorem. Its shortcoming consists in the fact that Noether's point of departure is the Lagrangian of the system under consideration, i.e. the strong restriction is imposed that the governing differential equations be in fact the Euler-Lagrange equations of a variational problem. Thus, specifically, dissipative systems, (since, in general, they do not possess a Lagrangian), cannot be treated by Noether's theorem and thus no systematic way exists to construct conservation laws. We succeeded [15], however, to establish a systematic procedure for constructing conservation laws which is not based on the Lagrangian, but rather directly on the differential equations of the system in question. Thus our procedure, which we later began to call the "Neutral Action" method, may be applied to a system regardless as to whether or not it possesses a Lagrangian and we showed later that our method, for systems with a Lagrangian, embodies the so-called Bessel-Hagen extension to Noether's theorem. In [15] several applications to dissipative systems have been worked out. But it turns out that even for non-dissipative systems our methodology is more advantageous as compared to the usage of a procedure based on Noether. This is illustrated with regard to non-homogeneous Bernoulli-Euler beams [16] with the following summary:

It is the purpose of this paper to construct conservation laws for the statics and dynamics of nonhomogeneous Bernoulli-Euler beams. To derive these conservation laws, we will use the newly proposed Neutral Action (NA) method (Honein et al., 1991, Phys. Lett., 155, 223-224; Chien, 1992, Conservation laws in nonhomogeneous and dissipative mechanical systems, Ph.D. Dissertation, Stanford University). The conservation laws derived should be useful characterizing concentrated defects, such as cracks and interfaces, in an otherwise smoothly nonhomogeneous beam.

Classically, Noether's first theorem (Noether, 1918, Transport Theory Stat. Phys. 1, 186-207) is available for construction of conservation laws for Lagrangian systems, such as a Bernoulli-Euler beam. However, since the NA method is applicable to dissipative as well as to Lagrangian systems, and since it encompasses Noether's method within the realm of Lagrangian systems, we choose to employ the NA method to achieve our purpose here. A comparison of these two methodologies, with an example illustrating the relative efficiency of the NA method over Noether's approach, will also be presented.

Conservation laws for non-homogeneous Mindlin plates were established [17]. Again, these laws should be useful in dealing with concentrated defects, such as cracks and interfaces at phase boundaries in otherwise smoothly non-homogeneous plates.

Conservation laws for linear viscoelasticity in one and two dimensions were constructed in [18] again providing potential tools for the study of fracture in such media.

Coupled fields were also considered and, specifically, several conservation laws for thermoelasticity and poroelasticity were established. Both, time-independent (uncoupled) and time-dependent versions of the theory in two dimensions were investigated. Advantage was taken due to the analogy between thermoelasticity and poroelasticity. In particular, a path-independent integral was derived, which represents, as we proved, the energy release rate accompanying crack growth [19].

In a related series of papers cracked beams and pipes were investigated further. Novel, accurate estimates of stress intensity factors for such systems were advanced in [20] and [21], including certain asymptotic expansions. Curved cracked beams were considered in [22].

Summarizing this section, it should be emphasized that conservation laws play an important role not only in defect-and fracture-mechanics, but also in numerical work (they can be built-in into various algorithms), in establishing existence and uniqueness theorems and in studying stability of systems.

References

[1] E. Honein, T. Honein, J. Kestin, and George Herrmann, "On Thermodynamic Modeling of Damage in Elastic Solids," *Nonlinear Thermodynamical Processes in Continua*, Ein gemeinsamer Workshop der TU Berlin und des Wissenschaftskollegs zu Berlin, Editors: W. Muschik and G.A. Maugin, Berlin, Heft 61, 1992, pp. 66-81.

[2] T. Honein, E. Honein, and G. Herrmann, "A Thermodynamically-Based Theory of Damage in Brittle Structures," *Proc. US-Europe Workshop on Fracture and Damage of Quasi-Brittle Materials*, Prague, 1994.

[3] T. Honein, E. Honein, and G. Herrmann, "Stress Analysis of Functionally Gradient Materials Based on a Heterogenization Procedure," *3rd International Symposium on Structural and Functional Gradient Materials,* October 1994, Lausanne, Switzerland.

[4] T. Honein and G. Herrmann, "A Circular Inclusion with Slipping Interface in Plane Elastostatics," *Micromechanics and Inhomogeneity, The Toshio Mura 65th Anniversary Volume,* pp. 127-136, Springer-Verlag New York, 1990a.

[5] T. Honein, E. Honein, and G. Herrmann, "Mono- and Poly-Circle Theorems in

Solids and Fluids," *Proceedings of the Eighth Symposium on Energy Engineering Sciences,* "Micro/Macro Studies of Multiphase Media," Conf-9005183, Argonne National Laboratory, Argonne, Illinois, pp. 190-196, 1990b.

[6] T. Honein, B. Honein, E. Honein, and G. Herrmann, "On Piezoelectric Circular Inclusions," *Mechanical Modelling of New Electromagnetic Materials,* R.K.T. Hsieh (Editor), pp. 259-266, Elsevier Science Publishers B.V., 1990c.

[7] T. Honein and G. Herrmann, "On Bonded Inclusions with Circular or Straight Boundaries in Plane Elastostatics," *Journal of Applied Mechanics,* **112**, pp. 850-856, 1990.

[8] E. Honein, T. Honein, and G. Herrmann, "On Two Circular Inclusions in Harmonic Problems," *Quarterly of Applied Mathematics,* Vol. L, No. 3, pp. 479-499, 1992.

[9] E. Honein, T. Honein, and G. Herrmann, "Further Aspects of the Elastic Field for Two Circular Inclusions in Antiplane Elastostatics," *Journal of Applied Mechanics,* **59**, pp. 774-779, 1992.

[10] G. Herrmann, "Application of the Heterogenization Methodology to the Analysis of Elastic Bodies with Defects," CISM Courses and Lectures No. 331, Modeling of Defects and Fracture Mechanics, edited by G. Herrmann, Springer-Verlag Publisher, pp. 166-206.

[11] G. Herrmann, "Micromechanics: Some Basic Methods and Current Trends," *Micromechanics of Concrete and Cementitious Composites,* edited by C. Huet, pp. 1-18, Presses Polytechniques et Universitaires Romandes, Lausanne, 1993.

[12] T. Honein, B.V. Honein, E. Honein, and G. Herrmann, "On the Interaction of Two Piezoelectric Fibers Embedded in an Intelligent Material," *ASME, Adaptive Structures and Material Systems,* AD-Vol.. 35, pp. 105-112, edited by G. P. Carman and E. Garcia, Book No. H00873, 1993.

[13] T. Honein, E. Honein, G. Herrmann, "Circularly Cylindrical and Plane Layered Media in Antiplane Elastostatics," *Journal Applied Mechanics,* **61**, pp. 243-249, 1994.

[14] E. Honein, T. Honein, and G. Herrmann, "Aspects of Heterogenization. *J. Appl. Mechs..*

[15] T. Honein, N. Chien, and G. Herrmann, "On Conservation Laws for Dissipative Systems," Ed. Elsevier Science Publishers B.V., Physics Letters A, Vol. 155, No. 4,5, May 1991, pp. 223-224.

[16] N. Chien, T. Honein, and G. Herrmann, "Conservation Laws for Nonhomogeneous Bernoulli-Euler Beams," *Int. J. Solids Structures*, Vol. 30, No. 23, pp. 3321-3335, 1993.

[17] N. Chien, T. Honein, G. Herrmann, "Conservation Laws for Non-Homogeneous Mindlin Plates," *Int. J. Engrg. Sci.*, 1994, in press.

[18] N. Chien, T. Honein, and G. Herrmann, "Conservation Laws for Linear Viscoelasticity," *ZAMP*, Vol. 44, 1993, pp. 44-52.

[19] N. Chien and G. Herrmann, "Conservation Laws for Thermo- or Poroelasticity", *J. Appl. Mechs.*, in press.

[20] H. Gao and G. Herrmann, "On Estimates of Stress Intensity Factors for Cracked Beams and Pipes," *Engineering Fracture Mechanics*, Vol. 41, No. 5, 1992, pp. 695-706.

[21] W. H. Müller, G. Herrmann, and H. Gao, "Elementary Strength Theory of Cracked Beams," *Theoretical and Applied Fracture Mechanics*, Vol. 18, 1993, pp. 163-177.

[22] W. H. Müller, G. Herrmann, and H. Gao, "A Note on Curved Cracked Beams," *Int. Journal of Solids and Structures,* Vol. 30, No. 11, 1993, pp. 1527-1532.

Publications

[1] E. Honein, T. Honein, J. Kestin, and George Herrmann, "On Thermodynamic Modeling of Damage in Elastic Solids," *Nonlinear Thermodynamical Processes in Continua*, Ein gemeinsamer Workshop der TU Berlin und des Wissenschaftskollegs zu Berlin, Editors: W. Muschik and G.A. Maugin, Berlin, Heft 61, 1992, pp. 66-81.

[2] T. Honein, E. Honein, and G. Herrmann, "A Thermodynamically-Based Theory of Damage in Brittle Structures," *Proc. US-Europe Workshop on Fracture and Damage of Quasi-Brittle Materials*, Prague, 1994.

[3] T. Honein, E. Honein, and G. Herrmann, "Stress Analysis of Functionally Gradient Materials Based on a Heterogenization Procedure," *3rd International Symposium on Structural and Functional Gradient Materials,* October 1994, Lausanne, Switzerland.

[4] T. Honein and G. Herrmann, "A Circular Inclusion with Slipping Interface in Plane Elastostatics," *Micromechanics and Inhomogeneity, The Toshio Mura 65th Anniversary Volume,* pp. 127-136, Springer-Verlag New York, 1990a.

[5] T. Honein, E. Honein, and G. Herrmann, "Mono- and Poly-Circle Theorems in Solids and Fluids," *Proceedings of the Eighth Symposium on Energy Engineering Sciences,* "Micro/Macro Studies of Multiphase Media," Conf-9005183, Argonne National Laboratory, Argonne, Illinois, pp. 190-196, 1990b.

[6] T. Honein, B. Honein, E. Honein, and G. Herrmann, "On Piezoelectric Circular Inclusions," *Mechanical Modelling of New Electromagnetic Materials,* R.K.T. Hsieh (Editor), pp. 259-266, Elsevier Science Publishers B.V., 1990c.

[7] T. Honein and G. Herrmann, "On Bonded Inclusions with Circular or Straight Boundaries in Plane Elastostatics," *Journal of Applied Mechanics,* **112**, pp. 850-856, 1990.

[8] E. Honein, T. Honein, and G. Herrmann, "On Two Circular Inclusions in Harmonic Problems," *Quarterly of Applied Mathematics,* Vol. L, No. 3, pp. 479-499, 1992.

[9] E. Honein, T. Honein, and G. Herrmann, "Further Aspects of the Elastic Field for Two Circular Inclusions in Antiplane Elastostatics," *Journal of Applied Mechanics*, **59**, pp. 774-779, 1992.

[10] G. Herrmann, "Application of the Heterogenization Methodology to the Analysis of Elastic Bodies with Defects," CISM Courses and Lectures No. 331, Modeling of Defects and Fracture Mechanics, edited by G. Herrmann, Springer-Verlag Publisher, pp. 166-206.

[11] G. Herrmann, "Micromechanics: Some Basic Methods and Current Trends," *Micromechanics of Concrete and Cementitious Composites,* edited by C. Huet, pp. 1-18, Presses Polytechniques et Universitaires Romandes, Lausanne, 1993.

[12] T. Honein, B.V. Honein, E. Honein, and G. Herrmann, "On the Interaction of

Two Piezoelectric Fibers Embedded in an Intelligent Material," *ASME, Adaptive Structures and Material Systems*, AD-Vol. 35, pp. 105-112, edited by G. P. Carman and E. Garcia, Book No. H00873, 1993.

[13] T. Honein, E. Honein, G. Herrmann, "Circularly Cylindrical and Plane Layered Media in Antiplane Elastostatics," *Journal Applied Mechanics,* **61**, pp. 243-249, 1994.

[14] E. Honein, T. Honein, and G. Herrmann, "Aspects of Heterogenization, *J. Appl. Mechs.*.

[15] T. Honein, N. Chien, and G. Herrmann, "On Conservation Laws for Dissipative Systems," Ed. Elsevier Science Publishers B.V., Physics Letters A, Vol. 155, No. 4,5, May 1991, pp. 223-224.

[16] N. Chien, T. Honein, and G. Herrmann, "Conservation Laws for Nonhomogeneous Bernoulli-Euler Beams," *Int. J. Solids Structures*, Vol. 30, No. 23, pp. 3321-3335, 1993.

[17] N. Chien, T. Honein, G. Herrmann, "Conservation Laws for Non-Homogeneous Mindlin Plates," *Int. J. Engrg. Sci.*, 1994, in press.

[18] N. Chien, T. Honein, and G. Herrmann, "Conservation Laws for Linear Viscoelasticity," *ZAMP*, Vol. 44, 1993, pp. 44-52.

[19] N. Chien and G. Herrmann, "Conservation Laws for Thermo- or Poroelasticity", *J. Appl. Mechs.*, in press.

[20] H. Gao and G. Herrmann, "On Estimates of Stress Intensity Factors for Cracked Beams and Pipes," *Engineering Fracture Mechanics*, Vol. 41, No. 5, 1992, pp. 695-706.

[21] W. H. Müller, G. Herrmann, and H. Gao, "Elementary Strength Theory of Cracked Beams," *Theoretical and Applied Fracture Mechanics*, Vol. 18, 1993, pp. 163-177.

[22] W. H. Müller, G. Herrmann, and H. Gao, "A Note on Curved Cracked Beams," *Int. Journal of Solids and Structures,* Vol. 30, No. 11, 1993, pp. 1527-1532.

[23] E. Honein, T. Honein, and G. Herrmann, "The Elastic Field of Two Circular Inclusions in Plane Elastostatics." To be published.

[24] E. Honein, T. Honein, and G. Herrmann, "The Effect of a Dislocation in a Fiber-

Reinforced Composite." To be published.