Basics of Reentry Vehicle Flight Dynamics

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Outline

- Introduction
- Atmosphere Model
- Earth Model
- Gravity Model
- Aerodynamics
- Trajectory Analysis

Forces and Moments

- A vehicle TRANSLATES, or changes **location**, from one point to another.
- And an object ROTATES, or changes its **attitude**.
- In general, the motion of any object involves both **translation** and **rotation**.
- The translations are in direct response to external FORCES. The rotations are in direct response to external torques or MOMENTS.
- The motion of a vehicle has **coupled translations** and **rotations**.
- Assume that the vehicle translates from one point to another as if all the mass of the vehicle were collected into a single point called the center of gravity (CG).
- We can describe the motion of the CG by using Newton's Laws of Motion.

Vehicle Motion

- *The motion of a vehicle in flight is affected primarily by:*
  - Atmosphere
  - Gravity
  - Earth Rotation
  - Aerodynamics (shape, asymmetries)
  - Propulsion

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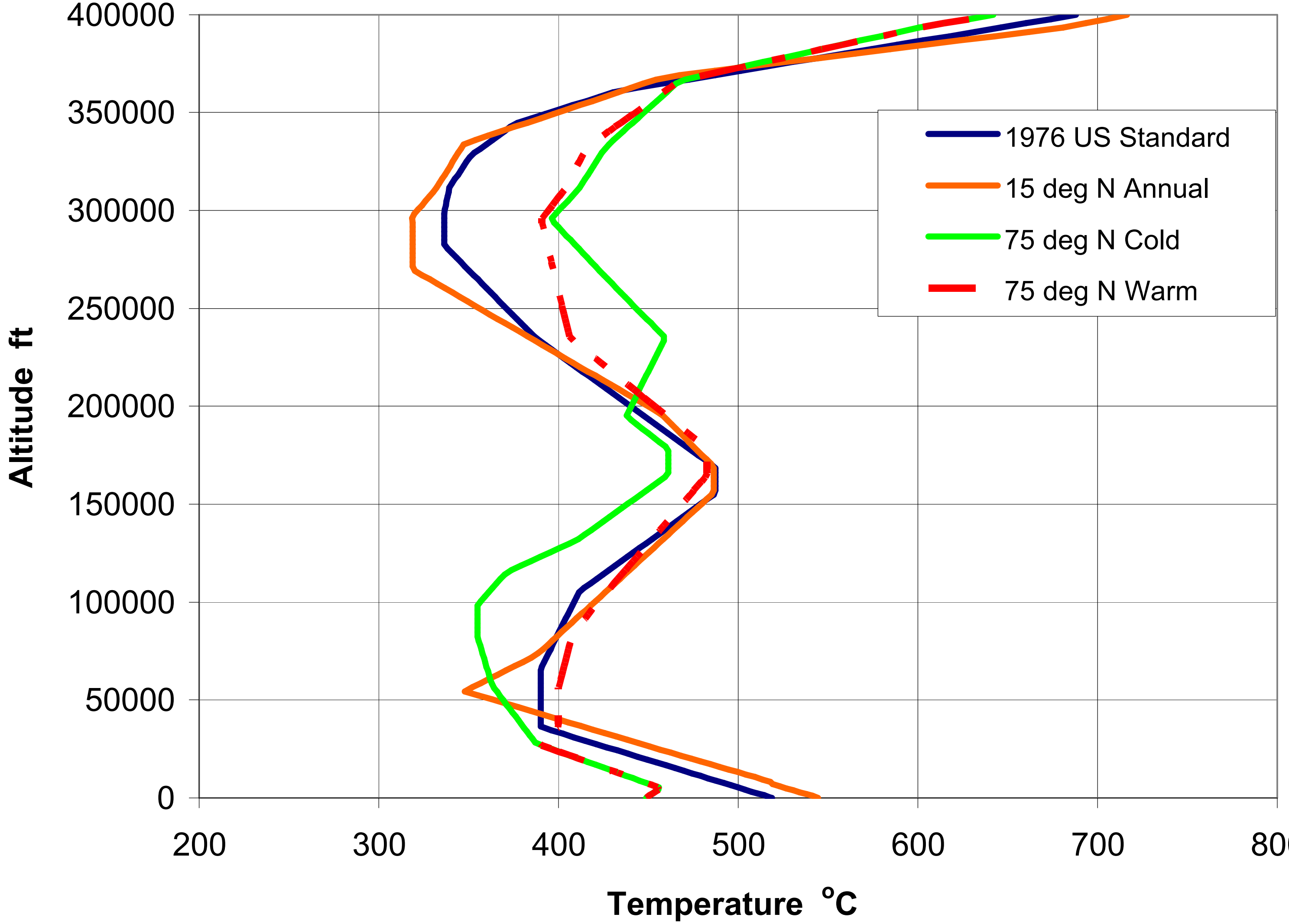
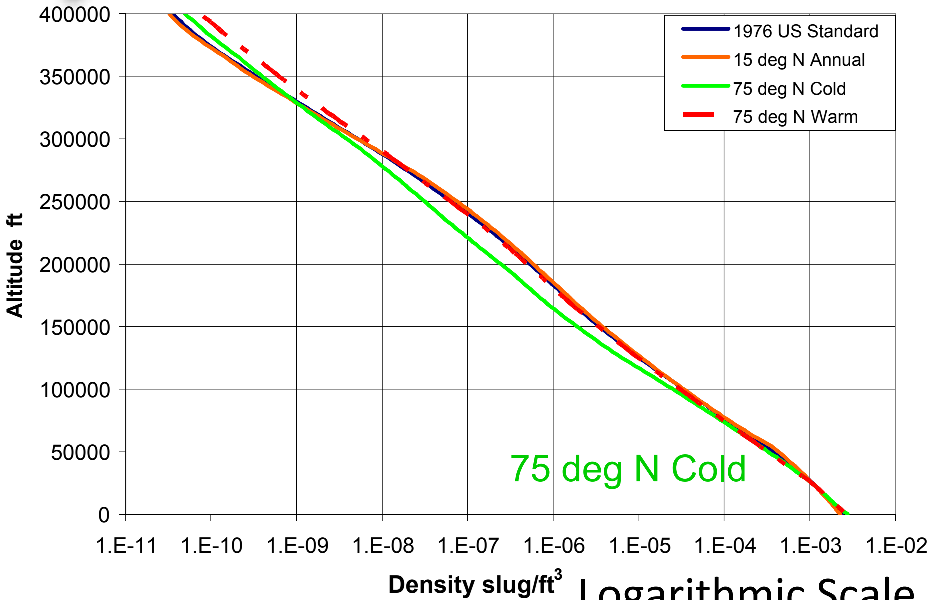
Atmosphere Models

- The 1976 U.S. standard atmosphere is an idealized mid-latitude representation of the atmosphere under year-round moderate solar activity.
- Other atmosphere models also represent average or monthly conditions for a given location and season.

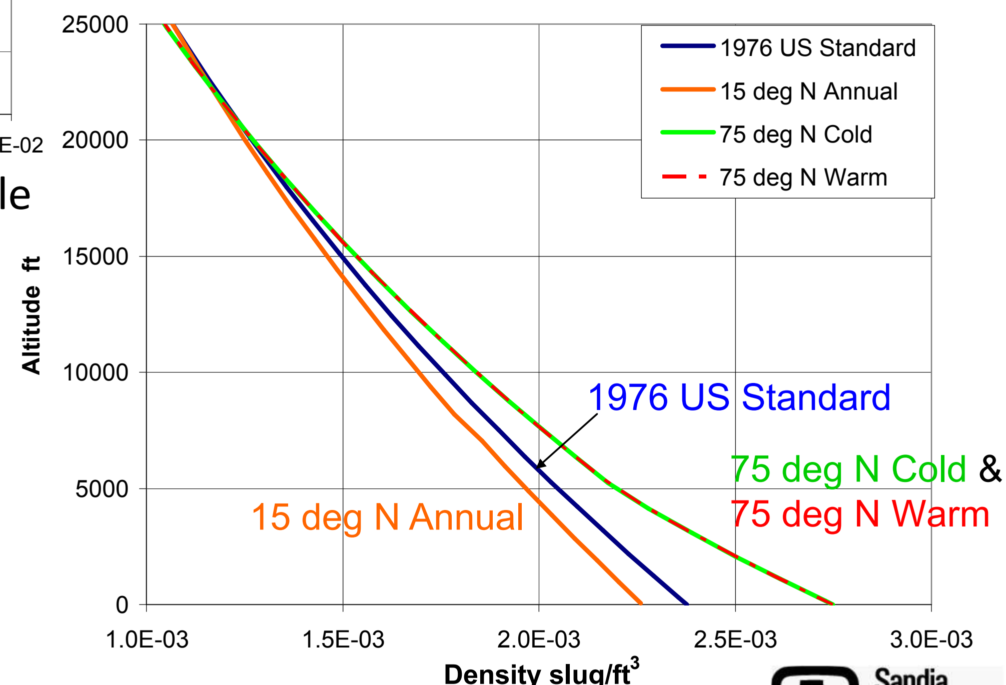
Representative Atmospheric Temperature Models



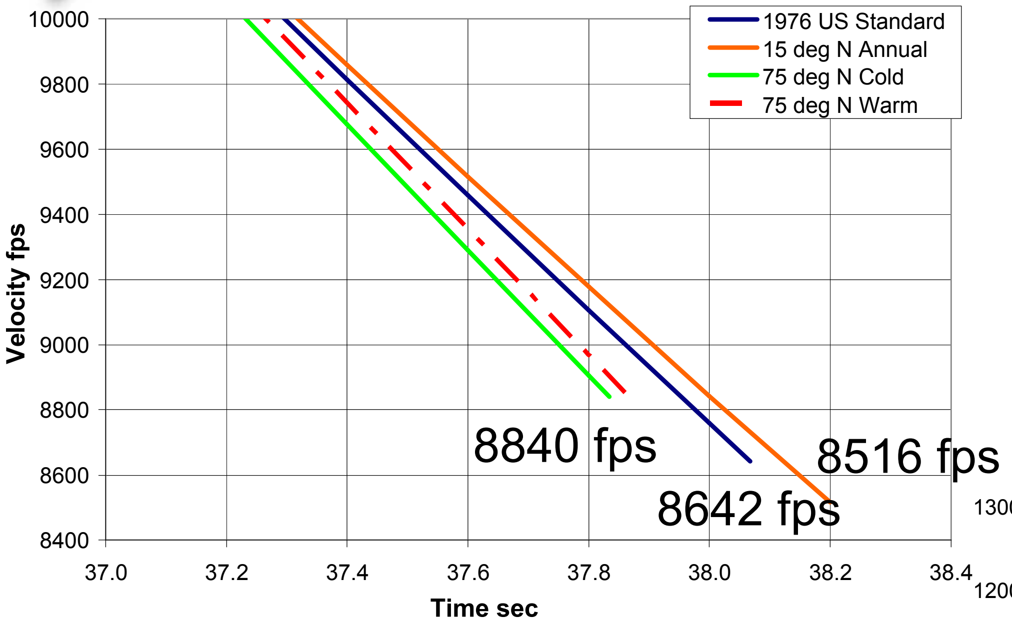
Representative Density Variation Throughout Reentry



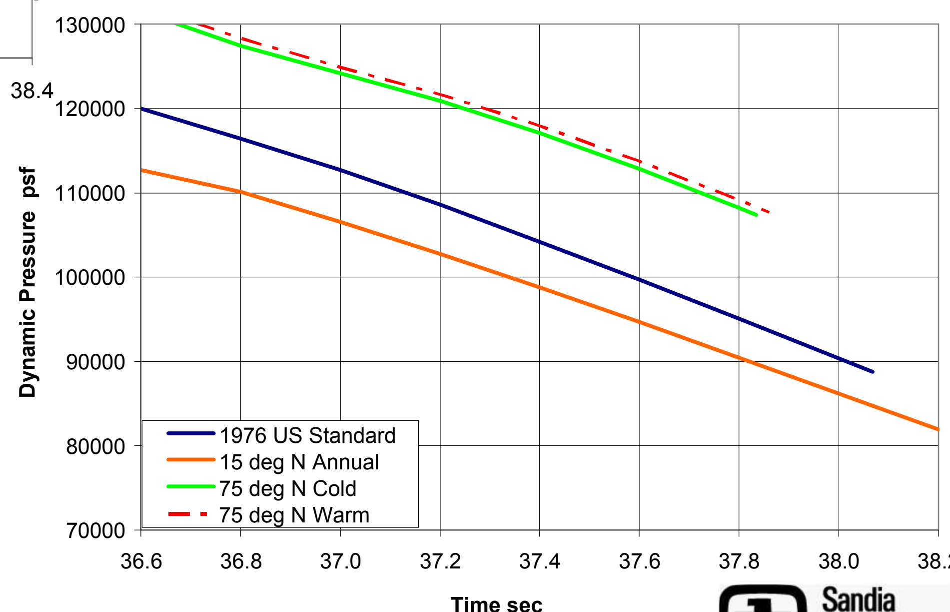
Logarithmic Scale



Velocity and Dynamic Pressure Variation with Atmosphere



Generic Reentry Vehicle Flight



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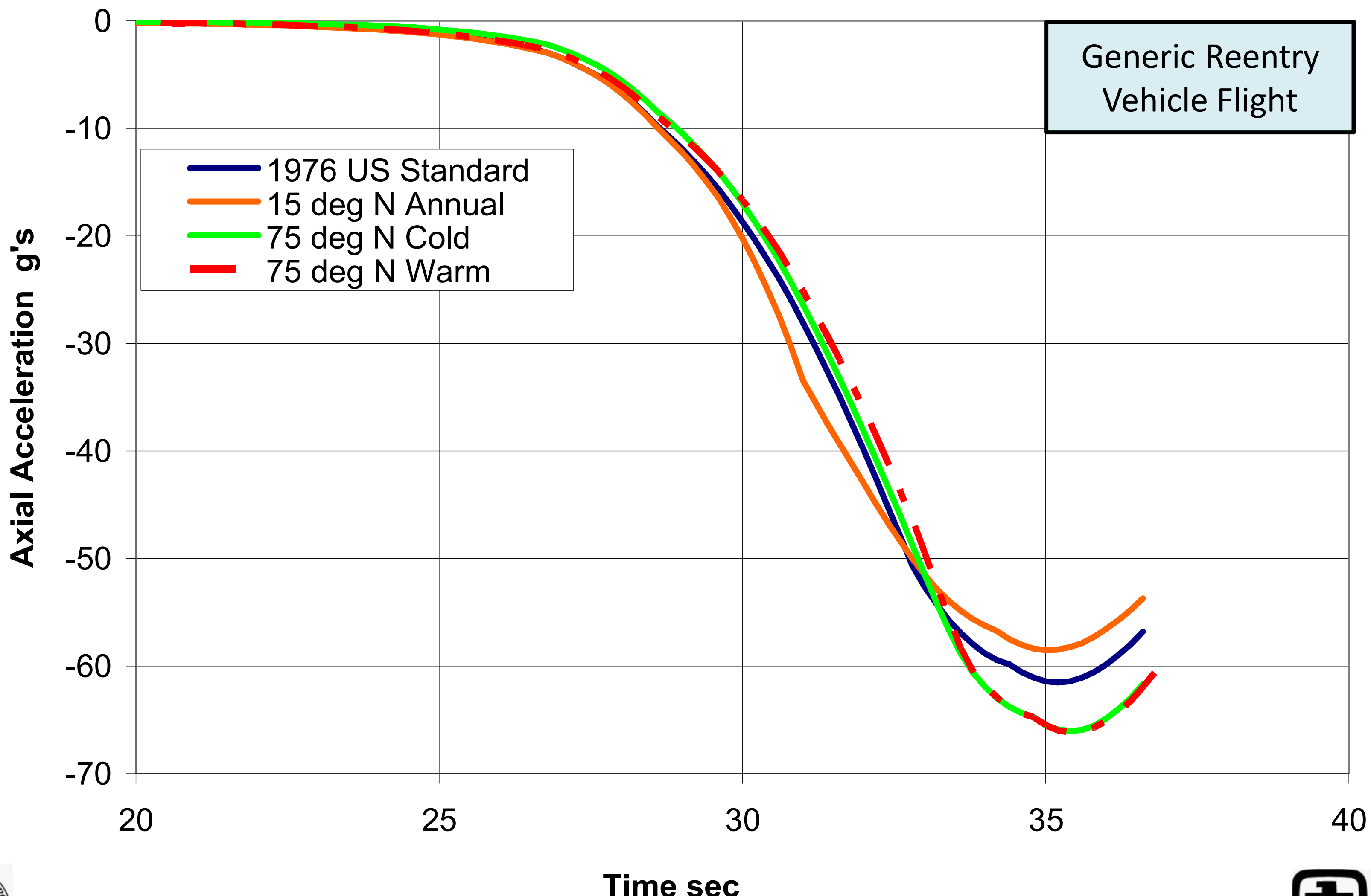
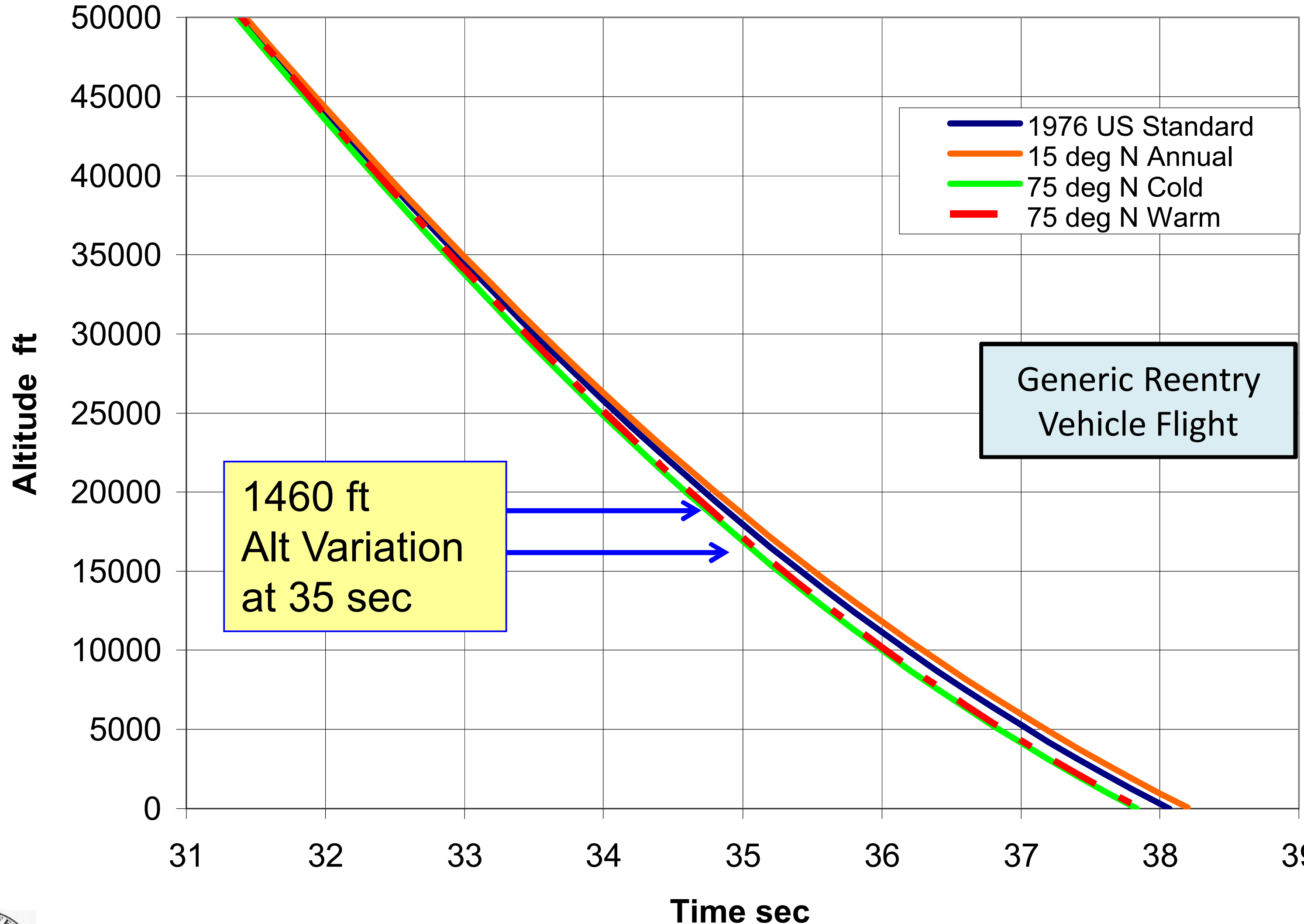
Representative Axial Acceleration Variation Throughout Reentry



Representative Time-Altitude History



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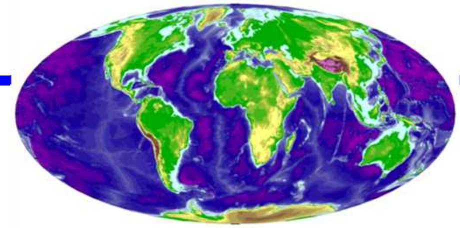
What is an Earth Model?



- An earth model consists of an earth or planet shape, a gravity model, and some additional parameters such as the earth rotation rate.
- The earth is assumed to be an ellipsoid with the equatorial radius greater than the polar radius. Its shape is given by an equatorial radius and either a polar radius, an eccentricity, or a flatness. If one of the latter parameters is provided, for example, eccentricity, the other two can be computed.

Simple Earth Models

- The simplest earth model is spherical with no gravity and no earth rotation. This is almost the same as not having an earth model.
- The next step for an earth model is to provide an earth rotation rate and a gravity constant in addition to the earth's equatorial and polar radii.

Atmosphere & Earth Constants

Parameter	Symbol	Metric Value	English Value
Sea Level Pressure	P_o	1.01325E5 N/m^2(Pascal)	2116.22 lb/ft^2
Sea Level Density	ρ_o	1.2250 kg/m^3	2.3769E-3 $slug/ft^3$
Sea Level Temperature	T_o	288.2 ^oK	518.7^oR
Sea Level Molecular Wt	M_o	0.0289644 kg/mol	28.9644 g/mol
Gas Constant for Air	R*	287.06 J/(kg - ^oK)	1716.5 $sec^2/(ft^2$ - $^oR)$
WGS84 Earth Equatorial Radius	$R_{E,}$ R_{EQ}	6,378,136.992 m	20,925,646.3 ft
WGS84 Earth Polar Radius	R_{NP}	6,356,752.316 m	20,855,486.6 ft
Sea Level Gravity	g_o	9.806649983 m/sec^2	32.1740485 ft/sec^2
WGS84 Earth Rotation Rate	ω_p	7.292115E-5 rad/sec	7.292115E-5 rad/sec

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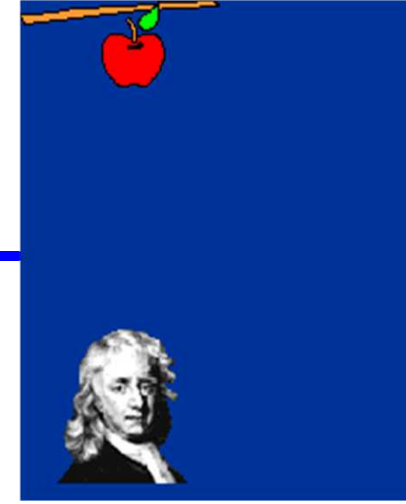
Gravity

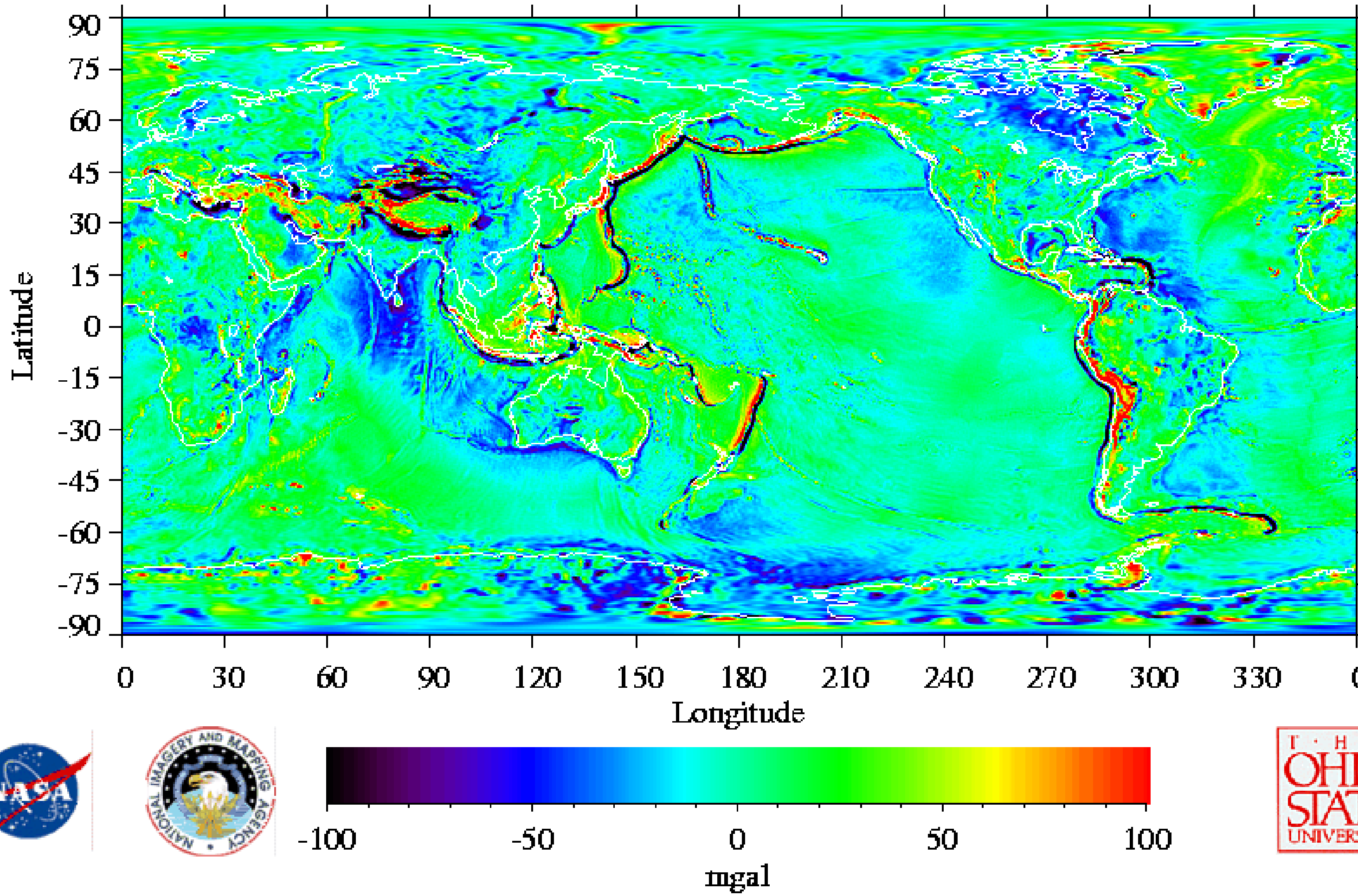


Gravity is Modeled as a Gradient of the Geopotential applied to the Earth.

The geopotential function can be divided into three parts. The first part corresponds to the potential derived from treating the earth as a point mass. When the spherical earth model is selected, it consists of just this one term. All other terms are neglected.

- The second part of the geopotential consists of terms which are functions of latitude only. These terms are called the zonal harmonics. The even zonal harmonics model the oblateness of the earth's shape and encompass what is called "normal gravity". The second-degree zonal harmonic, J2, is approximately 1000 times larger than any other term.
- The third part of the geopotential consists of the remaining terms that depend on both latitude and longitude.1
- The effects of higher-order terms (greater than the J2 term) on non-orbital trajectories are small.

30’ Mean Gravity Anomalies: EGM96 (Nmax=360)



Earth’s Geopotential

•In its complete form, the geopotential is commonly expressed in terms of spherical harmonics:

$$U = \frac{GM}{\left\|\vec{r}\right\|}\left\{1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left[\frac{\mathrm{Re}}{\left\|\vec{r}\right\|}\right]^{n} P_{n,m}\left(\sin\delta_{gc}\right)\left[C_{n,m}\cos m\lambda + S_{n,m}\sin m\lambda\right]\right\}$$

This is the standard form for the earth’s gravitational potential.

Cn,m and *Sn,m* can be related physically to regions on the earth's surface.

Earth gravitational models, created from satellite-tracking measurements, consist of values for the constants *GM*, *R*, *Cn,m*, and *Sn,m*.

J2 vs 8x8 Earth Model Differences

Trajectory Simulation for an RV Flying North from 500 kft on the Upleg of the Trajectory, through Apogee, to 400 kft (non-rotating earth)

Compare Parameters from 8x8 Earth Model to J2 Earth Model at Reentry (400 Kft).

Lat = Long = DR = CR = 0.0 at 400 kft

Parameter	8x8	J2	Difference
Time (sec)	0.015	0.000	0.015
Velocity (ft/s)	20000.20	20000.00	0.20
Longitude deg)	0.00081	0.000000	0.00081
Latitude (deg)	0.00006	0.000000	0.00006
Down Range (ft)	297.634	0.000	297.634
Cross Range (ft)	-21.993	0.000	-21.993

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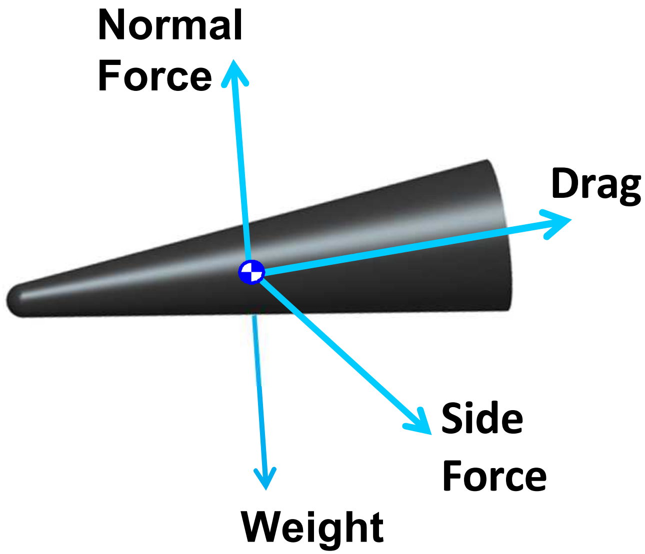
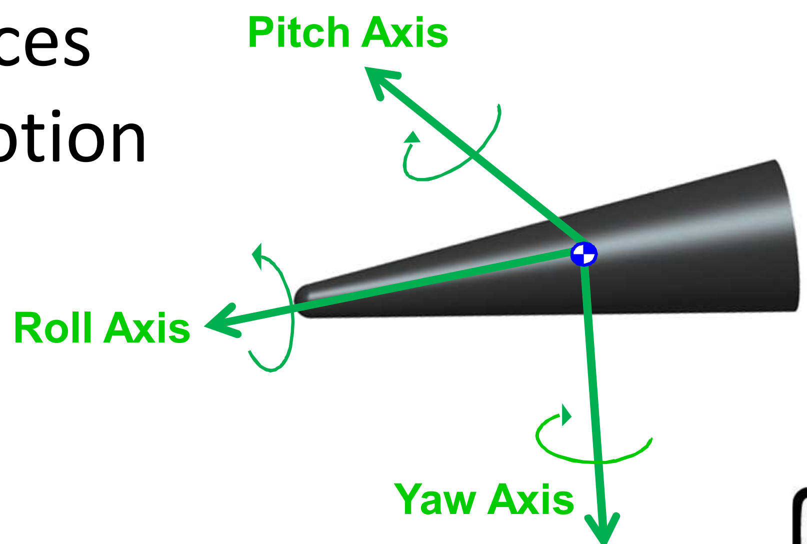
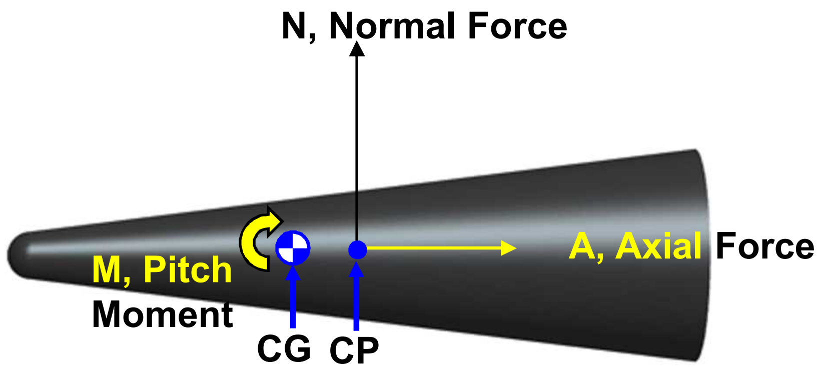
Aerodynamics

- Aerodynamics is the branch of fluid mechanics that deals with the motion of air and other gaseous fluids.
- Of interest are the forces acting on bodies in motion relative to such fluids.





Moment Reference Point and Center of Pressure



- **Moment Reference Point (MRP)**
  - In order to specify moments on a body, one must specify an MRP
  - MRP can be any convenient point, but normally, a point close to the cg is used to avoid loss of accuracy due to large transfer distances
  - For trajectory simulations, one computes moments about cg

- **Center of Pressure (cp)**
  - **CP** is the point on the body centerline through which the integrated forces act
    - Viscous forces don't contribute much to moments
    - Pressure forces dominate
  - At the CP, the moment due to pressure is zero
  - Relative to the MRP, $X_{cp} = M / N$
    - Assumes contribution of axial forces is negligible
    - This assumption can be erroneous if there are large asymmetrical drag devices

Force and Moment Coefficients

- Dynamic pressure

$$q = 0.5 \rho V_\infty^2$$

- Reference area

$$S = \pi \cdot r^2$$

- Normal force coefficient

$$C_N = \frac{F_N}{qS}$$

- Side force coefficient

$$C_Y = \frac{F_Y}{qS}$$

- Axial force coefficient

$$C_A = \frac{F_A}{qS}$$

- For moment coefficients, must also include a reference length - ***d***

- Pitch moment coefficient

$$C_m = \frac{m}{qSd}$$

- Yaw moment coefficient

$$C_n = \frac{n}{qSd}$$

- Roll Moment coefficient

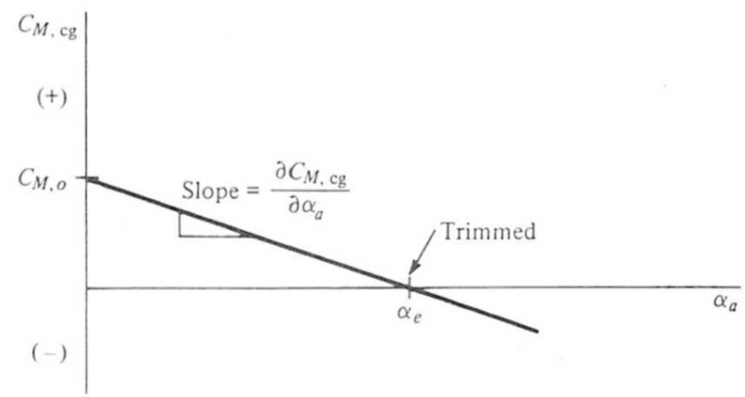
$$C_\ell = \frac{\ell}{qSd}$$

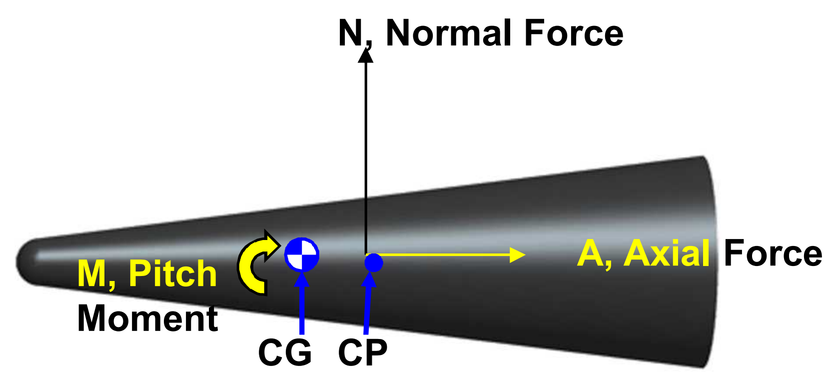
- Pressure coefficient

$$C_p = \frac{p - p_\infty}{q_\infty}$$

Requirements for Static Stability

- **Static Stability**: there must be a negative slope of the pitch moment about the cg vs. angle of attack curve at the equilibrium point
- **The CP (Center of Pressure) must be AFT of the CG (Center of Gravity)**
- One measure of static stability is **Static Margin**
  - $$SM = \frac{x_{cg} - x_{cp}}{L}$$
  - Rule of thumb: acceptable flight behavior for ballistic vehicles, SM ≈ 5% - 10%
- Most spherically-capped conic shapes are statically stable
- Fins and/or Flares are used on most missile/bomb shapes for static stability.

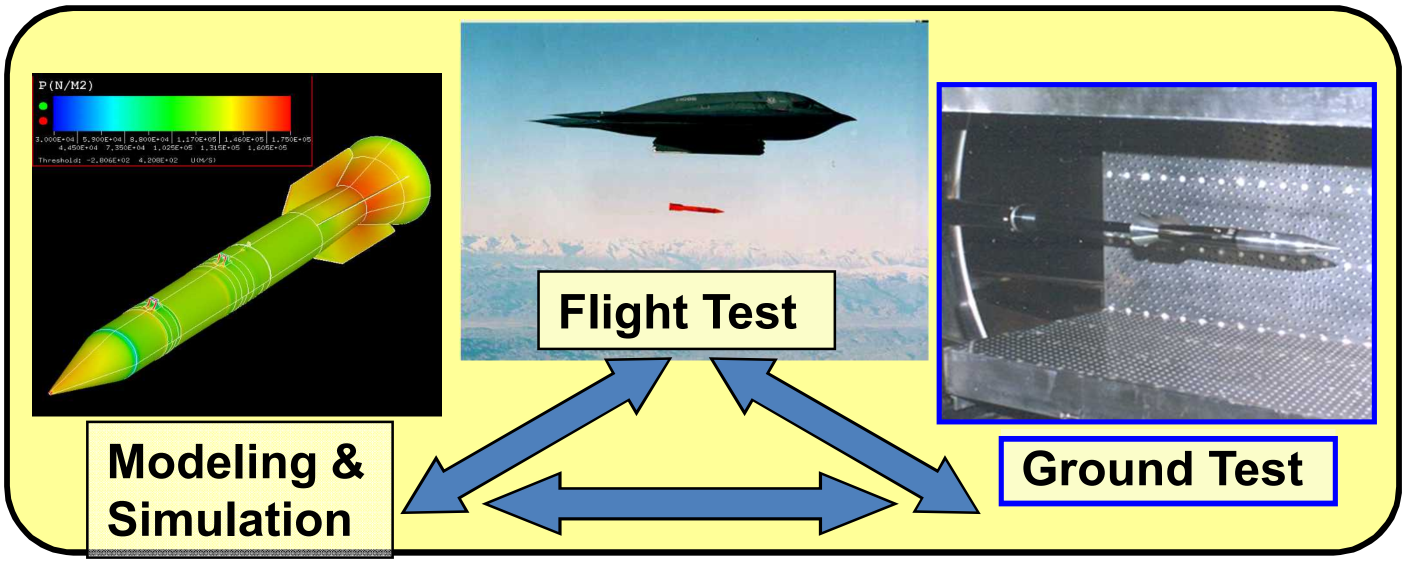




- **Note that stability and maneuverability are opposing concepts**

Normal Aerodynamic Design Sequence

- Semi-empirical methods/codes for preliminary design
- Computational Fluid Dynamics (CFD) for refined aerodynamics



- Wind Tunnel Testing for complicated shapes
- Aerodynamic Model creation with several methods
- Flight Test Data Analysis

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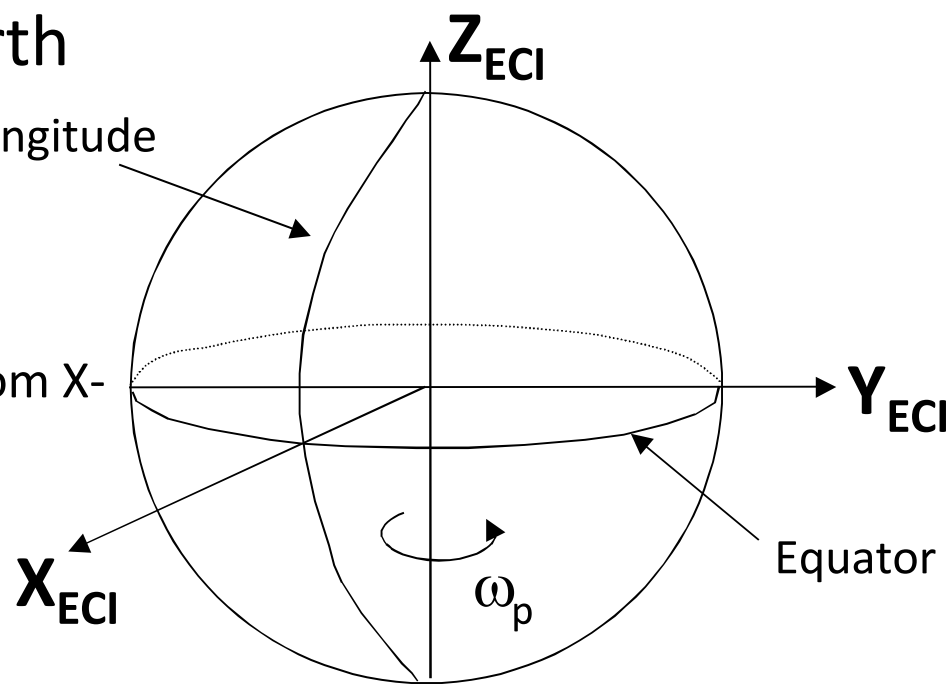
  - Terminology

Earth Centered Inertial Coordinate System

- Axes Fixed at Launch
- Does Not Rotate with Earth



X-ECI Aligned with Launch Longitude

Y-ECI Aligned along Equator 90º from X-ECI

Z-ECI Aligned with North Pole

Other Names: Inertial (I), Earth-Centered Inertial Cartesian (ECIC)

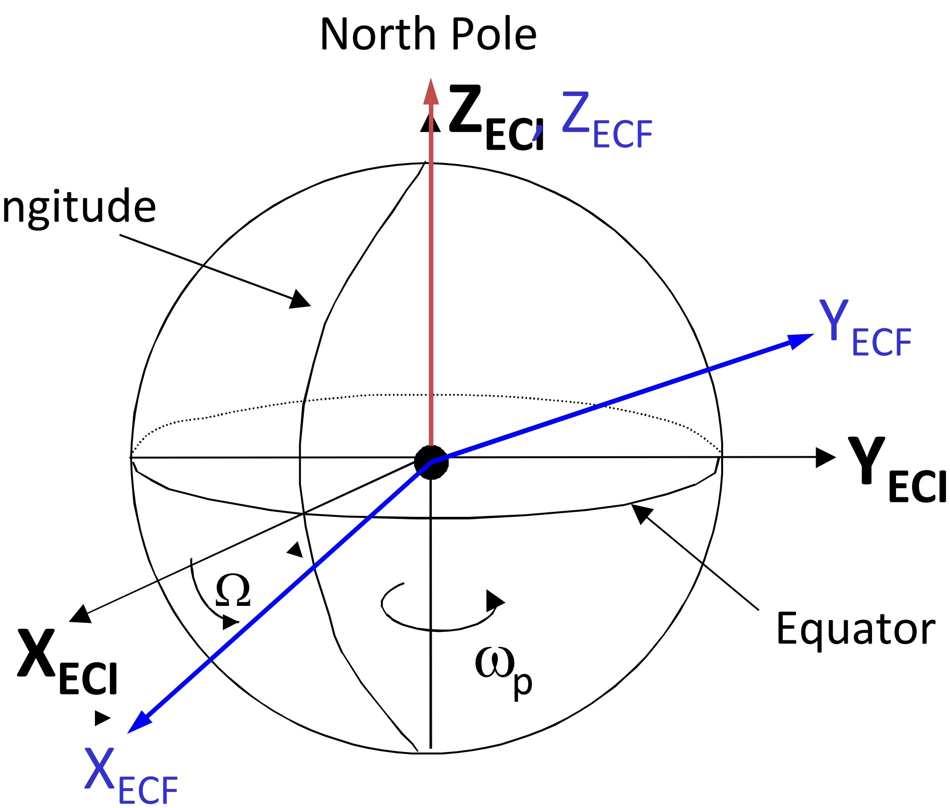
Earth-Centered, Fixed (ECF) Coordinate System

- Axes Fixed to Earth

- Rotates with Earth

X-ECF aligned with 0° Greenwich Meridian

Y-ECF aligned with 90°E Meridian



- Other Names: Earth-Fixed (EF), Earth-Fixed Geodetic (EFG), Earth-Centered Earth-Fixed Cartesian (ECFC)

Geodetic Coordinate System

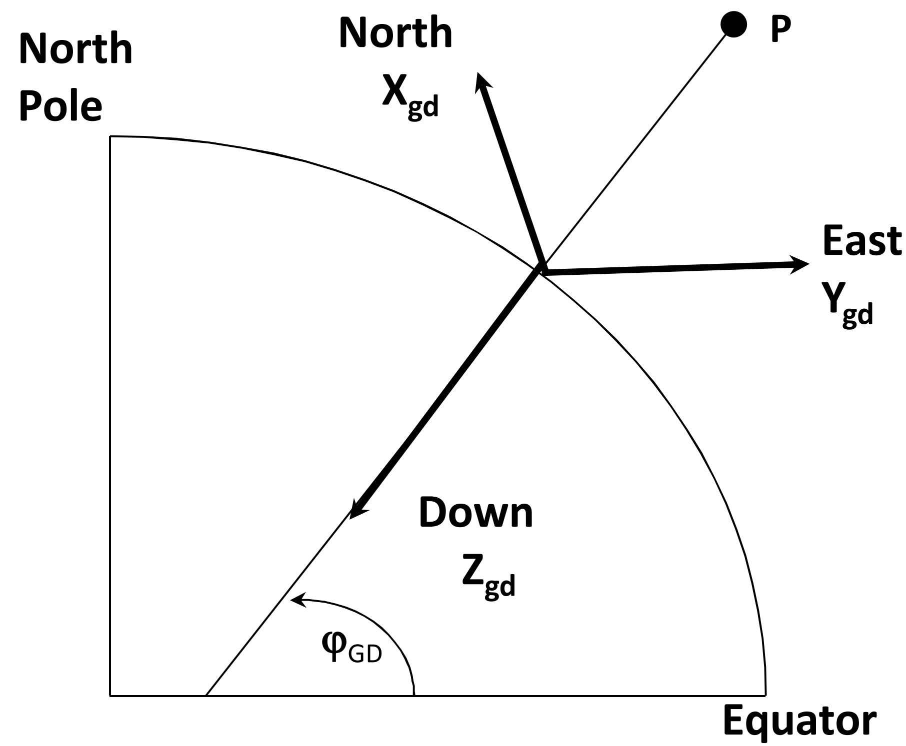
Origin at Local Horizon,

below the vehicle on a straight line perpendicular to the local horizon.

X-axis points North

Y-axis points East

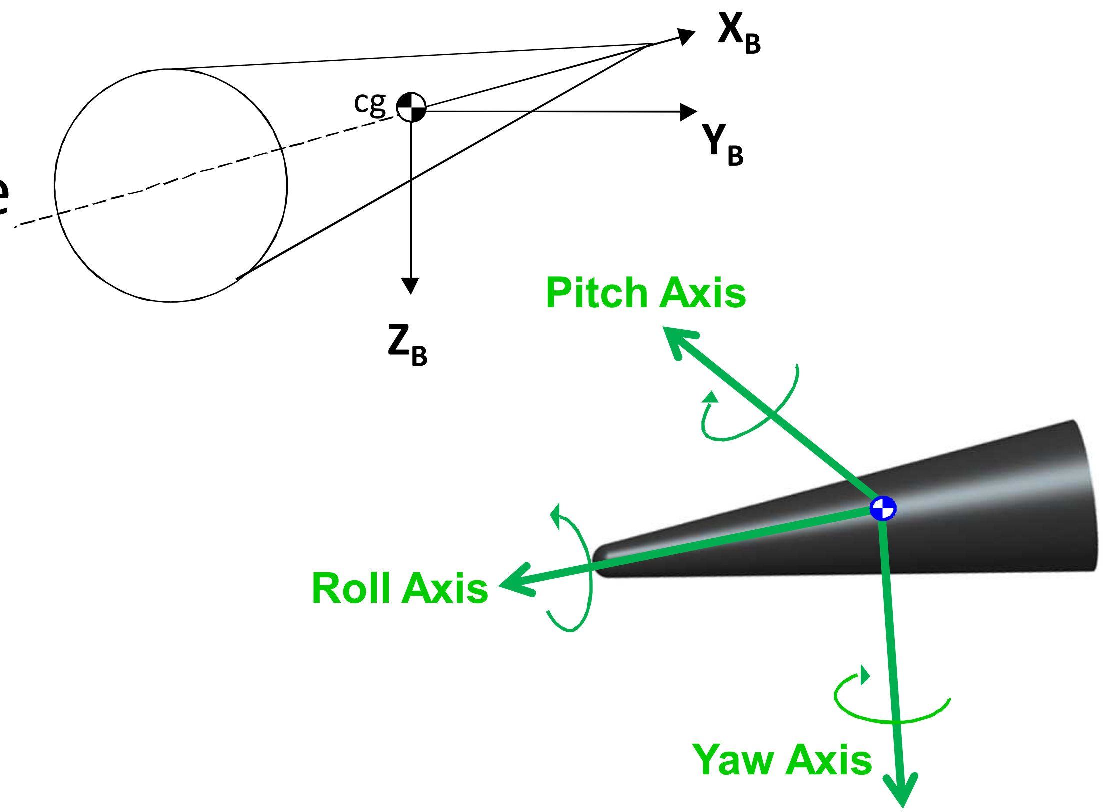
Z-axis points Down



Body Axes Coordinate System

Typically:

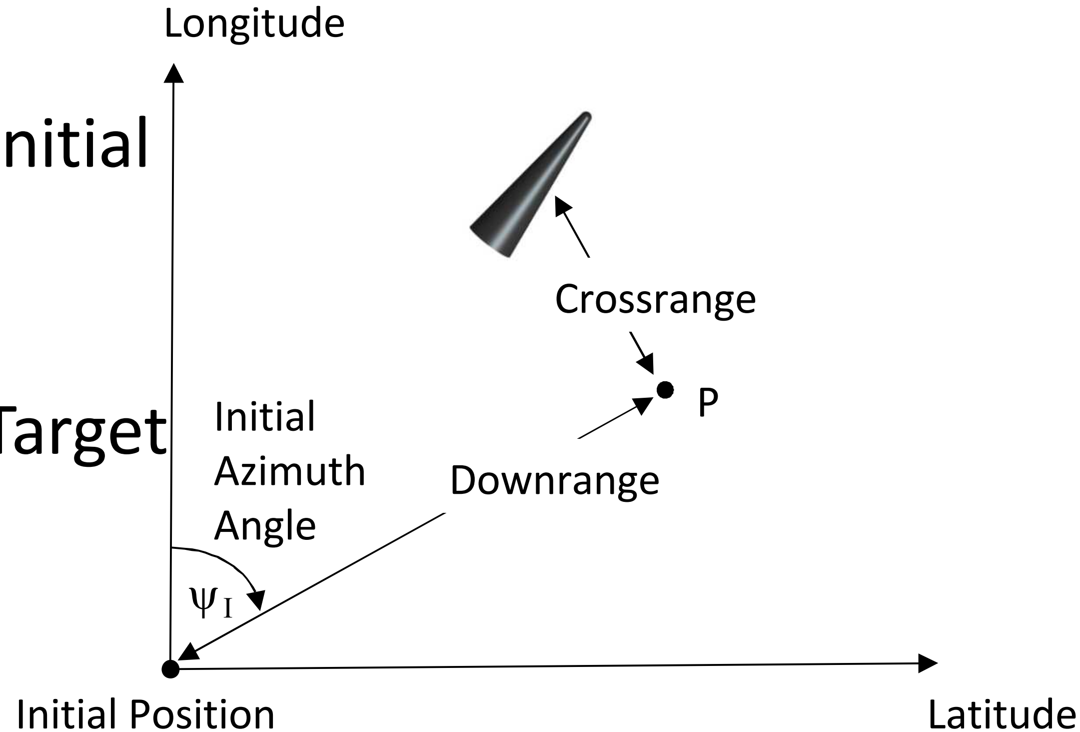
- X-axis out body nose
- Y-axis to the right
- Z-axis 90^{o} to Y-axis



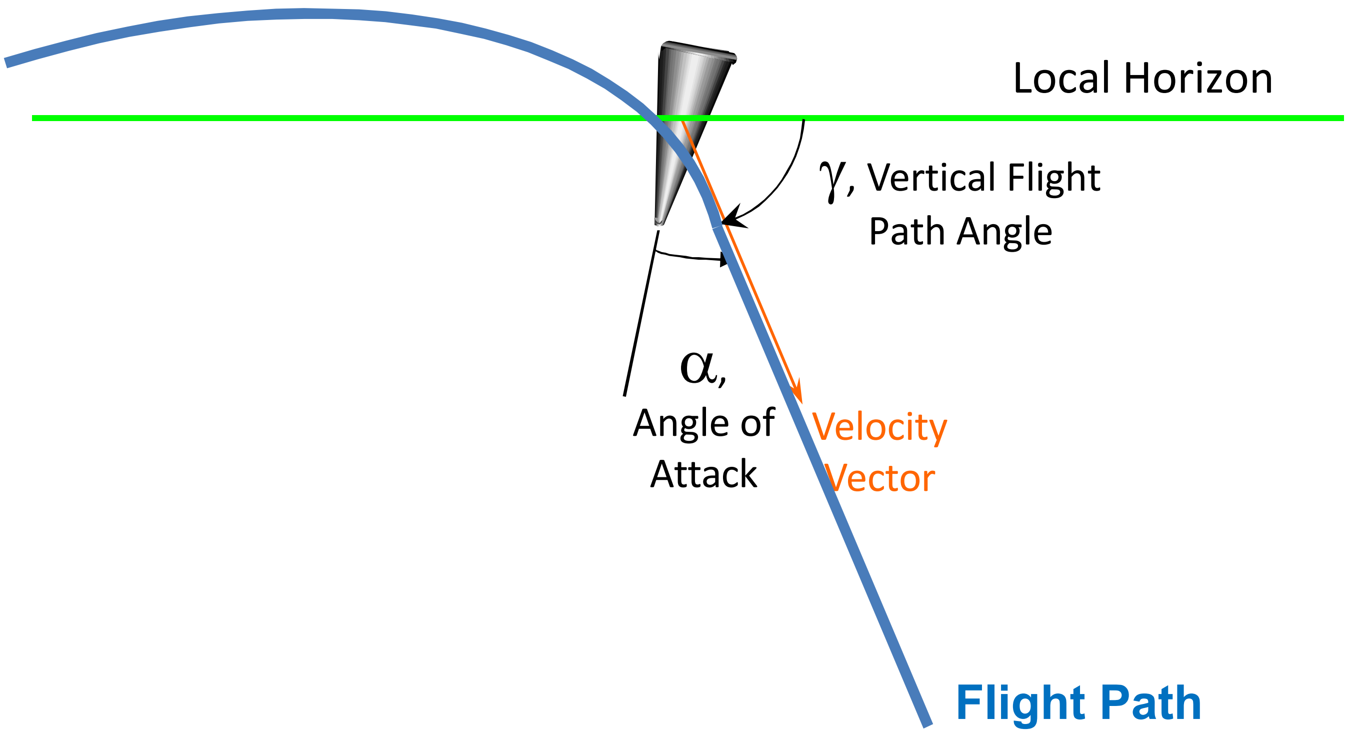
- Typically, Centered at CG. Can also be centered at Nose or Vehicle Base.

Range Axis Coordinates

- Downrange and Crossrange from Initial Position
- Downrange and Crossrange from Target Point
- Total Range = $\sqrt{DR^2 + CR^2}$



Body Orientation from Two Viewpoints

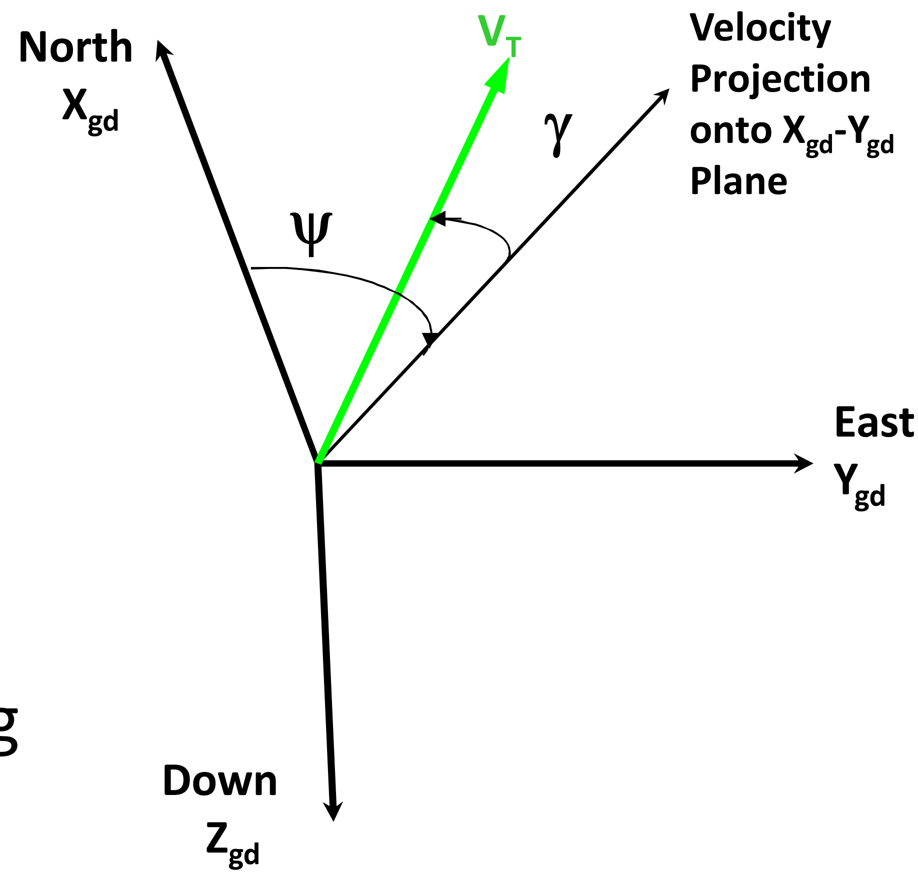


Flight Path Angles

Flight Path Angles give orientation of the trajectory path (velocity vector) wrt the local horizon

γ = Vertical Flight Path Angle, 'Gamma', 'flight path angle'

ψ = Horizontal Flight Path Angle, Azimuth, Heading



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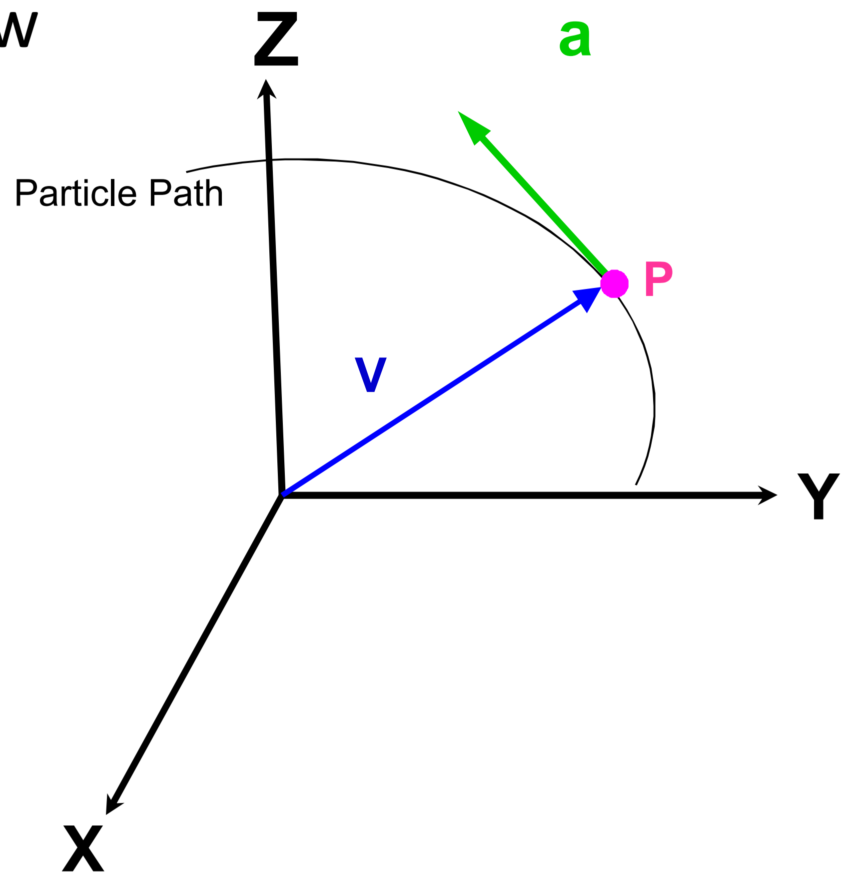
Kinematics of a Particle

- F = ma :   Newton's 2nd Law
- Fixed reference frame
- Velocity of a particle:

$$V = \frac{dr}{dt}$$

- Acceleration of a particle:

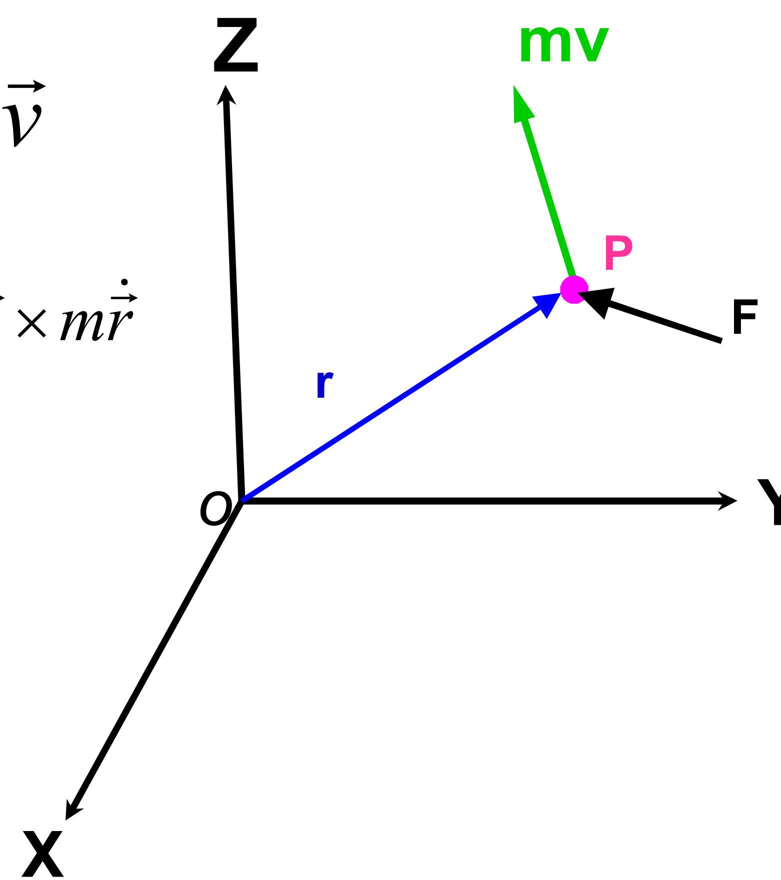
$$a = \frac{dv}{dt} = \frac{d^2 r}{d^2 t}$$



Angular Momentum

The Angular Momentum of the particle P, about point *O*, is given by:

$$\vec{H} = \vec{r} \times m\vec{v}$$



Differentiate with respect to time:

$$\dot{\vec{H}} = \vec{r} \times m\ddot{\vec{r}} + \dot{\vec{r}} \times m\dot{\vec{r}}$$

where $\dot{\vec{v}} = \ddot{\vec{r}}$

Since $\dot{\vec{r}} \times \dot{\vec{r}} = 0$

Then, $\dot{\vec{H}} = \vec{r} \times m\ddot{\vec{r}}$

Substitute Newton's 2nd Law: $\dot{\vec{H}} = \vec{r} \times \vec{F}$

The Moment, M of the Force F, is also known as: $\vec{M} = \vec{r} \times \vec{F}$

Therefore,

$$\boxed{\vec{M} = \dot{\vec{H}}}$$

Rotational Kinematics

- The 3 angular accelerations are computed from the 3 moment equations.
- These angular accelerations are integrated to give the angular velocities, ω_x, ω_y, ω_z

  - ω_x = p, vehicle roll rate , inertial angular rate about the body x-axis
  - ω_y = q, vehicle pitch rate , inertial angular rate about the body y-axis
  - ω_z = r, vehicle yaw rate , inertial angular rate about the body z-axis

6DOF Set of Equations

3 Force Equations : Used to Obtain Vehicle's Position and Velocity
3 Moment Equations : Used to Obtain Vehicles Angular Rates
9 Rotational Kinematic Equations: Used to Obtain Vehicle's Angular Orientation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_B = \begin{bmatrix} T_{ECI2B} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ECI}$$

How trajectory simulations vary from actual flights

- What atmosphere and wind conditions existed as the vehicle flew?
- How good is the aerodynamic model used in our trajectory simulation?
- For hypersonic vehicles, how well do we really know the ablation characteristics of the trajectory?

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Trajectory Simulation Components

- Equations of Motion
- Earth Model
- Atmosphere Model
- Mass Properties of Vehicle
- Aerodynamic Characteristics of Vehicle

Mach Number

- The ratio of the vehicle's speed to the speed of sound affects the forces on the vehicle, since the vehicle's aerodynamic characteristics (coefficients) change with Mach number.

- Mach number, $M = V/a$
  - ratio of the vehicle's speed to the speed of sound

Subsonic $0 < M << 1$

Transonic $M \sim 1$

Supersonic $1 < M < 5$

Hypersonic $M > 5$

Dynamic Pressure

Dynamic Pressure is proportional to Velocity-squared:

$$q = \frac{1}{2}\rho V^2$$

where ρ is atmospheric density at the desired altitude;

and V is vehicle total velocity

Aerodynamic Forces are Proportional to Dynamic Pressure:

$$F_x = C_x q S$$

where C_x is the aerodynamic coefficient in the x-direction;

q is the dynamic pressure; and,

S is the vehicle reference area (typically base area).

Ballistic Coefficient

Ballistic Coefficient is defined as:

$$\beta = \frac{W}{C_d A}$$

where:

W is vehicle weight (lb_f or N)

C_d is the vehicle Drag Coefficient (function of vehicle shape)

A is the vehicle Reference Area (normally Base area of reentry vehicle or rocket)

State Vector

- A State Vector gives vehicle’s position and velocity
- **Position Vector Information:**
  - Geodetic: Altitude, Geodetic Latitude, Longitude,
  - ECI: X, Y, Z Position
  - ECF: X, Y, Z Position
  - Geocentric: Radius to Earth’s Center, Geocentric Latitude, Longitude
  - Dwn/Crs: Downrange offset, Crossrange offset, Down offset (Used for Monte Carlo variations from Known Initial Condition)

State Vector, cont.

- **Velocity Vector Information:**
  - Geodetic: Velocity (or Mach Number), Vertical Flight Path Angle, Horizontal Flight Path Angle
  - Geodetic: North, East, Down Velocity
  - ECI: X, Y, Z Velocity
  - ECF: X, Y, Z Velocity
  - Geocentric: Velocity (or Mach Number), Vertical Flight Path Angle, Horizontal Flight Path Angle
  - Geocentric: North, East, Down Velocity
  - Dwn/Crs: Downrange Velocity offset, Crossrange Velocity offset, Down Velocity offset

Body Attitude Information

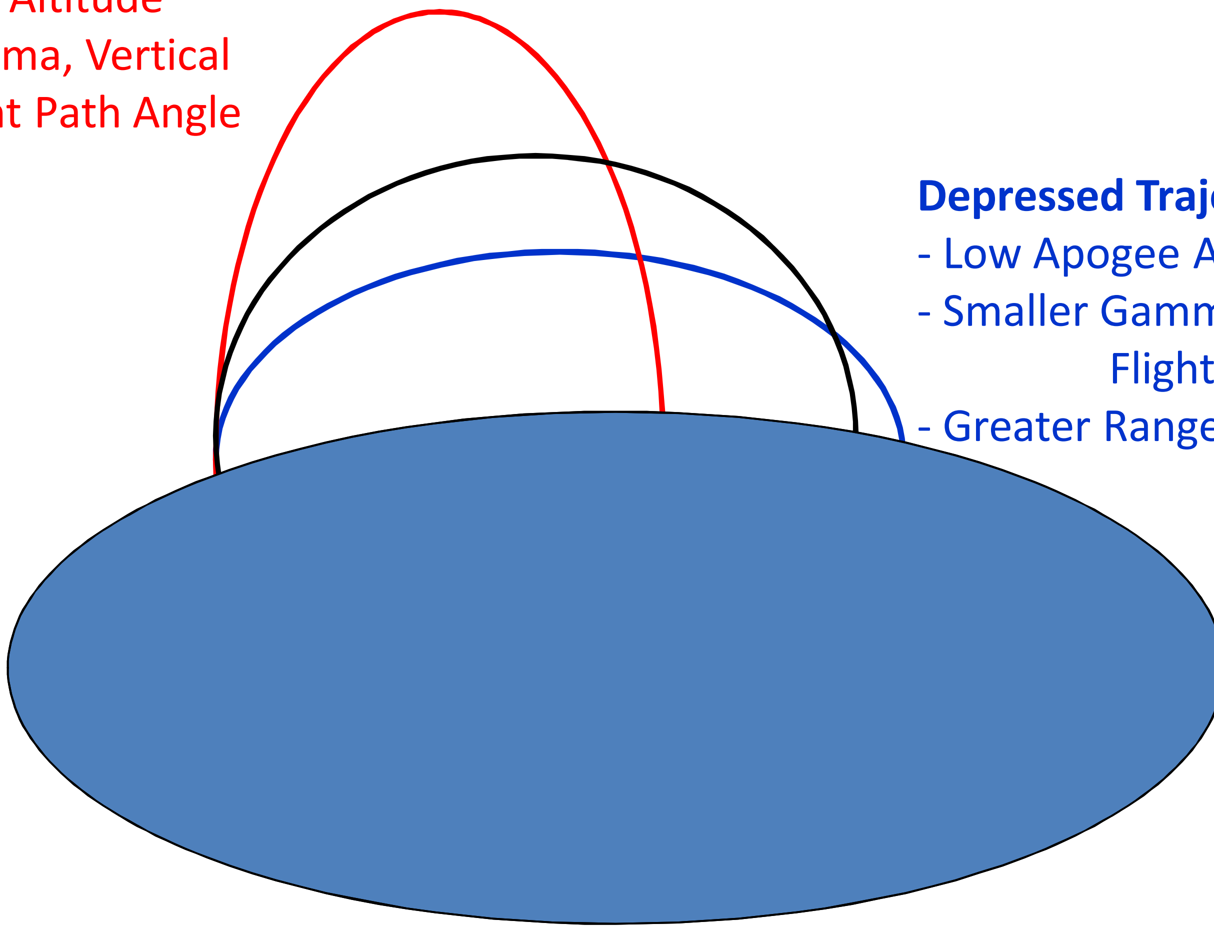
- Initial Body Attitude is initialized and computed for 6DOF Trajectories:
  - Geodetic or Geocentric: Yaw, Pitch, Roll Euler angles
  - Geodetic: Pitch Angle of Attack, Yaw Angle of Attack, Bank Angle
  - ECI: Yaw, Pitch, Roll Euler angles
  - ECF: Yaw, Pitch, Roll Euler angles

Vehicle Mass

- Vehicle Mass is given in the Initial Conditions
- A body may have a constant mass throughout a trajectory; or,
- A body may have a scheduled mass loss rate
  - propellant burned for a thrusting vehicle
  - heatshield material loss for a high speed vehicle
  - Loss/removal of vehicle appendages or spent motor
- Auxiliary equations determine current vehicle mass

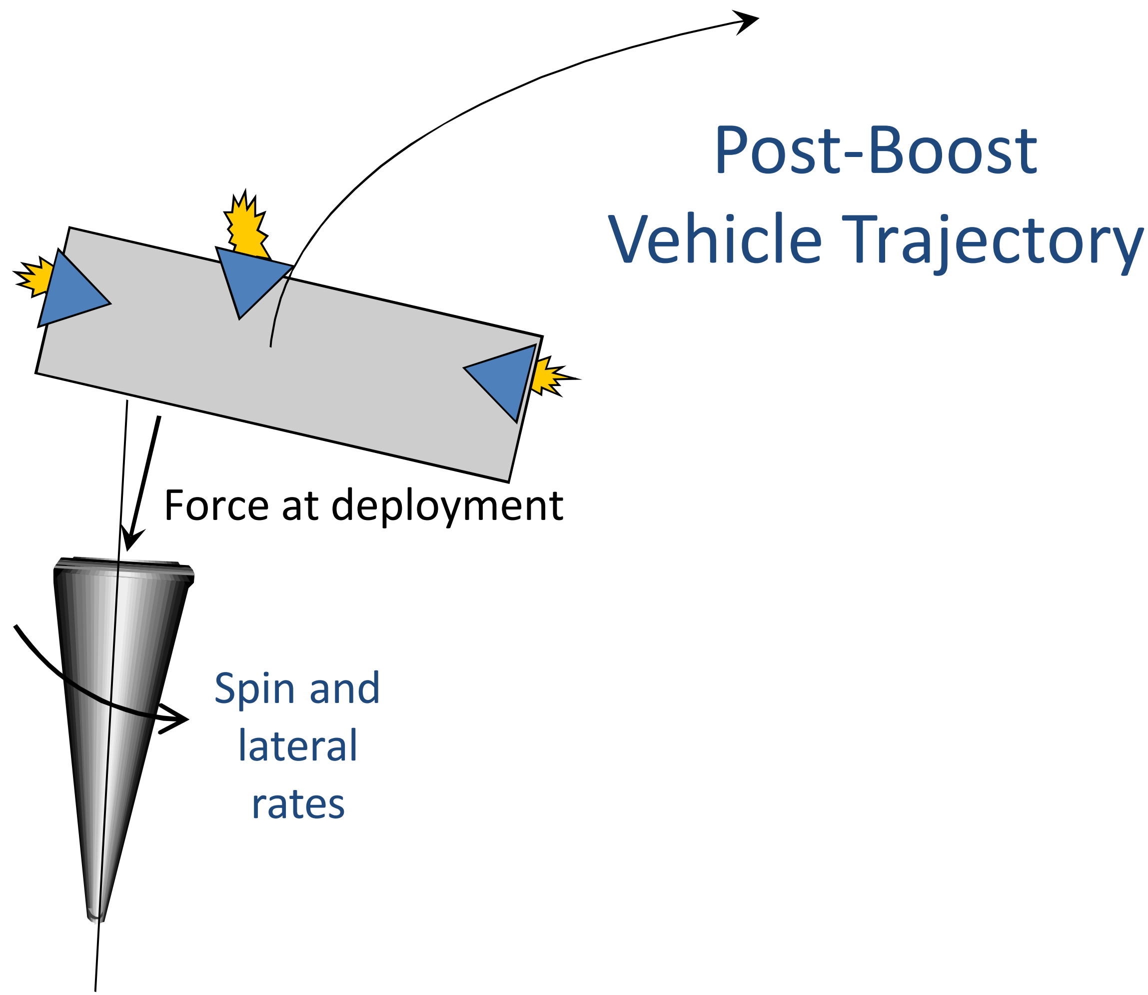
Trajectory Types

Lofted Trajectories
- High Apogee Altitude
- Greater Gamma, Vertical Flight Path Angle
- Less Range



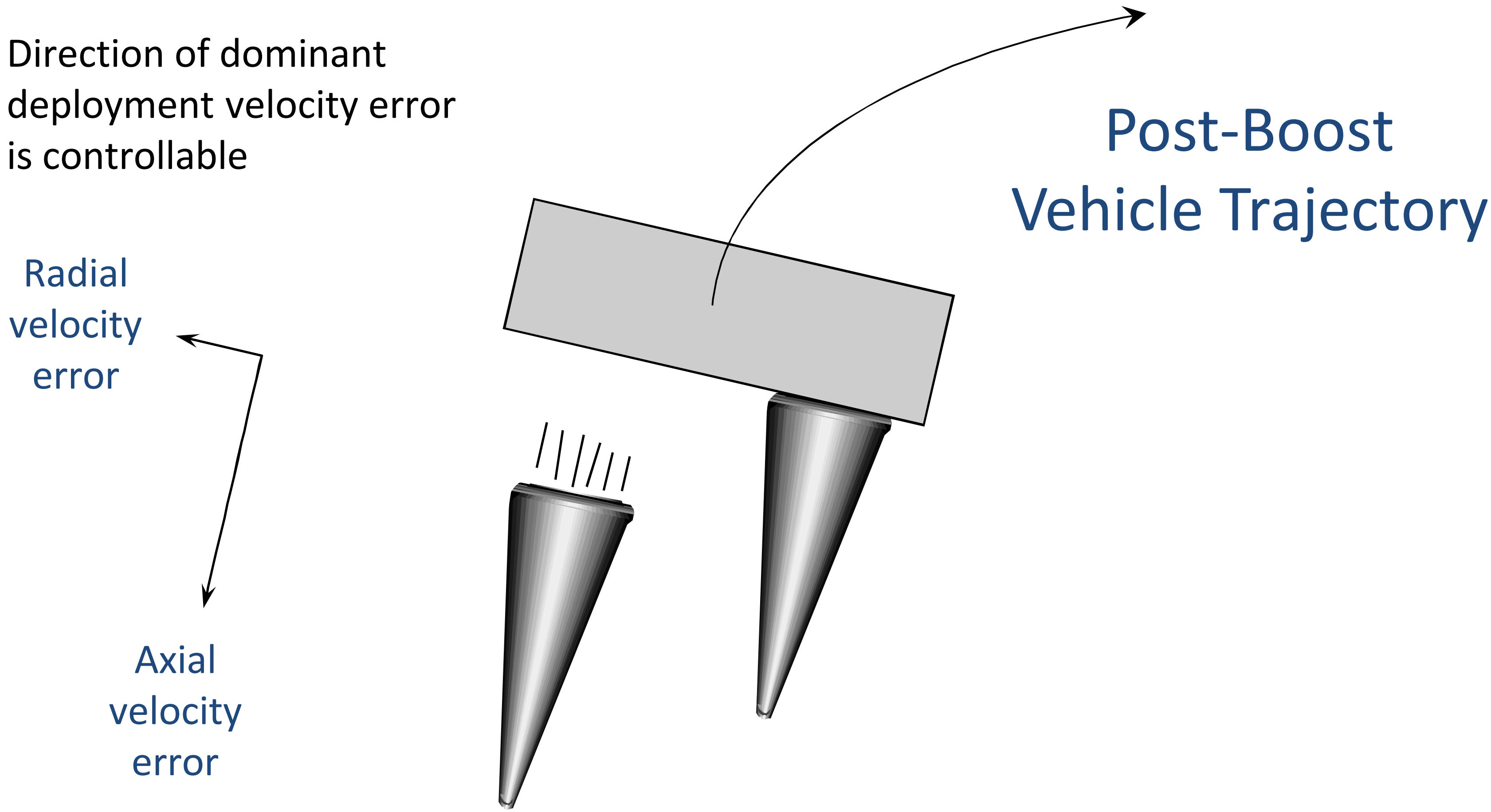
Depressed Trajectories
- Low Apogee Altitude
- Smaller Gamma, Vertical Flight Path Angle
- Greater Range

RV Deployment Errors



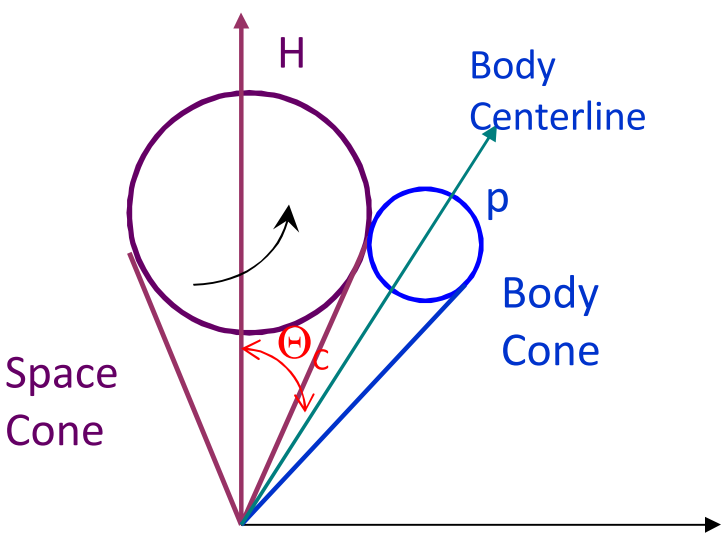
Deployment Velocity Errors

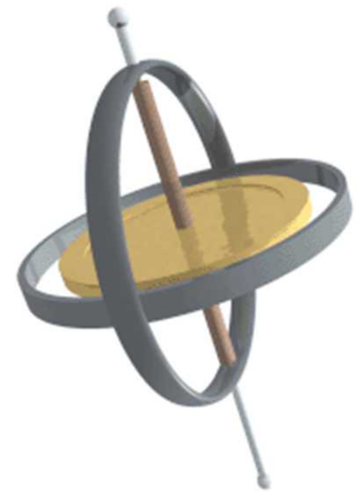
Direction of dominant deployment velocity error is controllable



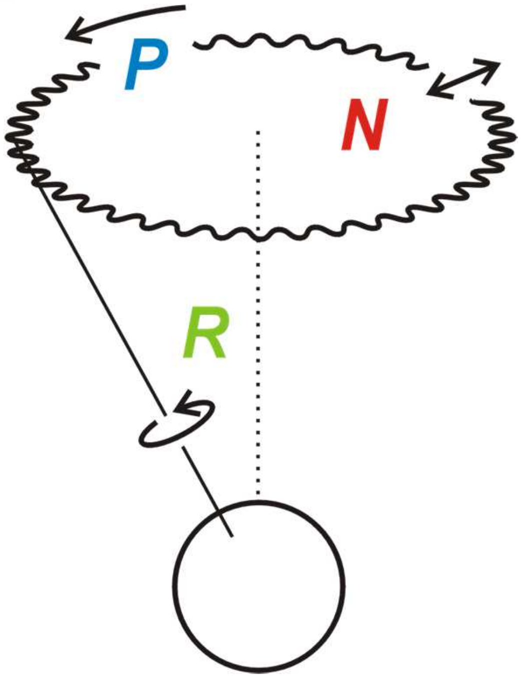
Vehicle Coning

- **Precession** refers to a change in the direction of the axis of a rotating body





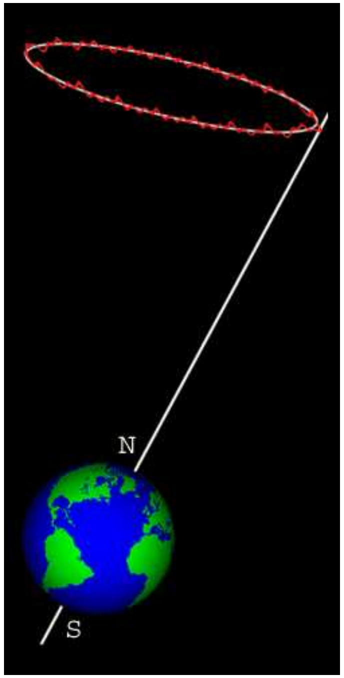
Vehicle Coning



•Rotation (green), Precession (blue) and Nutation (red) of the Earth

- **Nutation** is a slight irregular motion in the axis of rotation of a largely axially symmetric object, such as a gyroscope or a planet. (commonly called "wobble")

- In astronomy, nutation is a small irregularity in the precession of the equinoxes.



Exoatmospheric Rates

- Assumptions:
  - Zero Aerodynamic Forces
  - Symmetric Vehicle
  - Negligible Products of Inertia

p=vehicle roll rate

q = vehicle pitch rate

r = vehicle yaw rate

Ixx = Inertia about the body roll axis

I = Inertia about the body transverse axis

- Tipoff rate = ω_T = Body Transverse Rate

$$\omega_T = \sqrt{q^2 + r^2}$$

- Half Coning angle, Θ_C

$$\tan\theta_c = \frac{I\sqrt{q^2 + r^2}}{I_{xx} p}$$

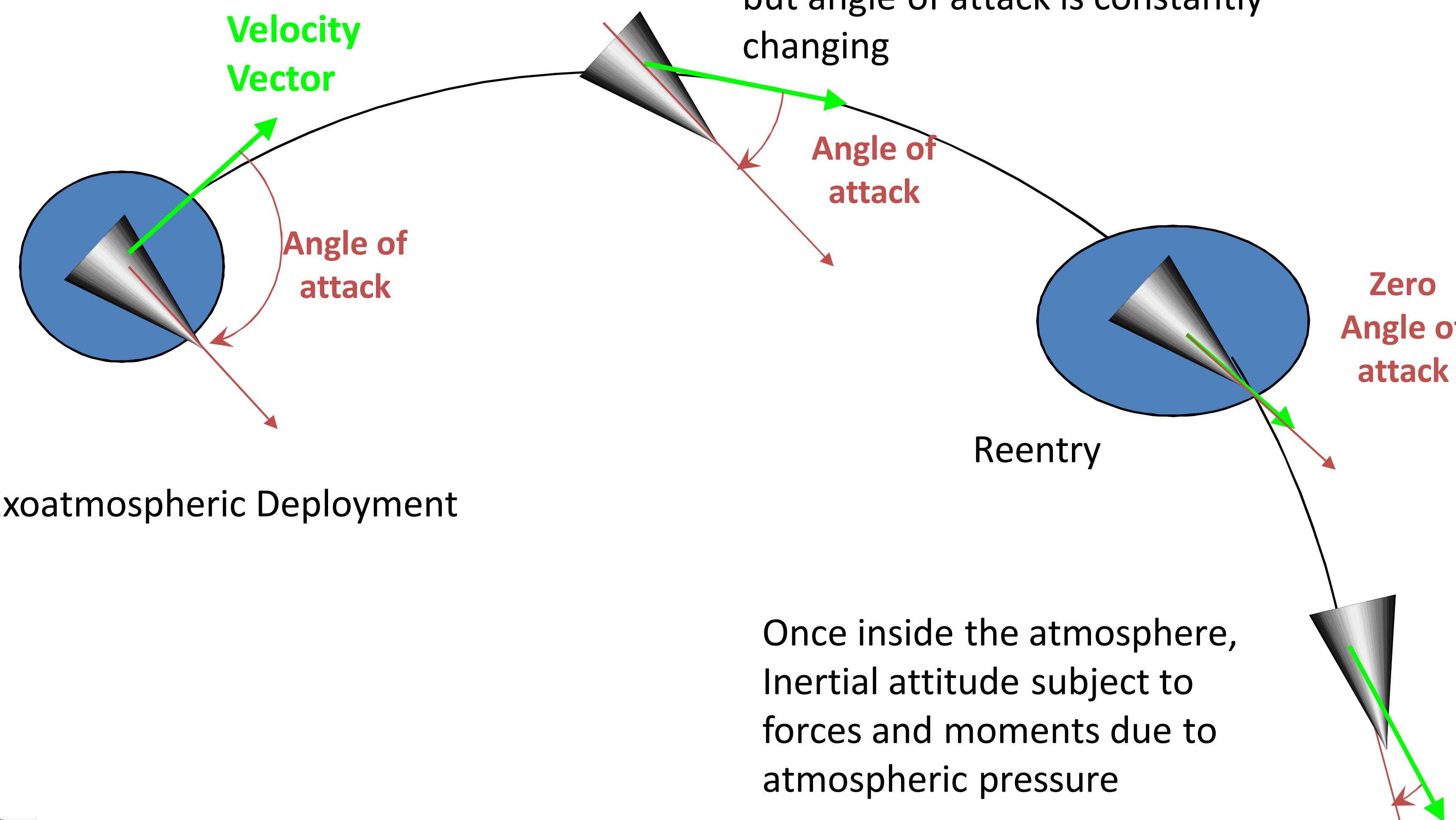
- Precession Rate, $\omega_{precession}$

$$\omega_{precession} = \frac{I_{xx} p}{I \cos\theta_c}$$

Constant Inertial Attitude

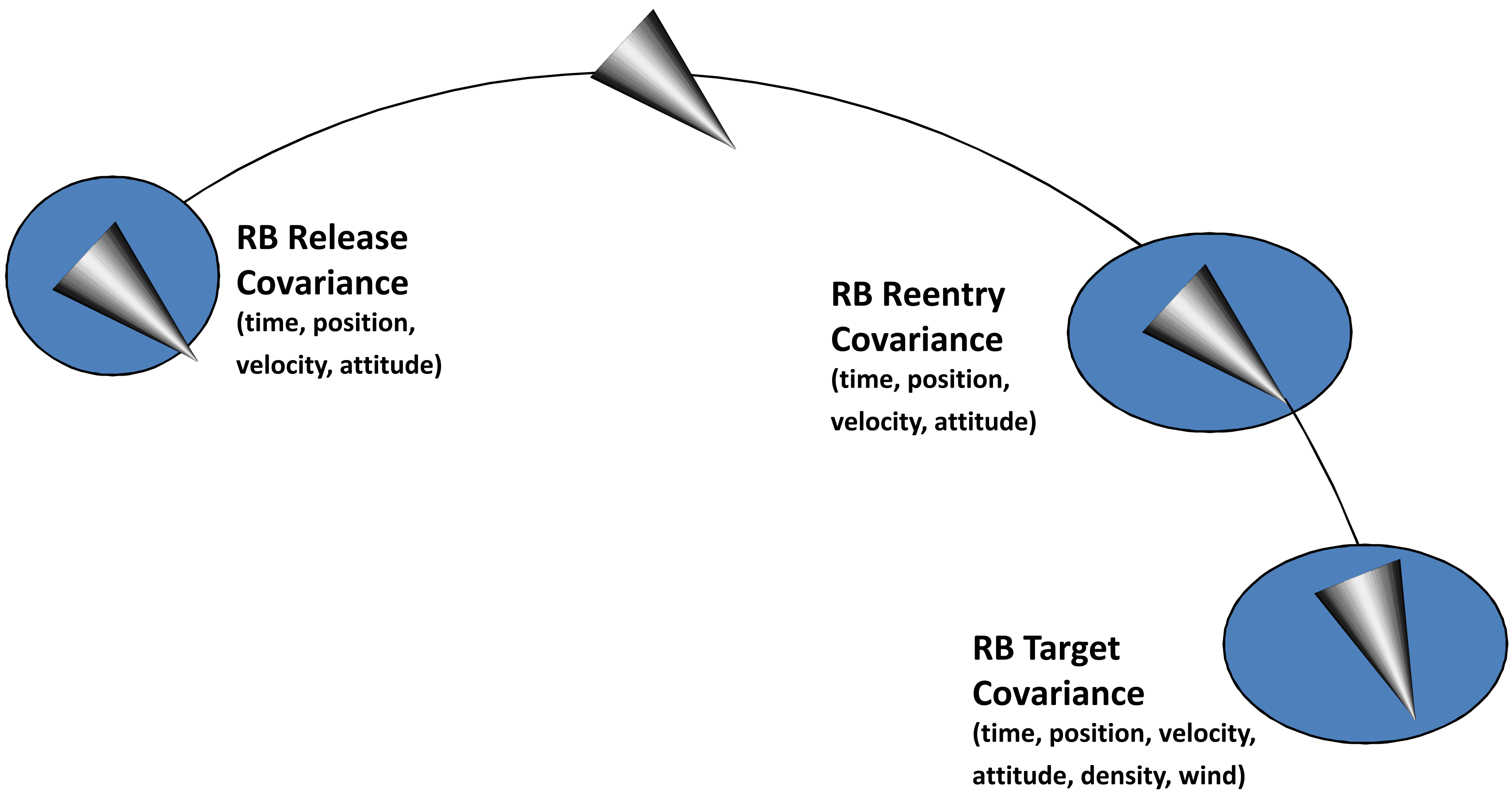
Inertial Euler Angles Remain Constant

Inertial Attitude remains constant, but angle of attack is constantly changing

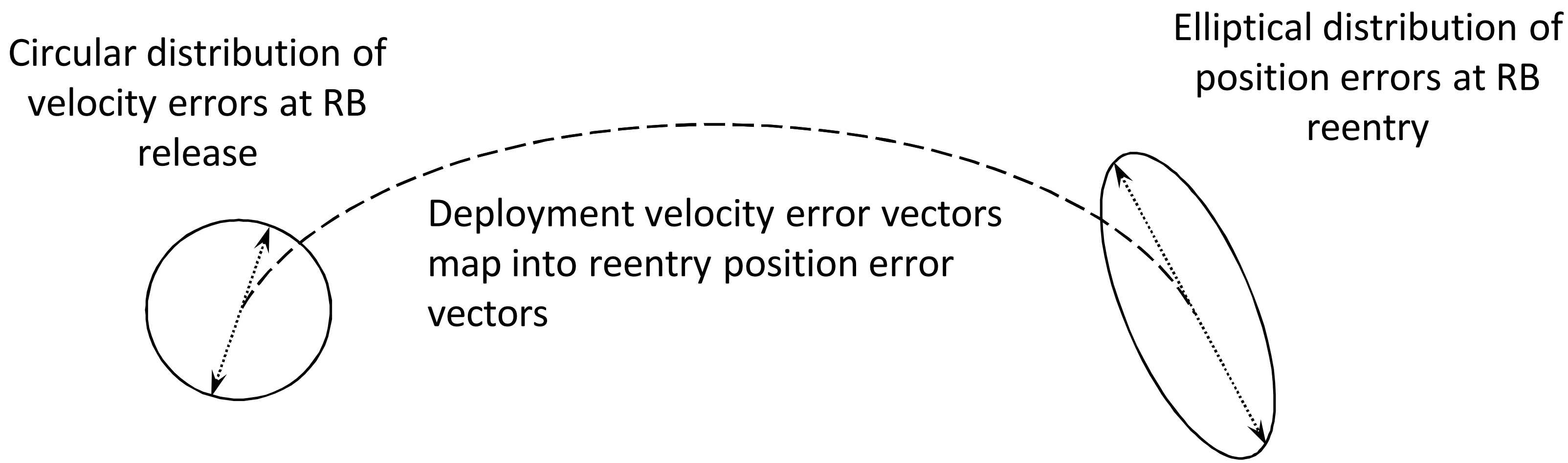


Once inside the atmosphere, Inertial attitude subject to forces and moments due to atmospheric pressure

Trajectory Deployment and Resulting Variations



Range Insensitive Axis

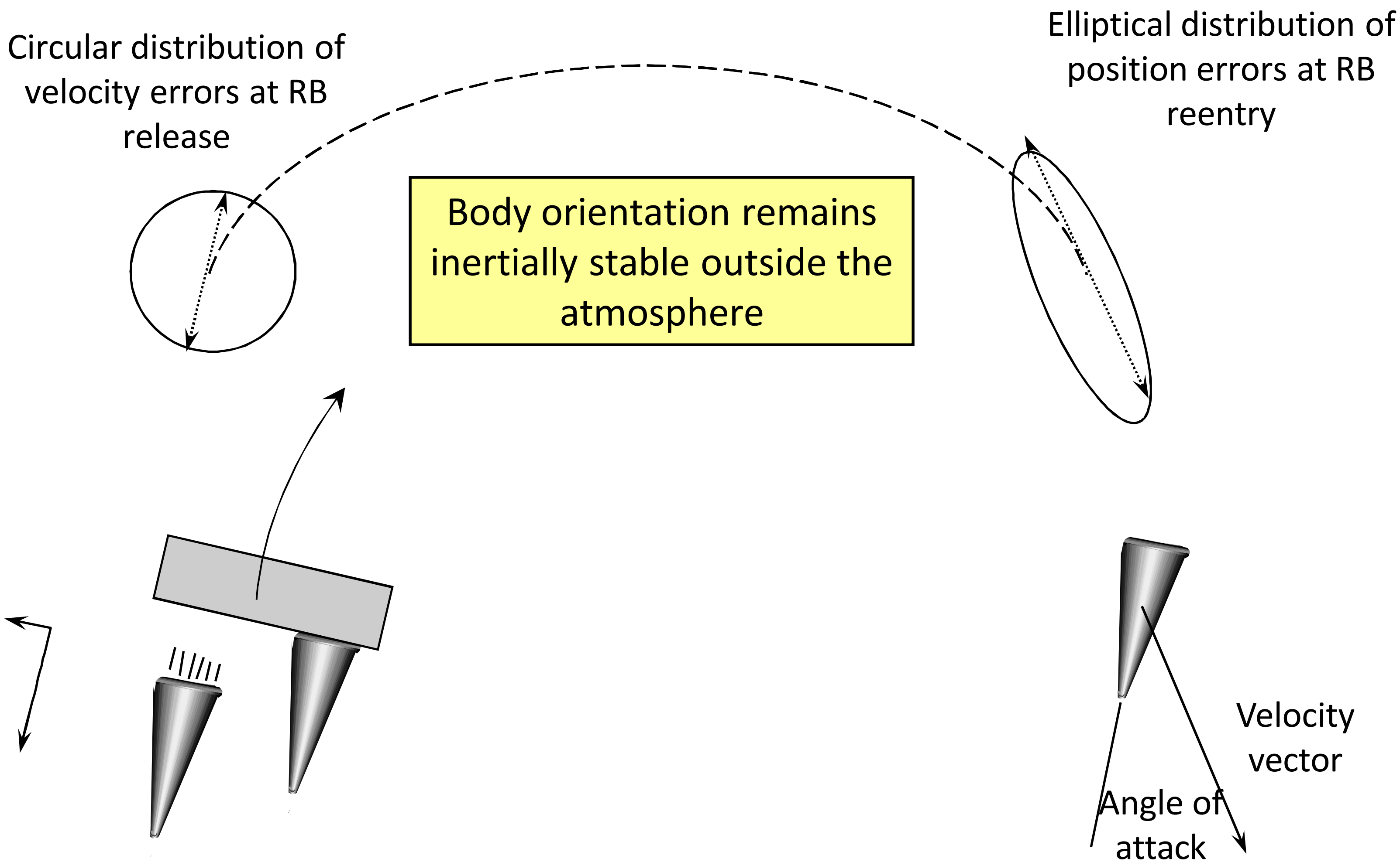


Two (opposite) velocity error directions which result in no displacement of the trajectory (except in time)

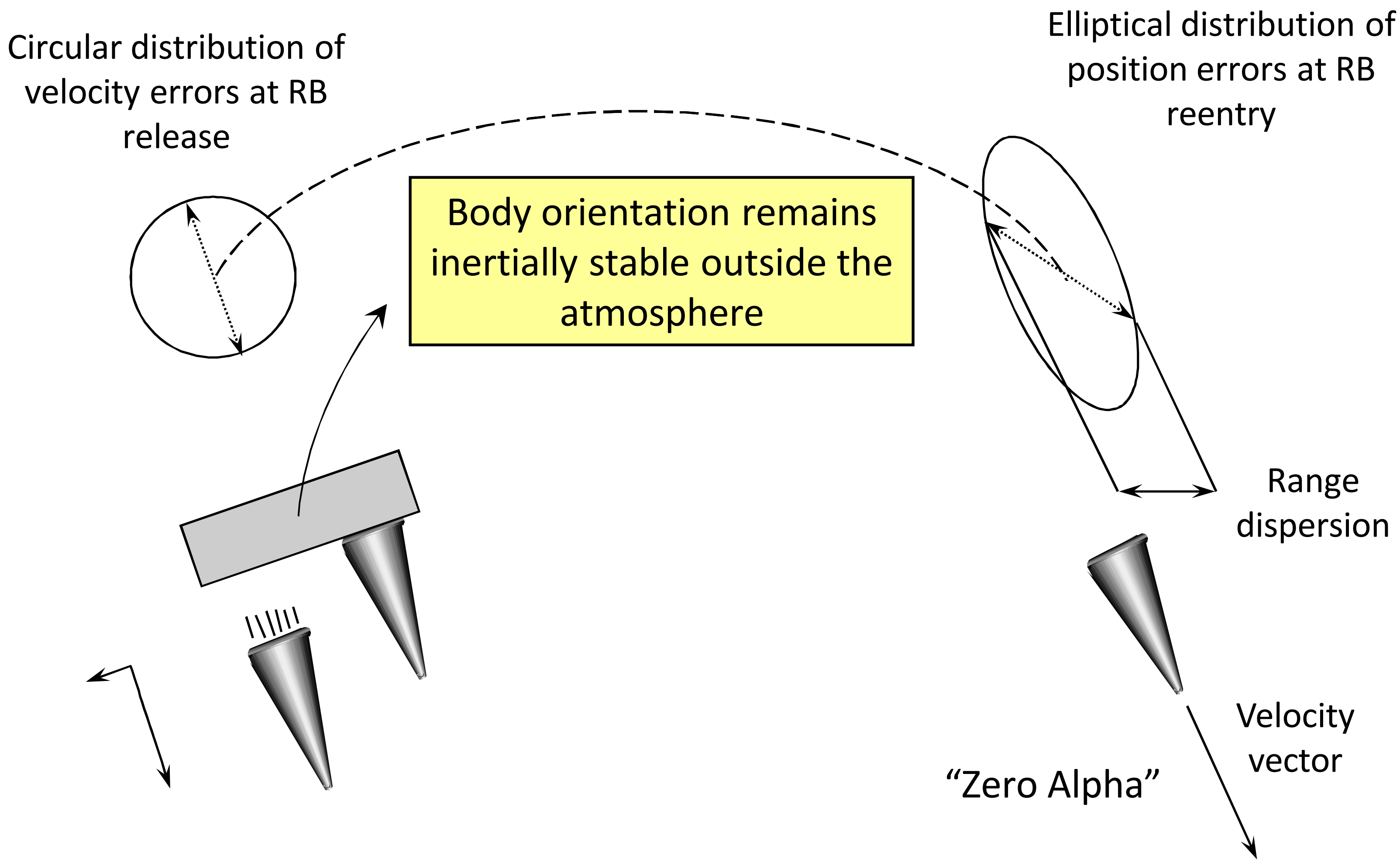
Velocity error directions form the “range insensitive axis”

Orientation of “range insensitive axis” relative to the velocity vector is trajectory dependent

Null Range Deployment



Zero Angle of Attack Deployment



Null Range vs. Zero Angle of Attack

- Selection of deployment direction is often based upon maximizing accuracy

- Zero Angle of Attack Deployment
  - Suffers from deployment-induced range errors

- Null Range Deployment
  - Suffers from aerodynamic variabilities associated with variations in angle-of-attack convergence

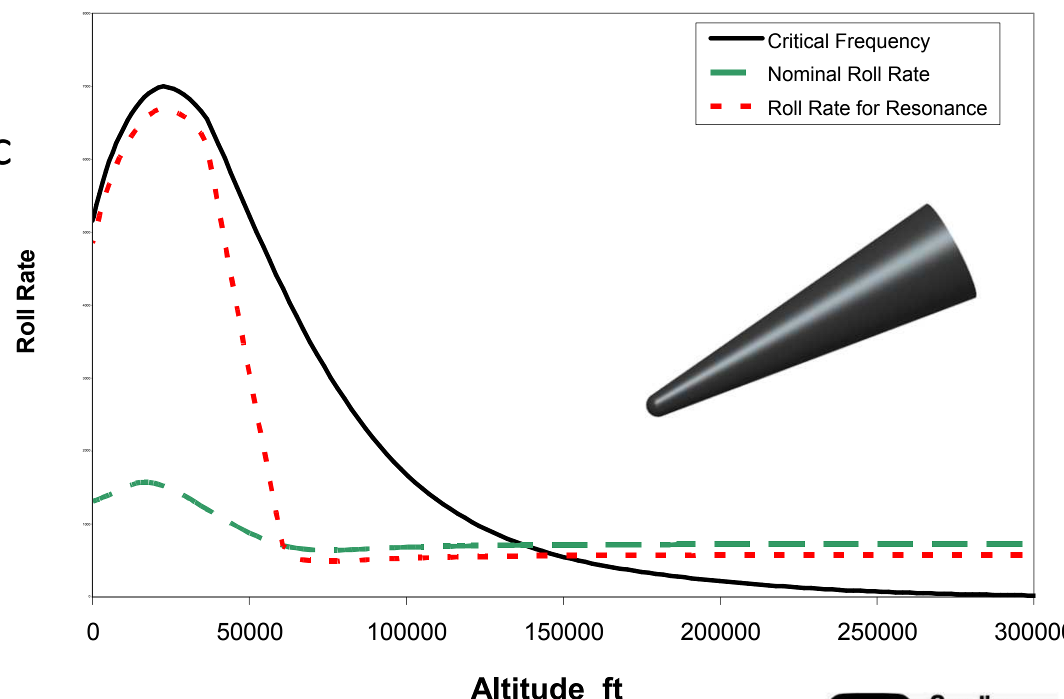
- For large deployment errors, null range deployment is preferred

- For small deployment errors, zero alpha deployment is preferred

Roll Behavior

- Typically, spin rate increases during reentry where it crosses the pitch/yaw natural frequency and then continues to increase rapidly.
- Small trim angles of attack induced by asymmetrical nosetip and/or heatshield ablation could amplify resonance when dynamic pressure is high. Large angular offset will likely cause greater dispersion, miss distance, and possible vehicle breakup.
- Vehicles that experience low altitude roll resonance are generally considered to be unstable.

When P_{cr} approaches P, roll resonance begins to initiate, and angle of attack increases.



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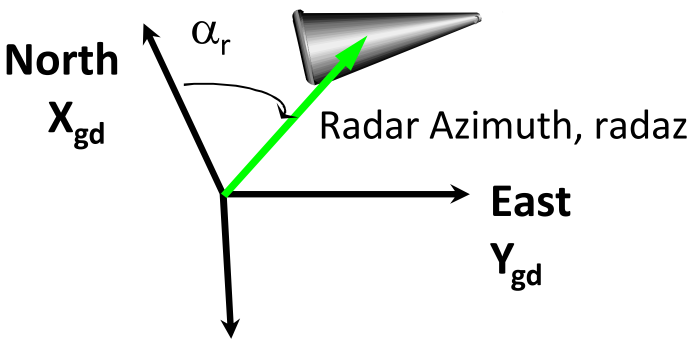
- Walt Wolfe, Sandia, retired

QUESTIONS?

Radar Tracking

α_r, Radar Azimuth Angle is Measured from North



Radar Range, Azimuth and Elevation Are the Position of a vehicle from the Radar Location

Radar Location is input to trajectory code for the Radar Antenna's Geodetic Latitude, Longitude and Altitude.