

A diffuse interface model of grain boundary faceting

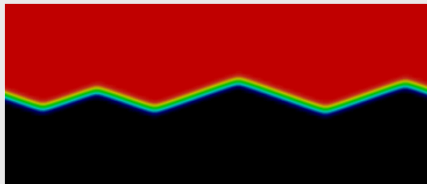
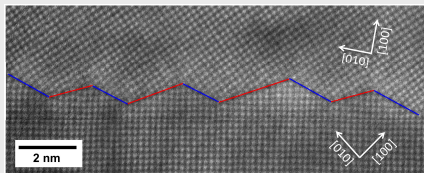
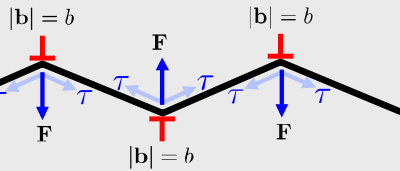
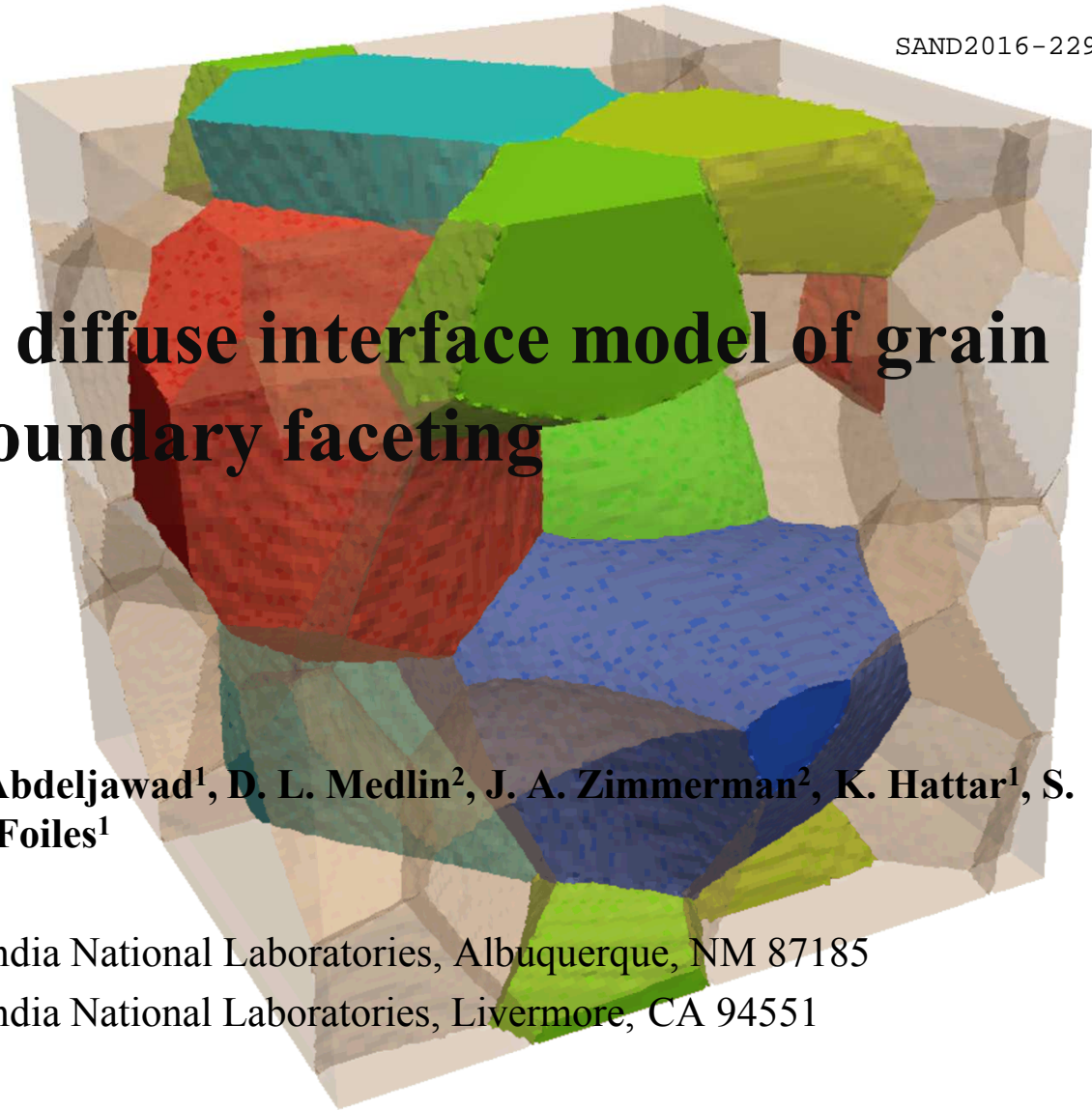
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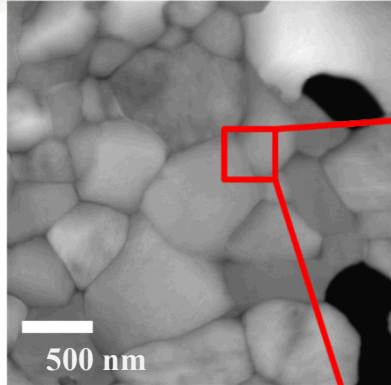


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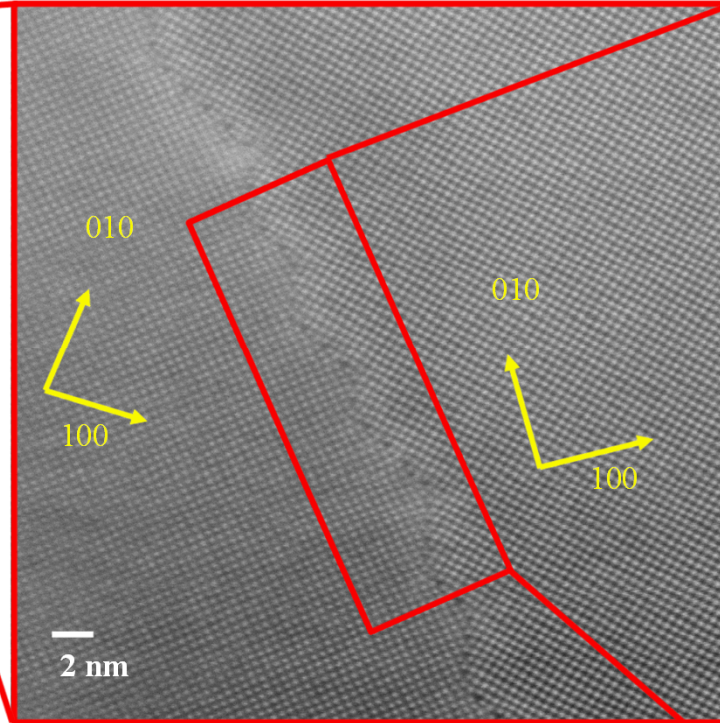
*Exceptional
service
in the
national
interest*

Grain Boundary (GB) Faceting

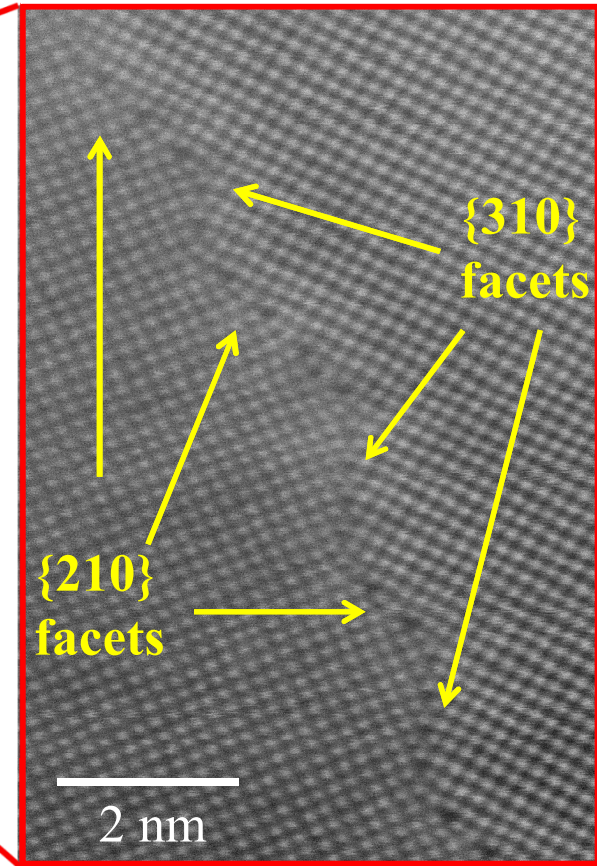
HAADF-STEM $\Sigma 5$ $\langle 001 \rangle$ GB in Fe



K. Hattar (1111)



Inclination from $\{310\}$: 26.3°



D. L. Medlin (8341)

Observations

- Thermal faceting into "hill-and-valley" morphology
- Boundary is faceted on $\{210\}$ and $\{310\}$ inclinations

Key Questions

- Thermodynamics of interface faceting (i.e., which GBs do facet?)
- Detailed examination via
 - Experimental characterization (HRSTEM)
 - Atomistic studies (DFT and MD)
 - Mesoscale framework (diffuse-interface model)
- Facet energetics and non-local interactions

Interface Thermodynamics

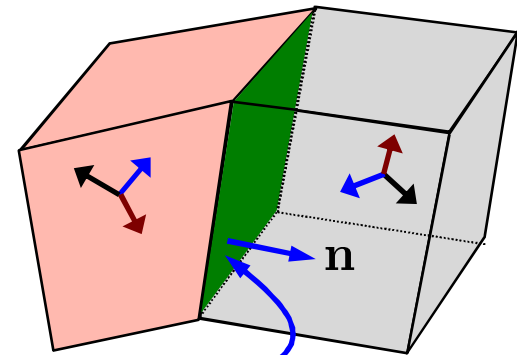
$$\gamma = \gamma_0 + f(\underbrace{\{\theta_1, \theta_2\}}_{\text{GB inclination}}, c, T, \underbrace{\{\phi_1 \Phi \phi_2\}}_{\text{GB misorientation}})$$

GB inclination
“GB plane normal”

GB misorientation
“crystallographic orientation
of abutting grains”

Interface stiffness: $\gamma + \frac{\partial^2 \gamma}{\partial \theta^2}$

Interface stress: $\tau_{ij} = \delta_{ij} \gamma + \frac{\partial \gamma}{\partial \epsilon_{ij}}$



GB plane: **n** normal unit vector or
 $\{\theta_1, \theta_2\}$ inclination angles

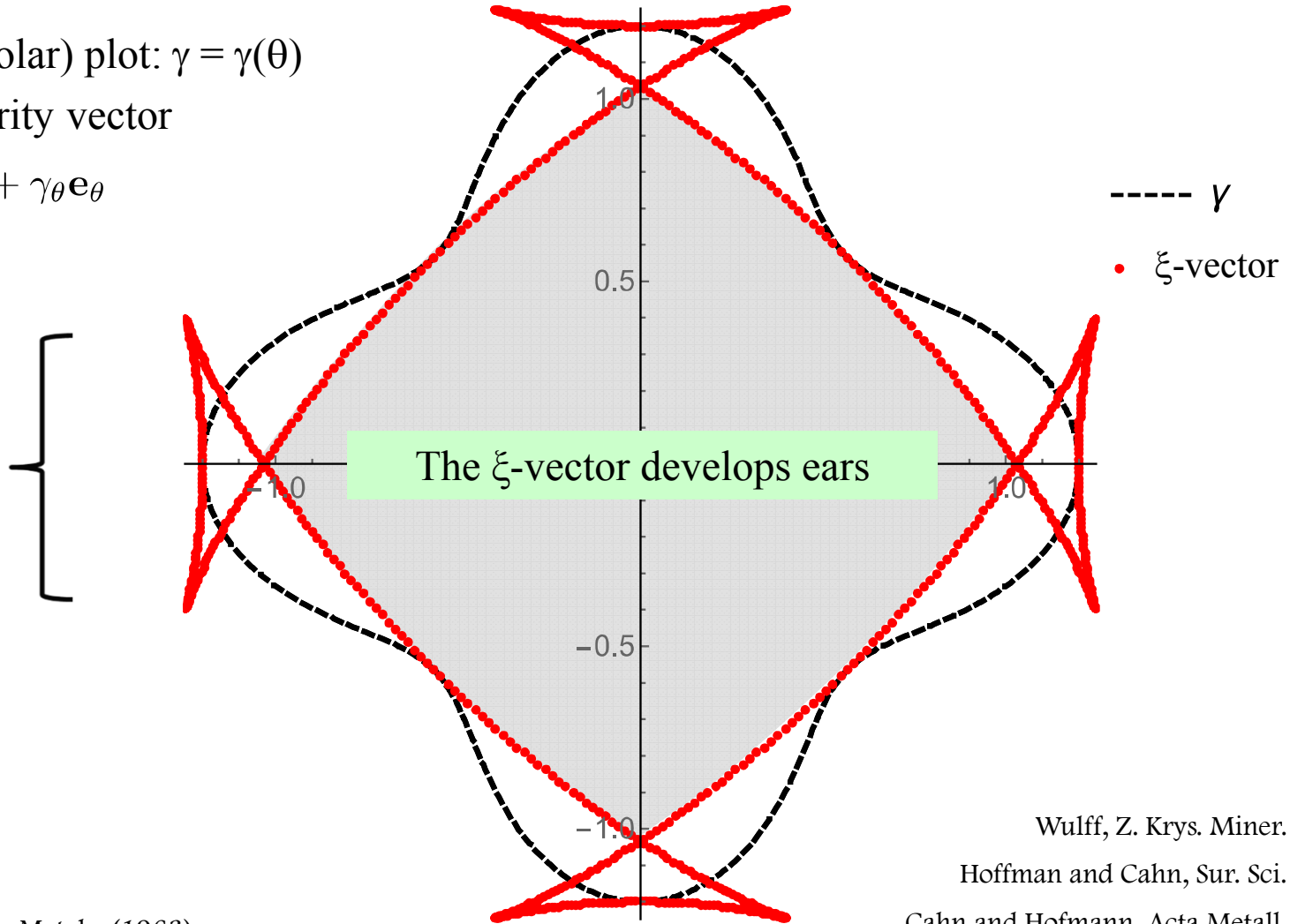
Faceting: Graphically

○ Wulff (polar) plot: $\gamma = \gamma(\theta)$

○ ξ -capillarity vector

$$\xi = \gamma \mathbf{e}_r + \gamma_\theta \mathbf{e}_\theta$$

An ear in
 ξ -vector



F. Frank, Amer. Soc. Metals (1963)

N. Cabrera, Sur. Sci. (1964)

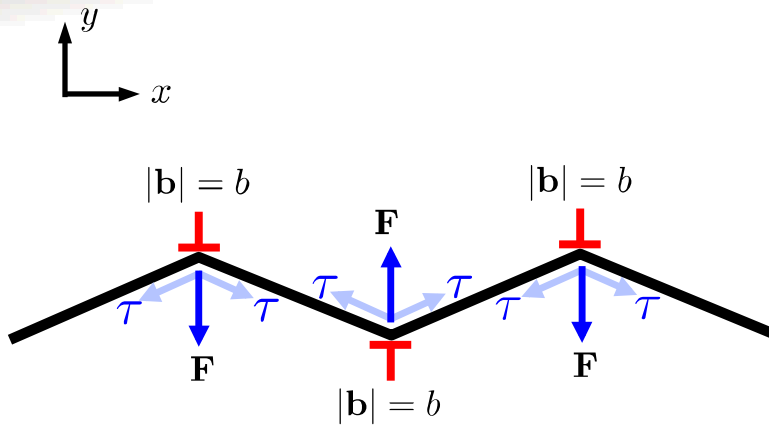
Wulff, Z. Krys. Miner. **34** (1901)

Hoffman and Cahn, Sur. Sci. **31** (1972)

Cahn and Hofmann, Acta Metall. **22** (1974)

Herring, Phys. Rev. **82** (1951)

Energetics of a Faceted GB



⊥ Dislocation with Burgers vector \mathbf{b} due to mismatch in translation vectors of interfaces

Dimitrakopoulos et al. Interface Sci. (1996)

\mathcal{T} : Interface stress. Discontinuity across junction leads to line forces

Hamilton et al. Phys. Rev. Lett. (2003)

- Interface anisotropy (high/low energy planes)
- Balance of interface stress (τ) at facet junctions leads to line forces $\mathbf{F} = \pm \mathbf{P}$
- Dislocation content at facet junctions and their elastic interactions

$$\mathcal{F}_{tot} = f_{inter} + f_{local} + f_{nonlocal}$$

↙
Anisotropic
interface

↓
Junction energy due to
dislocation and interface
stress

↘
Non-local interactions
between dislocations, and
point forces

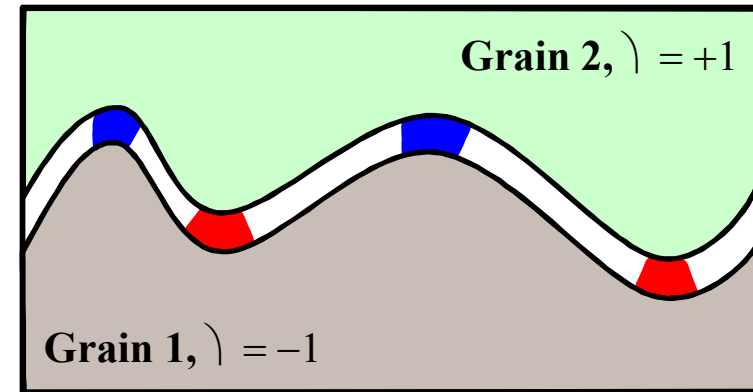
Phase Field Framework

■ Structural Order Parameter

- Mean curvature: $\kappa = \nabla \cdot \left(\frac{\nabla \phi}{|\nabla \phi|} \right) \sim \nabla^2 \phi$
- Higher order expansion:

$$\gamma = a_0(\theta) + a_2(\theta)\kappa^2 + a_4(\theta)\kappa^4 + \dots$$

Herring, Phys. Powd. Metall.
(1951)



■ $\nabla^2 \phi > 0$ ■ $\nabla^2 \phi < 0$

■ Total Energy

$$\mathcal{F}_{tot} = \int dr \left[A_H f(\phi) + \frac{\epsilon^2(\theta)}{2} |\nabla \phi|^2 + \Gamma_o (\nabla^2 \phi)^2 + \Gamma_1 \int_{|\mathbf{r}-\mathbf{r}'| > R_c} \frac{d\mathbf{r}' \nabla^2 \phi(\mathbf{r}) \nabla^2 \phi(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \right]$$

Bulk

Anisotropic GB
(Wulff, ξ -vector)

Local facet
junction energy

facet junction-junction
interactions

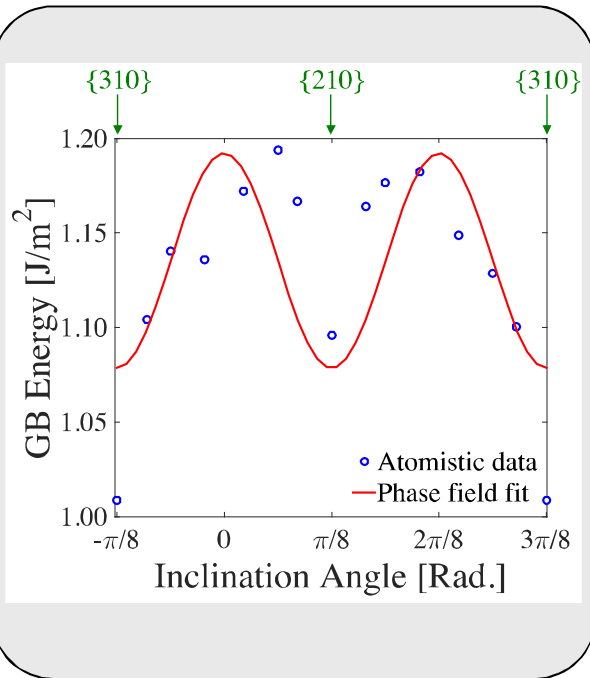
■ Dynamics

$$\frac{\partial \phi}{\partial t} = -L \left(\frac{\delta \mathcal{F}_{tot}}{\delta \phi} \right)$$

Einstein's razor: "Everything should be made as simple as possible, but no simpler"

Model Parameters: Atomistics

- The case for $\Sigma 5$ $\langle 001 \rangle$ tilt GB in BCC Fe



Proposed fit

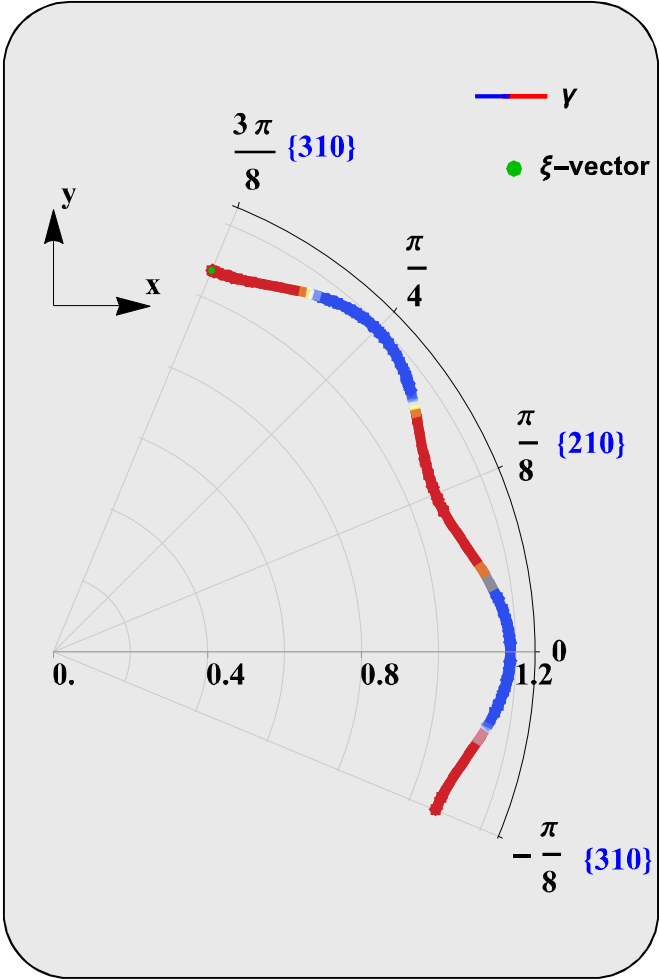
$$\epsilon(\theta) = \lambda [1 + \delta \cos(8\theta)]$$

Fit to atomistics

$$\lambda = 1.1$$
$$\delta = 0.05$$

Model Parameters

λ : Nominal GB energy
 δ : Strength of anisotropy
 Γ_0 : Junction energy
 Γ_1 : Non-local interactions

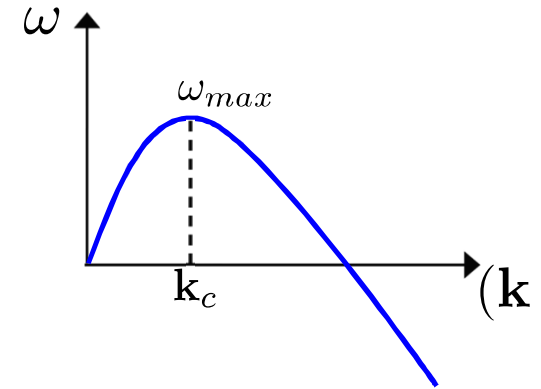
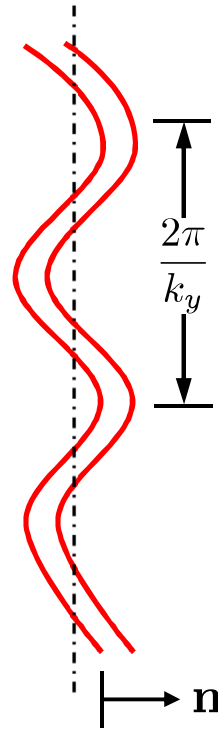


Linear Stability

- Linearize integro-differential Eq.
- Assume Fourier modes $\phi = \hat{\delta}_\phi \exp [i\mathbf{k} \cdot \mathbf{x} + \omega(\mathbf{k})t]$
- Dispersion relation $\omega(\mathbf{k}, \Gamma_o, \Gamma_1; \lambda, \delta)$

Γ_o : Junction energy
 Γ_1 : Junction interactions

$\omega(\mathbf{k}, \Gamma_o, \Gamma_1; \lambda, \delta)$
 $\omega > 0$: Growing
 $\omega < 0$: Decaying (unstable) (stable)



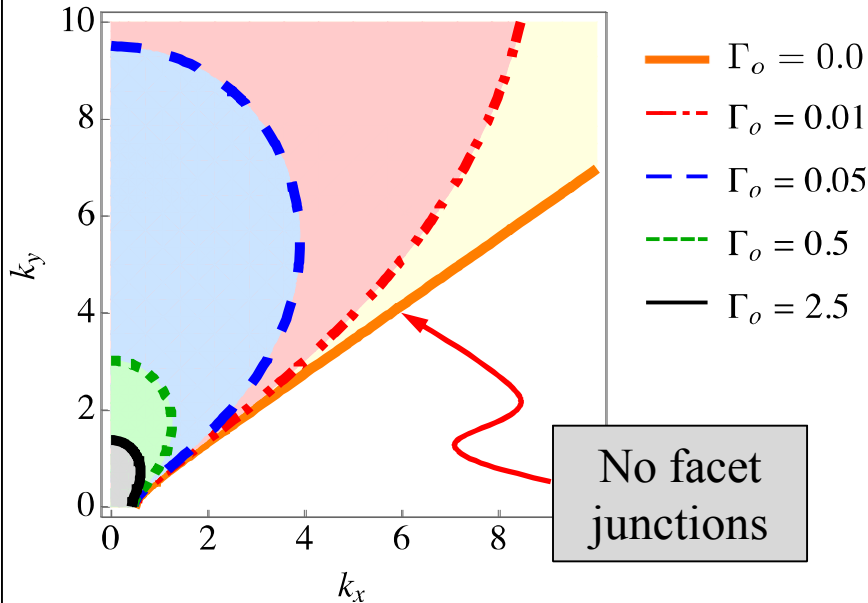
$$\omega(\mathbf{k}) = 4A_H - \lambda^2(1 + \delta) \left[\left(1 + \delta - \frac{4\pi\Gamma_1}{\lambda^2(1 + \delta)R_c} \right) k_x^2 + \left(1 - 63\delta - \frac{4\pi\Gamma_1}{\lambda^2(1 + \delta)R_c} \right) k_y^2 + \frac{2\Gamma_o}{\lambda^2(1 + \delta)} (k_x^2 + k_y^2)^2 \right]$$

Linear Stability (cont.)

Γ_o : Junction energy
 Γ_1 : Junction interactions

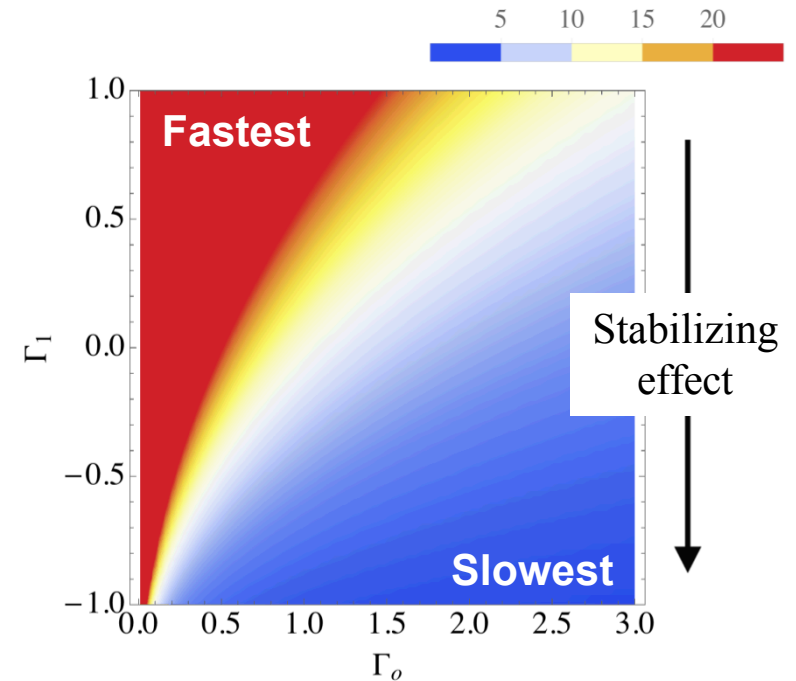
Facet junction energy with no interactions ($\Gamma_1 = 0$)

Shaded regions correspond to **unstable** perturbations



Maximum growth rate

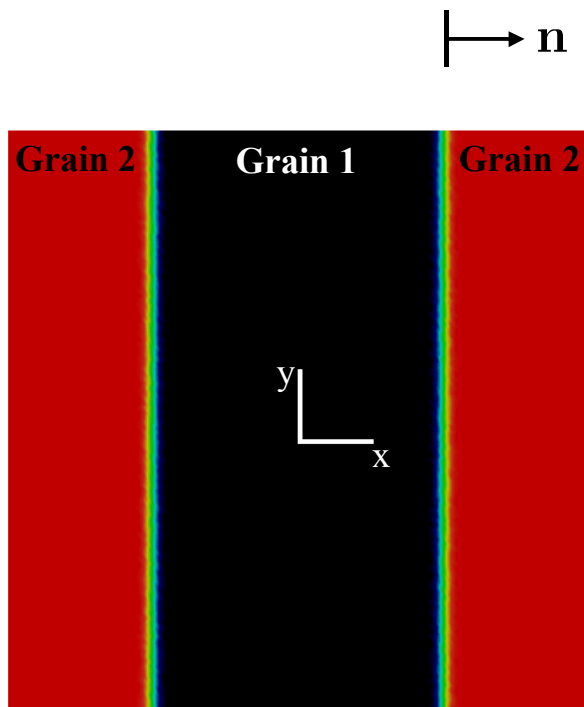
$$\omega_{max}(\mathbf{k}_c, \Gamma_o, \Gamma_1; \lambda, \delta)$$



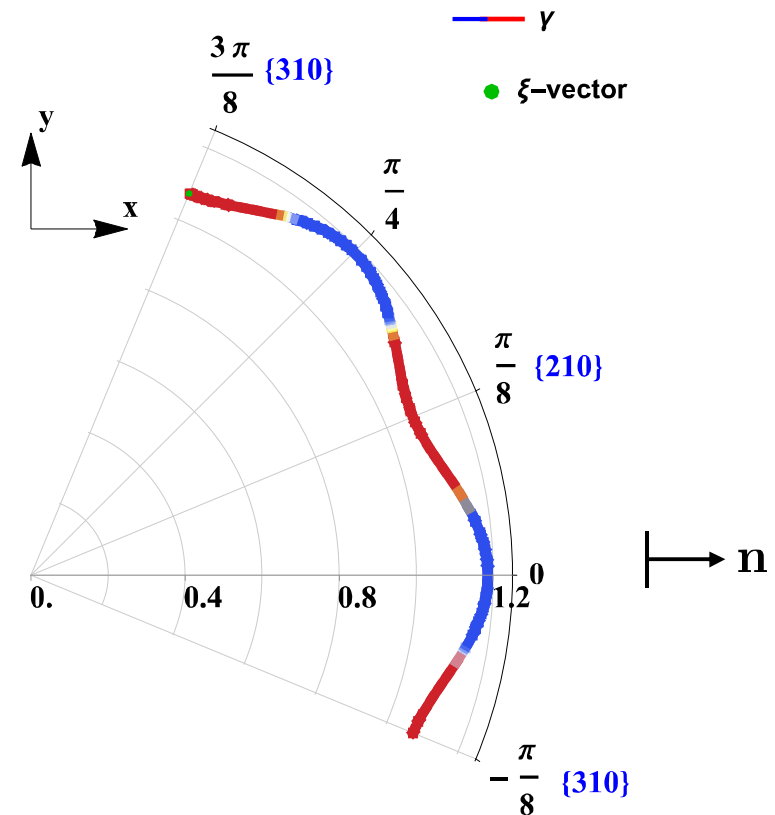
- Regions of instability decrease in size as facet junction energy increases
- Faceted GBs with large spacing between junctions

Results: A Two Grain Structure

- Vary the facet junction energy (Γ_o) with no junction interactions ($\Gamma_1 = 0$)
- Vary junction interactions (Γ_1) for a fixed junction energy



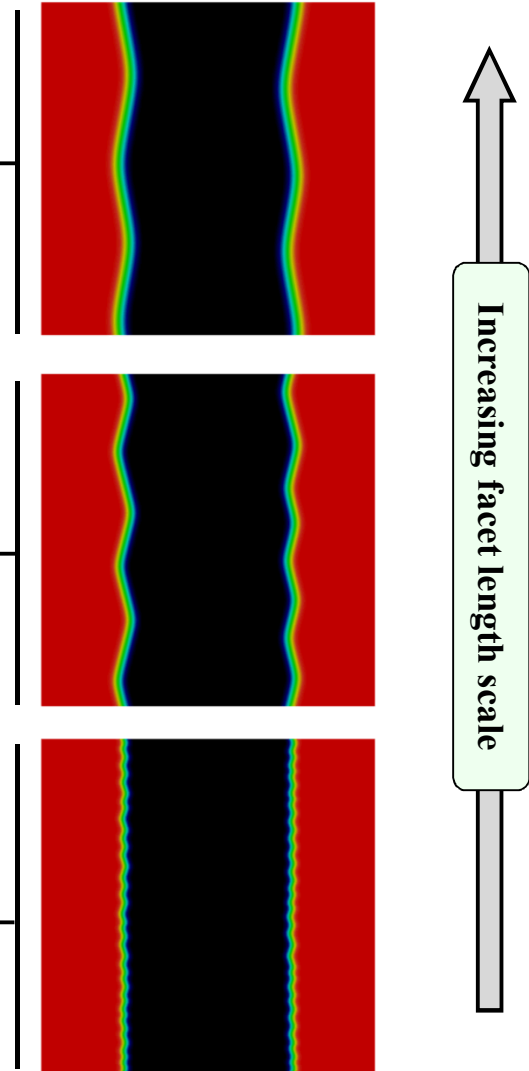
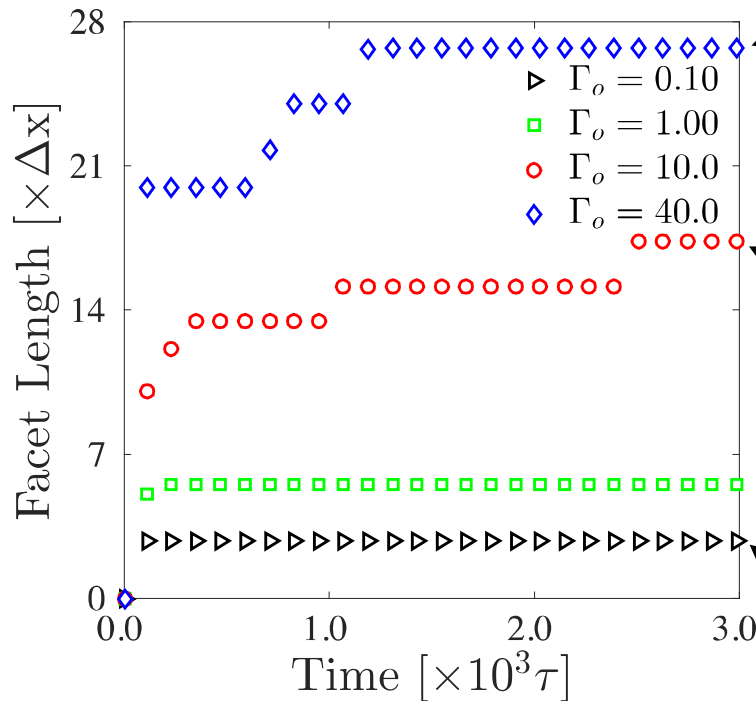
Critical quench



Results: Facet Junction Energy Γ_o

- A two-grain slab geometry

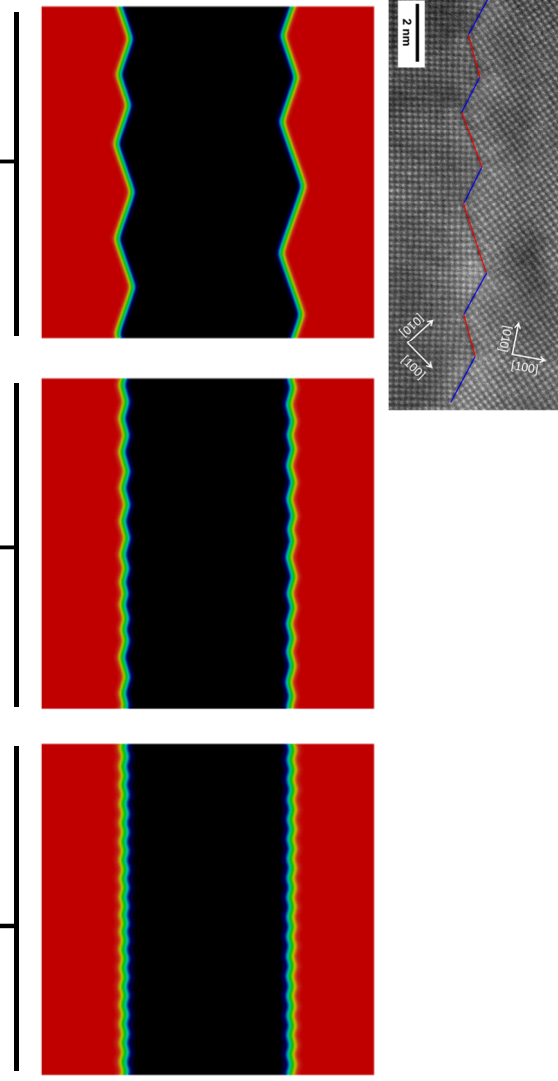
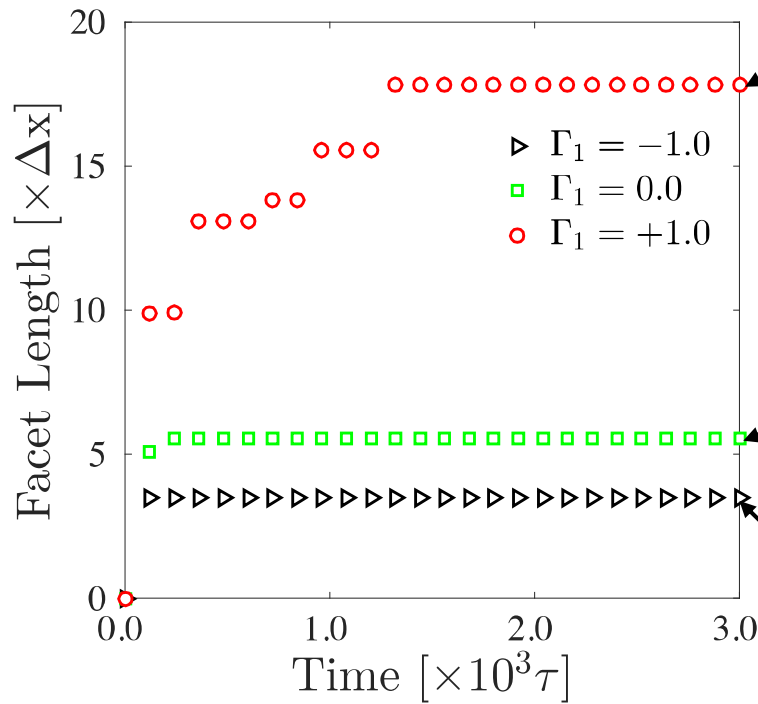
- Facet length scale increases with (Γ_o)



Γ_o : Junction energy

Results: Junction Interactions Γ_1

- A two-grain slab geometry
 - Negative (Γ_1) plays a stabilizing role



Γ_1 : Junction interactions

Conclusion

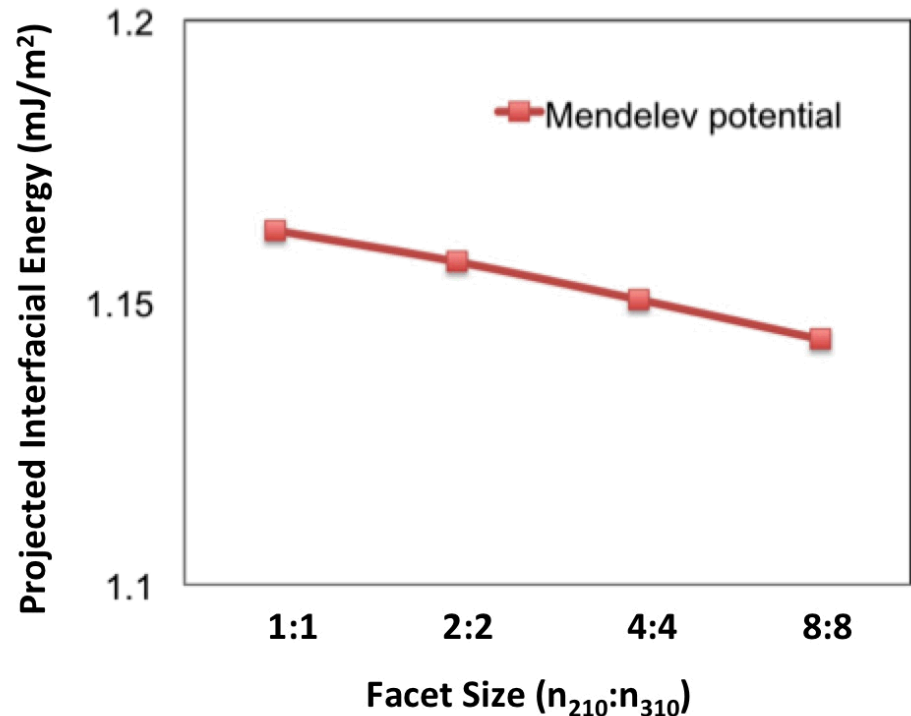
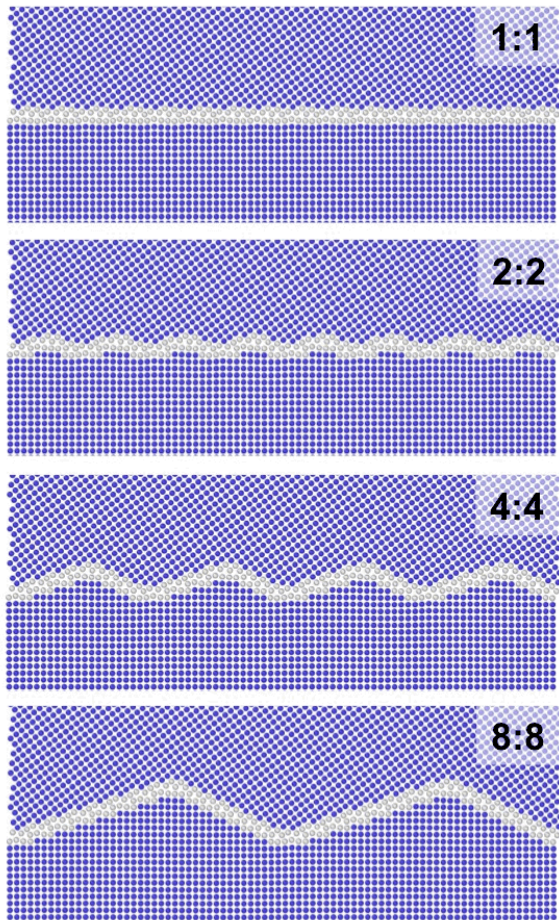
- A mesoscale framework that examines
 - GB anisotropy (non-convexity in the GB free energy)
 - Facet junction energy due to defect content
 - Non-local junction-junction interactions
- Faceting and subsequent coarsening driven by junction energetics
- Capable of examining any GB given the GB energy vs. inclination diagram

Thank you

Results: Facet Junction Energy Γ_0

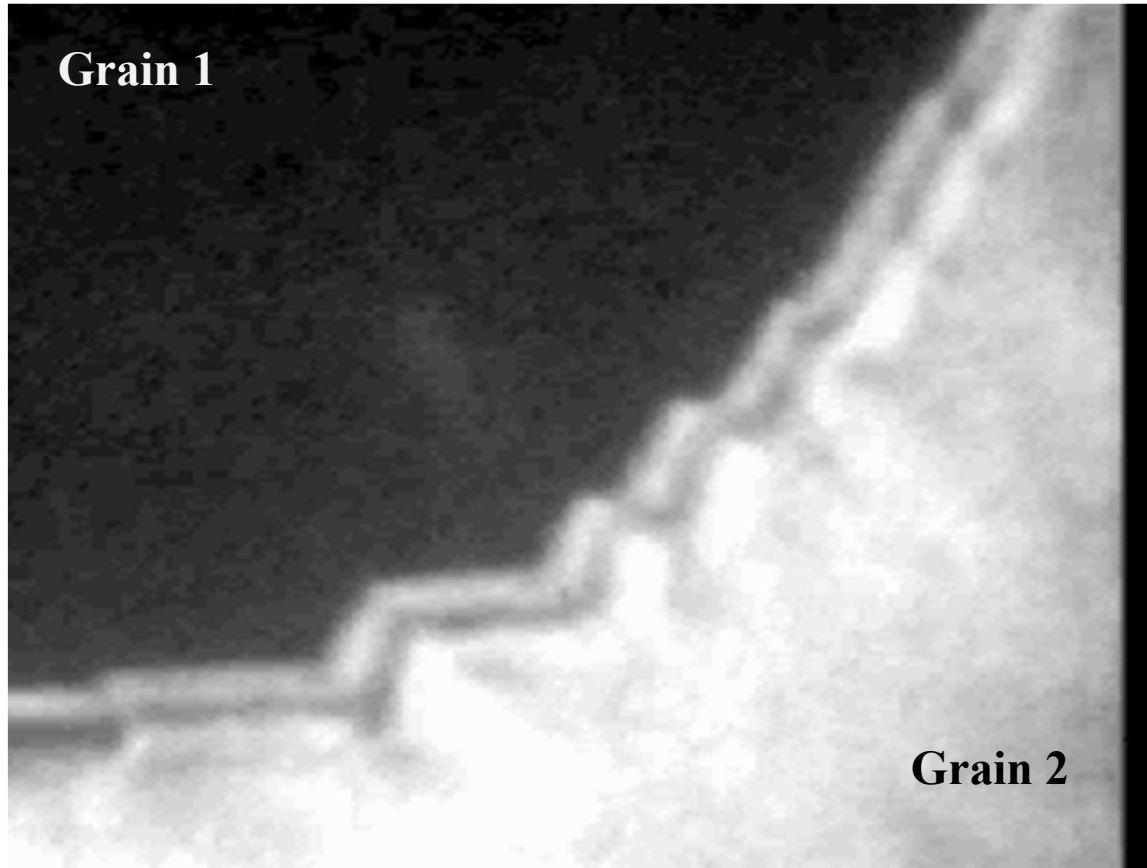
■ Atomistic Simulations

- Fix misorientation ($\Sigma 5$) and inclination (26.565° from $\{310\}$ plane)
- Vary number of facet junctions for a given system size



Energy is reduced with facet coarsening

Interface Faceting: Examples



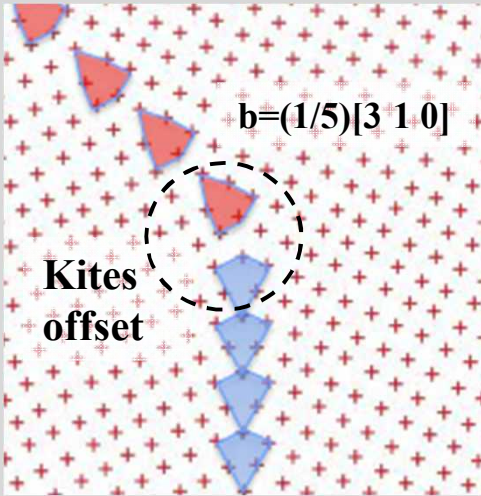
50 nm

D. L. Medlin and G. Lucadamo (2000)

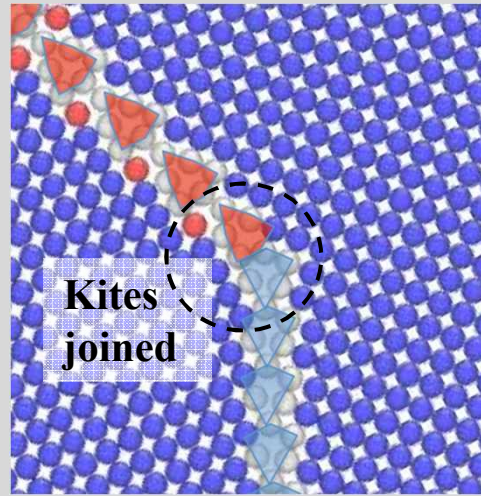
- Au films annealed at 500 °C
- $\Sigma 3$ GB
- $\{11\bar{2}\}$ facets and subsequent facet coarsening

There Is More To It Than This

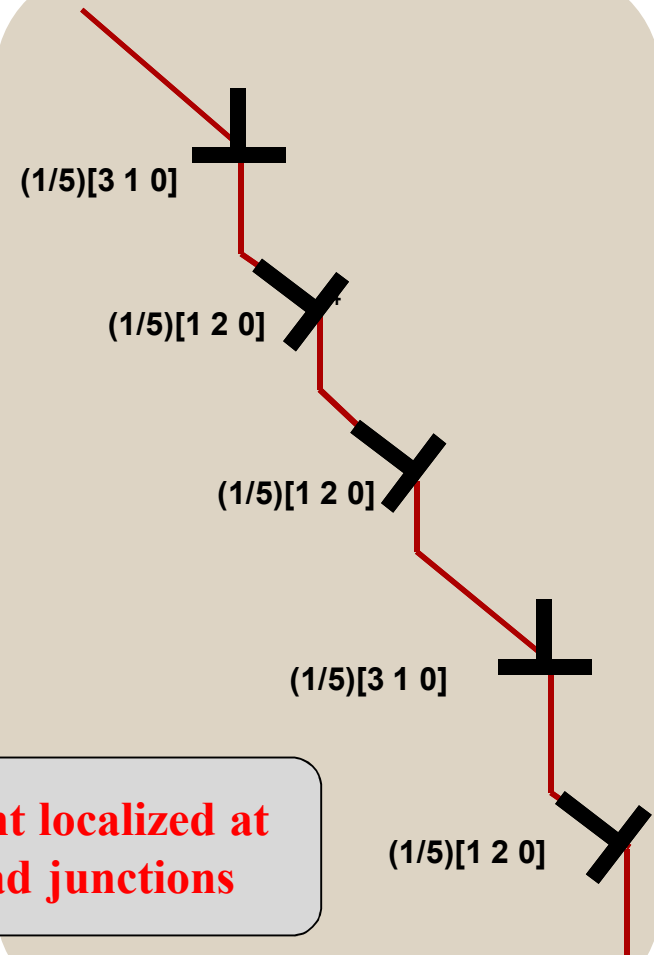
Experimental Junctions $b=(1/5)(120)$ and $(1/5)(310)$



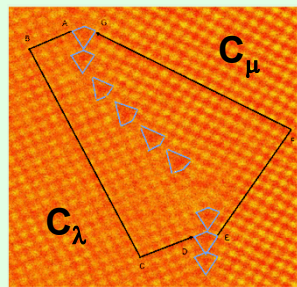
Relaxed Periodic Atomistic Structure



Defect Distribution



Interface circuit mapping



Path in μ crystal Path in λ crystal

$$\mathbf{b} = -(\mathbf{C}_\lambda + \mathbf{P}\mathbf{C}_\mu)$$

Burgers vector Re-express μ path in λ crystal coordinates.

Defect content localized at head-to-head junctions

Faceting: Mathematically

- Sharp interface law for GBs

$$v_n = M_{gb} \left(\gamma_{gb} + \frac{\partial^2 \gamma_{gb}}{\partial \theta^2} \right) \mathcal{K}$$

Herring, Phys. Powd. Metall. (1951)

Mullins, J. App. Phys. 27 (1956)

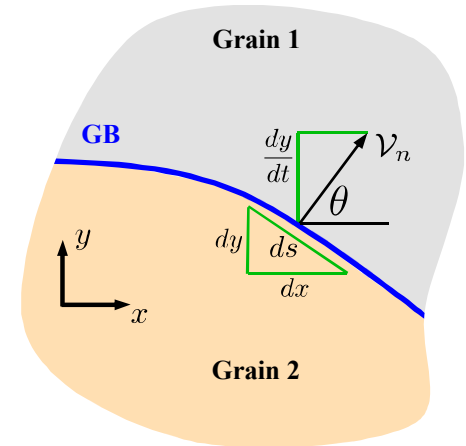
v_n : Normal velocity
 M_{gb} : GB mobility
 γ_{gb} : GB energy
 \mathcal{K} : Mean curvature

- Linearized

$$\frac{\partial y}{\partial t} = M_{gb} \left(\gamma_{gb} + \frac{\partial^2 \gamma_{gb}}{\partial \theta^2} \right)_o \frac{\partial^2 y}{\partial x^2}$$

When $\gamma_{gb} + \partial^2 \gamma_{gb} / \partial \theta^2 < 0$

Locally backward parabolic PDE



Thermodynamic instability of an initially planar surface with negative surface stiffness leads to faceting (hill and valley) structure