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ALEGRA Based Computation of Magnetostatic Configurations

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OUTLOOK

About ALEGRA MHD

Ellipsoid in electrostatic of magnetostatic field (exact solution)

Validation and Verification of ALEGRA MHD

Conclusion

V&V of ALEGRA MHD Code

Grinfeld, M., Niederhaus, J., and Porwitzky, A., Using the ALEGRA Code for Analysis of Quasi-Static Magnetization of Metals, ARL-TR-7415 SEP 2015.

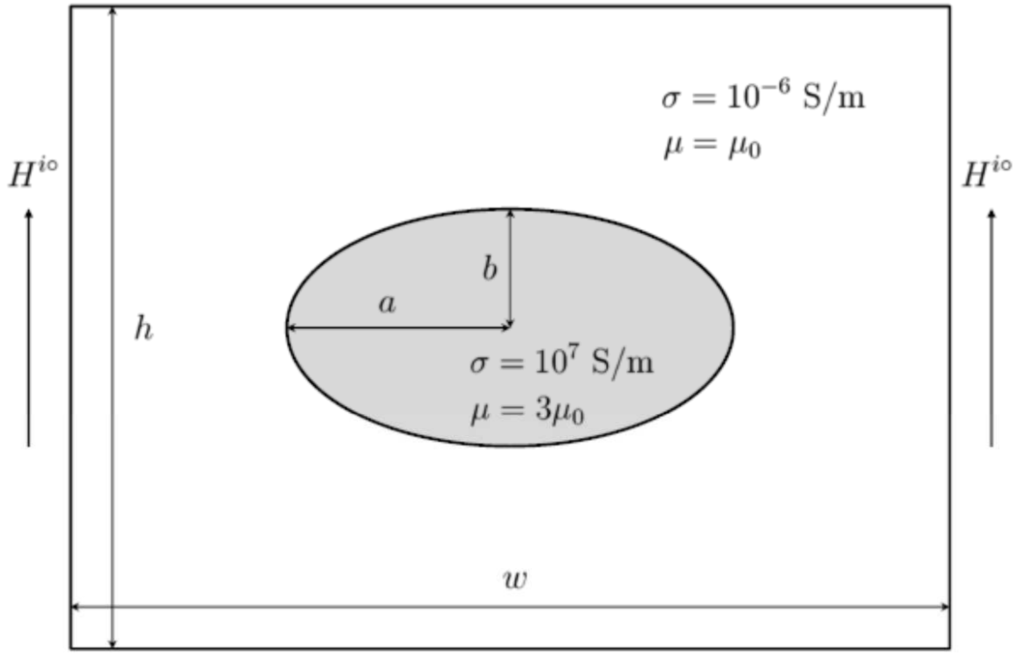

Fig. 1: Schematic diagram (not to scale) showing problem to be considered.

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Exact solution for elliptic inclusion

ARL

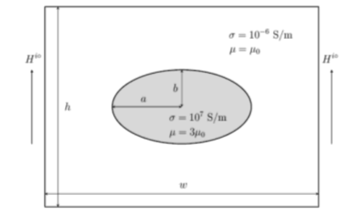

Fig. 1: Schematic diagram (not to scale) showing problem to be considered.

The boundary value problem

The PDE

$$\nabla_i\left(\mu H^i\right)=0, \quad H^i=-\nabla^i\eta \;\rightarrow \nabla_i\nabla^i\eta=0$$

The matrix/inclusion BC

$$[\eta]_-^+=\left[\mu\nabla^i\eta\right]_-^+ N_i=0$$

Conditions at infinity

$$\eta\rightarrow -H_i^{\circ}z^i \;\; at \;\; |z|\rightarrow\infty$$

The exact solution of BVP

Inside inclusion $\eta_-=-K_i z^i$

Inside matrix $\eta_+=A^i\nabla_i\Theta-H_i^{\circ}z^i,$

The Newtonian potential $\Theta=-\int_{\omega} d\omega^*\ln\left|\vec{z}-\vec{z}*\right| \quad \Theta_-(z)=C-\frac{1}{2}Y_{ij}z^i z^j$

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Exact solution for elliptic inclusion

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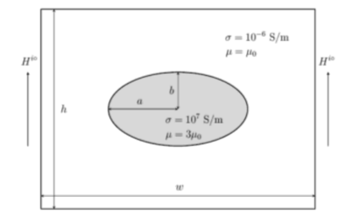

Fig. 1: Schematic diagram (not to scale) showing problem to be considered.

The boundary value problem

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The exact solution of BVP

Inside inclusion $\eta_- = -K_i z^i$ **Inside matrix** $\eta_+ = A^i\nabla_i\Theta - H_i^\circ z^i,$

The Newtonian potential $\Theta=-\int_\omega d\omega^*\ln\left|\vec{z}-\vec{z}^*\right|$ $\Theta_-\left(z\right)=C-\frac{1}{2}Y_{ij}z^i z^j$

Exact solution for elliptic inclusion

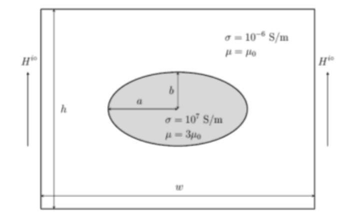

Fig. 1: Schematic diagram (not to scale) showing problem to be considered.

The Newtonian potential

$$\Theta = -\int_{\omega} d\omega^{*} \ln\left|\vec{z} - \vec{z}^{*}\right| \qquad \Theta_{-}(z) = C - \frac{1}{2} Y_{ij} z^{i} z^{j}$$

The magnetic field inside the inclusion

$$\begin{Vmatrix} K_1 \\ K_2 \end{Vmatrix} = \begin{Vmatrix} \dfrac{a_1 + a_2}{a_1 + \mu a_2} H_1^{\circ} \\ \dfrac{a_1 + a_2}{\mu a_1 + a_2} H_2^{\circ} \end{Vmatrix}$$

The Maxwell sub-system

$$z^{ijk}\nabla_j E_k = -\frac{1}{c}\frac{\partial E^i}{\partial t}, \quad z^{ijk}\nabla_j H_k = \frac{4\pi}{c} J^i,$$

$$\nabla_i E^i = 0, \quad \nabla_i H^i = 0$$

Hydrodynamics sub-system

$$\rho\left(\frac{\partial v^i}{\partial t} + v^j \nabla_j v^i\right) = -\nabla^i p + \rho q\left(E^i + z^{ijk} v_j H_k\right) - \rho v^i Q$$

$$\nabla_i v^i = 0$$

Boundary conditions

$$\left[E^i\right]_-^+ \tau_i = \left[H^i\right]_-^+ \tau_i = 0, \quad \left[D^i\right]_-^+ N_i = \left[B^i\right]_-^+ N_i = 0 \qquad v^j N_j = 0$$

V&V of ALEGRA MHD Code

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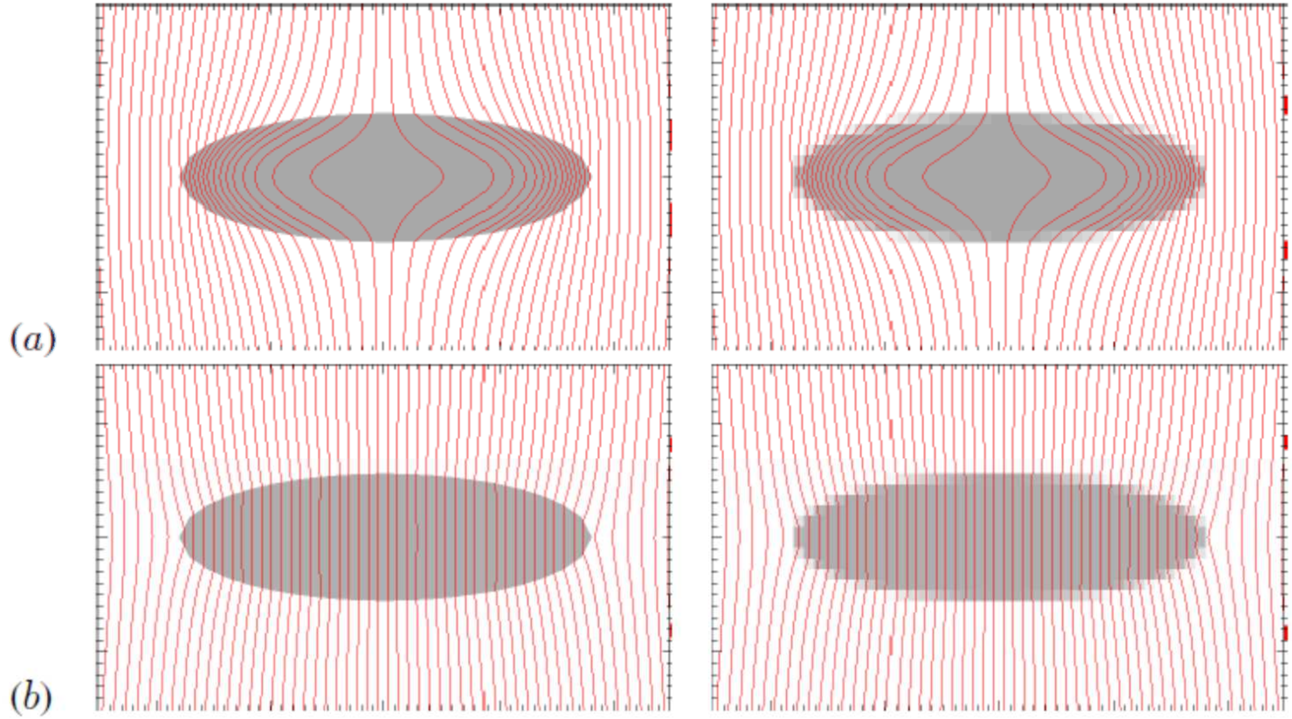
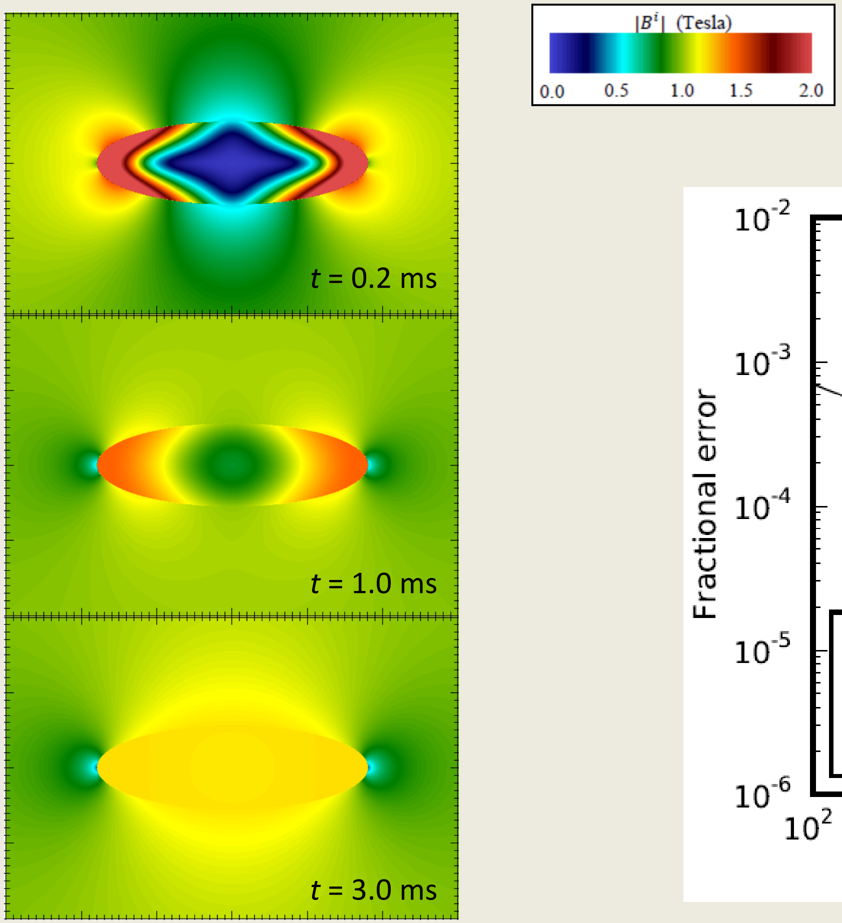


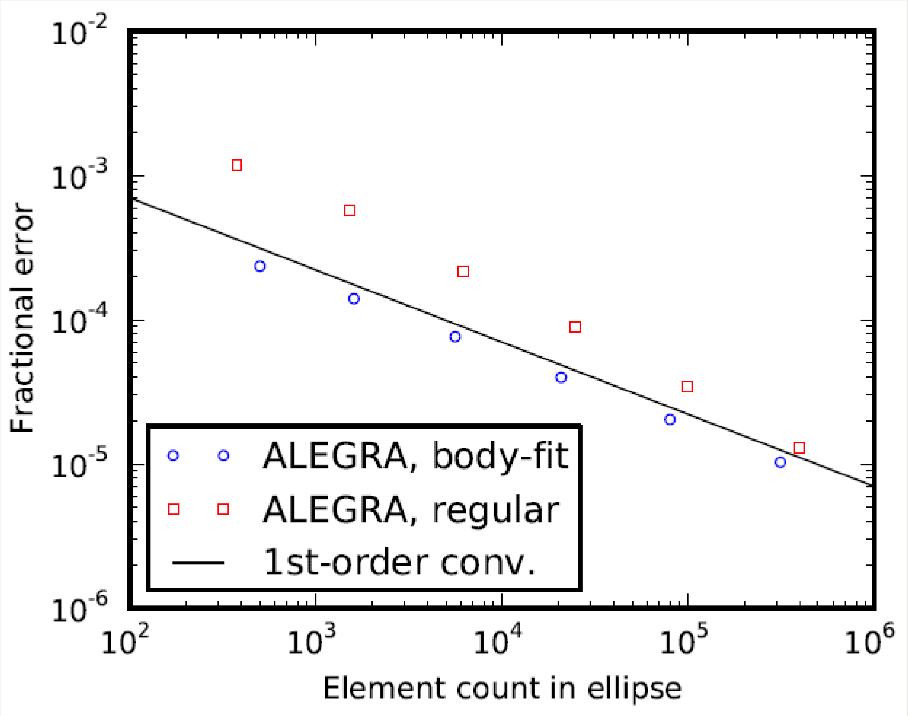
Fig. 3: Configuration of B^i field lines before a) and after b) after the equilibrium state is reached, for the body-fitted (left) and regular (right) mesh types. Simulation times: a) 0.2 ms, b) 3.0 ms.

Magnetostatics modeling: ellipse

Magnetic ellipse response to external field:



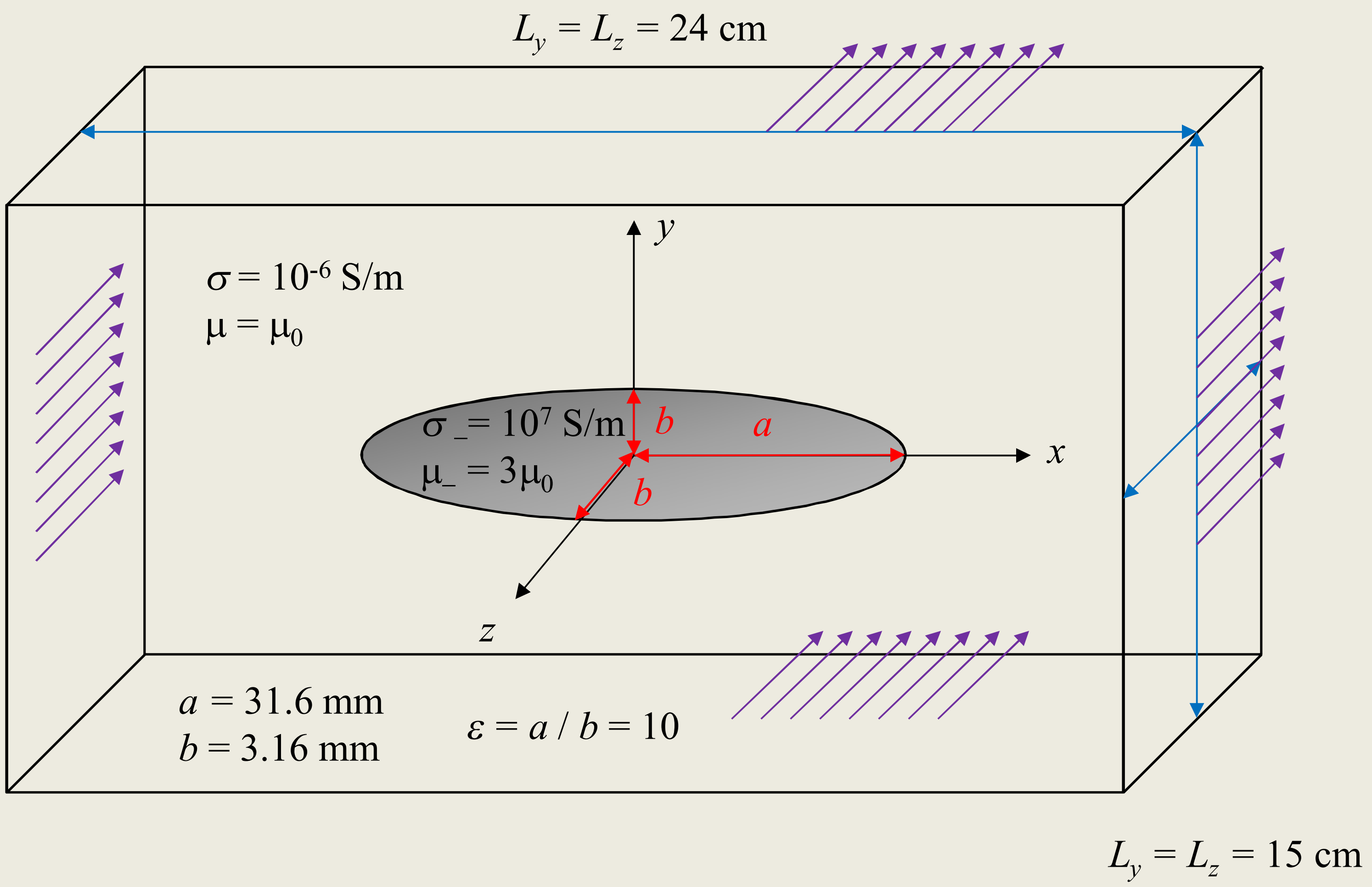
Convergence to analytic solution:



M. Grinfeld, J. Niederhaus, and A. Porwitzky, "Using the ALEGRA code for analysis of quasi-static magnetization of metals," U.S. Army Research Laboratory Technical Report ARL-TR-7415, 2015.

3D verification test

$$H^{\infty} = \frac{1}{\sqrt{3}\mu_0}(\hat{x} + \hat{y} + \hat{z})$$



Analytic solution

$$G = \frac{1}{2} Y_{axial} = 2\pi - \frac{2\pi\varepsilon}{(\varepsilon^2 - 1)^{3/2}} \left(\varepsilon\sqrt{\varepsilon^2 - 1} - \operatorname{arccosh} \varepsilon \right)$$

$$H_{||} = \left(1 + 2G \frac{\mu_{-} - 1}{4\pi} \right)^{-1} H_{||}^{\infty}$$

$$H_{\perp[2,3]} = \left(1 + (2\pi - G) \frac{\mu_{-} - 1}{4\pi} \right)^{-1} H_{\perp[2,3]}^{\infty}$$

$1 \rightarrow \text{axial } (x), \quad 2 \rightarrow \text{transverse } (y), \quad 3 \rightarrow \text{transverse } (z)$

$B_1 = \mu\_H_{||}$ $\quad B_2 = \mu\_H_{\perp 2}$ $\quad B_3 = \mu\_H_{\perp 3}$

$B_1 = 1.6645 \text{ T}$ $\quad B_2 = 0.8749 \text{ T}$ $\quad B_3 = 0.8749 \text{ T}$

Convergence study

N = elements spanning spheroid major axis

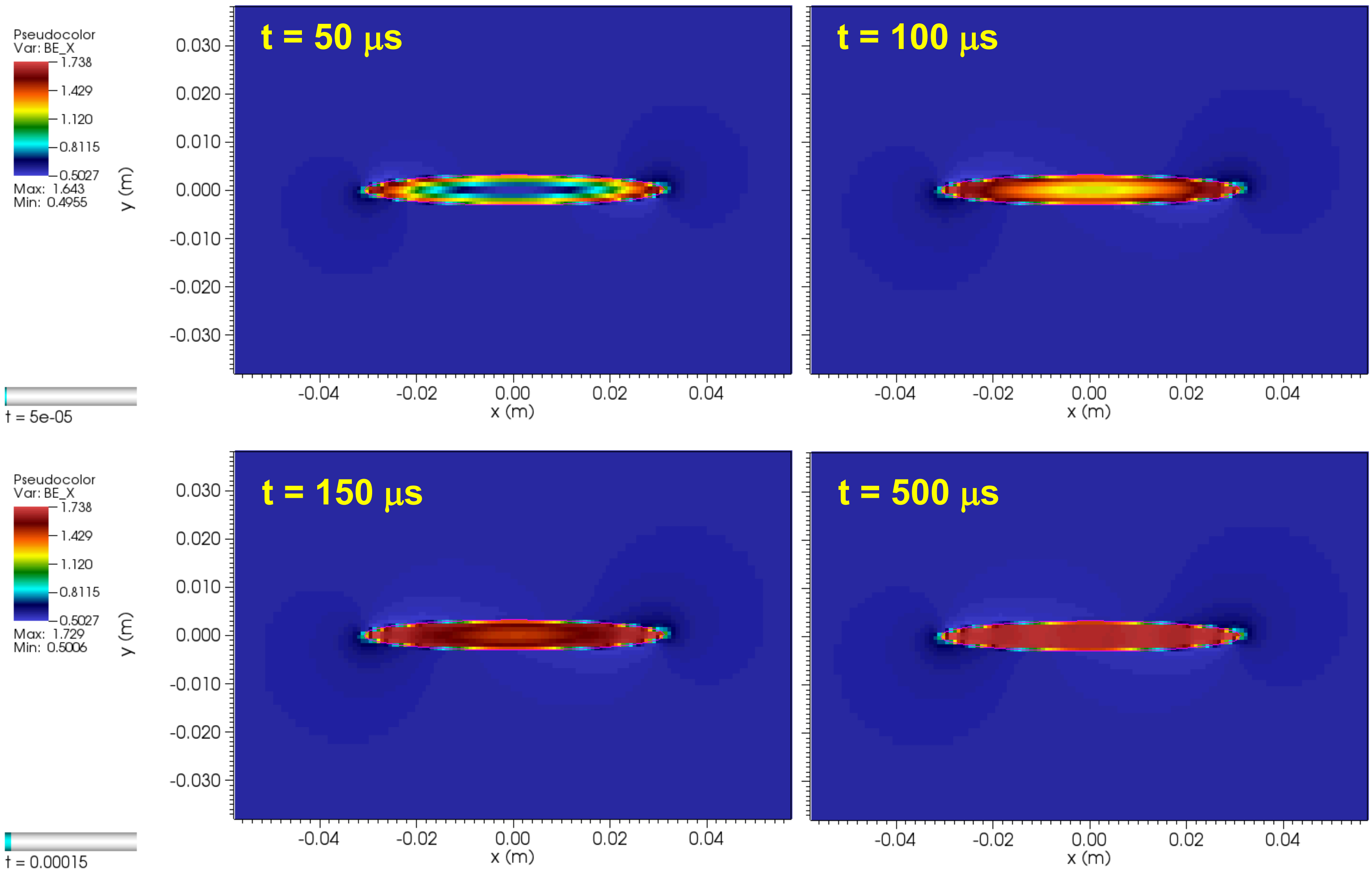
• N = 40	134,400 elements
• N = 80	1,075,200 elements
• N = 160	8,652,800 elements
• N = 320	70,090,384 elements

Mesh is rectangular, Eulerian, uniform in region of spheroid, graded by 4x near boundaries.

Error magnitude computed using the L2 norm relative to the exact solution:

$$e = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\frac{(B - B_{exact})^2}{B_{exact}}}$$

Time evolution (N = 80): B_x



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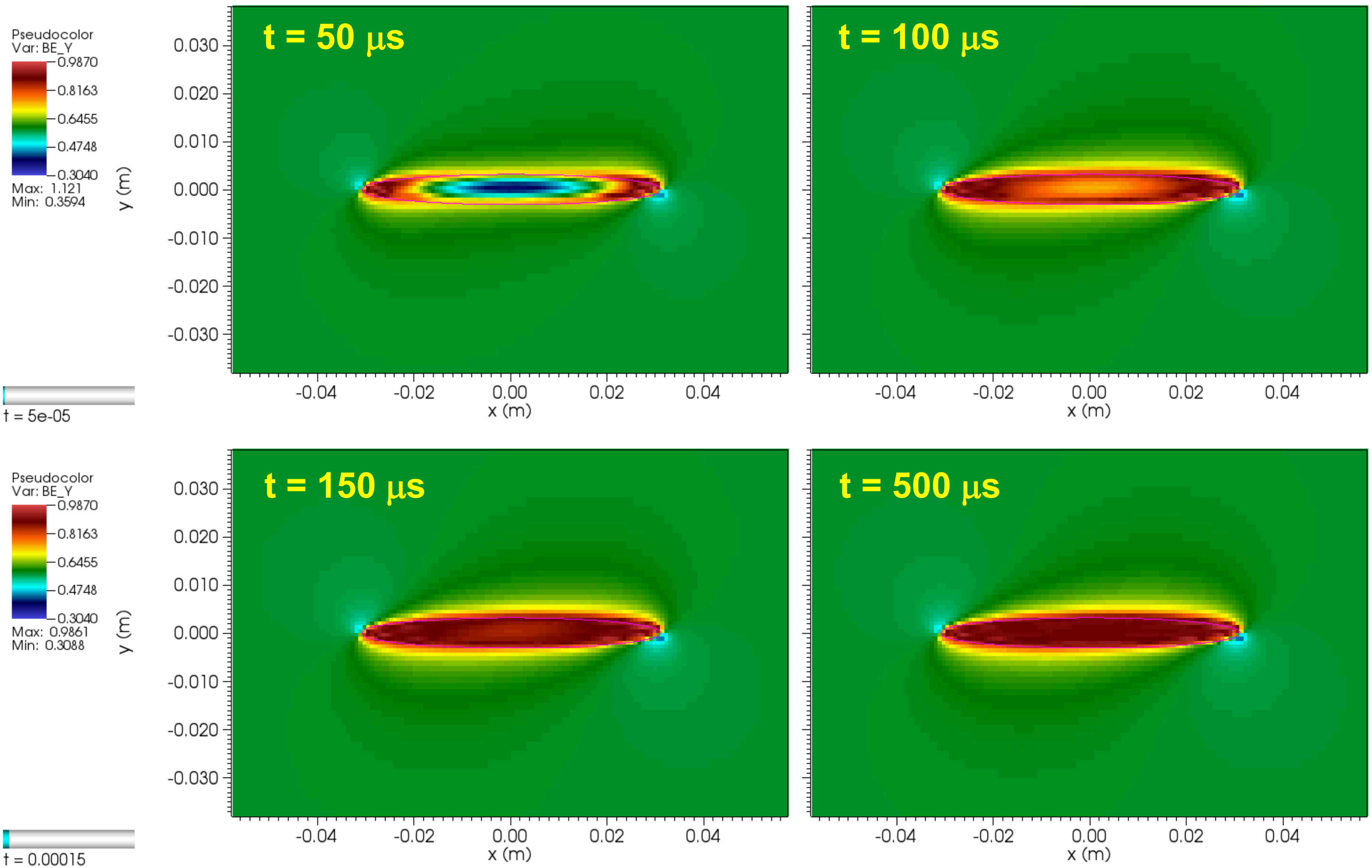
Time evolution (N = 80): B_y



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Time evolution (N = 80): B_z

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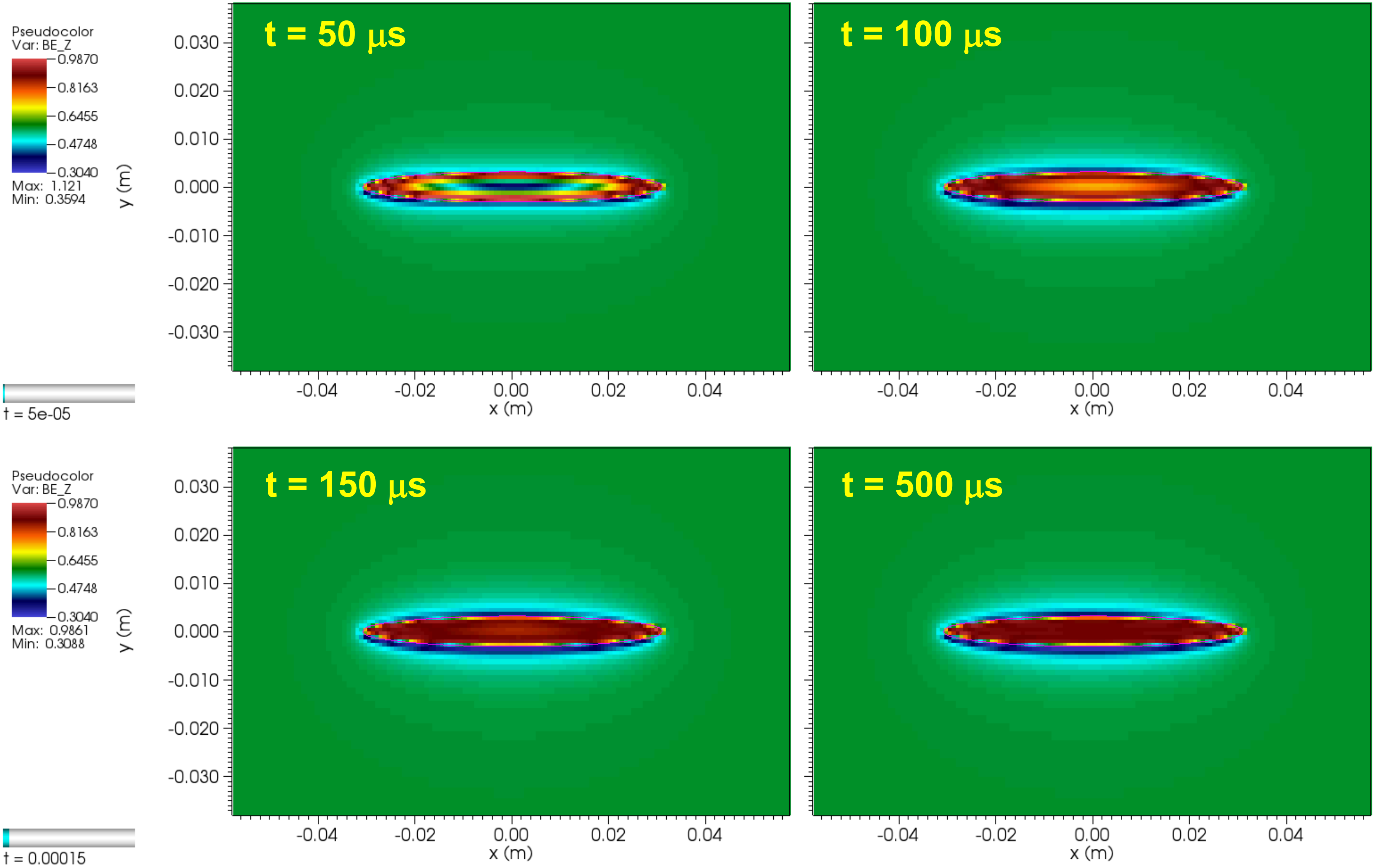
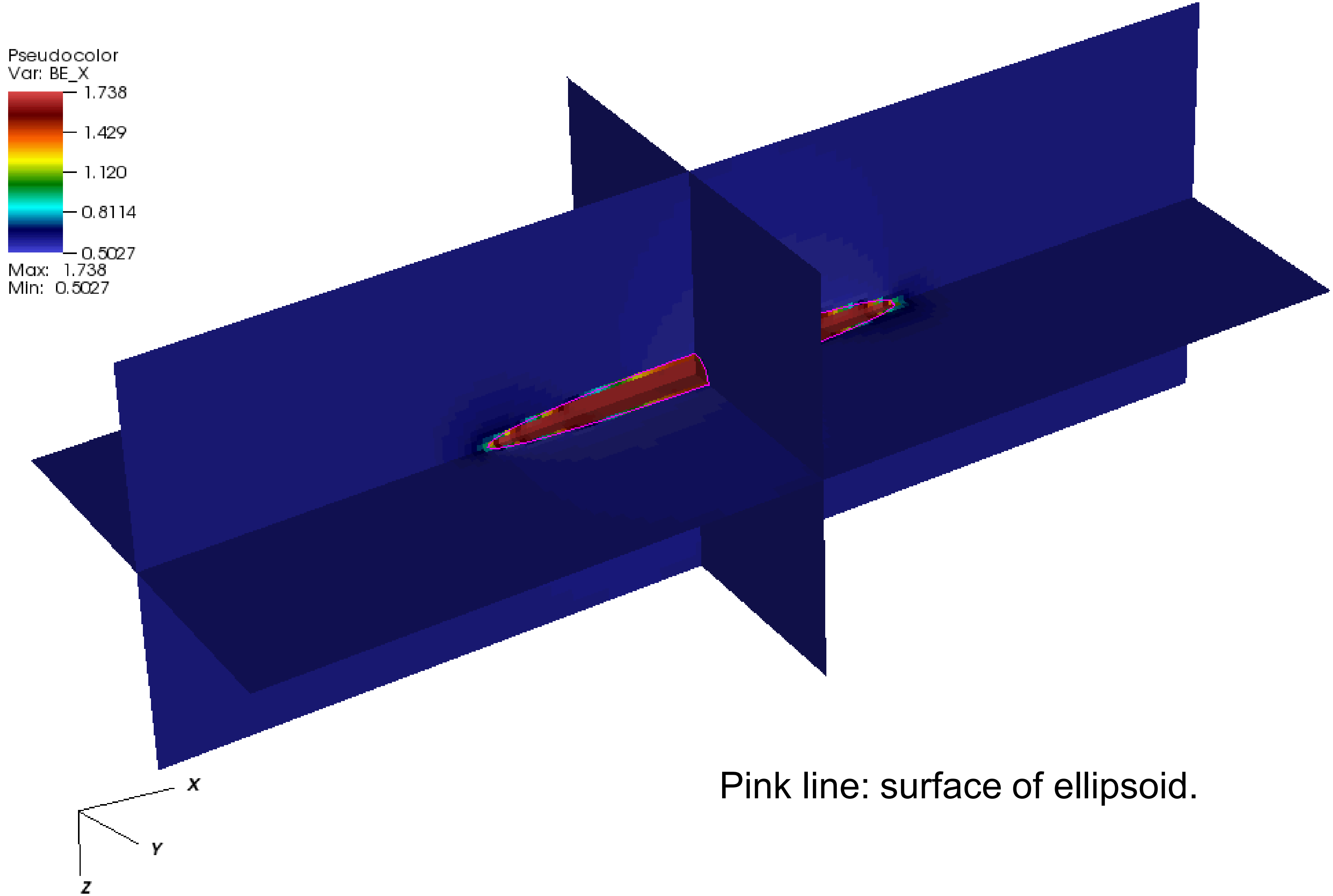


Solution at equilibrium: B_x



Pink line: surface of ellipsoid.

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Solution at equilibrium: B_y

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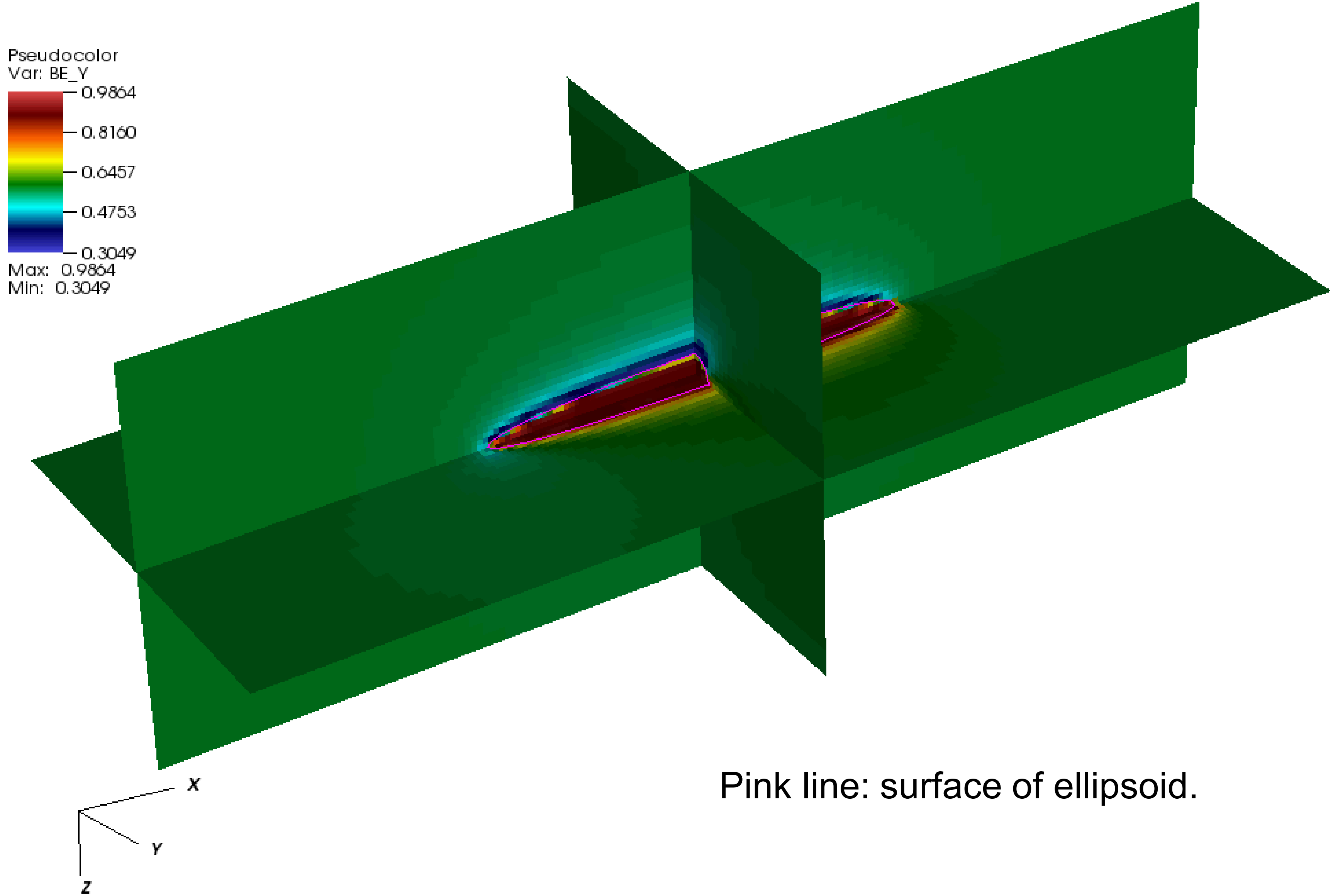
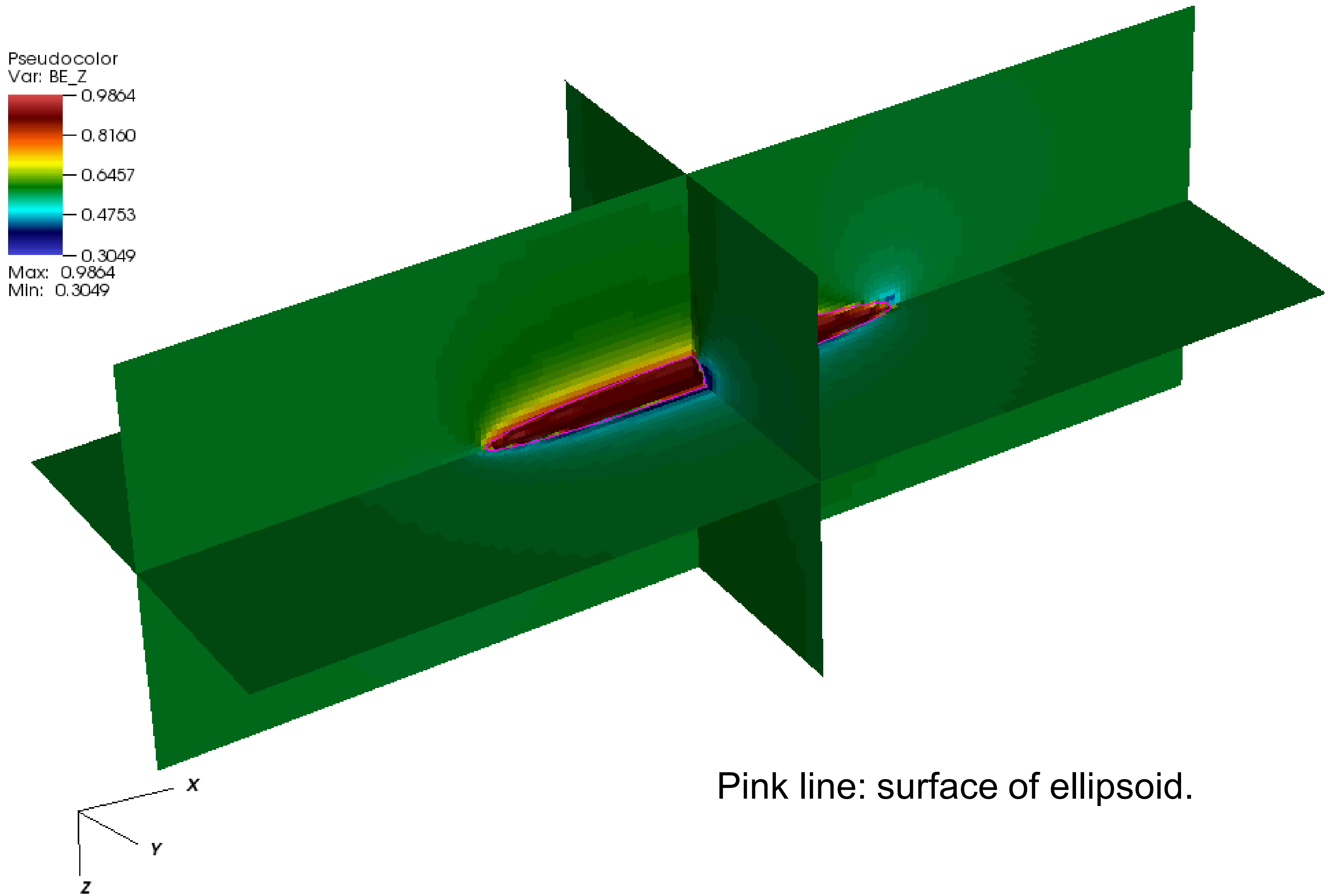


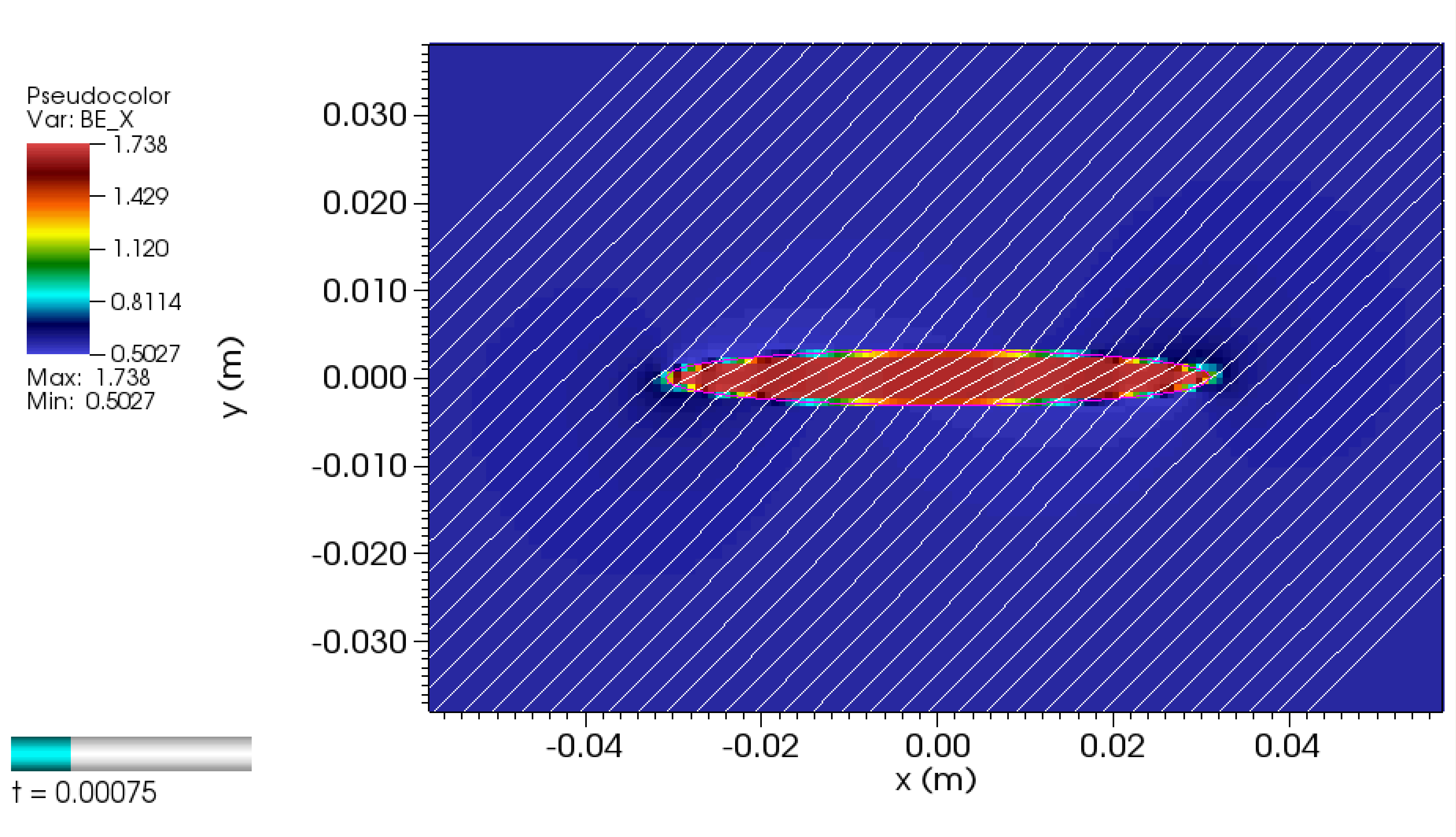
Pink line: surface of ellipsoid.

Solution at equilibrium: B_z

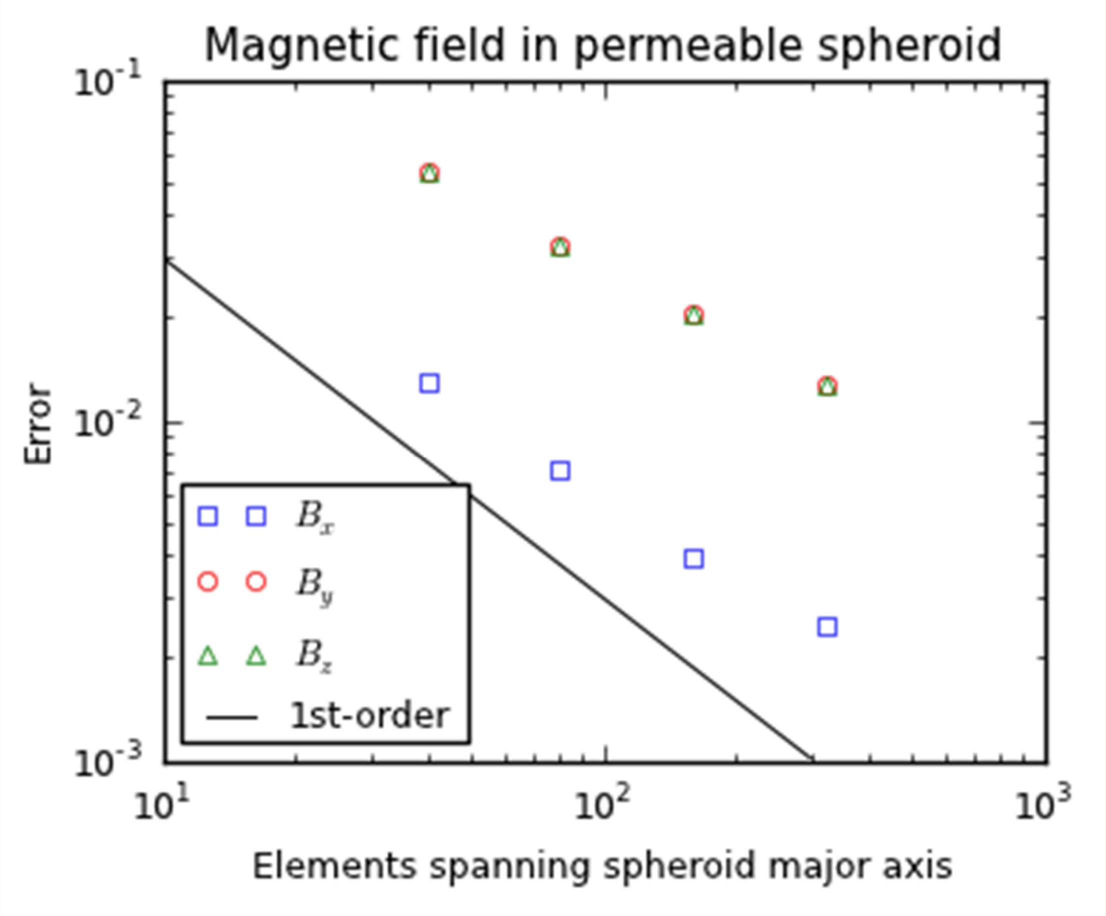


Pink line: surface of ellipsoid.

Magnetic field lines overlaid



Convergence trend



Error computed using the L2 norm relative to the exact solution, in single-material elements only.

Average rate for B_x: 0.8

Average rate for B_y and B_z: 0.7

Convergence trend

EXTRA SLIDES

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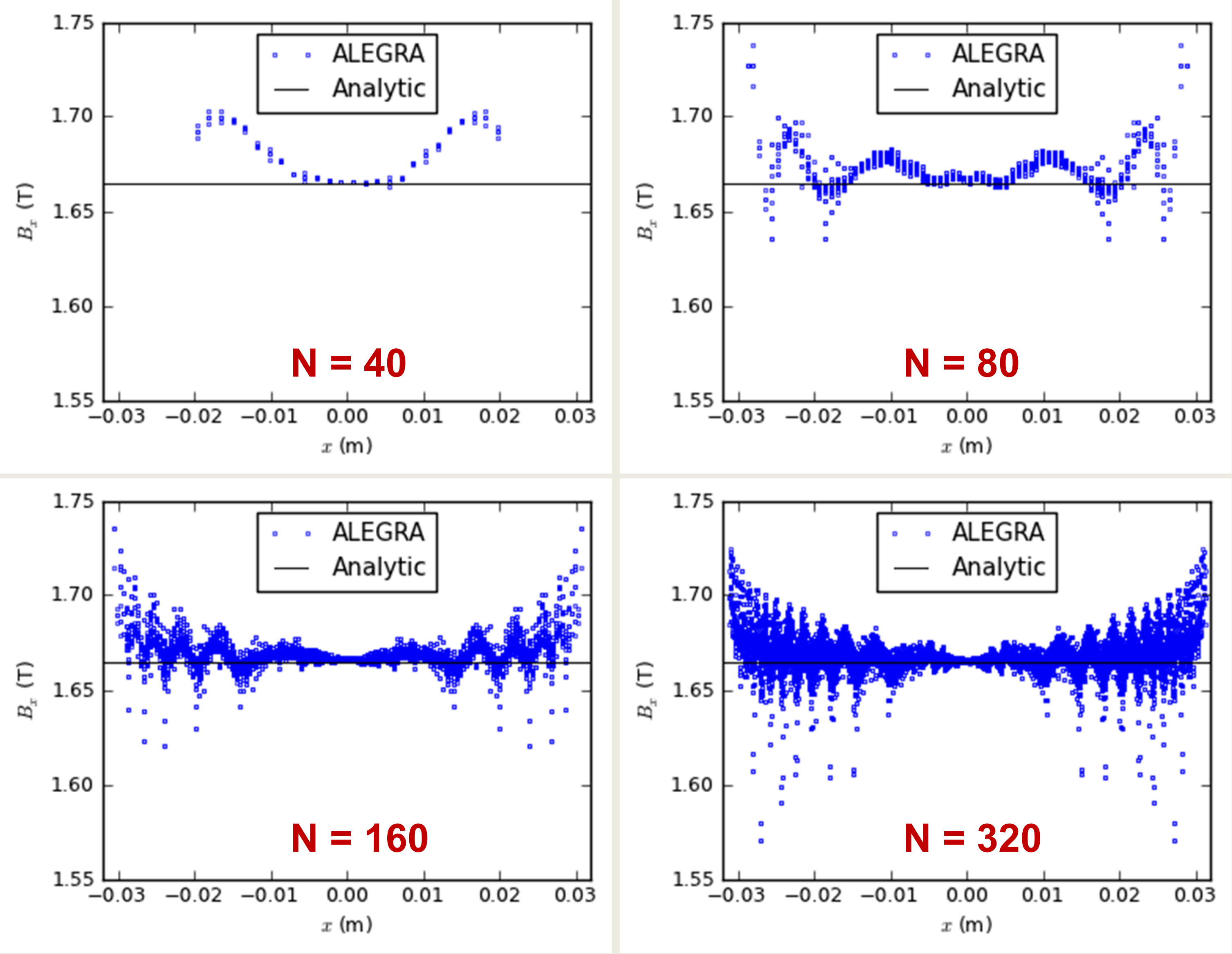
Scatter plots: B_x



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Scatter plots: B_y