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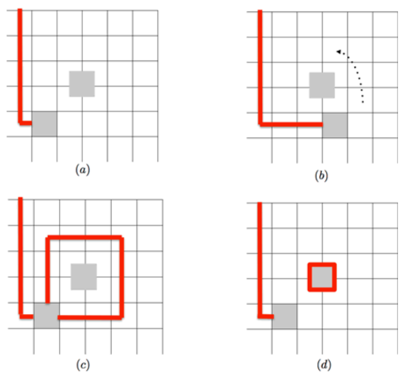
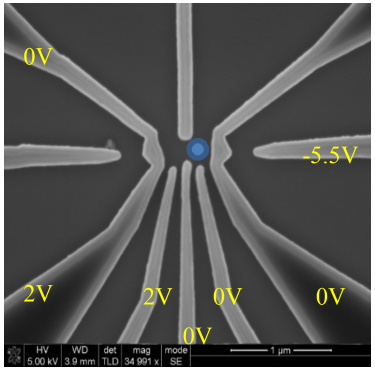
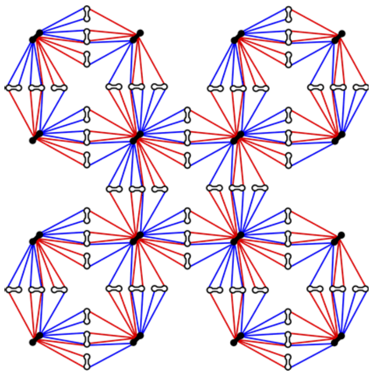






Universal fault-tolerant adiabatic quantum computing with quantum dots or donors

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

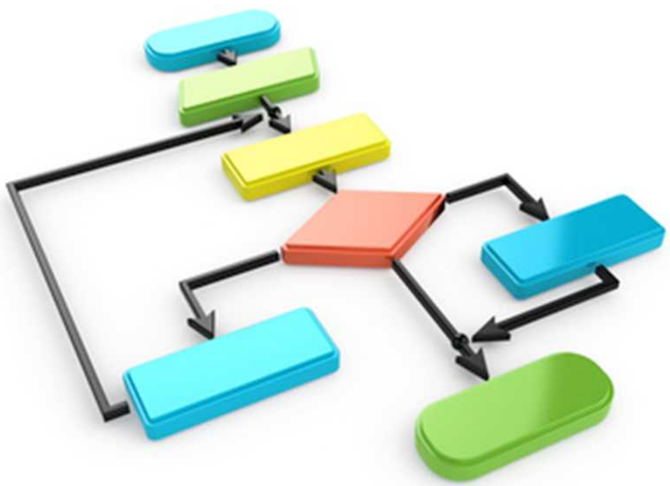
Andrew J. Landahl

Sandia National Laboratories

14 March 2016

Key Challenges to AQC

Universality

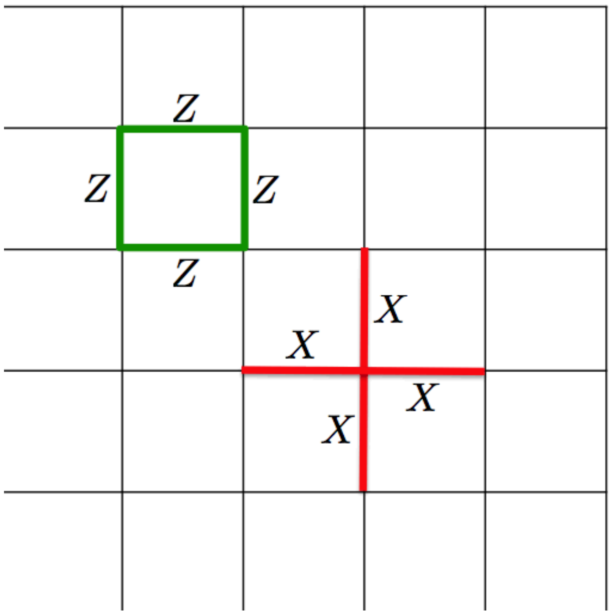


Fault Tolerance

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

The Surface-Code Hamiltonian

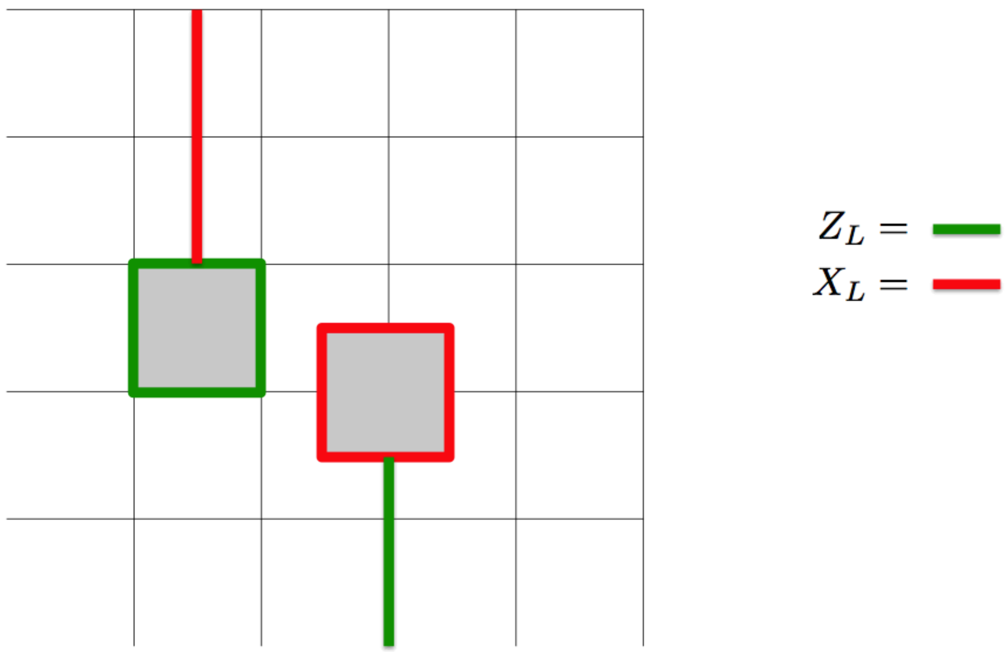
$$H_{surface\ code} = -\sum_s (X_N X_S X_E X_W)^{(s)} - \sum_p (Z_N Z_S Z_E Z_W)^{(p)}$$



1. **Does this enable universal FTAQC? (Yes.)**
2. **Can we mock this up with semiconductor qubits? (Sort of.)**

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Logical Qubits



- **Logical qubits:** Missing Hamiltonian terms ("punctures")
  - Each puncture doubles the ground state degeneracy
  - Gap is constant, independent of punctures' sizes

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Adiabatic Code Deformation

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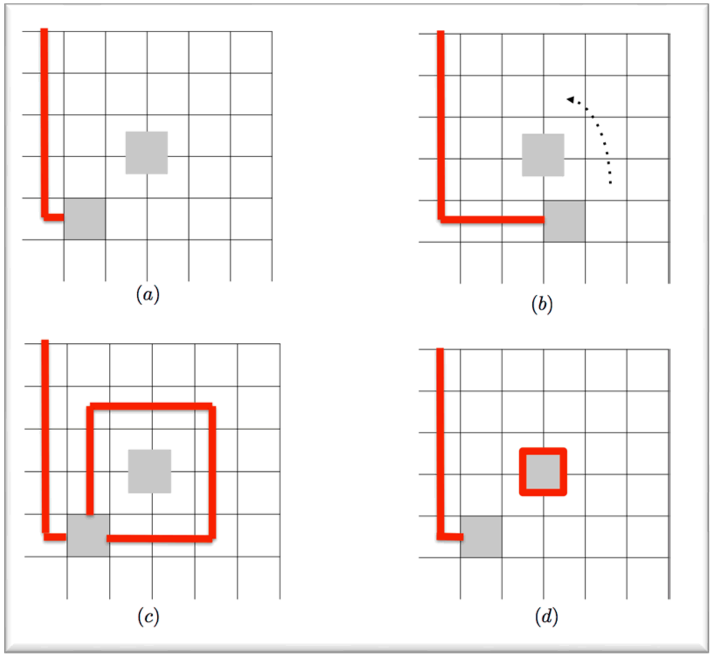
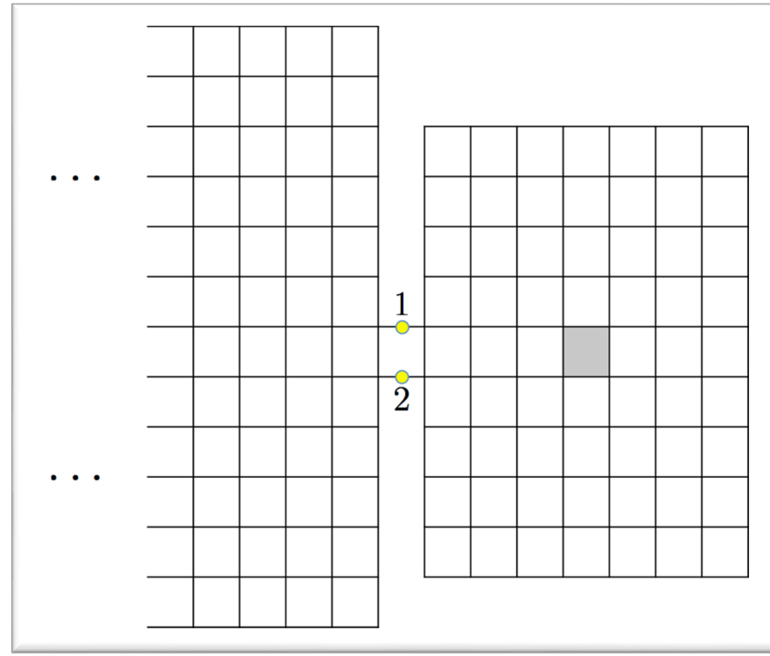
Braiding of punctures



Lattice surgery of patches

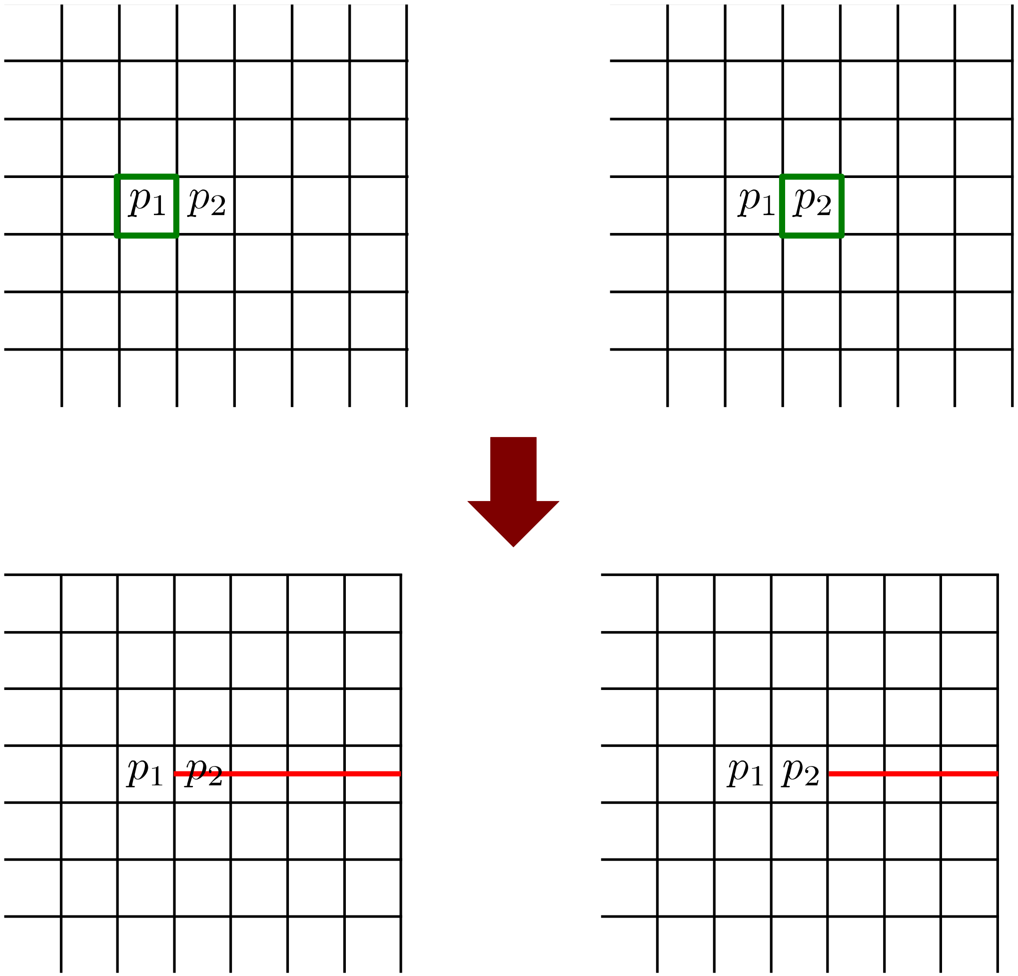


- $CNOT$ by **puncture braiding.**
- $M_X,\ M_Z$ by transversal **single-qubit measurements** on a patch.
- $|0\rangle,\ |+\rangle$ by **puncture creation** on a patch.
- $S|+\rangle,\ T|+\rangle$ by **state injection** on a patch.
- Active **syndrome measurement** commutes with AQC!

Universal!

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Adiabatic Puncture Creation



$$H_i = -\frac{\Delta}{2}(\overline{Z}_{p_1} + \overline{Z}_{p_2})$$

$$H_f = -\frac{\Delta}{2}\overline{X}_{p_1}\overline{X}_{p_2}$$

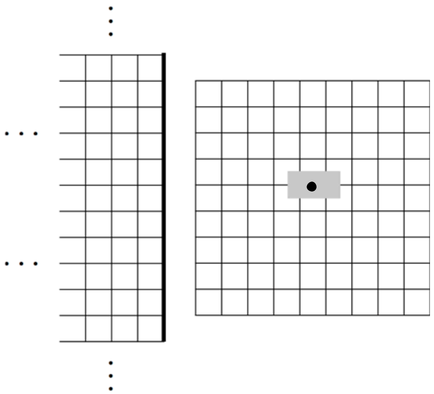
$\overline{Z}_{p_1}\overline{Z}_{p_2}$ is conserved!

Grow/shrink used for braiding/surgery operations are similar

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Adiabatic State Injection

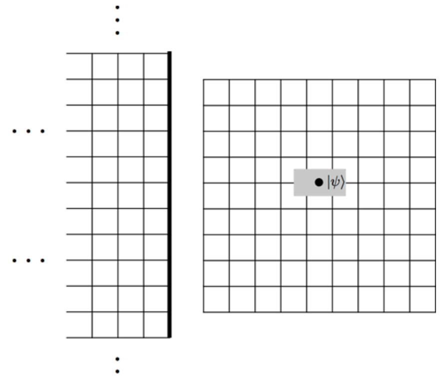
1. "Rough" logical $|+\rangle$



$-Z$ on center qubit ($|0\rangle$).

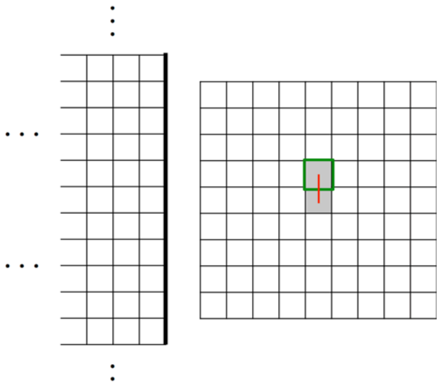
$$H(s) = (1-s)(-Z) + s(TH)Z(TH)^\dagger$$

2. "Rough" logical $T|+\rangle$



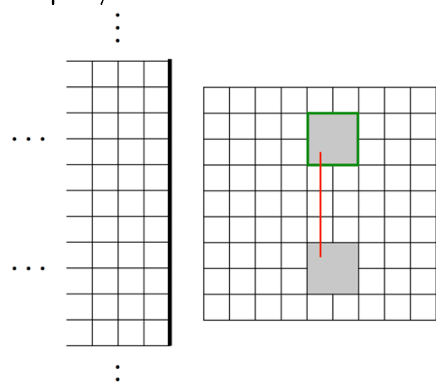
$-TXT^\dagger$ on center qubit ($T|+\rangle$).

3. "Smooth" logical $T|+\rangle$

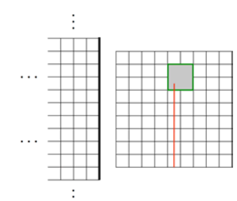


- X checks on, Z checks off.
- X on center qubit unaffected: is now logical X.

4. Grow $T|+\rangle$ into a large double puncture



- Exposed to errors during growth.



Fault Tolerance

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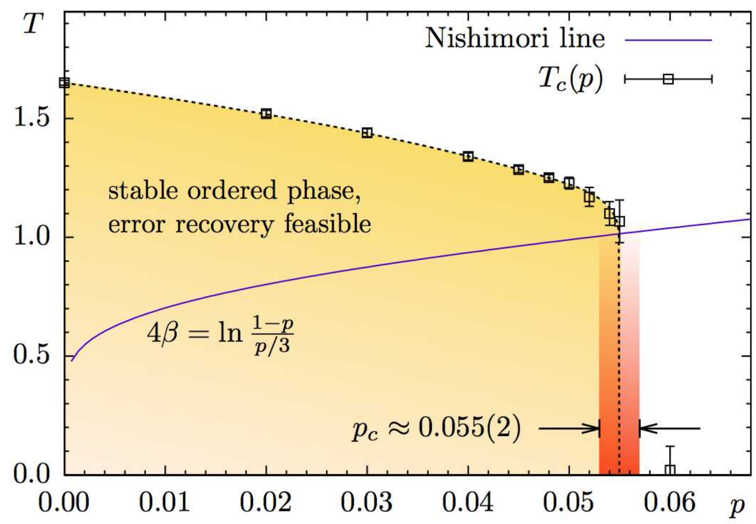
Phenomenological noise model

- Each "elementary operation" (e.g., syndrome-bit measurement, adiabatic evolution of one plaquette) fails with probability p. (Ideal + depolarization.)
- **Surface-code threshold:** 3.3% [1]
- **Color-code threshold:** 5.5% [2]
- AQC is fault-tolerant if "elementary operations" meet the relevant threshold!



Color-code threshold estimate from [2]

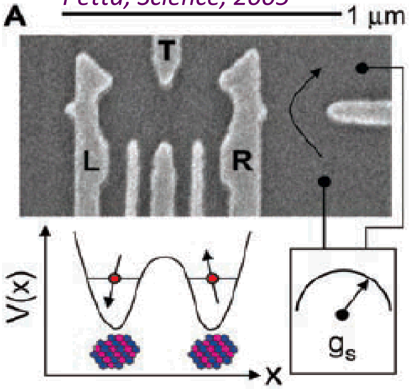
Open problems

1. How to synthesize four-body Hamiltonians in real hardware?
2. How to measure syndrome bits without a quantum circuit?
3. How to express threshold in experimentally relevant parameters?
   - (*E.g.*, coupling precision, clock jitter, bath coupling, non-adiabaticity, local perturbations, readout error, crosstalk, *etc*.)

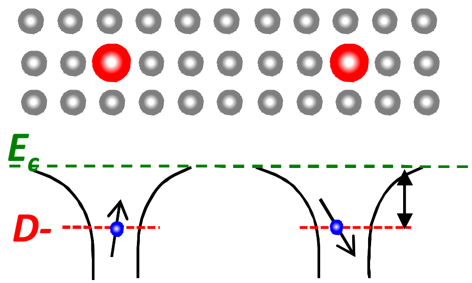
[1] Ohno *et al.*, Nucl. Phys. B **697**, 462 (2004). [2] Andrist *et al.*, PRA **85**, 050203(R) (2011).

Double-well silicon charge qubits

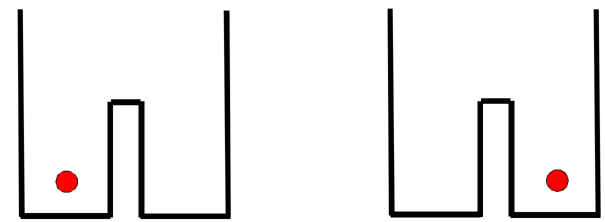
Petta, Science, 2005



Double quantum dot (DQD)



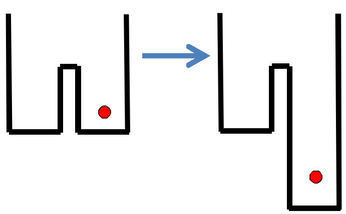
P donor pair



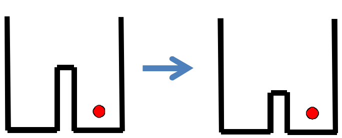
$|0\rangle = |1\rangle_L |0\rangle_R$ $\qquad$ $|1\rangle = |0\rangle_L |1\rangle_R$

Lousy T_2, but great for AQC!

One-qubit "interactions"

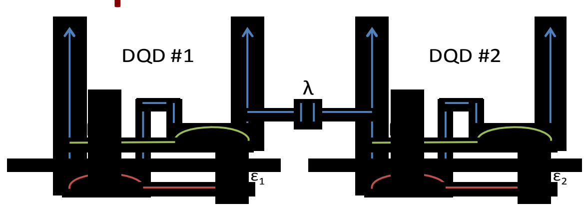


- *±Z* interaction

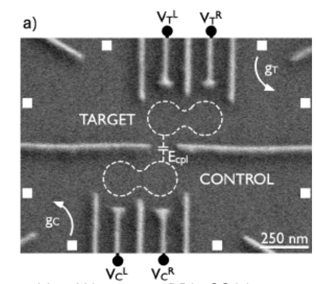


- *X* interaction

Two-qubit interactions

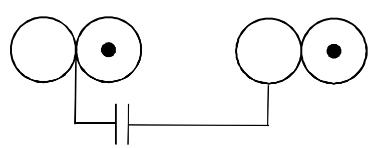


$\varepsilon_{1,2}$ = detuning
$\beta_{1,2}$ = tunnel barrier height
λ = capacitive coupling

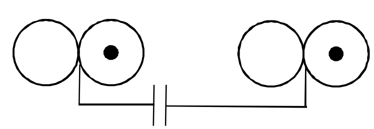


Van Weperen, PRL, 2011

- *±ZZ* interaction



- *±XZ* interaction (hard)



- *XX* interaction (VERY hard)

Avoid these!

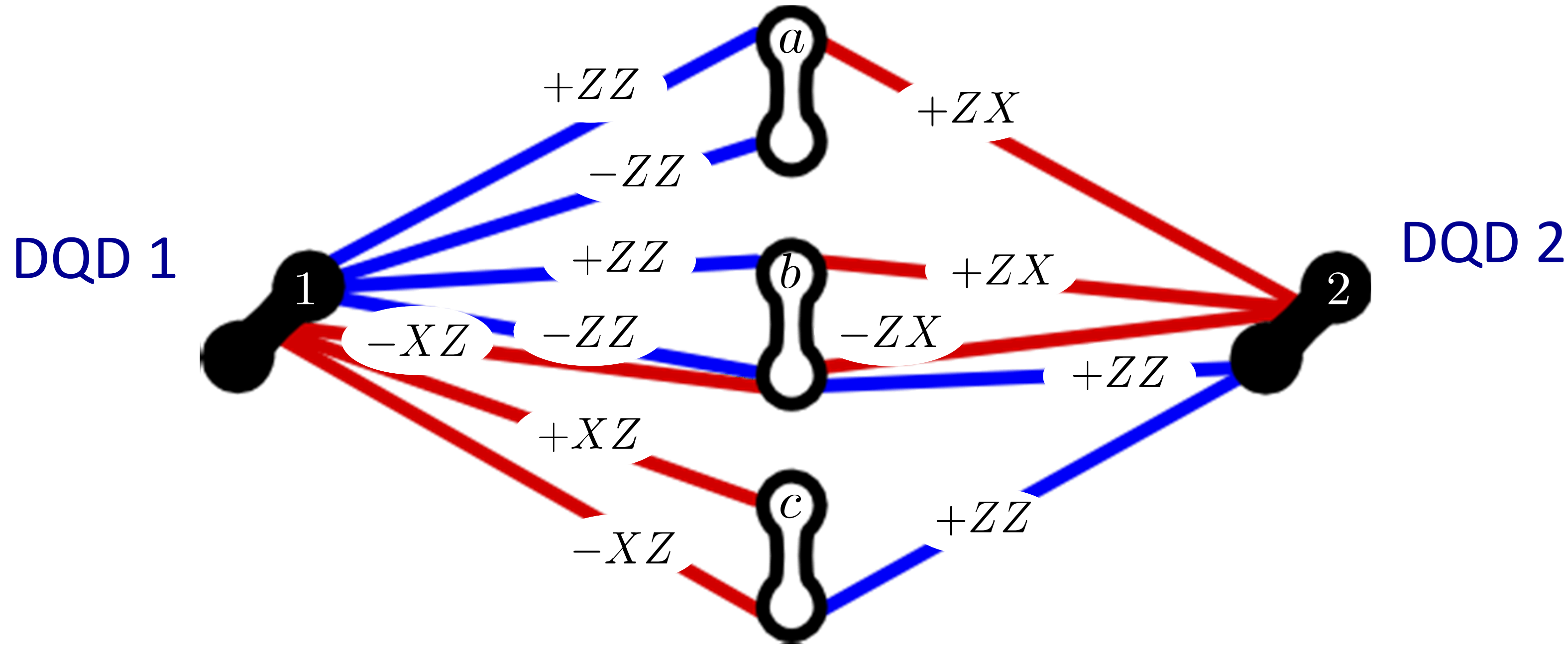
± XX Perturbative Gadgets

Second-order perturbation theory [1, 2, 3]

$$H_{ancilla} = \Delta \left(|-\rangle\langle -| \right)$$

$$H_{perturb} = H_{else} + \sqrt{\tfrac{\Delta}{2}} \left[X_2 Z_a + Z_2 Z_b + X_1 Z_b + Z_2 Z_c - \alpha Z_1 Z_b - \delta X_2 Z_b - (1+\gamma) Z_1 Z_c - (\beta + \alpha\delta) Z_1 Z_a \right]$$

$$H_{ancilla} + H_{perturb} \cong (\alpha ZZ + \beta ZX + \gamma XZ + \delta XX) \otimes |+++\rangle\langle +++|_{ancilla}$$



[1] Kempe, Kitaev, & Regev, SIAM J. Comput. **35**, 1070 (2006). [2] Oliveira & Terhal,QIC **8**, 900 (2008). [3] Jordan & Farhi, PRA **77**, 062329 (2008).

2D Architecture

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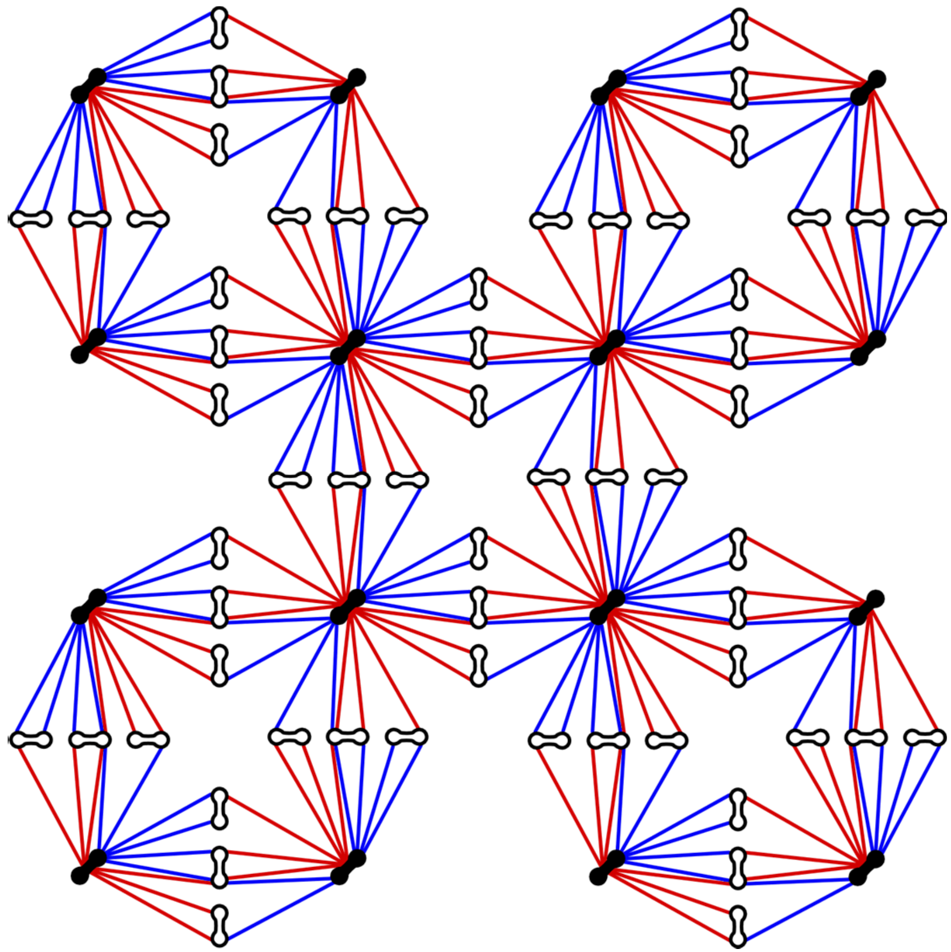
Tiles to any 2D nearest-neighbor qubit Hamiltonian.



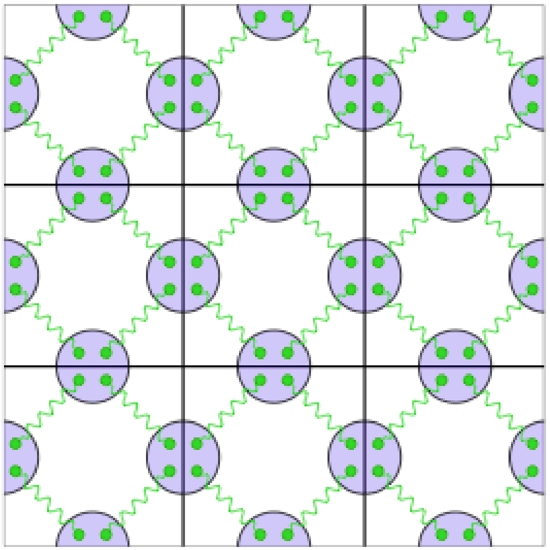
N.B. Diagram is planar: wires do not cross.

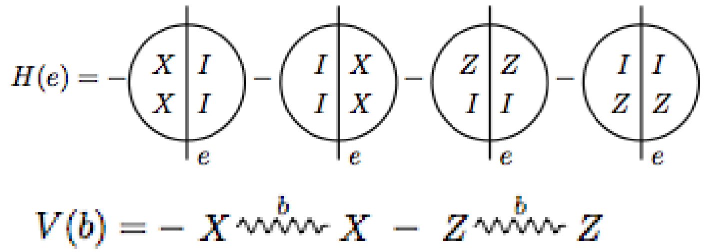
But how to realize 4-body interaction of surface-code Hamiltonian?

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Surface-Code Gadgets

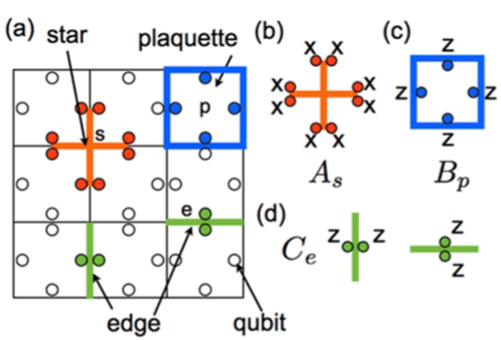
Perturbative [1]

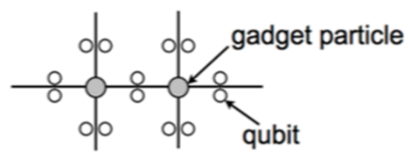




- **Pro:** Only *±ZZ*, *±XX* terms are needed.
- **Con:** Fourth-order perturbation theory required.

Non-perturbative [2]





- **Pro:** Non-perturbative!
- **Con:** 2-body terms between *d* = 4 qudits required.

[1] Brell *et al.*, NJP **13**, 053039 (2011). [2] Ocko & Yoshida, PRL **107,** 250502 (2011).

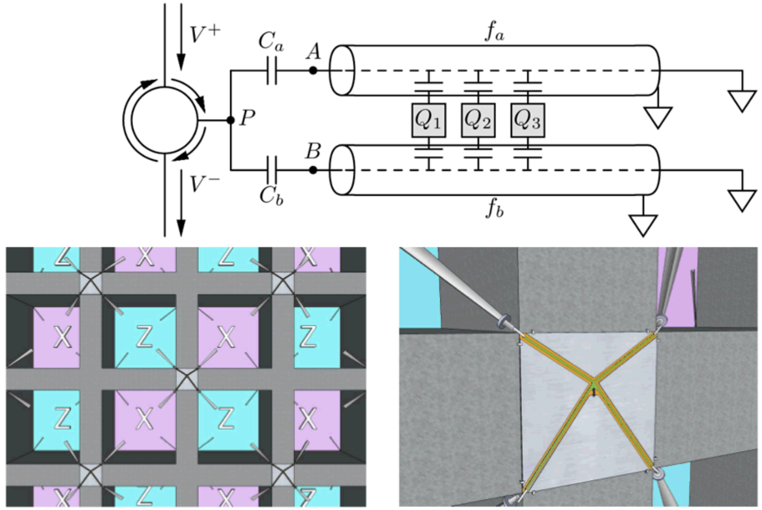
Syndrome Extraction

Approach 1: Use a quantum circuit [1]

- Turn Hamiltonian off during circuit. (Realistic?)
- **Circuit noise:** Depolarizing.
- **AQC noise:** Thermal, diabatic, local perturbations.
- $p_L < 10^{-15}$ @ $d = 11$ if, e.g., Ising perturbations obey $|h_i|,\ \|J_{ij}\|_1 < J \cdot 10^{-3}$.
- *Why use AQC if quantum circuits are available?*

Approach 2: Use specialized hardware

- Superconducting resonators coupled to Josephson junctions. [2] (Realistic?)
- Alternative: Engineered 4-body dissipation. [3] (Realistic?)

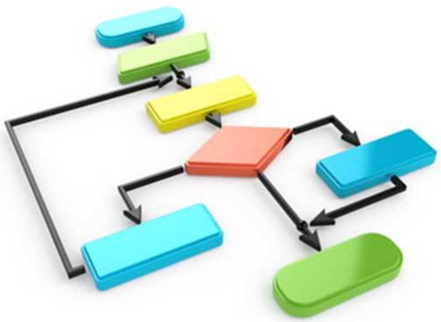


Proposed surface-code array from [2]

[1] Zheng & Brun, PRA **91**, 022302 (2015). [2] DiVincenzo & Solgun, NJP **15**, 075001 (2013).
[3] Herold *et al.*, arXiv:1511.05579 (2015).

Where to go from here

Universality



Fault Tolerance

1. Develop better layouts/approaches for realizing the surface-code Hamiltonian.
2. Develop better proposals for realizing syndrome extraction.
3. Numerically estimate the threshold against realistic noise.
4. Experimentally demonstrate a small AQC logical qubit.

The future lies beyond the world of Ising Hamiltonians.

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Backup Slides

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

How did we avoid "No-Go" theorems?

Non-Ising Hamiltonians

- Non-"stoquastic:" Avoids various no-go theorems.

Discretized "elementary" evolutions

- Can bound the error in each step.

Degenerate ground spaces

- Ground space includes both the solution and incorrect answers.
- Constant energy barrier between these.
- Active (commuting!) syndrome extraction and adaptive computation complements the energy barrier to enable fault tolerance. (No self-correction.)
- Akin to holonomic quantum computation, but at the logical (encoded) level.

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Putting it All Together

Surface-code dynamical evolutions

1. Adiabatic lattice preparation
2. Adiabatic lattice surgery
3. Adiabatic puncture preparation
4. Adiabatic puncture braiding
5. Adiabatic (magic-)state injection into a puncture
6. Non-adiabatic logical qubit measurements
7. Non-adiabatic syndrome measurements

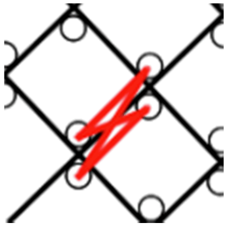
Fault-tolerant if each elementary evolution has a failure probability below the surface code threshold in the **phenomenological noise model** (**3.3%** for surface codes [1], **5.5%** for color codes [2]).

Errors are now sometimes suppressed in new ways:

- **Control errors:** Adiabaticity of evolutions
- **Qubit errors:** Constancy of gap; active error recovery
- **Measurement errors:** Repetition of syndrome extraction

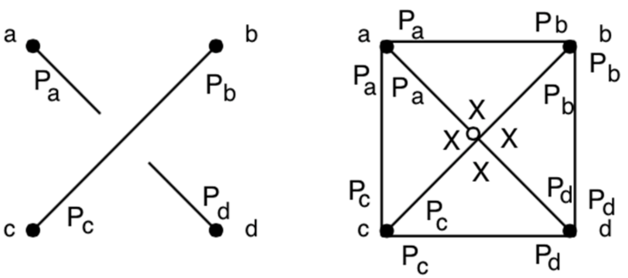
Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

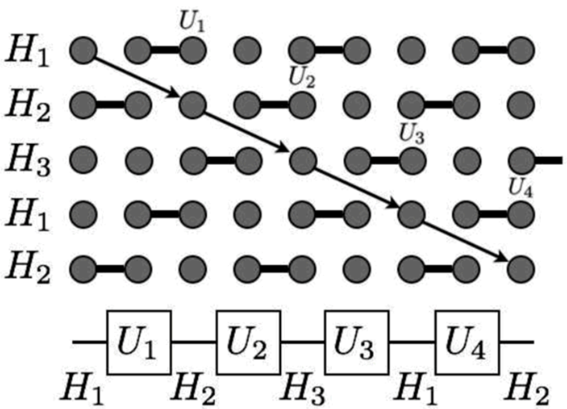
Progress to Date



Universality

- **[1] Oliveira-Terhal, [2] Lloyd-Terhal:** 2D grid (d = 2)
- **[3] Chase-Landahl:** 1D ring (d = 8)
- **[4] Nagaj-Wojcan:** 1D, translationally invariant (d = 10)
- **[5] Zanardi-Rasetti:** Holonomic quantum computing
- **[6] Bacon-Flammia, [7] Hen:** Teleported or controlled holonomic gates





Weaknesses

- Perturbative or non-planar interatctions
- HQC even less robust than traditional AQC

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Progress to Date

Fault Tolerance

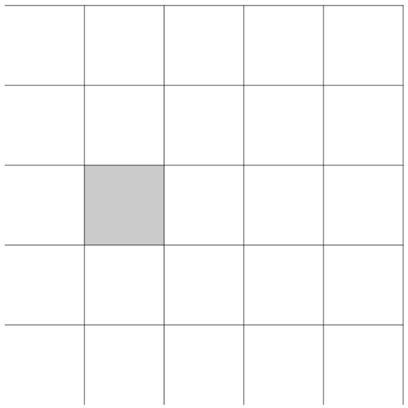
- **[1] Mizel:** Ground-State Quantum Computing
- **[2] Lidar:** Dynamical Decoupling
- **[3] Oreshkov *et al.*, [4] Zheng-Brun:** FTHQC
- **[5] Young *et al.***: No-go (p)theorems

Weaknesses

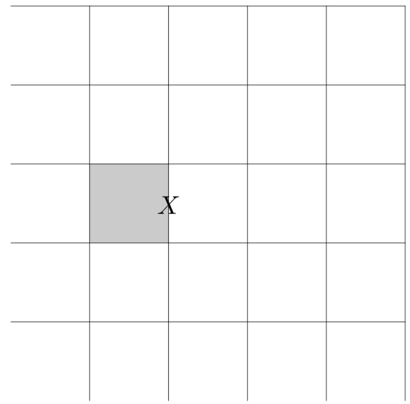
- GSQC not fault tolerant (yet?)
- DD intervals must become exponentially small
- FTHQC approaches are not spatially local

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

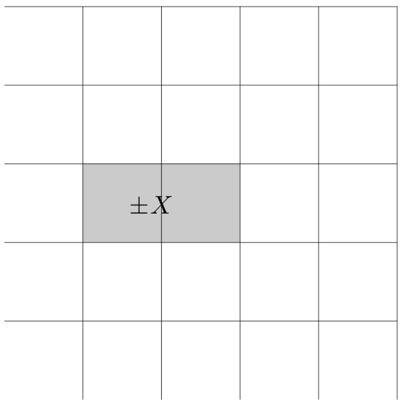
Code Deformation



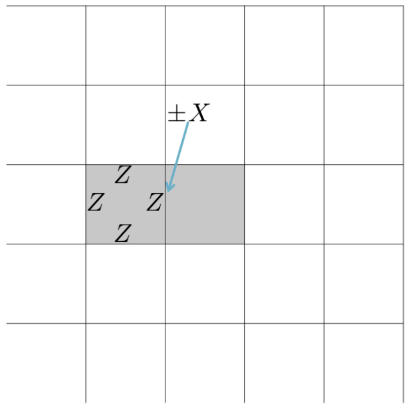
- *Z*-type puncture.



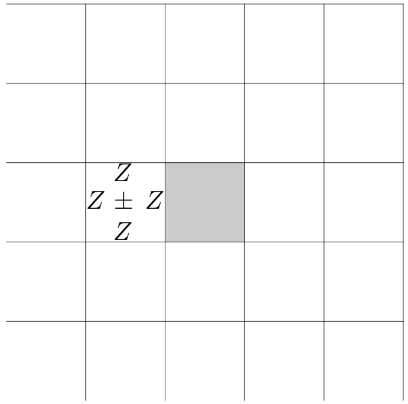
- Measure *X.*



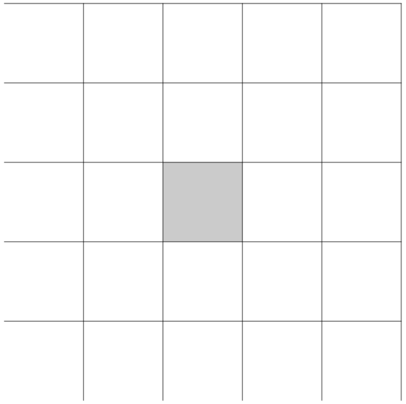
- ±*X* added to checks
- Puncture grows.



- Measure original *Z*-type puncture.



- ±(*Z*-type puncture) added to checks.
- Puncture shrinks.



- New *Z*-type puncture.

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A vision for AQC

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Adiabatic quantum computers that can run ANY quantum algorithm

1. **Algebraic and Number Theoretic Algorithms**
   1. Factoring
   2. Discrete-log
   3. Pell's Equation
   4. Principal Ideal
   5. Unit Group
   6. Class Group
   7. Gauss Sums
   8. Solving Exponential Congruences
   9. Matrix Elements of Group Representations
   10. Verifying Matrix Products
   11. Subset-sum
   12. Decoding

2. **Oracular Algorithms**
   13. Searching
   14. Abelian Hidden Subgroup
   15. Non-Abelian Hidden Subgroup
   16. Bernstein-Vazirani
   17. Deutsch-Jozsa
   18. Formula Evaluation
   19. Gradients, Structured Search, and Learning Polynomials
   20. Hidden Shift
   21. Pattern Matching
   22. Linear Systems
   23. Ordered Search
   24. Graph Properties in the Adjacency Matrix Model
   25. Graph Properties in the Adjacency List Model
   26. Welded Tree
   27. Collision Finding and Element Distinctness
   28. Graph Collision
   29. Matrix Commutativity
   30. Group Commutativity
   31. Hidden Nonlinear Structures
   32. Center of Radial Function
   33. Group Order and Membership
   34. Group Isomorphism
   35. Statistical Difference
   36. Finite Rings and Ideals
   37. Counterfeit Coins
   38. Matrix Rank
   39. Matrix Multiplication over Semirings
   40. Subset Finding
   41. Search with Wildcards
   42. Network Flows
   43. Electrical Resistance
   44. Machine Learning

3. **Approximation and Simulation Algorithms**
   45. Quantum Simulation
   46. Knot Invariants
   47. Three-manifold Invariants
   48. Partition Functions
   49. Adiabatic Algorithms
   50. Quantum Approximate Optimization
   51. Quantum Approximate Optimization
   52. Zeta Functions
   53. Weight Enumerators
   54. Simulated Annealing
   55. String Rewriting
   56. Matrix Powers

50+ algorithms: http://math.nist.gov/quantum/zoo

…and that can be made arbitrarily reliable with polylogarithmic overhead

Input:
- Ideal quantum algorithm Q
- n qubits
- T time
- ε desired simulation precision

Output:
- Imperfect *adiabatic* quantum computation Q'
- $n = \text{poly}(n, \log(1/\varepsilon))$ qubits
- $T = \text{poly}(T, \log(1/\varepsilon))$ time

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Going beyond QUBO

New ways of thinking are needed

Non-stoquastic instead of stoquastic Hamiltonians

- Stoquastic: *H* can be made real with non-positive off-diagonal elements in some basis.

  For example, the Transverse Ising Model (TIM) is stoquastic:

$$H = \sum_{1 \leq i \leq n} h_i X_i + g_i Z_i + \sum_{1 \leq i < j \leq n} g_{ij} Z_i Z_j$$

Discretized primitive adiabatic evolutions instead of one monolithic adiabatic evolution

- Allows us to quantify the error in AQC as “error per evolution”

Degenerate instead of non-degenerate ground spaces

- Not all ground states have to be “the” answer.
- Loosen up and let QECCs put energy barriers between ground states.

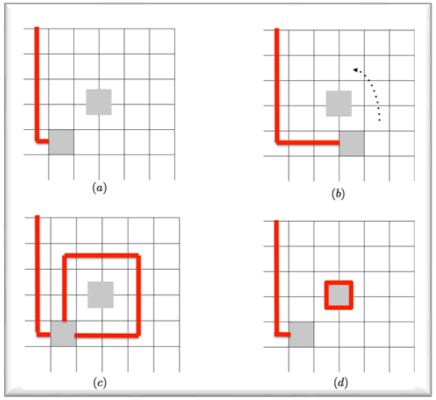
Measurements and adiabatic evolutions can co-exist

- Measurement of QEC checks will not disrupt adiabatic evolutions of logical information.
- True even for non-commuting checks in gauge QEC codes.

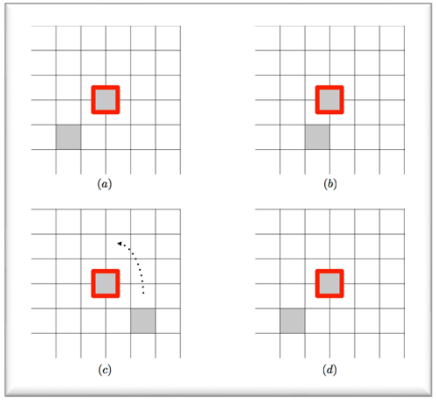
Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Universal logical gate set

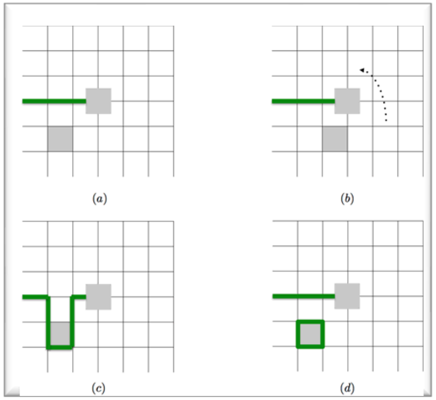
- Logical $CNOT$ from rough-smooth puncture "braiding."



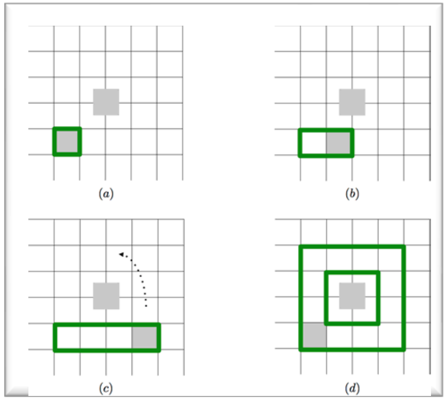
$XI \leftrightarrow XX$



$IX \leftrightarrow IX$

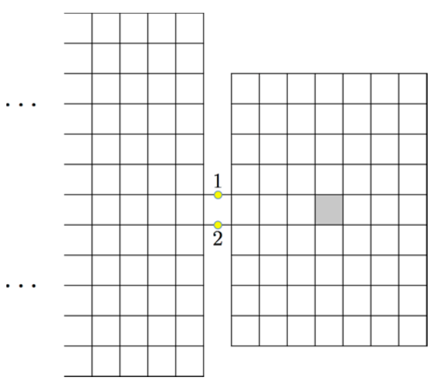


$IZ \leftrightarrow ZZ$



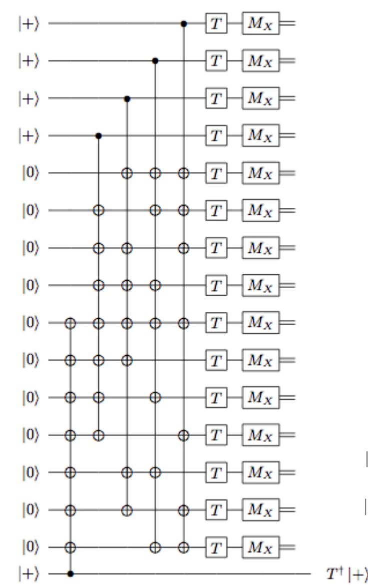
$ZI \leftrightarrow ZI$

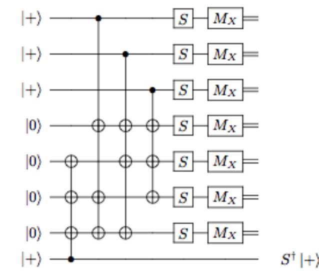
- "Lattice surgery" to isolate punctures:

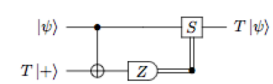


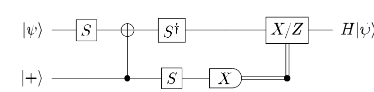
- *E.g.*, M_{X_1}, M_{X_2} isolates a smooth puncture

- $|0\rangle,\ |+\rangle$ by syndrome measurement on isolated punctures
- $M_X,\ M_Z$ by transversal measurement on isolated punctures
- $S|+\rangle,\ T|+\rangle$ states by state injection + logical Clifford distillation









Construction in context

A note to Holonomic Quantum Computing fans [1]:

- The evolutions are holonomic at the ***LOGICAL*** level, but not at the physical level.

What about the "no-go" (p)theorems that Robin Blume-Kohout reported on at last year's AQC? [2]

- The (p)theorems only apply when all ground states hold "the" answer.

How does this relate to fault-tolerant "ground-state quantum computing"? [3]

- This is a completely different construction. GSQC still assumes any ground state is a solution.

[1] Zanardi & Rasetti, Phys. Lett. A **264**, 94 (1999). [2] Young *et al.*, PRX **3**, 041013 (2013). [3] Mizel, arXiv:1403.7694 (2014).

Construction in context

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How does this relate to similar papers?

- [4] **Lidar FTAQC:** DD intervals become exponentially close together
- [5] **Oreshkov *et al.* FTHQC:** No energy gap for errors; exposed to thermal noise.
- [6] **Flammia & Bacon's Adiabatic Gate Teleportation:** On physical, not logical qubits.
- [7] **Zheng & Brun FTHQC against thermal noise:** Spatial locality issues not considered.

- [8] **Zheng & Brun FTHQC with surface codes:** Generated after our paper
  - Variations on some of our elementary evolutions. (Notably, Hadamard by twist cuts.)
  - Hamiltonian turned off to use circuits for syndrome extraction. (Why AQC?)
  - Threshold estimated for detailed noise model:
    - During circuits: depolarizing noise
    - During AQC: thermal bath, local perturbations, nonadiabaticities
    - Rough estimates that it is more qubit-efficient than FT Q. Circuits, BUT the analysis assumes native 4-body interactions.

[4] PRL **100**, 160506 (2007). [5] PRA **80**, 022325 (2009). [6] PRL **103**, 120504 (2009). [7] PRA **89**, 032317 (2014). [8] PRA **91**, 022302 (2015).

Adiabatic lattice preparation

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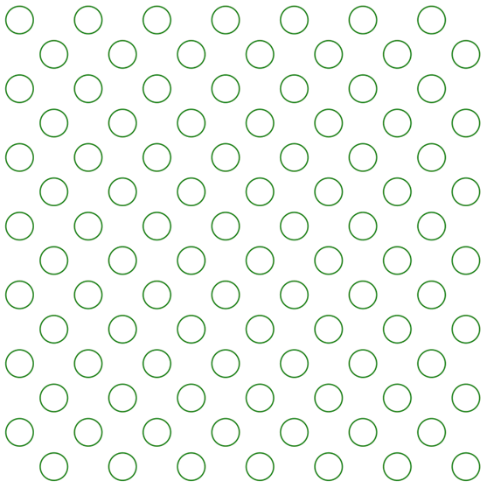
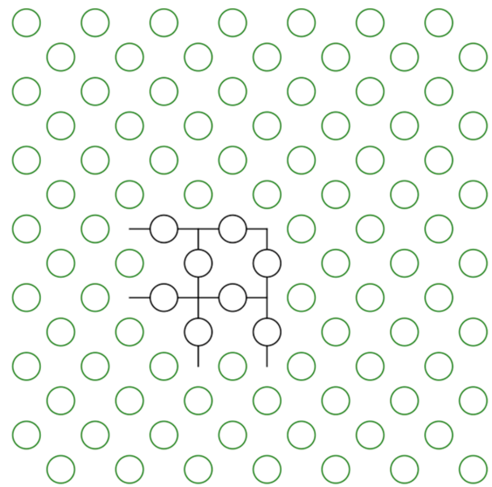
Grow the lattice step-by-step

$$H_i = -\Delta \sum_j Z_j$$

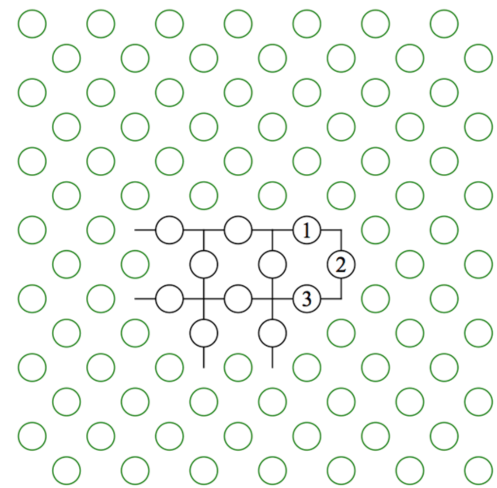
$$H(t) = \left(1 - \frac{t}{T}\right) \sum_{j \in \mathcal{Q}} (-\Delta Z_j) + \frac{t}{T} \sum_{S \in \mathcal{G}} \left(-\frac{\Delta}{2} S\right) + \frac{t}{T} \sum_{j \notin \mathcal{Q}} (-\Delta Z_j)$$



Isolated qubits



2 x 2 surface code



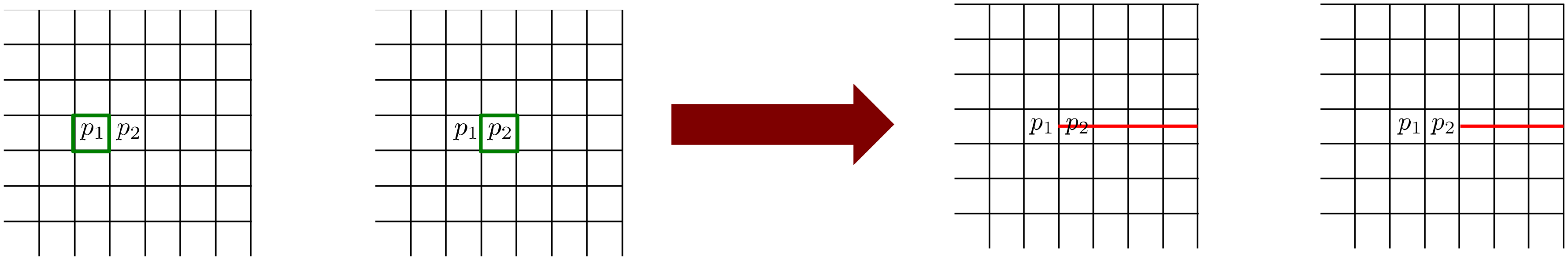
Add plaquettes one by one

- Gap is kept constant by adding terms one by one
- Time to prepare lattice grows with code size

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Adiabatic puncture preparations

Smooth $|0\rangle$ and rough $|+\rangle$ double-puncture preparations

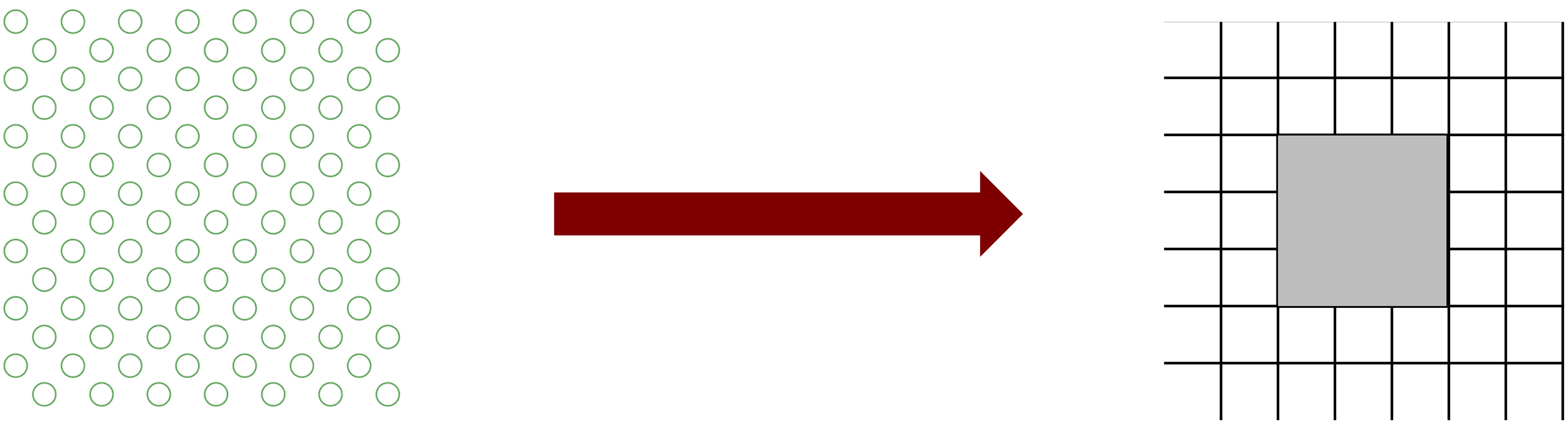


$$H_i = -\frac{\Delta}{2}(\overline{Z}_{p_1} + \overline{Z}_{p_2})$$

$\overline{Z}_{p_1}\overline{Z}_{p_2}$ is conserved!
Z errors are meaningless!

$$H_f = -\frac{\Delta}{2}\overline{X}_{p_1}\overline{X}_{p_2}$$

Smooth $|+\rangle$ and rough $|0\rangle$ double-puncture preparations



$$H_i = -\Delta \sum_j X_j$$

Qubits in punctured region not adiabatically evolved

Cesare *et al.*, Phys. Rev. A 92, 012336 (2015)

Adiabatic puncture grow/shrink

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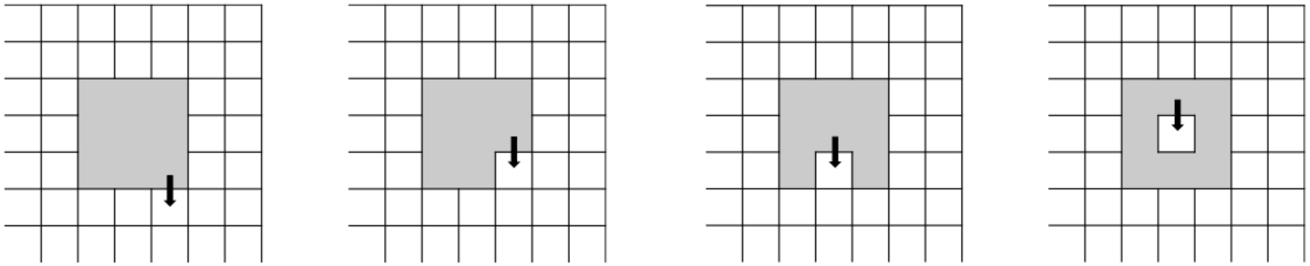
Four cases to consider: 1, 2, 3, or 4 edges bordering puncture interior



In each case, turn off the plaquette $-Z$ check while turning on $-X$ on interior qubit(s)