SAND2016-1605C

The Derivation of Appropriate Laboratory Vibration Test Durations and Number of Shock Hits from Non Stationary Field Test Data

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ABSTRACT

It is often necessary to qualify systems to shock and vibration field environments that vary with time (i.e., non-stationary). The techniques used to define the magnitude of the test specification are typically based on an estimate of the Maximum Predicted Environment (e.g., maxi-max envelop, statistical model, etc.). However, given this definition of the test levels, when the field environment is both long in duration and non-stationary, it is clearly neither appropriate nor practical to set the duration of a laboratory vibration test or the number of hits for a laboratory shock test equal to the durations and hits for the total service life. The purpose of this paper is to present an approach for defining the appropriate vibration test durations and number of shock hits associated with the P99/90 Maximum Predicted Environment laboratory test levels using a power law fatigue model to generate the same amount of damage that was observed during a road test.

INTRODUCTION

This study was intended to identify the appropriate duration of a laboratory random vibration test and the appropriate number of laboratory shock "hits" needed to replicate the fatigue damage incurred during a cross country road trip when using the Maximum Predicted Environment (MPE) to define the test levels. The underlying assumption is that the intensity of the field environment varies significantly over time and that the MPE levels represent a conservative upper bound estimate of the field environment.

DESCRIPTIONOF FIELD TEST

The data used in this study came from a 600-mile road trip on a commercial carrier. The payload consisted of two palletized mass mock packages (one package strapped loosely to the pallet and one tightly strapped). Tri-axial accelerometers were attached directly to each package. Because

* Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000

the packages were rigid, the response data are considered to be representative of the inputs to the packages. The results presented in this paper are for the tightly strapped package.

Approximately 11 hours of high speed interstate highway data were available for this study. The high speed interstate data was used to create the "large ensembles" used in this study. The low speed rural highway data was used to create the "small ensembles" used in this study. A "skyline plot" consisting of the peak response for consecutive 4096 point blocks of data was created for the tightly mounted vertical accelerometer response. The skyline plot was used to identify which time segments represented shock environments and which ones represented vibration environments.

The vibration events were divided into 30 second segments for this analysis. Some segments were deemed to be out of family by visual inspection so the 11 hour interstate ensemble produced 1102 segments (551 minutes). Acceleration Spectral Densities (ASDs) were computed for each vibration segment. The ASDs were converted to 1/6th octave bandwidth resolution so as to reduce the effects of variance error on the analysis.

Shock Response Spectra (SRS) were computed for each distinct "shock" time segment. The SRS presented in this paper were computed using a Maxi-Max Absolute Acceleration (MMAA) algorithm with a 3% critical damping ratio. Each SRS was assigned an effective "duration" of "1".

DERIVATION OF MAXIMUM PREDICTED ENVIRONMENTS

Two different models were used to generate the P99/90 (99% probability of occurrence with 90% confidence) MPE spectra depending on the size of the data ensemble. If there were not sufficient data (N < 200) then a Lognormal Tolerance Limit model (LNTL) was used [1]. If sufficient data were available (N > 200) then an Empirical Tolerance Limit (ETL) model was used [1]. Appendix A provides a brief description of both methods.

DERIVATION OF UNBIASED ENSEMBLES

In order for the following fatigue models to be both realistic and conservative it is crucial for the ensembles to be defined so as to span the full range of the expected environment.

For the smaller sized ensembles (N < 200) simply using the "as measured" spectra most likely won't contain extreme events. Therefore, the solution for small ensembles was to use a parametric statistical model to generate suitable ensembles of realizations. For cases where the MPE spectra was generated using the LNTL model, regardless of size, an adaptation of that model described below was used to generate the realizations.

The first step in generating LNTL realizations was to generate the "log" mean and "log" standard deviation based on the measured ensemble. Equation (1) was used to compute the log mean, μ_S, for the "as measured" ensemble.

$$\mu_S = mean(\log 10(S_j) \tag{1}$$

The standard deviation for a normally distributed ensemble, σ_S, can be computed as the difference between P99 response and the sample mean divided by 2.33 as shown in equation (2). The appropriate estimate of the P99 level is the P99/90 MPE spectra generated by the LNTL model.

$$\sigma_S = (log(S_{MPE}) - \mu_S)/2.33 \qquad (2)$$

Once the log mean and log standard deviation were generated, they were used to create ensembles of realized spectra of any size. This came in handy for cases where the "as measured" ensemble was deemed to be smaller than the anticipated ensemble (for example - if it was desired to double the trip distance).

For cases where the ETL model was used to generate the MPE spectra (i.e., large ensembles), realizations were generated by sampling the "as measured" ensemble with replacement.

DETERMINATION OF 90% CONFIDENCE LEVELS

However, just because a synthesized ensemble was created using the appropriate statistical model, it still might not be conservative. Therefore, bootstrap sampling techniques were used to improve the confidence in the results. Bootstrap sampling simply refers to the generation of multiple realizations of the ensemble using the models described in the previous section. Given the desired 90% confidence level, 200 realized ensembles were generated (the goal was to have a minimum of 20 values exceed the chosen confidence level). The fatigue damage compression model described later in this paper was applied to all 200 sample ensembles.

Once the equivalent durations were computed, the next step was to generate a Distribution Free (DF) Probability Response Function (PDF) of the maximum compressed durations for the 200 bootstrap realizations. A DFPDF is created by simply rank ordering the durations and plotting them against a vector ranging from 0.5/200 to (200-0.5)/200 in increments of 1/200 (this assumes that we do not know the 0 or 1 values of the PDF). The 90% highest value of the compressed duration was used as the best estimate of the MPE duration.

POWER LAW FATIGUE TIME COMPRESSION MODEL

This study is based on the assumption that both the random vibration and the shock environments encountered during normal truck transport will induce fatigue failures, as opposed to first passage failures (i.e., it is assumed that transportation shocks are simply transient random events). Therefore, a Minor's Rule power law fatigue model was used to characterize the data. As an aside, while any given shock has too few cycles to meet the criteria for a Minor's Rule fatigue model, which is a large cycle fatigue model, if one considers that the shocks and the random vibration environments represent a continuous environment the large cycle criterion does apply.

In its simplest form, Minor's Rule states that a material's fatigue failure envelope can be defined as a straight line with a negative slope on a log-log plot of applied stress, σ, versus cycles to failure, N. For the purpose of this study, that formula has been rewritten in terms of duration, T, and acceleration amplitude, G, in equation (3) and ASD spectra, S, in equation (4). The slope of the straight line, which is defined as the fatigue exponent, *b*, is based on material properties that can be looked up in any standard reference on fatigue failure.

$$(G_2/G_1) = (T_1/T_2)^b \tag{3}$$

$$(S_2/S_1) = (T_1/T_2)^{2b} \quad (4$$

For this study we chose b=0.15 (which is consistent with ductile material properties). Recalling that the dB ratio of responses is defined as $20\log_{10}(G_2/G_1)$ and $10\log_{10}(S_2/S_1)$, this specific choice of the fatigue scale factor equates to a decade reduction in the equivalent total duration for each 3dB increase in test level.

Based on this compression damage model, the basic approach used in this study for estimating the equivalent compressed duration associated with the MPE spectra can be explained as a four step process.

1) Identify the ratio between each individual spectrum in the ensemble and the MPE spectra. Because the spectral content for the individual spectra are not exactly like that of the MPE spectra, this ratio varies as a function of frequency.
2) Insert the ratio into the applicable power law model, eqn (3) or eqn (4), to compute the compressed durations associated with the individual spectra.
3) The total duration is then computed as the sum of the ensemble of the individual compressed durations for each frequency.
4) The final compressed duration is defined to be the maximum of the frequency dependent durations.

Figure 1 presents an example of the application of the fatigue compression model for a large ASD ensemble (1102 segments / 551 minutes). For this example the "as measured" data were used directly (i.e., no bootstrap sampling). The top plot in Figure 1 shows the ASDs for the raw ensemble, the MPE, and a typical event (denoted as TYP). The middle plot shows the corresponding compressed durations for the ensemble and the typical event, as well as the reference duration for each event (30 seconds). As one would expect when using the 99% probability of occurrence to define the MPE, several events exceed the MPE levels. The same fatigue scaling relationship that allows for significant compression of spectra that are smaller than the MPE spectra results in time "expansion" for spectra that are greater than the MPE spectra. The bottom plot shows the final composite compressed duration as a function of frequency along with the maximum duration.

The compression ratio for this case is ≈10:1. This is reasonably consistent with the Mil-Std 810 guidance [2], which recommends testing using the most severe spectra for 1 hour per 1000 miles (a ratio of ≈15:1 if one assumes a driving speed of 65mph.

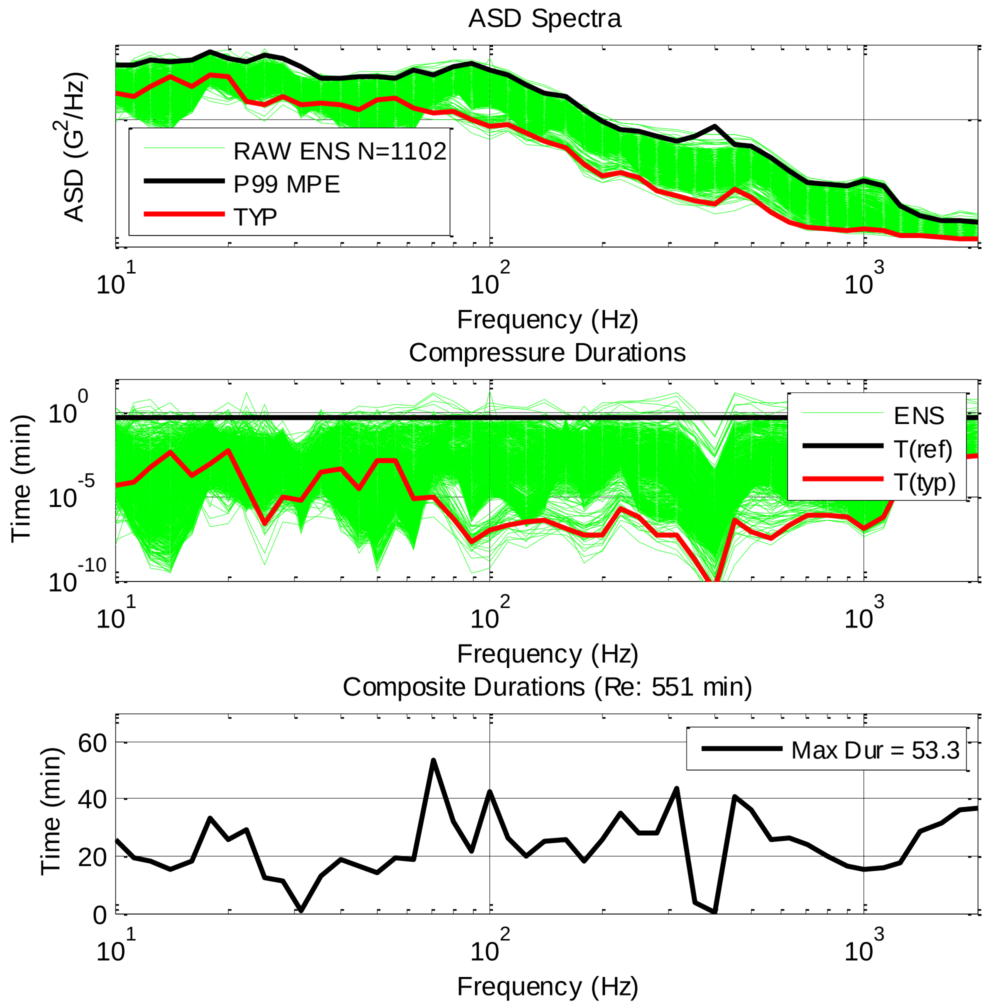

Figure 1: Deterministic Example of Time Compression

CASE STUDIES

The following cases are intended to show how the process works for large and small shock and vibration ensembles and to present some of the challenges encountered in the data.

1) ETL analysis for a large vibration ensemble versus the MPE spectra (720 minute target life).
2) LNTL analysis for a small vibration ensemble versus the MPE spectra (147 minute target life).
3) LNTL analysis for a large shock ensemble versus the MPE spectra (134 hit target life).
4) LNTL analysis for a small shock ensemble versus the MPE spectra (6 hit target life).
5) ETL analysis for a large vibration ensemble versus the test specification (720 minute target life).

CASE 1: ETL LARGE VIBRATION

Figure 2 presents the results for the ETL bootstrap analysis of the large ASD ensemble shown in Figure 1. The top plot in Figure 2 compares the MPE spectra against the "as measured" ensemble. The middle plot compares the MPE spectra against a typical bootstrap ensemble realization (for this example 1440 segments were generated for each bootstrap ensemble). The bottom plot shows the Distribution Free (DF) Probability Distribution Function (PDF) for the maximum durations associated with each of the 200 bootstrap realizations. The 90% confidence value is ≈102 minutes. As a point of reference, the equivalent compressed duration for the deterministic case shown in Figure 1 is 70 minutes (53.3*1440/1102).

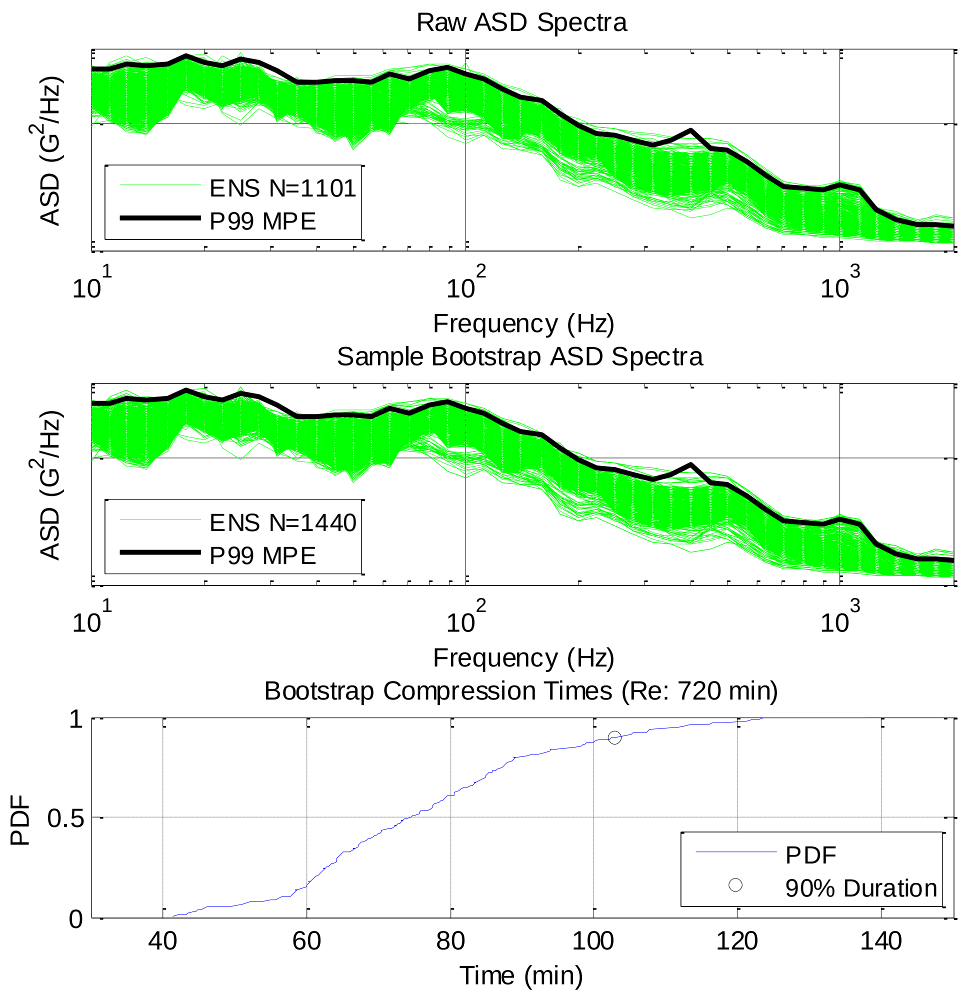

Figure 2: Large Vibration Ensemble Empirical Tolerance Model vs MPE

The most important observation for this test case is that the compression ratio is a strong function of how well the MPE spectrum follows the envelope of the ensemble. While a certain number of exceedances are to be expected, a significant number of minor outliers in the ensemble or a couple of major outliers can skew the results. Therefore, it is recommended that either the outliers be removed from the ensemble or the MPE spectra be adjusted to better envelop the outliers. Case 3 shows an example of this latter approach.

CASE 2: LNTL SMALL VIBRATION

Figure 3 shows the results for a LNTL bootstrap analysis of a small vibration ensemble. The reader will note that the "as measured" ensemble is hardly "in family". As a result, the P99/90 MPE spectrum was generated using the upper half of the family of ASDs. However, since the mean and standard deviation were derived based on the entire ensemble, the bootstrap realizations fills in gap between the two halves of the ensemble. The compression ratio is ≈12:1.

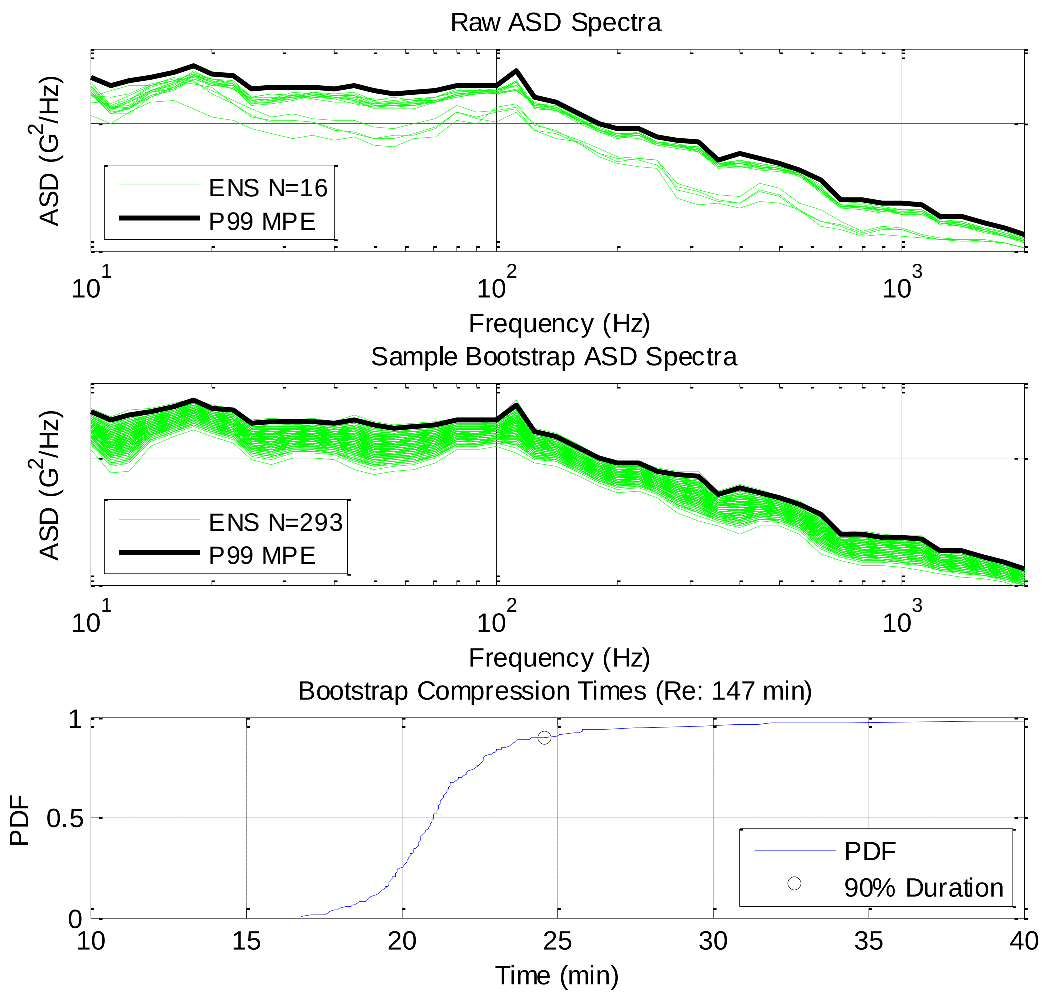

Figure 3: Small Vibration Ensemble Lognormal Tolerance Model vs MPE

CASE 3: LNTL LARGE SHOCK

Figure 4 shows the results for a LNTL bootstrap analysis of a relatively large shock ensemble. The resulting compression ratio was a relatively modest 3.5:1. However, without further study it is not obvious what should be considered a "typical" compression ratio for shocks.

This case presents an example of one way in which we dealt with responses that exceed the MPE spectra. The reader will note that the MPE spectrum appears to be well above the as measured ensemble. In fact it was artificially increased so as to envelop two outliers that we felt should not be a part of the LNTL MPE calculation but we wanted to be a part of the defining the resulting test specification. By developing the bootstrap realizations using this "enveloped" MPE spectra we still generate spectra that exceed the MPE spectra. However, had we not forced the MPE spectrum to envelop these outliers the predicted number of "compressed" hits would actually have been greater than the starting value of 134.

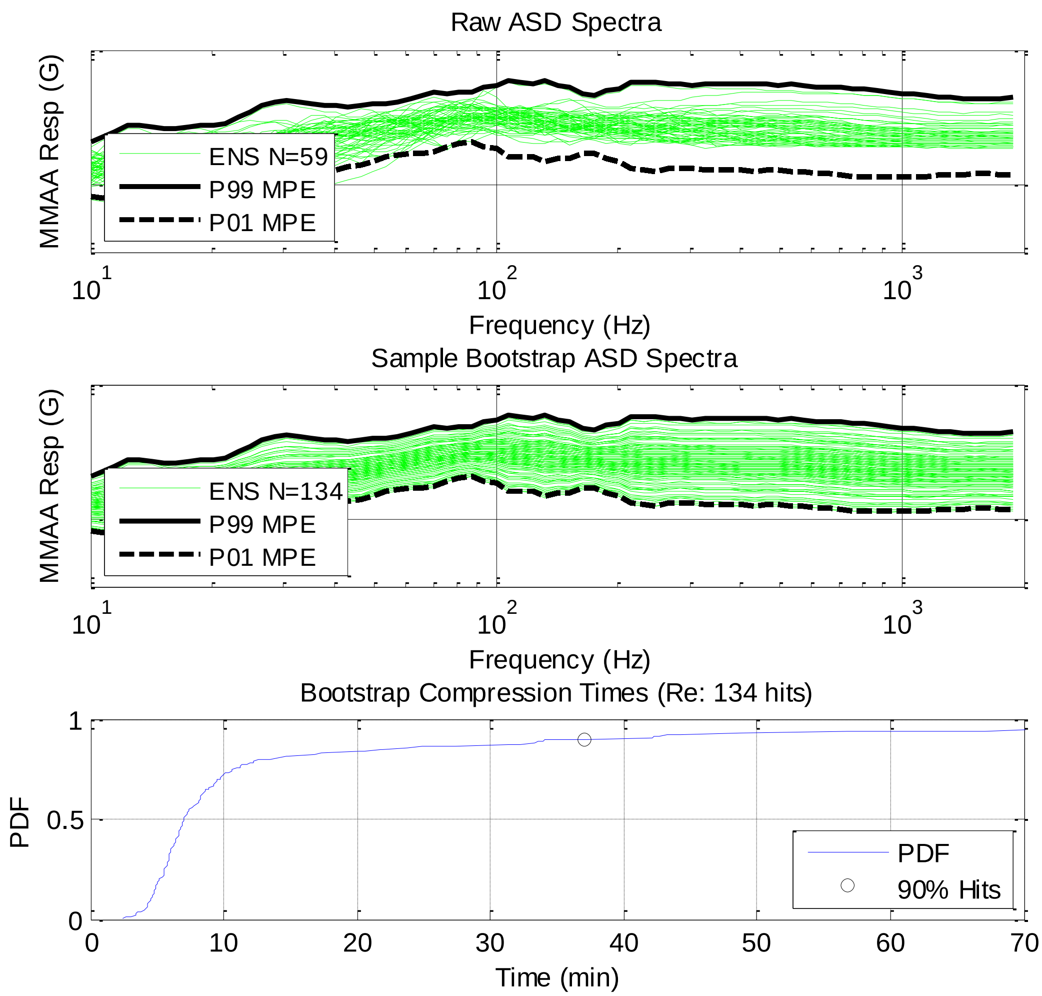

Figure 4: Large Shock Ensemble Lognormal Tolerance Model vs MPE

CASE 4: LNTL SMALL SHOCK

Figure 5 shows the results for a LNTL bootstrap analysis of a small shock ensemble. The main point for this example is that we are generating a very small number of realizations (8). As one would expect for such a small number of spectra, only a fraction of the bootstrap ensembles will include a realization that exceeds the MPE spectra. In view of this situation, it is reassuring that the resulting compression ratio (4:1) is comparable to the corresponding large shock ensemble compression ratio generated in Case 3 (3.5:1).

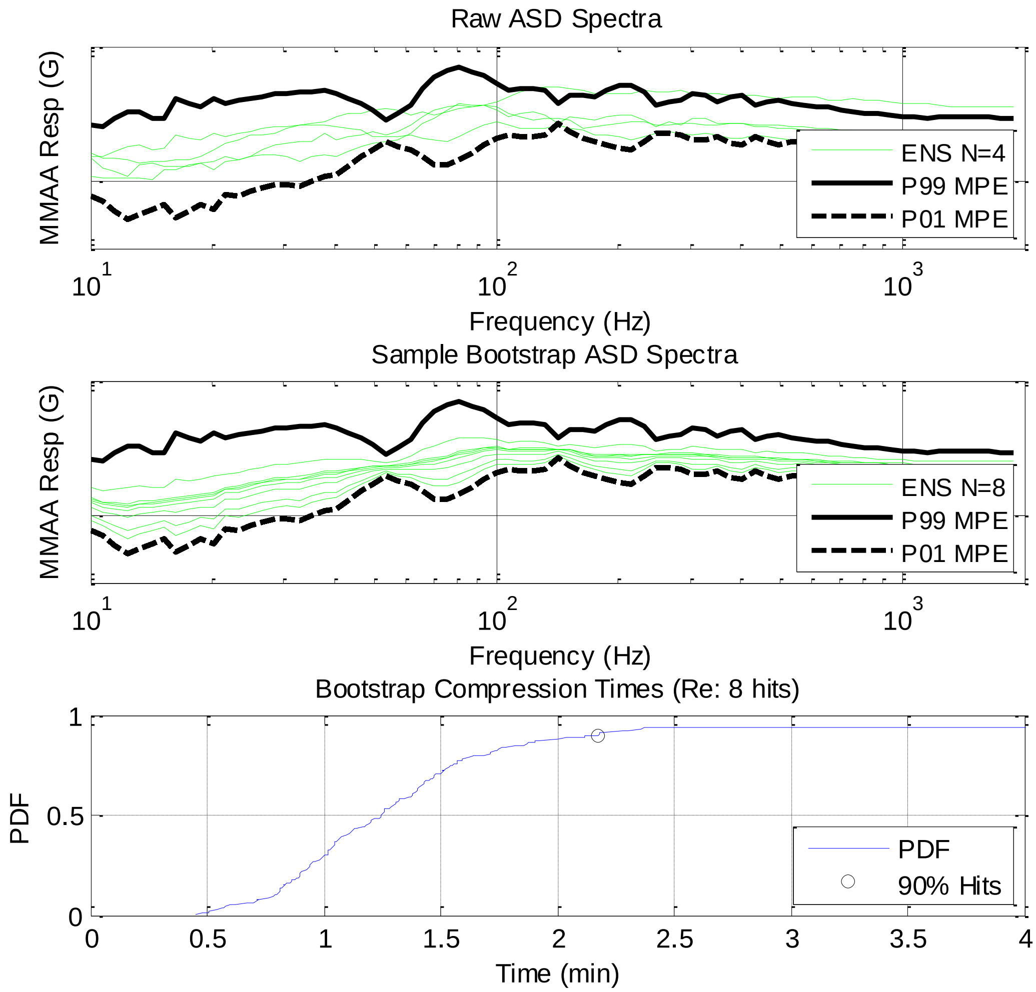

Figure 5: Small Shock Ensemble Lognormal Tolerance Model vs MPE

CASE 5: COMPRESSION REFERENCED TO TEST SPECIFICATION

Until now all of the cases have used the raw MPE spectra as the reference for the test time compression analyses. However, it is common to introduce margin when deriving a test specification (either intentional or inadvertent) and this margin can be used to affect an increase in the time compression. The degree of additional compression depends on the correlation between the test specification and the underlying ensemble.

Figure 6 shows the results for the ETL model of the large vibration ensemble evaluated against the corresponding test specification (the underlying ensemble is the same as in Case 1). The compressed duration is ≈42 minutes as compared to the 102 minute duration computed using the MPE spectra as shown in Case 1. As an aside, the test specification can always be tailored upwards as deemed appropriate to introduce more test time compression.

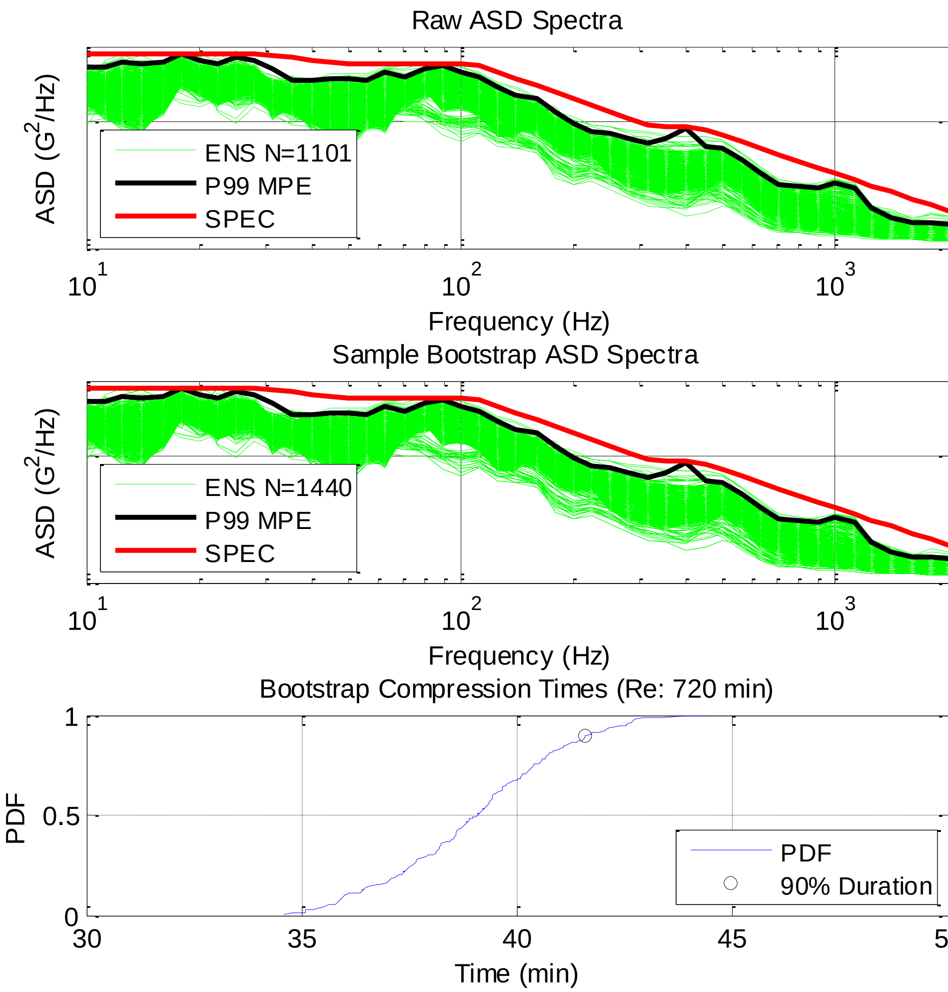

Figure 6: Large Vibration Ensemble Empirical Tolerance Model vs Test Specification

SUMMARY AND CONCLUSIONS

In general the models presented in this paper represent an approach for estimating the effective vibration test duration and/or number of shock hits for a laboratory test based on the MPE response spectra.

The main lesson learned was that the relationship between the reference spectrum (MPE or test specification) and the underlying ensemble has a strong influence on the effective compression ratio between the field and laboratory durations. The result of this is that ensembles including large numbers of spectra that exceed the MPE spectra and/or a few spectra that significantly exceed the MPE spectra exhibit lower time compression ratios. Given this fact, it was also shown that minor adjustments in the MPE spectra and/or the corresponding test specification can result in significant additional time compression.

One area for improvement would be to look into the use of other statistical models for generating the realizations for the small vibration ensembles and for the shock ensembles. For example, some sources indicate that an exponential distribution is useful for modeling shocks.

REFERENCES

[1] NASA-HDBK 7005, December 4, 2000; "Dynamic Environmental Criteria".

[2] MIL-STD 810G, January 1, 2000; "Environmental Engineering Considerations and Laboratory Tests".

APPENDIX A: OVERVIEW OF MPE MODELS

The LNTL model assumes that the log of the data , $x=log_{10}(X)$, is normally distributed and hence uses the log mean, μ, and log standard deviation, σ, of the ensemble to predict the P99/90 value as shown in equation (5).

$$x_{99}=\mu+k_{99}\sigma \tag{5}$$

The confidence level is accounted for by the fact that the "*k*" factor is a function of the sample size. It is recognized that the data are probably not lognormal distributed but identifying the true distribution for small sample sets is difficult so this assumption was considered acceptable for the purposes of this study.

The ETL model simply rank orders the available data and identifies the 99% highest value. Bootstrap sampling with replacement was used to develop the 90% confidence levels for the data.

The decision to use 200 events as the dividing line between using the LNTL and ETL models was based on the assumption that having at least 2 points above the P99 MPE level demonstrated that the ensemble was reasonably large enough to produce a predictive measure of the 99% response level.