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Development of a Single Input Multiple Output (SIMO) Input Derivation Algorithm for Oscillatory Decaying Shocks

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Abstract

During shaker shock testing of a complex system it may be desirable to match a Shock Response Spectra (SRS) at one location while controlling the test at a different location. Further, it may be desirable to match SRSs at multiple locations. This paper describes an algorithm for deriving an optimum shaker shock input such that a weighted combination of the responses for multiple locations is matched with respect to the field data measured at those locations. This work assumes the shock environment is characterized by a SRS. Since the SRS is a nonlinear transformation of the underlying acceleration waveform, the optimization process will be based on the decayed sine synthesis algorithms developed by David Smallwood.

I. Introduction

In traditional shaker shock testing, the shaker input is computed to match the requirement at the input to the component being tested and either no attempt is made to match the response at other locations of interest or it is done manually. If no attempt is made to match the responses then what the response locations experience during the shaker test may be very different and a gross over test compared to the field measurement. If an attempt is made to match the responses then it is a manual attempt by guessing what change in the input would create an intended change in the response of the locations of interest. Often a step in the intended direction of one response location may mean two steps away at another response location. This can be a difficult and slow process to compute a shaker input to produce suitable responses at multiple locations of interest. This algorithm automates this process to achieve a weighted 'best fit' input. It is recognized that with only one input a perfect solution to achieve the intended response at every point of interest is impossible. Rather only a 'best fit' is possible based on the weighted combination of responses.

II. Smallwood's Method to Compute Shaker Input for a Single Output

This method considers the shaker input a sum of decaying sinusoids as shown in equation 1.

$$A(t) = \sum_{i=1}^{N} A_i e^{-\lambda_i w_i t} \sin(w_i t + \varphi_i) \qquad (1)$$

where

A = acceleration amplitude of sinusoid
w = frequency of sinusoid
φ = delay of sinusoid
λ = decay rate of sinusoid
t = time
N = number of sine tones

The objective of Smallwood's algorithm [1] is to produce a set a decayed sinusoids that will produce an SRS that will suitably match the reference SRS. The reference SRS location is typically at the input to the test article on the shaker head. The test is controlled at this point.

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Both Smallwood's original optimization and the SIMO method optimize only the amplitude and leave all other decayed sine parameters as set originally by the user. Smallwood's algorithm works by starting with the lowest frequency sine tone, iterating on the amplitude of that sine tone and then moving to the next higher frequency sine tone until the maximum frequency sine tone is reached. The algorithm steps from low frequency to high frequency because the low frequencies can affect the higher frequencies while the high frequencies only weakly affect the low frequencies.

The user must enter a guess of the sine tone amplitudes and set the other decayed sine parameters to be used throughout the optimization. For the test cases in section IV, the parameters are as follows: Four sine tones per octave were used, the decay was set such that all the sine tones decay at the same rate, and no delay was set for any sine tones. Also the SRS was calculated with 3% critical damping.

Once a guess of the initial sine tone parameters is made, the algorithm first calculates the SRS of the initial guess input and calculates the error with respect to the reference SRS at the lowest sine tone (see equation 2).

$$Error = \frac{SRS(f) - SRS_{ref}(f)}{SRS_{ref}(f)} \tag{2}$$

Then, in an effort to determine how sensitive the SRS is to a change in the decayed sine amplitude, the amplitude is perturbed and a new SRS is computed. Then the sensitivity is computed by equation 3.

$$Sensitivity = \frac{A(f) - A_{perturb}(f)}{SRS(f) - SRS_{perturb}(f)} \tag{3}$$

The sensitivity is used to compute the new amplitude in equation 4.

$$New\ Amplitude = 0.8 * sensitivity * \left(SRS_{reference} - SRS_{pre-perturb}\right) + |A_{current}| \tag{4}$$

With the new decayed sine amplitude, the SRS can be recomputed and the error at the first decayed sine tone can be recomputed. If the error is more than a defined tolerance then the amplitude is iterated on. If the error is less than the defined tolerance then the algorithm steps to iterate on the amplitude of the next decayed sine tone. When stepping to the next sine tone, the new time history is computed using the previous, lower frequency, sine tones.

Once a full sweep is done the compensating pulse is calculated. The compensating pulse is necessary to ensure that the acceleration, velocity, and displacement begin and end at 0. The next major iteration, starting at the lowest frequency sine tone, takes into account the contribution of all the higher frequency sine tones and the compensating pulse. Generally, two to three major iterations through all sine tone frequencies produces a good match between the reference SRS and the SRS produced from the decayed sine parameters.

III. Method to Compute Shaker Input for Multiple Outputs

The method to compute the shaker input when accounting for multiple output locations is very similar to the single output location explained in the previous section. The major difference is that transfer functions and the decayed sine input are convolved to determine the response at the multiple output locations. The frequency domain convolution equation is shown in equation 5.

$$X_o(\omega) = H_{xq}(\omega)Q(\omega) \tag{5}$$

where

$X_o(\omega)$ = Fourier transform of response at output location
$H_{xq}(\omega)$ = frequency response function
$Q(\omega)$ = Fourier transform of decayed sinusoid input excitation

The SRS error at the sine tone frequency is computed with equation 6. Each individual response location error is weighted based on the user input weighing function.

$$Weighted\ Error = \frac{\sum \frac{\mu_i \left(SRS_i(f) - SRS_{ref_i}(f)\right)}{SRS_{ref_i}(f)}}{\sum \mu_i} \tag{6}$$

The sensitivity of the SRS to the decayed sine amplitude is computed using equation 6. Similar to the error calculation, the sensitivity incorporates the multiple output locations and user input weighting function.

$$Sensitivity = \frac{A(f) - A_{perturb}(f)}{\left(\frac{\sum \mu_i \left(SRS_i(f) - SRS_{perturb_i}(f)\right)}{\sum \mu_i}\right)} \tag{7}$$

The new decayed sine amplitude is computed using equation 8.

$$New\ Amplitude = c * Sensitivity * \left(\frac{\sum \mu_i \left(SRS_i(f) - SRS_{ref_i}(f)\right)}{\sum \mu_i}\right) + A(f) \tag{8}$$

Similarly to the method with one output, once the new decayed sine amplitude is computed, the SRS can be recomputed and the error at the first decayed sine tone can be recomputed. If the weighted error is more than a defined tolerance then the amplitude of the sine tone is iterated on. If the weighted error is less than the defined tolerance then the algorithm steps to iterate on the amplitude of the next decayed sine tone. Once a full sweep is done the compensating pulse is calculated.

IV. Test Case

The method was tested on a mass, spring, damper system referred to hereafter as the ‘toy model’ shown in figure 1.

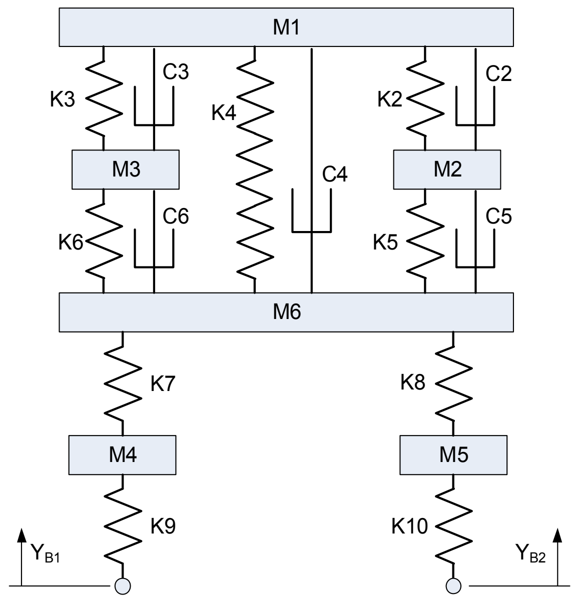

Figure 1: Babuska 'Toy Model'

The idea is to put an experimental input into the toy model input, then compute the response at the target locations (M1, M2, and M3). The responses are entered into the algorithm as 'target' SRS along with the transfer functions and then the algorithm attempts to compute an input that matches the target SRS. Then comparisons can be made between the target and predicted SRS and the experimental and algorithm computed input. The experimental input is shown in figure 2.

There are three test cases that will be examined in the following section. The first test case uses an experimentally obtained input to compute the target SRS. Then the algorithm uses the target SRS and transfer functions from the toy model to compute an 'optimized' input such that the SRS at the three points of interest match the target SRS.

Test Case 1: Single Input, Equal Weighting Outputs, Coupled System

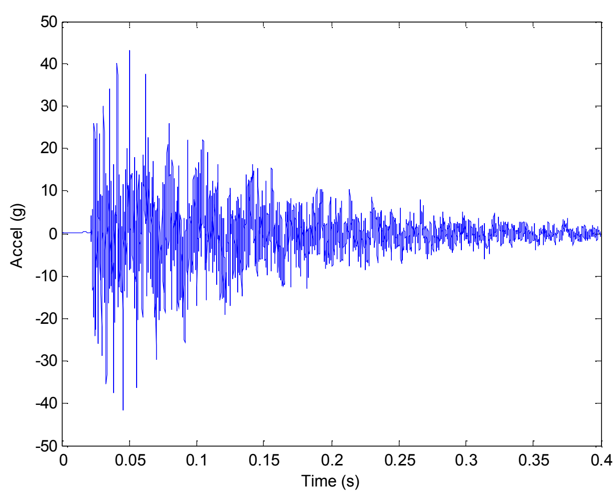

Figure 2: 'Experimental' Input

Figure 3 shows a comparison between the predicted and target SRS. The algorithm computed optimized input produced SRS that show good agreement with the target SRS. The algorithm was run assuming an equal weighting between all three response locations. The results in figure 3 illustrate this. For example, at 1000 Hz, M1 matches very closely while M2 is low and M3 is equally high.

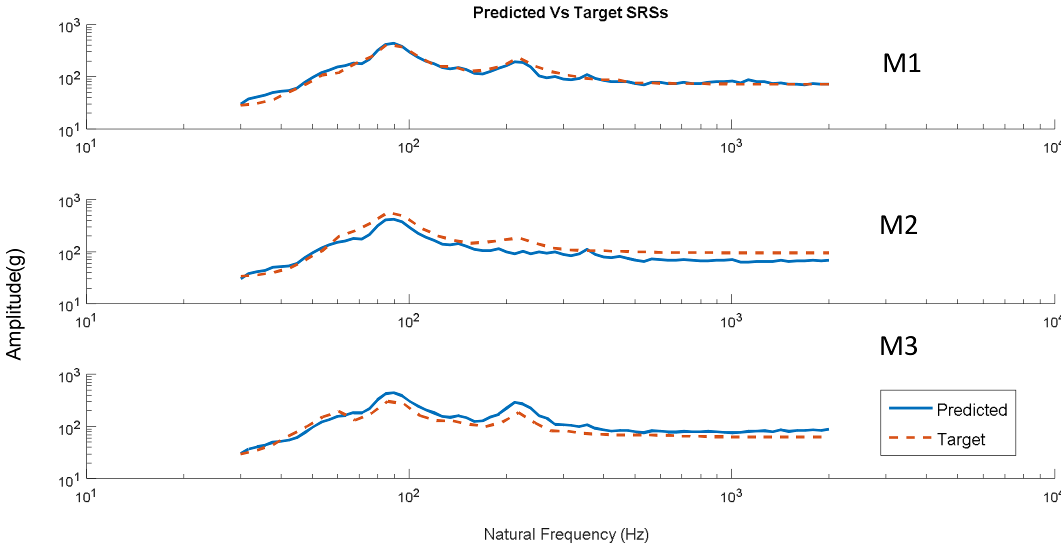

Figure 3: Comparison of Target and Predicted SRS

Figure 4 shows the algorithm computed optimized input. The time histories are noticeably different. The inputs are not optimized in the algorithm and it is noted that different time histories can produce a very similar SRS.

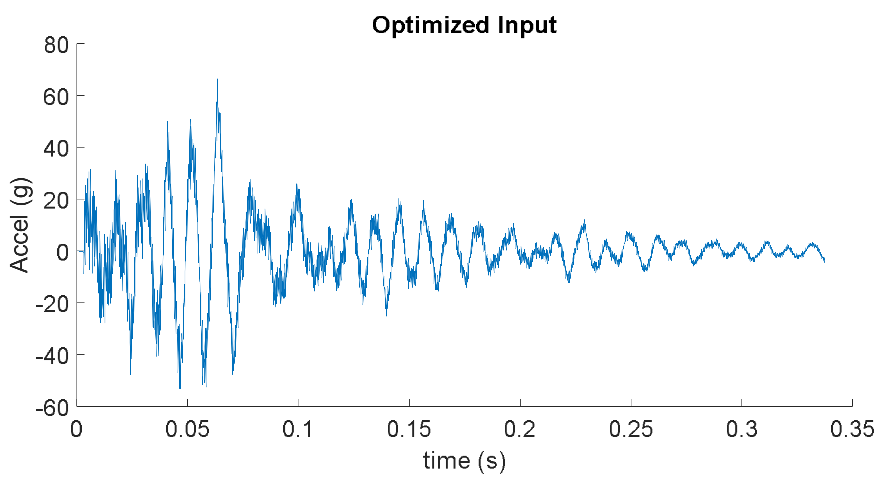

Figure 4: Algorithm Generated Optimized Input

Test Case 3: Decoupled System

It is often the case that the test spec or target SRS are obtained from a field test that has different boundary conditions and different excitation than what is possible on a shaker shock test. The second test case will replicate this scenario by using different boundary conditions than what is used by the algorithm to compute the optimized input. A second input to the toy model will also be used to compute the target SRS whereas the shaker has only one input.

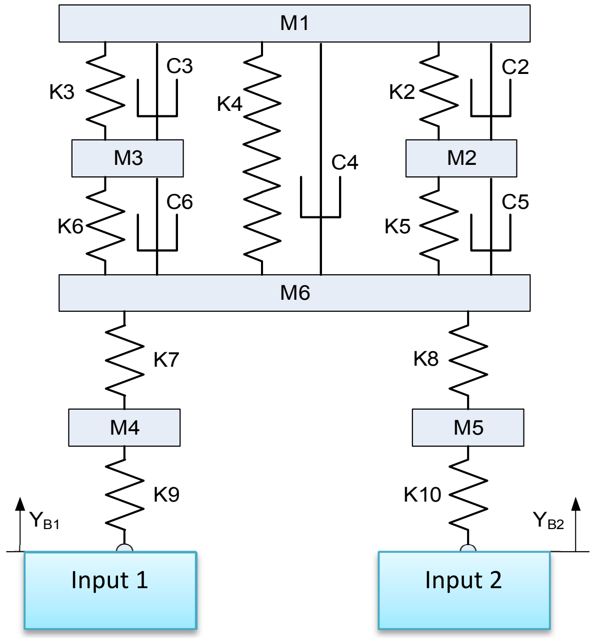

Figure 5: Toy model used to compute the target SRS

The decoupled system uses two inputs to compute the predicted SRS. Input 1 is the same input used in the last test case. The second input is shown in figure 6. With the two inputs and the toy model in figure 5 the target SRS are computed.

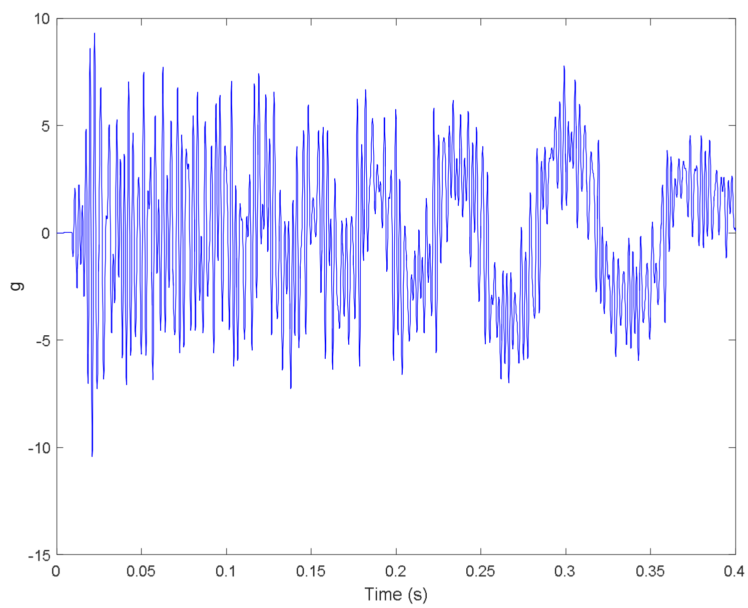

Figure 6: Input 2

A subset of the toy model used to compute the target SRS is used by the algorithm to compute the optimized input (see figure 7). The mass, spring, and damper values have not changed.

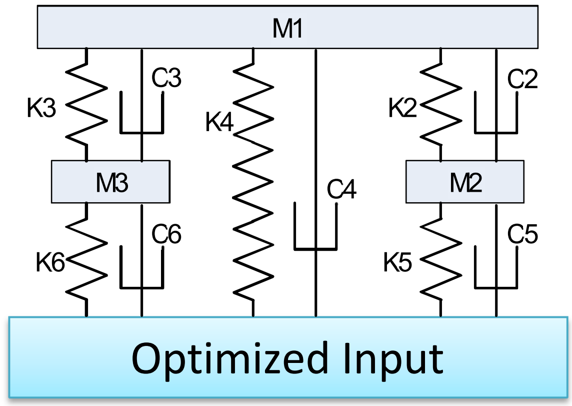

Figure 7: Toy model used to compute optimized input

Figure 8 shows how well the predicted SRS match the target SRS. There is not as good agreement as in the first test case particularly with mass 1. This is expected with the more difficult (realistic) scenario of test case 2.

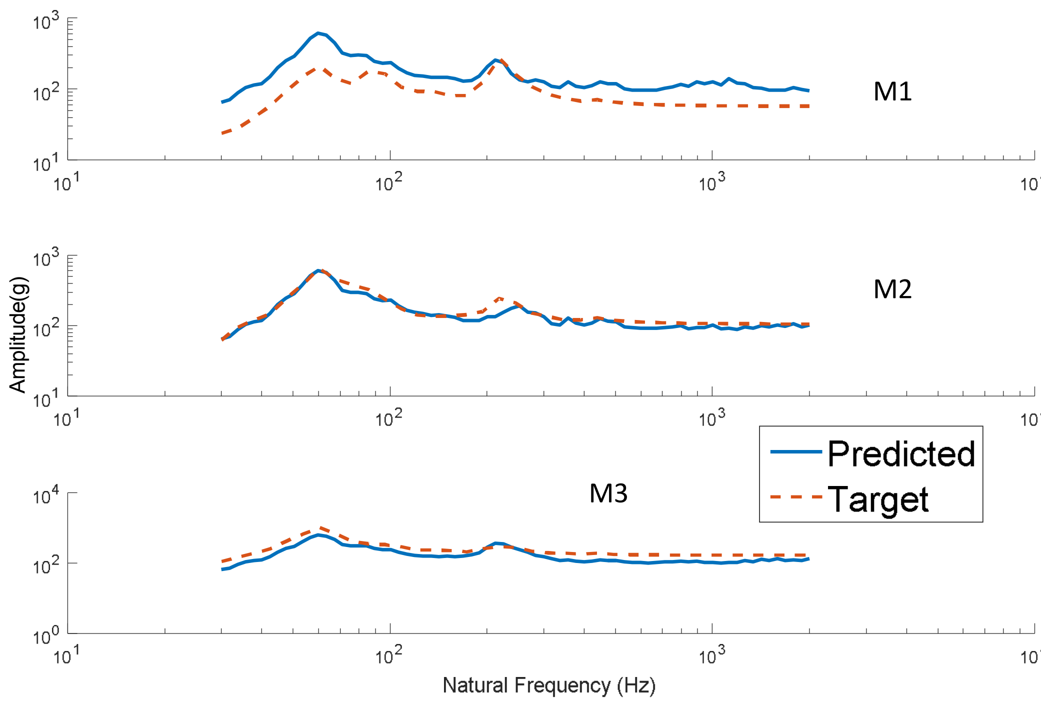

Figure 8: Predicted vs Target SRS

The case 2 optimized input is shown in figure 9. The time history is noticeably different than the time history used to produce the target SRS. It is roughly 2X higher in amplitude. The inputs are not optimized in the algorithm and it is noted that different time histories can produce a very similar SRS.

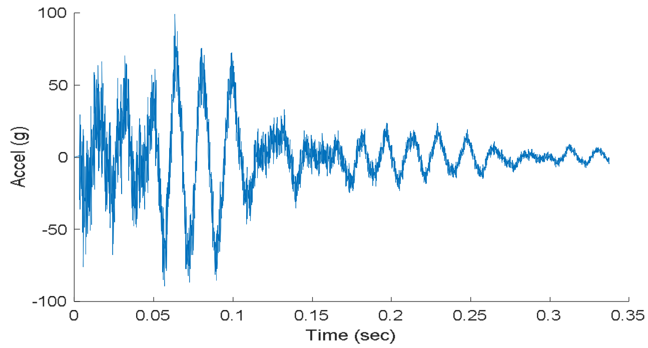

Figure 9: Optimized Input

V. Conclusions

This paper described a method to compute a shaker shock input such that a weighted combination of the responses for multiple locations is matched with respect to the field data measured at those locations. The simplified test case demonstrates that this can dramatically speed up a process that was previously manually iterative. However, there remains much to be done to prove out this method for different systems. For example, if the system does not respond over a particular bandwidth, the algorithm will increase the input amplitude at that frequency in an attempt to match the reference SRS. This would result in a high input at that frequency and show an under test compared with reference SRS. This is a limitation that can be seen in the single output method as well. Also, it may be useful to have a response limited weighting scheme where the objective is to prevent an over test at all locations. This would mean that each response location SRS could be equal to or less than the reference SRS at each frequency.

References

[1] Smallwood DS. An Improved Recursive Formula for Calculating Shock Response Spectra. The Shock and Vibe Bulletin, May 1981