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A Combinatorial Model of Dentate Gyrus Sparse Coding and Pattern Separation

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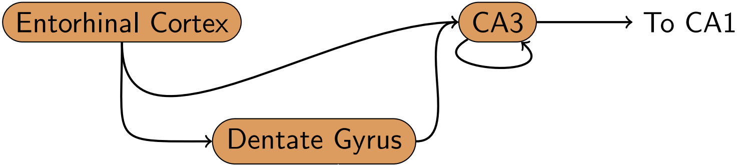
Dentate Sparse Coding Basics



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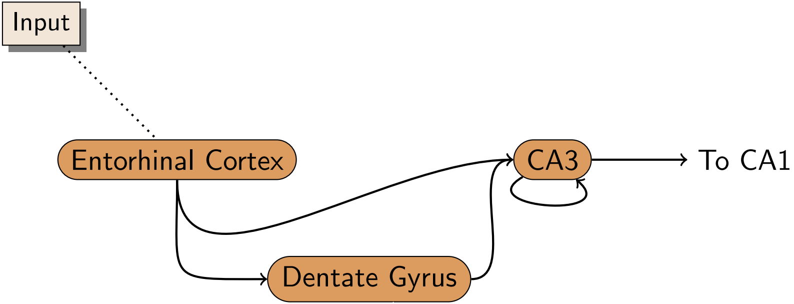

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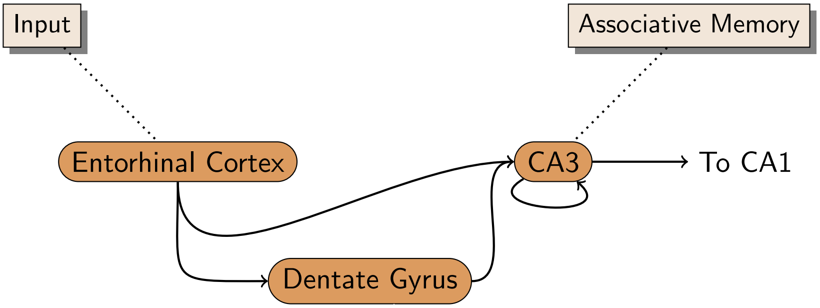

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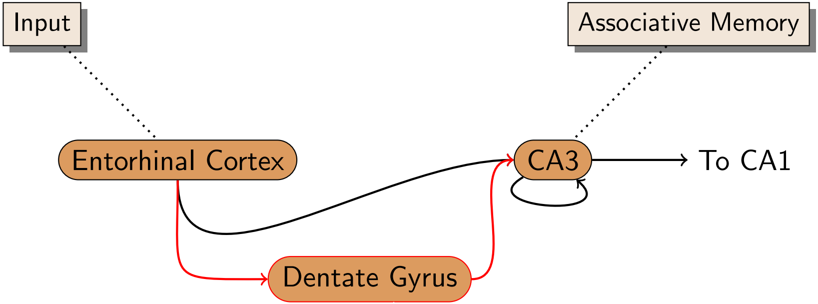

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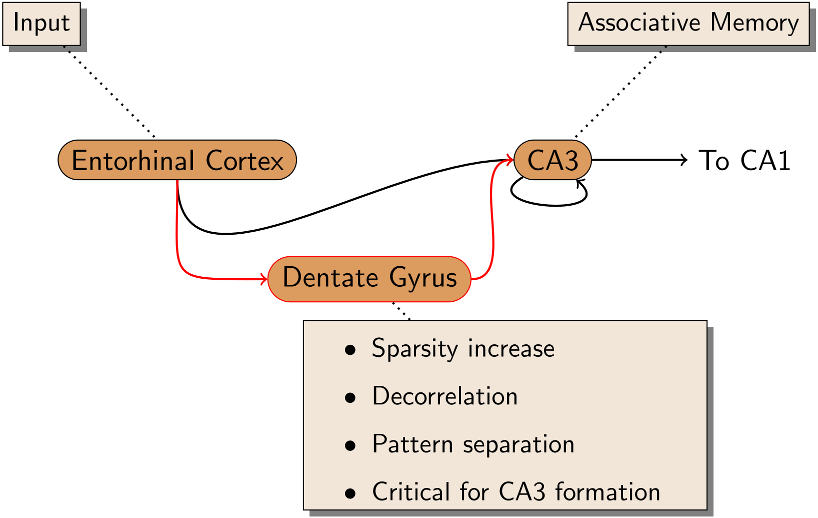

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- With $f(\alpha) = 0$ for $\alpha < 1$, $f(\alpha) = 1$ for $\alpha \geq 1$, the map

$$F = fA_\Delta$$

is a sparse coding of $\{0,1\}^n$ into $\{0,1\}^k$, where $k = |\Delta|$.

Example

Let $n = 4$, $\Delta = \{\{1,2\},\{2,3\},\{2,3,4\}\}$. Then,

$$A_\Delta = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \end{bmatrix}.$$

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First simplification: Assume Δ is composed of all p-sized subsets for some $1 < p \leq n$.

Some Basic Results

Proposition

If $||x_i||_1 = q_i$*, then* $||y_i||_1 = \binom{q_i}{p}$*, and that* F *maps* $E = \{||x||_1 \geq p\}$ *injectively into a subset* $F(E)$ *of* $\{0,1\}^k$ *where* $k = \binom{n}{p}$*.*

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Denote $n - ||x - x'||_1 = r$, $||x||_1 = q$, $||x'||_1 = q'$, $\delta_1 = q - r$, $\delta_2 = q' - r$. *We have*

$$d(F(x), F(x')) = \delta_1 \binom{q-1}{p-1} + \delta_2 \binom{q'-1}{p-1}.$$

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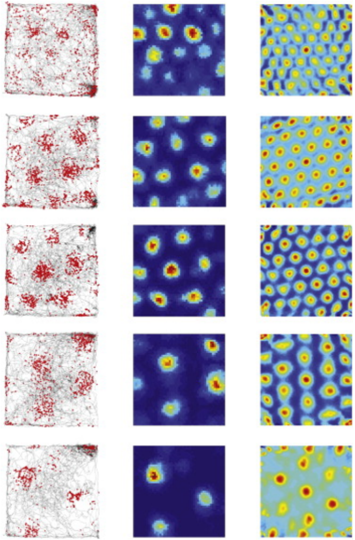
  - More realistic behavior
  - Generalize from assumption that Δ comprises p-sized subsets
  - Model's reaction to these constraints measure suitability
- Two types of generalization:

Concept	Method	Biology
Mimic Input Structure	Prune Δ	Grid Cells
Adaptive Fidelity	Mixed Coding	Adult Neurogenesis

Pruning Δ

If the distribution of the input data is known, we can remove entries from Δ and still maintain good fidelity.



Giocomo, Moser and Moser

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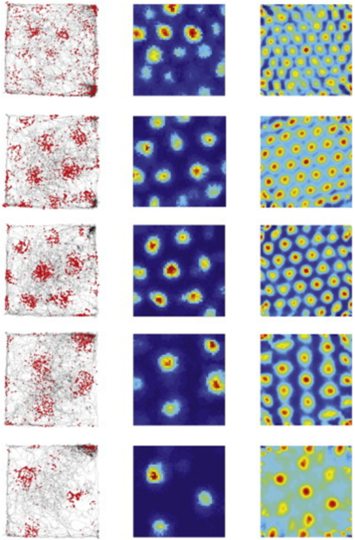

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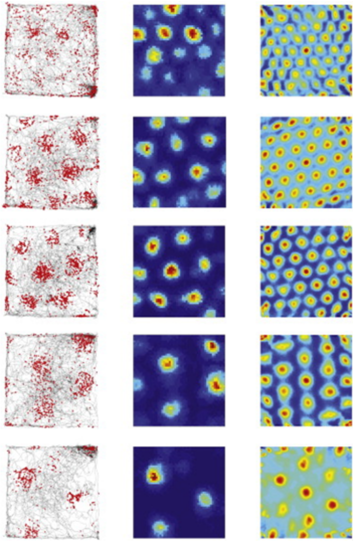
Example

Grid cells in EC encode spatial information using modular code. For $\lambda_0, \ldots, \lambda_T$ relatively prime,

$$x \mapsto (x \bmod \lambda_0, \ldots, x \bmod \lambda_T)$$

Chinese Remainder Theorem gives uniquely represented position.



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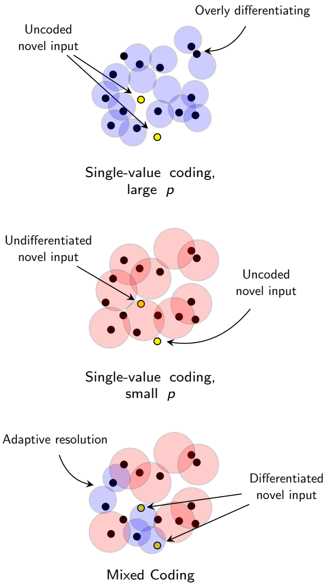
Pruning Δ

- Take input space group isomorphic to grid cell input
- Remove inadmissible η_i from Δ
- Dramatically reduces target dimension k
- Size and activity within literature expectations

Mixed Coding

Example

- The brain experiences adult neurogenesis—the development of new neurons throughout life.
- Young neurons = broadly tuned
- Old neurons = tightly tuned
- Mixed coding increases information capacity.



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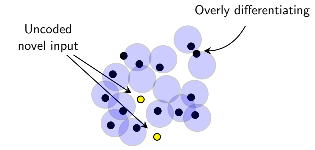
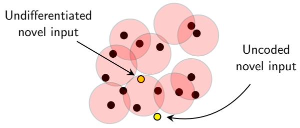


Single-value coding, large p



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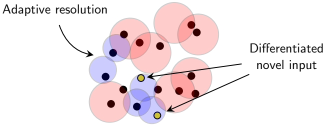

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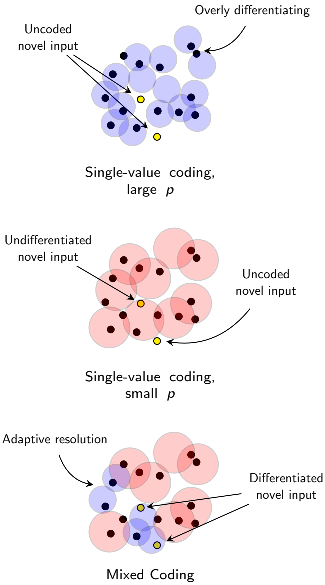

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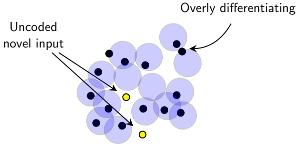
Conditions exist to guarantee sparsity and decorrelation.



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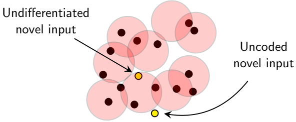

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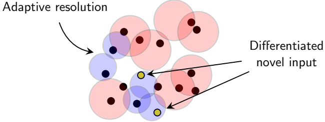

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