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Scope, Complexity, Options, Risks, Excursions (SCORE) Version 3.0 Mathematical Description

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Abstract

The purpose of the Scope, Complexity, Options, Risks, Excursions (SCORE) model is to estimate the relative complexity of design variants of future warhead options. The results of this model allow those considering these options to understand the complexity tradeoffs between proposed warhead options. The core idea of SCORE is to divide a warhead option into a well-defined set of scope elements and then estimate the complexity of each scope element against a well understood reference system. The uncertainty associated with estimates can also be captured. A weighted summation of the relative complexity of each scope element is used to determine the total complexity of the proposed warhead option or portions of the warhead option (i.e., a National Work Breakdown Structure code). The SCORE analysis process is a growing multi-organizational Nuclear Security Enterprise (NSE) effort, under the management of the NA-12 led Enterprise Modeling and Analysis Consortium (EMAC), that has provided the data elicitation, integration and computation needed to support the out-year Life Extension Program (LEP) cost estimates included in the Stockpile Stewardship Management Plan (SSMP).

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NOMENCLATURE

EMAC	Enterprise Modeling and Analysis Consortium
FPU	First Production Unit
LEP	Life Extension Program
NSE	Nuclear Security Enterprise
NWBS	National Work Breakdown Structure
SCORE	Scope, Complexity, Options, Risks, Excursions
SME	Subject Matter Expert
SSMP	Stockpile Stewardship Management Plan

1. INTRODUCTION

This document describes the mathematics behind the Scope, Complexity, Options, Risks, Excursions (SCORE) model developed for the Enterprise Modeling and Analysis Consortium (EMAC). The purpose of SCORE is to estimate the relative complexity of design variants of future warhead options. The results of this model allow stakeholders to understand complexity tradeoffs between the proposed warhead options. The core idea of SCORE is to first divide a warhead option into well-defined pieces of work (e.g., nuclear explosive package), each with a distinct scope. These pieces are termed scope elements, or simply "elements", and typically connect back to National Work Breakdown Structure (NWBS) codes. For each element, a well understood reference system is identified, against which comparisons of complexity can be made. The combination of these elements and their associated reference systems is referred to as the reference basis. A key feature of SCORE is that the reference basis can be a hybrid of multiple systems, which allows for estimates of complexity to be made against the most applicable reference system. Weights based on the cost of the defined reference basis are applied to each element to represent the scale to which each contributes to the reference basis. This ensures that the complexity of each element is normalized to the appropriate value. A weighted summation of the relative complexity of each element can be used to determine the total complexity of the warhead option or some portion of the warhead option.

SCORE Version 3.0 includes two features that were not part of the 1.X versions of the model. First, it includes sampling to estimate the distribution of warhead design option complexity. Since the development of SCORE Version 1.0, Subject Matter Experts (SMEs) have been able to provide complexity estimates (i.e., "estimates") that include uncertainty (i.e., minimum, most likely, and maximum values). However, in previous versions of SCORE, only minimum, maximum, and mean output complexity scores (i.e., "scores") were calculated. The inclusion of sampling allows the probability distribution function of warhead design option complexity to be estimated.

The second feature new to Version 3.0 includes the concept of leveraging within excursions. Excursions represent pairs of warhead design options considered together. This could be because both design options will be executed within the same timeframe or there may be overlap in the capabilities or resources that are used to accomplish them. Leveraging is intended to be used when two different warhead design options have overlaps or commonalities. As a result, the overall complexity for both warhead options may be less than the sum of the parts. The inclusion of leveraging allows SMEs to provide multipliers that reduce the complexity of pairs of elements when they are executed as part of an excursion. Sampling is also used to approximate the uncertainties associated with excursions.

No changes were made to the complexity score calculation between SCORE Version 3.0 and Version 2.0. The changes between these two versions only impacted the EMAC SCORE factor calculation, which is documented elsewhere [1].

This document is divided into two sections. Section 2 describes the inputs to the model. Section 3 describes the methodology that is used to calculate model outputs.

1.1. Key Terms

Key terms will be defined as follows before further detail of the process and the mathematical model are explored:

Scope Element (element) – The lowest level at which scope of work is defined. Each element is associated with a NWBS code, a type (e.g., nuclear), and a category for grouping purposes (e.g., radiation case). Multiple elements can belong to the same NWBS code.

Phase – The development and production life of a weapon system is divided into phases. Phases associated with production must account for the number of units produced.

Reference System – A weapon system that can be used as the basis of comparison for a subject matter expert (SME) for a specific element. Reference systems must have an understood cost and associated scope of work.

Reference System Cost – The costs associated with an element, broken out by phase, for a specific reference system. These data are used for creating relative weights when combining elements that have the same NWBS code. An element reference cost can come from a weapon system cost or SME cost estimate.

Reference Basis – The hybrid of reference systems against which complexity is measured. It consists of a collection of elements with a reference system identified for each element. SCORE does not require that all elements use the same reference system, which allows SMEs to make complexity estimates against the "best available" reference system.

Design Choice – A specific design implementation for an element. Multiple design choices can be defined for each element.

Complexity Estimate (estimate) – As compared to a "best available" reference system, relative estimates of complexity for a design choice are given by SMEs and include low, most likely, and high estimates. Values are given for each combination of element, design choice, and phase combination.

Warhead Option – A warhead design variant under consideration. A warhead option is defined by the design choices that are selected for each element, as well as the production quantities associated with each element.

Excursion – A pair of warhead options that are analyzed as a single unit. In some cases, for example, development or production aspects of warhead options can be leveraged to reduce the overall complexity of executing both programs

Complexity Score (score) – Native output of the SCORE model which allows for relative comparison of complexity between multiple systems, warhead options of the same system, or between excursions.

2. INPUT DATA AND ASSOCIATED CALCULATIONS

SCORE estimates the complexity of a warhead option by identifying a reference basis that is a collection of elements, measuring the complexity of each of these elements for the proposed warhead option relative to a reference system, and using a weighted sum to produce an overall estimate of complexity for warhead options or excursions. The following subsections describe the inputs that are necessary to create SCORE outputs.

2.1. Scope Elements

SCORE requires a collection of elements that collectively define the reference basis. Let,

I:	The number of scope elements.
$i = 1,2,\ldots,I$:	An index on the scope elements.

2.2. Categories

SCORE uses the generic structure "categories" to create model outputs. SCORE outputs are created by combining subsets of elements for a proposed warhead option. The category structure is used to represent the elements that belong to a given output. A category may contain multiple elements. For example, there may be a category called "Type" that contains two members "Nuclear Components" and "Non-Nuclear Components". For this category, each element would be assigned to one of these two categories. Based on this input, the SCORE model will create two sets of outputs for the "Type" category, one that considers the combined complexity of nuclear elements and one that considers the combined complexity of non-nuclear elements. A second category could be the "Fifth Level NWBS Code". In this case, the members of that category would be a valid list of NWBS codes (e.g., 1.1.1.X.1, 1.1.1.X.2, etc.). This would allow analysts to view outputs by NWBS code. Further, to capture the total complexity of a design, a category called "System Level Complexity" could be created that contains a single member called "Total". All elements would belong to the "Total" member of this category. Let,

K:	The number of categories.
$k = 1,2,\ldots,K$:	An index on the categories.
$J(k)$:	The number of category members in category k.
$j = 1,2\ldots,J(k)$:	An index on the category members for category k.
$\alpha_{i,k} = j \in 1,2\ldots,J(k)\ \forall i,k$:	Integer that identifies that scope element, i, belongs to category member, j, of category, k. All scope elements must belong to exactly one valid category member for each category.

2.3. Phases

While time is not explicitly captured in SCORE, the concept of phases is used to capture variations in complexity throughout the lifecycle, or "phase", of the effort. Let,

P: The number of phases.

$p = 1,2\ldots,P$: An index on the phases.

Phases can be defined as one of two types: ones where the complexity depends upon the quantity of units produced and ones which do not. For example, normally the complexity of a production phase is tied to the quantity of units being produced but the complexity of the design phase is not. Let,

$\beta_p \in \{0,1\}$: Binary input that identifies that the complexity score for phase, p, will be adjusted for production quantities by the score calculation.

2.4. Reference Systems and Reference Basis

SCORE determines the complexity of a proposed warhead option by comparing the complexity of the proposed option to the complexity of a reference system for each element. SCORE allows analysts to create a reference basis by identifying a reference system for each element. Let,

R: The number of reference systems.

$r = 1,2\ldots,R$: An index on the reference systems.

When creating a reference basis, the analyst must specify which reference system will be used for each element *i.* Let,

$\gamma_{i,r} \in \{0,1\}$: Binary input that identifies that the scope element, *i*, and the reference system, *r*, belongs to the reference basis.

All elements must use exactly one reference system in the reference basis, as enforced in Constraint (1).

$$\sum_r \gamma_{i,r} = 1 \quad \forall i \tag{1}$$

For each element, phase, and reference system a cost value must be provided. All cost data should be provided in same year dollars to avoid skewing the outputs due to inflation.

$C_{i,p,r} \in R_{\geq 0}$: EMAC reference cost for a given scope element *i,* phase *p*, and reference system *r.*

These cost values are used to create a weight for each element *i* and phase *p*. This weighting, represented by $W_{i,p}$, represents the fraction of cost that element *i* contributes to the reference basis in phase *p*. It is calculated using Equation (2) below.

$$W_{i,p} = \frac{\sum_{r} \gamma_{i,r} \cdot C_{i,p,r}}{\sum_{i} \sum_{p} \sum_{r} \gamma_{i,r} \cdot C_{i,p,r}} \quad \forall i,p \tag{2}$$

Finally, for each element *i* and reference system *r*, the number of units that were built need to be specified. Let,

$N^{Build}{}_{i,r} \in N_0$:	Reference quantity for a given scope element *i* built in reference system *r*.

2.5. Design Choices and Complexity Estimates

The inputs above define the structure of the problem and the reference basis to be used. Design choices are used to specify strategies for addressing each element *i* in proposed warhead options. Let,

$L(i)$:	The number of design choices under consideration for scope element *i*.
$l = 1,2 \ldots, L(i)$:	An index on the design choices for scope element *i*.

For each element *i* and phase *p*, it is necessary to specify the complexity of design option *l*. Let,

$F^{Low}{}_{i,p,l} \in N_0$:	Low complexity estimate for a given scope element *i*, phase *p*, and design choice *l*.
$F^{ML}{}_{i,p,l} \in N_0$:	Most likely complexity estimate for a given scope element *i*, phase *p*, and design choice *l*.
$F^{High}{}_{i,p,l} \in N_0$:	High complexity estimate for a given scope element *i*, phase *p*, and design choice *l*.

For each design choice, element, and phase, analysts must obtain three estimate values that represent relative scope complexity – lowest, most likely, and highest. Complexity estimates (or "estimates") are represented as a random variable in the model. Three estimates are obtained so that a probability distribution can be populated, which captures this randomness. Uncertainty in estimates is assumed to follow a triangular distribution. This distribution is defined by a low, most likely, and high parameter.

It is assumed that a value of 100 corresponds to the baseline complexity of the reference basis. A relative scope estimate of 100 for a given element and phase implies that the complexity of the design choice would be the same as the reference basis. Similarly, a value of 200 would imply that the design choice is twice as complex.

2.6. Warhead Options

A warhead option represents a system design that is under consideration and consists of two key components. First, a warhead option requires that a design choice be specified for each element. Second, a production quantity must be defined for each element. Together, these inputs define what the warhead is and how many units of production will be required for each element within the warhead option. The model can consider multiple warhead options. Let,

M:	The number of warhead options.
$m = 1,2\ldots,M$:	An index on the warhead options.

Each warhead option must have a valid design choice for each element. Let,

$D_{i,m} = l \in 1,\ldots,L(i)$:	Input that identifies that the design choice, *l*, is being used for scope element, *i*, in warhead option, *m*.

Next, the production quantity for each element *i* in each warhead option *m* needs to be identified. Let,

$Z_{i,m}$:	Production quantities associated with a given scope element *i* and warhead option *m.*

While a single input is sufficient to specify the production quantity, capturing how the quantity is derived is better for both understanding and traceability. Therefore, SCORE allows either the production quantity for an element to be specified directly or specified through a series of inputs to arrive at this value. The production quantity for each element is derived using a three step process. First, the number of systems to be produced is determined. Second, the number of components of each element per system is defined. Finally, the number of overbuilds for each element is defined.

To derive the system production quantity, production can be divided into production that supports active and inactive systems. The method for determining the number of active systems is shown first. Let,

$v_m \in N_0$:	Number of active ready warheads for warhead option, *m.*
${v^{Spares}}_m \in N_0$:	Number of active hedge and logistics warheads for warhead option, *m.*
${v^{Percent}}_m \in R_{\geq 0}$:	Fraction of active hedge and logistics warheads for warhead option, *m.*
$\vartheta_m \in \{0,1\}$:	Binary that indicates that the ${v^{Percent}}_m$ term will be used, instead of ${v^{Spares}}_m$, to calculate the number of active hedge

and logistics warheads relative to active ready for warhead option, *m*.

For a given warhead option, the number of active units to produce, V_m, can be calculated using Equation (3). Note that the number of additional hedge and logistics units can be determined using a percentage of the ready units or a fixed value but not both.

$$V_m = \left(1 + \vartheta_m \cdot v^{Percent}{}_m\right) \cdot v_m + v^{Spares}{}_m \cdot (1 - \vartheta_m) \tag{3}$$

Similar logic is used to calculate the number of inactive units to be produced. Let,

$x_m \in N_0$: Number of inactive hedge warheads for warhead option, *m*.

$x^{Spares}{}_m \in N_0$: Number of inactive logistics and reserve warheads for warhead option, *m*.

$x^{Percent}{}_m \in R_{\geq 0}$: Fraction of inactive logistics and reserve warheads relative to inactive hedge warheads for warhead option, *m*.

$\chi_m \in \{0,1\}$: Binary that indicates that the $x^{Percent}{}_m$ term will be used, instead of $x^{Spares}{}_m$, to calculate the number of inactive logistics and reserve warheads for warhead option, *m*.

For a given warhead option, the number of inactive units to produce, X, can be calculated using Equation (4) below.

$$X_m = \left(1 + \chi_m \cdot x^{Percent}{}_m\right) \cdot x_m + x^{Spares}{}_m \cdot (1 - \chi_m) \tag{4}$$

The preceding inputs and equations define the total number of systems that need to be built for each warhead option. The next step is to determine the number of components that need to be produced for each element. Let,

$\delta_i \in \{0,1\}$: Binary that indicates that the active quantities be used in the build quantity calculation for scope element, *i*.

$\varepsilon_i \in \{0,1\}$: Binary that indicates that the inactive quantities be used in the build quantity calculation for scope element, i.

$t_i \in N$: Multiplier for production quantities associated with a given scope element, *i* (i.e., the number of units per system).

$y_i \in N_0$: Number of overbuilds for a given scope element, *i*.

$y^{Percent}{}_i \in R_{\geq 0}$: Fraction of overbuilds for a given scope element, *i*.

$\psi_i \in \{0,1\}$: Binary that indicates that the $y^{Percent}{}_i$ term will be used, instead of y_i, to calculate the number of overbuilds for a given scope element, *i*.

$z_{i,m} \in N_0$: Number of production quantities associated with a given scope element *i* and warhead option *m*. (i.e., direct specification of quantity, not derived).

$\pi_{i,m} \in \{0,1\}$: Binary that identifies that a value has been directly specified for production quantities for scope element, *i*, and warhead option, *m* and should not be derived.

Given these inputs, the production quantity of element *i* for warhead option *m*, $Z_{i,m}$, can be calculated using Equation (5).

$$Z_{i,m} = [\pi_{i,m} \cdot z_{i,m}] + \left[(1 - \pi_{i,m}) \cdot \left(t_i \cdot \left(1 + \psi_i \cdot y^{Percent}{}_i\right) \cdot (\delta_i \cdot V_m + \varepsilon_i \cdot X_m) + y_i \cdot (1 - \psi_i)\right)\right] \quad (5)$$

The first term of Equation (5) allows the analyst to directly input the production quantity for each element. The second term allows the analyst to calculate the production quantity from the number of systems that will be built. Either of these methods can be used but not both. In the second term, the analyst can decide which combination of active and inactive units should be included in the calculation. The result of this summation is multiplied by the number of components per system, t_i, to determine the number of components to build (*versus* the number of systems). This value can be increased by either a percentage or a defined number of additional units but not both.

2.7. Leveraging Options and Excursions

Leveraging options are used to define pairs of systems that make sense to analyze simultaneously. Let,

B: The number of leveraging options.

$b = 1,2,\ldots,B$: An index on the leveraging options.

For each leveraging option, it is necessary to define the pair of warhead options that belong to the leveraging option. Let,

$B_b^1 = m \in 1,\ldots,M$: Input that identifies the first warhead option *m* that is part of leveraging option *b*.

$B_b^2 = m \in 1,\ldots,M$: Input that identifies the second warhead option *m* that is part of leveraging option *b*.

When two warhead options are considered as part of a leveraging option, it may be possible to realize a reduction in the complexity of elements for each warhead option. SCORE makes the following assumptions about leveraging:

- Leveraging is modeled using a leveraging multiplier for each element and phase.
- The leveraging multiplier is used to adjust the low, most likely, and high estimates.
- The leveraging multiplier is a function of the number of years separating two warhead options.
- The sequencing of the warhead options is not considered.
- The leveraging multiplier is applied equally to both warhead options.

The relationship between the leveraging multiplier and the number of years separating two warhead options is determined through three user inputs. Let,

$\tau^{Full}_{b,i} \in N_0$:	Input that identifies the maximum number of years that the two warhead options in leveraging option *b* can be separated by while still maintaining full leveraging for scope element, *i*.
$\tau^{None}_{b,i} \in N_0$:	Input that identifies the minimum number of years that the two warhead options in leveraging option *b* can be separated after which there will be no leveraging for scope element, *i*.
$d_{b,i,p} \in [0,1]$:	Leveraging multiplier for scope element *i,* in phase *p* for leveraging option *b* when the warhead options are separated by $\tau^{Full}_{b,i}$ years or less.

The leveraging multiplier is assumed to follow a piecewise linear relationship. When the number of years between warhead options is less than or equal to $\tau^{Full}_{b,i}$, there is full leveraging. In this case, the complexity of a given element and phase would be reduced by multiplying the baseline complexity and the leveraging multiplier $d_{b,i,p}$. When the number of years between warhead options is greater than or equal to $\tau^{None}_{b,i}$, there is no leveraging. In this case, the leveraging multiplier takes a value of one. When the number of years between warhead options is between $\tau^{Full}_{b,i}$ and $\tau^{None}_{b,i}$, linear interpolation between $d_{b,i,p}$ and one is used to determine the value of the leveraging multiplier. In cases where $\tau^{Full}_{b,i} \geq \tau^{None}_{b,i}$, the $\tau^{Full}_{b,i}$ value should be ignored.

The Figure 1 below illustrates this idea for the case when $\tau^{Full}_{b,i} = 1$, $\tau^{None}_{b,i} = 5$, and $d_{b,i,p} = 0.6$. In the case where there is one year or less separating the two warhead options, the complexity of elements *i* in phase *p* of both warhead options in leveraging option *b* would be sixty percent of the complexity as compared to the case when the number of year separating the two warhead options was five years or more.

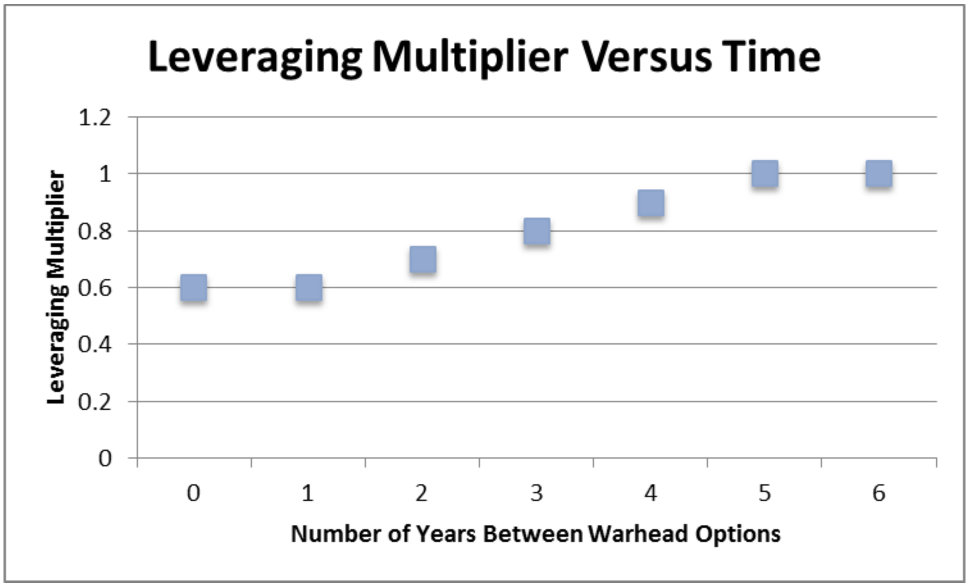

Figure 1. Illustrative example showing the relationship between the leveraging multiplier and the number of years between warhead options.

Note that for a given element and leveraging option the leveraging multiplier can vary by phase, but the time parameters apply to all phases. It is important that the leveraging multiplier be allowed to change by phase since leveraging may be possible in some phases (e.g., Systems Engineering and Integration and Production Development) and not possible in other phases (e.g., production). The time parameters are not collected by phase in order to reduce the burden of the data collection process.

Excursions are combinations of options to be considered in the context of stockpile planning with specific start dates for each warhead option. Leveraging options describe a time-based relationship between two warhead options. Excursions define start dates for each warhead option within a leveraging option. Let,

E:	The number of excursions.
$e = 1,2,\ldots,E$:	An index on the excursions.

For each excursion, it is necessary to define the leveraging option that will be used. Let,

$h_e \in 1,2,\ldots,B$:	The leveraging option that is used for excursion *e*.

Finally, the start dates for each warhead option in an excursion need to be defined. SCORE allows for uncertainty in the start date of the two warhead options associated with an excursion. Let,

$a_e^1 \in N$:	The earliest start year value for the first warhead option in excursion *e*.

$a_e^2 \in N$: The earliest start year value for the second warhead option in excursion *e*.

$\sigma_e^1 \in N$: The latest start year value for the first warhead option in excursion *e*.

$\sigma_e^2 \in N$: The latest start year value for the second warhead option in excursion *e*.

It is assumed that the start year for both the first and second warhead option in an excursion follow a discrete uniform distribution whose bounds are based on the respective earliest and latest start year values. The latest start year values must be greater than or equal to the earliest start year values, as indicated Equations (6) and (7) below.

$$a_e^1 \leq \sigma_e^1 \tag{6}$$

$$a_e^2 \leq \sigma_e^2 \tag{7}$$

3. OUTPUT DATA AND ASSOCIATED CALCULATIONS

This section describes how SCORE outputs are calculated. Recall, the outputs of the model are termed “scores” and represent the complexity of various parts of proposed warhead options. The resulting scores are an indication of relative complexity of proposed warhead options against a reference system (which could be a combination of several reference systems). SCORE uses a flexible, data-driven approach that allows users to customize the types of outputs that are calculated. There are three main options available for calculating outputs. The first option is related to assumptions about the uncertainty associated with estimates. One of two assumptions can be used to calculate outputs. The first approach assumes that the estimates provided by SMEs define a triangular distribution. The second approach does not assume that the estimate uncertainty is defined by a particular probability distribution. This approach only assumes that the mean complexity is a weighted combination of the estimates provided by the SMEs. The user is able to select the preferred approach in SCORE Version 3.0, but the triangular distribution approach is recommended because it makes full use of the probabilistic information available.

The second option determines how the calculation will address production phases. Outputs can either represent the complexity associated with full production of a warhead or the complexity up to and including the first production unit (FPU). In the full production case, the calculation accounts for both the change in the complexity of production and the number of units to be produced (as compared to the reference system). In the FPU case, only the complexity associated with producing a single system is accounted for. It is important to note that the calculation currently assumes that all units that are produced have the same complexity. In reality, the first production unit may be more complex than the subsequent units. Given this, it is recommended that the full production option be used. Let,

$\epsilon \in \{0,1\}$:	Binary that identifies that outputs will be calculated through full production.

The third option determines if outputs will be calculated for individual warhead options or excursions (pairs of warhead options). Subsections 3.1 and 3.2 describes each of these approaches, respectively.

Regardless of how the three options above are set, a variety of metrics can be calculated. For example, the total complexity of an option or the complexity of an option by phase can be calculated. The complexity of logical groups of elements can also be calculated for an option (e.g., nuclear *versus* non-nuclear components). The complexity of these groups can also be broken out by phase.

All SCORE outputs are calculated using Equation (8). The term $S^{\cdot}_{i,p,m}$ represents the complexity score for phase p of element i for warhead option m. The dot in the superscript of the S and F terms is a wildcard that represents the value being calculated. Four values can be used for this wildcard: minimum, most likely (ML), maximum, and sample. The minimum, most likely (ML), and maximum $F^{\cdot}_{i,p,l}$ values are the three estimates provided by the SMEs. The sample value is explained below.

$$S^{\cdot}{}_{i,p,m,\epsilon} = [W_{i,p}] \cdot \left[\frac{F^{\cdot}{}_{i,p,D_{i,m}}}{100}\right] \cdot \left[\beta_p \cdot \frac{\epsilon \cdot Z_{i,m} + (1-\epsilon) \cdot t_i}{\sum_r \gamma_{i,r} \cdot N^{Build}{}_i} + (1-\beta_p)\right] \quad (8)$$

Recall that the first term in Equation (8) represents the fraction of cost that element i contributes to the reference basis in phase p. The second term is the estimate divided by 100. This term represents the relative difficulty of an element and phase compared to the baseline. Recall that the baseline difficulty is assumed to be 100. The third term represents the relative complexity associated with production. In the case where the phase under consideration is not adjusted for the build quantity, such as non-production phases (e.g., Systems Engineering and Integration), the third term of Equation (8) takes a value of one. When build quantities are considered, such as production phases, there are two subcases: complexity through full production and complexity through FPU. In the full production case, this term is the ratio of the number of units to be built for the element compared to the reference basis. When only the complexity up to FPU is desired, this term is equal to the number of components required in a single system (t_i) divided by the total number of components that were built in the reference basis. This represents the component production quantities associated with producing a single unit of the system.

The product of the second and third terms of Equation (8) represents the total change in complexity when the complexity of the task being performed and the production quantities are combined. The weight term (i.e., first term of Equation (8)) is multiplied by this product to normalize the complexity scores to the reference basis. This is necessary since each element and phase combination does not contribute equally to the overall complexity score. For example, an element may be 10 times more difficult than the reference basis for a given phase, but its overall impact to the complexity of a warhead option will be limited if that element and phase only represented a small portion of the reference basis.

It is important to distinguish the difference between a complexity estimate and a complexity score. An estimate is an input to the model that is based on SMEs assessment of the difficulty of an element for a given phase compared to a reference system. A score is an output of the model that is normalized to the reference basis and can be used to compare the complexity of two different warhead options when they are based on the same reference basis.

3.1. Warhead Option Outputs

This subsection describes how to calculate outputs for warhead options. As mentioned above, these outputs can be calculated using one of two assumptions: a weighted mean (Subsection 3.1.1) or random variables that follow a triangular distribution (Subsection 3.1.2). Outputs that assume estimates are random variables are a more accurate representation of the results. However, outputs using a weighted mean are available for comparison to previous analyses. For a warhead option, it is assumed that the complexities of all element and phase combinations are independent.

3.1.1. Weighted Mean Outputs

Three types of outputs are produced in the weighted means case: minimum, maximum, and mean complexity scores. For outputs that use a weighted mean, no specific distribution is identified for the estimate. The only assumption that is made is that the mean of this distribution is given by Equation (9), which puts more emphasis on the most likely value than the minimum or maximum, similar to a PERT 3-point approximation. The bar over the *F* term is used to indicate that the result is a weighted mean.

$$\bar{F}^{Mean}{}_{i,p,D_{i,m}} = \frac{F^{Low}{}_{i,p,D_{i,m}} + 4 \cdot F^{ML}{}_{i,p,D_{i,m}} + F^{High}{}_{i,p,D_{i,m}}}{6} \tag{9}$$

Equation (10) can be used to find the mean complexity score of an element and phase for a warhead option.

$$\bar{S}^{Mean}{}_{i,p,D_{i,m},\epsilon} = \frac{S^{Low}{}_{i,p,D_{i,m},\epsilon} + 4 \cdot S^{ML}{}_{i,p,D_{i,m},\epsilon} + S^{High}{}_{i,p,D_{i,m},\epsilon}}{6} \tag{10}$$

The minimum, maximum, and mean complexity scores can be calculated for warhead options, categories, and category members. Let,

$O^{Low}{}_{j,k,m}$: Low complexity score output for category member *j*, for category *k*, for warhead option *m*.

$\bar{O}^{Mean}{}_{j,k,m}$: Mean (weighted) complexity score output for category member *j*, for category *k*, for warhead option *m*.

$O^{High}{}_{j,k,m}$: High complexity score output for category member *j*, for category *k*, for warhead element *m*.

$O^{Low}{}_{j,k,m,p}$: Low complexity score output for category member *j*, for category *k*, for warhead option *m*, in phase *p*.

$\bar{O}^{Mean}{}_{j,k,m,p}$: Mean (weighted) complexity score output for category member *j*, for category *k*, for warhead option *m*, in phase *p*.

$O^{High}{}_{j,k,m,p}$: High complexity score output for category member *j*, for category *k*, for warhead option *m*, in phase *p*.

SCORE calculates the low, high, and weighted mean complexity scores for each category member, of each category, for each warhead option. These results are calculated by phase and over all phases. To calculate each of these outputs, the model uses the category concept introduced in Subsection 2.2 of this document. These inputs are used to identify all of the elements and phases that are associated with each output. These outputs are created by summing the low, high, and weighted mean complexity scores for each applicable element and phase. The high and low complexity scores for each element and phase are computed using Equation (8). The complexity scores for the mean values are computed using Equation (10). The complexity scores for each element and phase can be calculated for both the complexity through FPU and the complexity through full production cases.

3.1.2. Triangular Distribution Outputs

For the second set of outputs it is assumed that estimates are random variables that follow a triangular distribution. In this case, the low, mostly likely, and high estimates provided as input by SMEs are used as the parameters for this distribution, and outputs can be provided that show any percentile of the complexity scores. These percentiles are obtained through Monte Carlo sampling. Four types of outputs are provided: minimum, maximum, mean, and percentiles. The minimum and maximum complexity scores are calculated analytically. The mean is also calculated analytically since an expression exists for the mean of a triangular distribution. In this case, the mean complexity score can be calculated using Equation (11).

$$S^{Mean}{}_{i,p,D_{i,m},\epsilon} = \frac{S^{Low}{}_{i,p,D_{i,m},\epsilon} + S^{ML}{}_{i,p,D_{i,m},\epsilon} + S^{High}{}_{i,p,D_{i,m},\epsilon}}{3} \tag{11}$$

Sampling is used to generate complexity score percentiles. Refer to Law and Kelton for a discussion of how to generate samples from a triangular distribution [2]. This reference also describes how to generate samples when a left or right triangular distribution is used (i.e., $F^{Low}{}_{i,p,l} = F^{ML}{}_{i,p,l}$ or $F^{High}{}_{i,p,l} = F^{ML}{}_{i,p,l}$). In the case where the low, mode, and high estimates are equal or no low and high values are provided, the most likely value should be used for all samples.

In order to perform sampling, the user must specify the number of samples that need to be obtained. Let,

N:	The number of samples to draw.

Additionally, the user must specify the number of percentiles they wish to capture and the associated percentile values. Let,

Y:	The number of percentiles.
$y = 1,2,\ldots,Y$:	An index on the percentiles.
$\rho_y \in (0,1)$:	The percentile value associated with percentile *y*.

Finally, four new types of outputs are required. Let,

$O^{Mean}{}_{j,k,m}$:	Mean complexity score output for category member *j*, for category *k*, for warhead option *m*.
$O^{Percentile}{}_{j,k,m,y}$:	y^{th} percentile complexity score output for category member *j*, for category *k*, for warhead option *m*.

$O^{Mean}{}_{j,k,m,p}$: Mean complexity score output for category member j, for category k, for warhead option m, in phase p.

$O^{Percentile}{}_{j,k,m,p,y}$: y^{th} percentile complexity score output for category member j, for category k, for warhead option m, in phase p.

The approach for calculating the minimum, maximum, and mean outputs is essentially the same as the weighted mean case. The only difference is that the weighted mean calculation is replaced by the mean calculation in Equation (11).

The model uses the following sampling approach to compute the complexity score percentiles. First, the model generates N sample complexity scores for each element, phase, and design choice. When the same design choice is used in multiple warhead options, common samples are used to reduce unnecessary variance. Next, the output values are calculated. This is achieved by calculating complexity scores for each set of element samples using the Category inputs data to specify which element's sample complexity scores should be combined to produce the aggregated score for each output. This results in a collection of N sample complexity scores for each of the outputs. Finally, the desired percentiles are estimated for each output using the sample values.

3.2. Excursion Outputs

This subsection describes how to calculate the outputs for excursions. Similarly to the previous sections (3.1.1 and 3.1.2), these outputs can be calculated assuming one of two metrics: a weighted mean approach (Subsection 3.2.1) or random variables that follow a triangular distribution (Subsection 3.2.2).

3.2.1. Weighted Mean Outputs

When weighted means are used, the excursion outputs are similar to warhead option outputs (Subsection 3.1.1). The minimum, maximum, and mean complexity scores are calculated for each category, and category member, however the outputs are now calculated by excursion. Let,

$O^{Low}{}_{j,k,e}$: Low complexity score output for category member j, for category k, for excursion e.

$\bar{O}^{Mean}{}_{j,k,e}$: Mean (weighted) complexity score output for category member j, for category k, for excursion e.

$O^{High}{}_{j,k,e}$: High complexity score output for category member j, for category k, for excursion e.

$O^{Low}{}_{j,k,p,e}$: Low complexity score output for category member j, for category k, for excursion e, in phase p.

$\bar{O}^{Mean}{}_{j,k,p,e}$: Mean (weighted) complexity score output for category member j, for category k, for excursion e, in phase p.

$O^{High}{}_{j,k,p,e}$: High complexity score output for category member j, for category k, for excursion e, in phase p.

SCORE calculates the low, high, and weighted mean complexity scores for each category member, of each category, for each excursion. These results are calculated by phase and totaled over all phases. To calculate the maximum and minimum outputs, the model first determines the maximum and minimum number of years that the two warhead options in the excursion could be separated by. When the number of years separating the warhead options is minimized, leveraging will be maximized. When the number of years separating the warhead options is maximized, leveraging will be minimized. For each element and phase of each warhead option in the excursion, the high and low complexity scores are computed using Equation (8). Once the complexity scores for each element and phase of each warhead option is calculated they are added together to determine the combined complexity score for each element and phase of the excursion. Using the number of years separating the warhead options and the leveraging function described in the Leveraging Options section (2.7), the leveraging multiplier can be calculated for an element. This is multiplied by the sum of the complexity scores for each element and phase to determine the leveraged complexity score. Once these final complexity scores are computed for each element and phase of the excursion, they can be added in various combinations to produce all of the desired outputs (e.g., total, by phase, by NWBS, etc.).

The calculation of the weighted mean requires the consideration of complexity uncertainty and schedule uncertainty. The key step is to compute the mean complexity score for each element and phase in the excursion, considering both types of uncertainty. Once these values are calculated, they can be added in different combinations to compute the desired outputs. The first step is calculating and summing the mean element and phase values in each warhead option using Equation (10). The next step is to determine the probability mass function for the number of years separating the two warhead options. For each number of years separating the warhead options, the leveraging multiplier is then calculated. The final step is to compute a weighted sum using the probability of the warhead options being separated by a number of years, the associated leveraging multiplier, and the baseline complexity score. Once the mean complexity score for each element and phase has been computed, they can be combined additively to produce the desired outputs.

3.2.2. Triangular Distribution Outputs

When triangular distributions are used, the leveraging option outputs are similar to the sampling-based warhead option outputs (Subsection 3.1.2). The minimum, maximum, mean, and percentile complexity scores are calculated for each category, and category member, however the outputs are now calculated by excursion. Let,

$O^{Mean}{}_{j,k,e}$: Mean complexity score output for category member j, for category k, for excursion e.

$O^{Percentile}{}_{j,k,e,y}$: y^{th} percentile complexity score output for category member j, for category k, for excursion e.

$O^{Mean}{}_{j,k,p,e}$: Mean complexity score output for category member j, for category k, for excursion e, in phase p.

$O^{Percentile}{}_{j,k,p,e,y}$: y^{th} percentile complexity score output for category member j, for category k, for excursion e, in phase p.

The approach for calculating the minimum, maximum, and mean outputs is essentially the same as the weighted mean case for excursions. The only difference is that the weighted mean calculation is replaced by the calculation in Equation (11).

The sampling approach used to compute complexity score percentiles for excursions differs from the approach used for warhead options because it also considers schedule uncertainty and leveraging. The first step of this process is to produce three sets of sample complexity scores using samples from a triangular distribution. The first two sets are independent sample complexity scores for each element, phase, and design choice. A set is created for each of the two warhead options in the excursion. It is assumed that when there is no leveraging between the elements and phases common to both warhead options in the excursion, the sample complexity scores in each warhead option are independent. The third set of samples is the combined complexity scores for the elements and phases in each warhead option that are part of the excursion. It is assumed that the complexity scores for the elements and phases common to the two warheads in the excursion have perfect positive correlation when leveraging exists between them. The second step is to generate sample start dates for each of the warheads in the excursion. The start dates for each warhead option are sampled from a uniform discrete distribution. The third step is to generate leveraging multipliers. The number of years between the sampled start dates determines the leveraging multipliers for each element and phase, as described in the Leveraging Options section (2.7). The final step is to calculate the complexity score outputs. This is done by adding the appropriate collection of element and phase complexity scores multiplied by the leveraging multiplier. When no leveraging exists (i.e., the multiplier is one) the independent complexity score samples are used. When there is leveraging (i.e., the multiplier is less than one) the correlated complexity score samples are used. This process is repeated until N sets of sample outputs have been created. This process results in a collection of sample complexity scores for each output from which the percentiles of interest can be calculated.

4. CONCLUSIONS

SCORE allows for comparisons of relative complexity to be made between proposed design variants of future warhead options. The bottom-up nature of this approach allows SMEs to divide the scope associated with a warhead option into manageable elements that can be compared to known reference systems. The use of three point estimates of complexity allows for uncertainty to be captured in both the SME provided inputs and model outputs. The SCORE model and analysis process benefits the Nuclear Security Enterprise (NSE) by generating quantitative complexity outputs that capture uncertainty and can be traced back to clearly stated assumptions and input values.

5. REFERENCES

1. J. L. Gearhart, J. N. Samberson, S. Shettigar, J. M. Jungels, K. M. Welch, and D. A. Jones, *Scope, Complexity, Options, Risks, Excursions (SCORE) Factor Mathematical Description*, SAND2017-2280. Sandia National Laboratories, Albuquerque, NM, January 2017.

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