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Scope, Complexity, Options, Risks, Excursions (SCORE) Factor Mathematical Description

Jared L. Gearhart, Jonell N. Samberson, Subhasini Shettigar, John M. Jungels, Kimberly M. Welch, Dean A. Jones

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

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Jared L. Gearhart, Dean A. Jones
Operations Research and Computational Analysis
Sandia National Laboratories
P.O. Box 5800, M.S. 1188
Albuquerque, New Mexico 87185-1188

Jonell N. Samberson, Subhasini Shettigar, Kimberly M. Welch
Nuclear Security Enterprise and Cost Analysis
Sandia National Laboratories
P.O. Box 5800, M.S. 0431
Albuquerque, New Mexico 87185-0431

John M. Jungels
Interactive Systems Simulation and Analysis
Sandia National Laboratories
P.O. Box 5800, M.S. 1004
Albuquerque, New Mexico 87185-1004

Abstract

The purpose of the Scope, Complexity, Options, Risks, Excursions (SCORE) model is to estimate the relative complexity of design variants of future warhead options, resulting in scores. SCORE factors extend this capability by providing estimates of complexity relative to a base system (i.e., all design options are normalized to one weapon system). First, a clearly defined set of scope elements for a warhead option is established. The complexity of each scope element is estimated by Subject Matter Experts (SMEs), including a level of uncertainty, relative to a specific reference system. When determining factors, complexity estimates for a scope element can be directly tied to the base system or chained together *via* comparable scope elements in a string of reference systems that ends with the base system. The SCORE analysis process is a growing multi-organizational Nuclear Security Enterprise (NSE) effort, under the management of the NA-12 led Enterprise Modeling and Analysis Consortium (EMAC). Historically, it has provided the data elicitation, integration, and computation needed to support the out-year Life Extension Program (LEP) cost estimates included in the Stockpile Stewardship Management Plan (SSMP).

CONTENTS

FIGURES

TABLES

NOMENCLATURE

BCR	Baseline Cost Review
EMAC	Enterprise Modeling and Analysis Consortium
LEP	Life Extension Program
NNSA	National Nuclear Security Administration
NSE	Nuclear Security Enterprise
NWBS	National Work Breakdown Structure
PD	Process Development
SCORE	Scope, Complexity, Options, Risks, Excursions
SE&I	Systems Engineering and Integration
SME	Subject Matter Expert
SSMP	Stockpile Stewardship and Management Plan
T&Q	Test and Qualification

1. INTRODUCTION

The Scope, Complexity, Options, Risks, Excursions (SCORE) model has two main purposes:

(1) The SCORE model can generate relative complexity scores (i.e., "scores") that represent the relative complexity of design variants of future warhead options [1]. These scores allow stakeholders to understand complexity tradeoffs between the proposed warhead options.

(2) The SCORE model can also be used to create Enterprise Modeling and Analysis Consortium (EMAC) SCORE factors. EMAC SCORE factors, or simply "factors", are similar to complexity scores, except that they represent the complexity of a National Work Breakdown Structure (NWBS) code relative to one weapon system (or "system") of interest, referred to as the base system.

Factors have primarily been used as inputs to the Cost Estimation Model owned and operated by the National Nuclear Security Administration's (NNSA's) Office of Cost Policy and Analysis (NA-143), which is used to support early planning estimates for the Stockpile Stewardship and Management Plan (SSMP). The factors provided to NA-143 represent the relative complexity of the weapon systems under consideration relative to a base system chosen by NA-143 for each NWBS element. However, factors have also been used to support site studies not related to the SSMP.

This document describes how the SCORE model calculates EMAC SCORE factors. Section 2 provides an overview of the factor calculation using notional examples. Section 3 describes the mathematical details of the factor calculation.

1.1. Key Terms

The following key terms will be defined as follows before further detail of the process and the mathematical model are explored. The terms below apply to both score and factor calculations:

Scope Element (element) – The lowest level at which scope of work is defined. Each element is associated with a NWBS code, a type (e.g., nuclear), and a category for grouping purposes (e.g., radiation case). Multiple elements can belong to the same NWBS code.

Phase – The development and production life of a weapon system is divided into phases. Phases that are associated with production are not included in the factor calculations at this time per the preference of NA-143.

Reference System – A weapon system that can be used as the basis of comparison for a subject matter expert (SME) for a specific element. Reference systems must have an understood cost and associated scope of work.

Reference System Cost – The costs associated with an element, broken out by phase, for a specific reference system. This data is used for creating relative weights when combining elements that have the same NWBS code. An element reference cost can come from a weapon system cost or SME cost estimate.

Design Choice – A specific design implementation for an element. Multiple design choices can be defined for each element.

Complexity Estimate (estimate) – As compared to a "best available" reference system, relative estimates of complexity for a design choice are given by SMEs and include low, most likely, and high estimates. Values are given for each combination of element, design choice, and phase combination.

Warhead Option – A warhead design variant under consideration. A warhead option is defined by the design choices that are selected for each element, as well as the production quantities associated with each element.

The following terms are specific to calculating factors:

Base System – The specific reference system against which the complexity of a proposed warhead option is measured against for EMAC SCORE factors.

Reference Conversion Factor – Value that allows an estimate for an element to be transformed into an estimate that is relative to a base system or SME estimate.

EMAC SCORE Factor (factor) – Measure of relative complexity as compared to the base system for an NWBS code. This is the result of the factor calculation.

2. OVERVIEW OF EMAC SCORE FACTOR CALCULATION METHODOLOGY

This sections provides an overview of the EMAC SCORE factor (or "factor") calculation methodology and example factor calculations for six key cases. All factor calculations require several preliminary steps to condition the input data. The factor calculation leverages the same inputs that are used to calculate complexity scores (i.e., "scores"). When calculating scores, a single set of input data is used to estimate the complexity of a proposed system or several variants of a proposed system. However, the factor calculation requires that several sets of SCORE input data be linked together to establish a chain from the system under consideration to the base system.

The first step of the factor calculation methodology is to identify the systems that are relevant to the analysis being performed. At a minimum, a factor analysis requires the system under consideration and a base system. In some cases, it may not be practical to directly compare the system being analyzed to the base system. In these cases, intermediate reference systems can be added. Any number of intermediate reference systems can be used, as long as a reference chain between the system under consideration and the base system can be established. The examples shown in this section will consider notional data for a base system (W76-1), an intermediate reference system (B61-12), and the system under consideration (IW-1). Some portions of the IW-1 will reference the B61-12 while others will reference the W76-1 directly. Figure 1 shows the relationship for these three systems in this notional example.

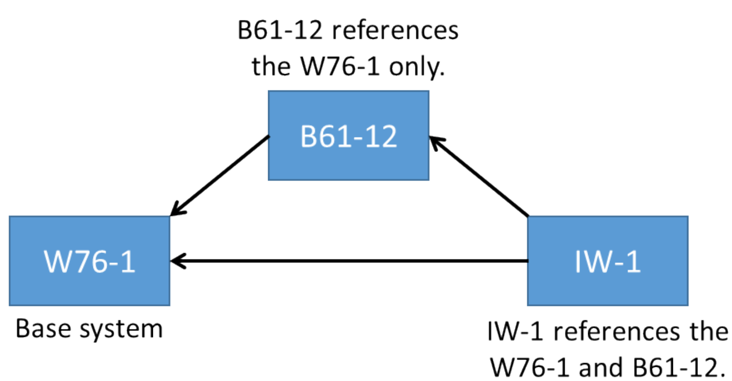

Figure 1. Notional example demonstrating the hierarchical nature of reference system chains for EMAC SCORE factor calculations.

The next steps are to identify all of the scope elements (i.e., "elements") that are relevant to a given analysis, assign each element to an NWBS code, and connect elements across the systems that are part of a factor analysis to the base system. The basic approach for calculating factors is to divide the complexity of the system under consideration by the complexity of the base system. If all elements that are a part of the base system are not included in the calculation, the complexity of the base system (i.e., the value in the denominator) will be incorrect, which will make the final calculation incorrect. Again, all elements from the base system must be included in a factor analysis whether or not they are used by the system under consideration. In other words, we are comparing a proposed warhead option to the "full scope" of the base system. If necessary, additional elements that are not part of the base system can be added to the system under consideration by referring to them as "new scope".

Table 1 shows eight notional elements that will be used to demonstrate factor calculations. These elements are assigned to one of five hypothetical NWBS codes. The first two columns of Table 1 show a NWBS code and the name of each element assigned to it. The third column indicates that seven of the eight elements are a part of the base system (i.e., the W76-1). Component B is a new scope element that is part of the B61-12 and IW-1 but not the W76-1. The last two columns further define the reference chains associated with each element.

In this example, there are several types of reference chains. The simplest type of reference chain occurs when the system under consideration directly references the base system. This is the case for the Systems Engineering, Widget A, and Widget 2 elements. Widget B and Component A represent the case where the system under consideration references an intermediate system, the B61-12 in this example, which then references the base system. Widget 1 represents the case where an element that is part of the base system but not part of the system under consideration. Component B represents the case where an element that is part of the system under consideration is not part of the base system. Finally, Component 1 represents the case where a reference exists for an element under consideration, but the SME providing a complexity estimate (or "estimate") chooses to define their own reference against which to provide estimates.

Table 1. Summary of the scope elements used for SCORE factor calculation example.

NWBS Code	Element Name	Included in W76-1?	B61-12 Reference System	IW-1 Reference System
1.X.1	Systems Engineering	Yes	W76-1	W76-1
1.X.2	Widget A	Yes	W76-1	W76-1
1.X.2	Widget B	Yes	W76-1	B61-12
1.X.3	Widget 1	Yes	W76-1	N/A
1.X.3	Widget 2	Yes	W76-1	W76-1
1.X.4.1	Component A	Yes	W76-1	B61-12
1.X.4.1	Component B	No	New Scope	B61-12
1.X.4.2	Component 1	Yes	W76-1	New Scope

These preliminary steps ensure that the input data is formatted in a manner that supports the factor calculation, which can be performed separately for each NWBS code. The following five subsections (2.1 through 2.5) show how factors are calculated for each of the five NWBS codes listed in the table above. Each of these groups of elements has different characteristics that highlight different aspects of the calculation. Finally, a sixth subsection (2.6) is included that shows how data from NWBS codes 1.X.4.1 and 1.X.4.2 can be aggregated to produce a factor for NWBS code 1.X.4. Throughout these example cases, three life cycle phases are considered: System Engineering and Integration (SE&I), Test and Qualification (T&Q), and Production Development (PD). When performing the SCORE process, SMEs provide low, most likely, and high estimates where an estimate of 100 indicates the complexity of that phase for an element as referenced to the system of choice is exactly equal. Similarly, an estimate of 200 would indicate the scope complexity is twice that of the reference system. For clarity, single point estimates of complexity, or most likely estimates, will be used in these examples. Section 3 describes how the three point estimates captured during the SCORE process can be used to approximate factor uncertainty. To be clear, all data shown in this report is notional.

2.1. Factor Case #1: Direct Reference to Base System (NWBS 1.X.1)

Factor Case #1 shows how factors are calculated when elements directly reference the base system. In this case, the NWBS code of interest is 1.X.1. Only a single element, Systems Engineering, belongs to this NWBS code, and it directly references the W76-1 base system. Table 2 summarizes the relevant input data. For each phase, the reference cost from the W76-1 and the SME estimates for the IW-1 are provided.

Table 2. Summary of input data for Factor Case #1.

Element Name	Phase	W76-1 Reference Cost ($ Millions)	IW-1 Estimates
Systems Engineering	SE&I	20	120
	T&Q	3	100
	PD	7	90
NWBS 1.X.1 Total		**30**	

For the Systems Engineering element, the above inputs indicate that the complexity of the SE&I phase for the IW-1 is estimated to be 20 percent greater than the complexity of the SE&I phase for the W76-1 (i.e., 1.2 or 120/100). Similarly, the complexity is estimated to be equal for the T&Q phase (i.e., 1.0) but 10 percent less for the PD phase (i.e., 0.9). The only remaining step is to combine the estimates for each of these phases into a single value. This is done using a weighted summation where the weights are based on the reference cost data from the base system. In this example, SE&I has a weight of 0.67 since it accounts for $20 million of the $30 million spent on 1.X.1 in the base system (i.e., 67 percent). Given this, the factor for NWBS code 1.X.1 can be calculated using the following equation.

$$\{1.X.1\ Factor\} = 1.2 * \frac{20}{30} + 1.0 * \frac{3}{30} + 0.9 * \frac{7}{30} = 1.1$$

The equation above states that the factor is a weighted summation of the estimates for each phase. In this example, the complexity of the IW-1 is 10 percent greater that the complexity of the W76-1 for NWBS code 1.X.1.

It is important to note the role that the reference cost data plays in factor calculations. For all factor calculations, cost data is used to established weights for a weighted summation of elements and phase estimates. Since element costs are always divided by the total reference costs for the NWBS code under consideration, the calculation does not rely on absolute costs. Given this, the factor calculation is less sensitive to uncertainty in absolute element costs as long as the relative costs between elements are stable.

2.2. Factor Case #2: Chained System References (NWBS 1.X.2)

Factor Case #2 shows how factors are calculated when elements reference an intermediate reference system as opposed to the base system. It also shows how factors are calculated when multiple elements belong to the same NWBS code. In this case, the NWBS code of interest is 1.X.2. Widget A and B are the two elements that are assigned to this NWBS code. Like the Systems Engineering element considered in Subsection 2.1, Widget A references the W76-1 directly. Widget B, however, uses the B61-12 as the reference system. Since the W76-1 is the base system in this example, a reference chain needs to be established to connect Widget B in the IW-1 to Widget B in the W76-1. Table 3 summarizes the input data that is relevant for this example. The first four column headings of this table are the same as Table 2. The fifth column contains the estimates of the B61-12 relative to the W76-1 for Widget B. Since Widget A references the W76-1 directly, the estimates in column five for this element are not needed for the calculation.

Table 3. Summary of input data for Factor Case #2.

Element Name	Phase	W76-1 Reference Cost ($ Millions)	IW-1 Estimates	B61-12 Estimates
Widget A	SE&I	30	120	-
	T&Q	10	140	-
	PD	8	50	-
Widget B	SE&I	25	150	180
	T&Q	15	140	200
	PD	12	50	100
NWBS 1.X.2 Total		**100**		

The factor for this NWBS code can be calculated using the equation below. The first line of the equation corresponds to Widget A and uses the approach described in Subsection 2.1. The second line of the equation corresponds to Widget B. In this part of the calculation, estimates must be chained together in order to estimate the complexity relative to the W76-1. Consider the SE&I phase. The data implies that the IW-1 complexity is 1.5 times greater that the B61-12, which is in turn 1.8 times greater than the W76-1. By multiplying these two values, the implication is that the complexity of the IW-1 for SE&I is 2.7 times greater than the W76-1. We refer to the value that allows an estimate for an element to be transformed into an estimate that is relative to a base system, or SME estimate, as a conversion factor. In other words, 1.8 is the conversion factor for SE&I to make the connection from the intermediate system (B61-12) to the base system (W76-1).

$$\{1.X.2\ Factor\} = 1.2 * \frac{30}{100} + 1.4 * \frac{10}{100} + 0.5 * \frac{8}{100}$$

$$+ 1.5 * 1.8 * \frac{25}{100} + 1.4 * 2.0 * \frac{15}{100} + 0.5 * 1.0 * \frac{12}{100} = 1.7$$

In this case, the factor is a weighted summation of the estimates for each phase adjusted by the appropriate conversion factor. The complexity of the IW-1 is 70 percent greater that the complexity of the W76-1 for NWBS code 1.X.2. If there were more than one intermediate reference system, the estimates divided by 100 for the intermediate systems can be multiplied together to determine the conversion factor.

2.3. Factor Case #3: Excluding Reference System Elements (NWBS 1.X.3)

Factor Case #3 shows how factors are calculated when an element that is part of the base system is not included in the system being analyzed. In this case, the NWBS code of interest is 1.X.3. The two relevant elements are Widget 1 and 2. Both of these elements are used in the W76-1, but only Widget 2 is used in the IW-1. Table 4 summarizes the input data that is relevant for this example. In this case, there are no estimates for Widget 1 since it is not used in IW-1 (i.e., estimates are "not applicable" or "N/A").

Table 4. Summary of input data for Factor Case #3.

Element Name	Phase	W76-1 Reference Cost ($ Millions)	IW-1 Estimates
Widget 1	SE&I	10	N/A
	T&Q	10	N/A
	PD	2	N/A
Widget 2	SE&I	30	130
	T&Q	10	150
	PD	8	110
NWBS 1.X.3 Total		**70**	

The equation below shows how the factor is calculated in this case. Since there is no intermediate reference system, the calculation is similar to Case #1. However, the element Widget 1, which is not a part of the IW-1, must be included in the calculation because the factor represents the complexity relative to the all elements that were a part of the W76-1. Note that although all of the estimates for Widget 2 are greater than 100, the value of the factor is less than 1. This is due to the fact that the scope associated with Widget 1, which represented about one-third of the W76-1, is not part of IW-1.

$$\{1.X.3\ Factor\} = 0.0 * \frac{10}{70} + 0.0 * \frac{10}{70} + 0.0 * \frac{2}{70}$$

$$+ 1.3 * \frac{30}{70} + 1.5 * \frac{10}{70} + 1.1 * \frac{8}{70} = 0.9$$

2.4. Factor Case #4: Adding New Scope Elements (NWBS 1.X.4.1)

Factor Case #4 shows how factors are calculated when elements that were not part of the base system are added to the system under consideration. This feature is necessary since there is no guarantee that the base system will contain all of the necessary elements. The NWBS code of interest is 1.X.4.1. The two relevant elements are Components A and B. Each of these IW-1 elements use the B61-12 as the reference system. However, Component B represents new scope or work that was not part of the W76-1 and therefore does not have base system reference cost data. In this case, cost data from the reference system, the B61-12, will be used in the factor calculation. Table 5 summarizes the input data that is relevant for this example.

Table 5. Summary of input data for Factor Case #4.

Element Name	Phase	W76-1 Reference Cost ($ Millions)	B61-12 Cost Estimate ($ Millions)	IW-1 Estimates	B61-12 Estimates
Component A	SE&I	20	-	140	120
	T&Q	25	-	100	120
	PD	5	-	90	130
Component B	SE&I	-	15	100	100
	T&Q	-	5	100	100
	PD	-	5	100	100
NWBS 1.X.4.1 Total		**50**			

The following equation shows how factors are calculated when new scope is added. The first line of this equation accounts for the complexity of Component A. The W76-1 reference cost data is used to establish the weights and conversion factors that are used to convert IW-1 estimates that are relative to the B61-12 to estimates that are relative to the W76-1. The second line shows how new scope is accounted for. For each phase, the B61-12 reference cost is divided by the W76-1 total cost for the NWBS code being considered to determine the weight. The sum of the three terms in the second line of the equation is 0.5. This represents the complexity that is added to the overall factor by adding Component B. In this case, the cost of Component B was $25 million. If this scope is combined with Component A from the W76-1, which had a reference cost of $50 million that means there is a 50 percent increase overall.

$$\{1.X.4.1\ Factor\} = 1.4 * 1.2 * \frac{20}{50} + 1.0 * 1.2 * \frac{25}{50} + 0.9 * 1.3 * \frac{5}{50}$$

$$+ 1.0 * 1.0 * \frac{15}{50} + 1.0 * 1.0 * \frac{5}{50} + 1.0 * 1.0 * \frac{5}{50} = 1.9$$

More specifically, the key idea when adding new scope elements is that the calculation must reflect the scope of work that is additional to what was done in the base system. All of the terms in the equation above use the total cost of the base system for NWBS code 1.X.4.1 (i.e., $50 million) in the denominator. Again, the $25 million reference cost that is associated with Component B was not part of the base system and therefore cannot be included in the denominator. The cost in the numerator of each term corresponds to the reference cost at the end of each reference chain. Since Component A can be traced back to the W76-1, the W76-1 reference costs are used to determine the associated weights. Since Component B was a new element in the B61-12, the B61-12 reference cost data is used to determine the associated weights.

When adding new scope elements, it is important that the correct reference cost data is used. There are typically two sources for this data. The first source is actual cost data or high-quality cost estimates from the system being referenced. For example, data from a baseline cost review (BCR) of a particular weapon system. The second source of reference cost data is directly from a SME. That is, the SME provides scope of the proposed work and creates a cost estimate for the new scope of work. Regardless of the source of this cost data, it is important to ensure all cost data is normalized to same-year dollars.

2.5. Factor Case #5: Replacing Existing Elements (NWBS 1.X.4.2)

Factor Case #5 shows how factors are calculated when an element that was part of the base system is replaced by a new scope element in the system under consideration. This occurs in cases where the base system and the system under consideration contain the same element, but the underlying work that was associated with that element in the base system cannot or should not be used for generating estimates. For example, the base system scope might not be fully understood by the SMEs generating estimates, or the scope of the reference system is drastically different from the element in the system under consideration. Let us consider a case where a new design will be used for a component in a proposed system, but that same component was directly reused in the base system. Given this, the proposed work could be significantly larger than the work in the base system (e.g., 25 times more difficult), making it difficult for SMEs to accurately quantify the relative increase in work. Attempting to quantify a large scope difference is risky and prone to estimation errors. In this case, the SMEs can choose to define the work scope and cost estimate for the element for the proposed system. This new scope cost estimate is used in the factor calculation and the description of the work is used by the SMEs to generate estimates.

The approach for calculating factors with this methodology is shown for NWBS code 1.X.4.2, which contains a single element (Component 1). Table 6 summarizes the input data that is relevant for this example. In this case, the reference cost for Component 1 is $7 million. Let us assume Component 1 was directly reused for the base system. Next, assume the IW-1 team will be redesigning Component 1 and has chosen to provide a cost estimation for this new scope of work.

Table 6. Summary of input data for Factor Case #5.

Element Name	Phase	W76-1 Reference Cost ($ Millions)	SME Cost Estimate ($ Millions)	IW-1 Estimates
Component 1	SE&I	3	50	100
	T&Q	2	40	100
	PD	2	35	100
NWBS 1.X.4.2 Total		**7**		

The following equation shows how factors are calculated in this case. In each term, the total reference cost for this element in the base system is used in the denominator since this is the system the proposed work is being compared to while the values in the numerator correspond to SME cost estimates. The increase in complexity is accounted for by relative differences in cost.

$$\{1.X.4.2\ Factor\} = 1.0 * \frac{50}{7} + 1.0 * \frac{40}{7} + 1.0 * \frac{35}{7} = 17.9$$

Here, the estimates for the IW-1 were all equal to 100. However, the calculation above can accommodate cases where these values are not equal to 100. For example, assume that for the IW-1 two different design choices are being considered for Component 1. In both case, it would make sense to the compare these options to the SME defined option, but the estimates for one design may be larger to account for the added scope.

2.6. Factor Case #6: Aggregating Factors (NWBS 1.X.4)

Factor Case #6 shows how factors are calculated at higher-level NWBS codes using data that was defined at a more detailed level of NWBS codes. In Factor Cases #4 and #5, factors were calculated for NWBS codes 1.X.4.1 and 1.X.4.2, respectively. In this example, the NWBS code of interest is 1.X.4. Table 7 is a combination of Table 5 and Table 6 and summarizes the input data that is relevant for this example.

Table 7. Summary of input data for Factor Case #6.

Element Name	Phase	W76-1 Reference Cost ($ Millions)	Reference Cost Estimate ($ Millions)	IW-1 Estimates	B61-12 Estimates
Component A	SE&I	20	-	140	120
	T&Q	25	-	100	120
	PD	5	-	90	130
Component B	SE&I	-	15	100	100
	T&Q	-	5	100	100
	PD	-	5	100	100
Component 1	SE&I	3	50	100	-
	T&Q	2	40	100	-
	PD	2	35	100	-
NWBS 1.X.4 Total		**57**			

When calculating a factor for a higher-level NWBS code, the scope elements from each detailed NWBS code need to be consolidated. Once this done, the same calculations described in previous factor cases can be used. The major difference between this calculation and the ones for Factor Cases #4 and #5 is that the value in the denominator has changed to $57 million to represent the base cost for NWBS code 1.X.4.

$$\{1.X.4\ Factor\} = 1.4 * 1.2 * \frac{20}{57} + 1.0 * 1.2 * \frac{25}{57} + 0.9 * 1.3 * \frac{5}{57}$$

$$+ 1.0 * 1.0 * \frac{15}{57} + 1.0 * 1.0 * \frac{5}{57} + 1.0 * 1.0 * \frac{5}{57}$$

$$+ \quad 1.0 * \frac{50}{57} + \quad 1.0 * \frac{40}{57} + \quad 1.0 * \frac{35}{57} = 3.9$$

In this case, the factor is 3.9. Recall that the factors for NWBS codes 1.X.4.1 and 1.X.4.2 were 1.9 and 17.9, respectively. Despite the fact that the factor for NWBS code 1.X.4.2 was large, this calculation accounts for the fact that it only represented \$7 million out of the \$57 million in the base system.

3. FACTOR CALCULATION EQUATIONS

This section describes the equations that are used to calculate EMAC SCORE Factors. It assumes that the reader is familiar with the calculations used in the SCORE model, which are outlined in detail in the SCORE v3.0 Mathematical Description Document [1]. The following notation is taken from the SCORE document. Let,

$i = 1, 2, \ldots, I$:	An index on the scope elements, where I is the number of elements.
k_n:	The category index that represents NWBS codes.
$j = 1,2 \ldots, J(k_n)$:	An index on the NWBS codes in the NWBS.
$\alpha_{i,k_n} \in 1,2 \ldots, J(k_n)$:	Integer that identifies that scope element, i, belongs to NWBS code, j, of category, k_n.
$p = 1,2 \ldots, P$:	An index on the phases, where P is the number of phases.
$\beta_p \in \{0,1\}$:	Binary input that identifies that the complexity score for phase, p, will be adjusted for production quantities.
$r = 1,2 \ldots, R$:	An index on the reference systems, where R is the number of reference systems.
$C_{i,p,r} \in R_{\geq 0}$:	EMAC reference cost for a given scope element i, phase p, and reference system r.

In addition to the above inputs, several additional inputs are required to calculate factors. First, the base reference system and the reference system that will represent new scope elements must be defined. Let,

$r^B \in R$:	The reference system index which corresponds to the base system.
$r^S \in R$:	The reference system index which corresponds to the SME new scope reference system.

The next set of inputs relates to the system under consideration. Let,

$F^*_{i,p}$:	The complexity estimate for element i, in phase p, for the system under consideration.
$\bar{r}_i \in R$:	The reference system for element i of the system under consideration.

In the score calculation, multiple warhead options can be analyzed by generating various combinations of element design choices. The factor calculation described in this section assumes that factors for a specific warhead option are being generated. As outlined above, the inputs $F^*_{i,p}$ represent the estimate for the option under consideration. The wildcard in the superscript can be replaced with various measures of complexity (e.g., low, most likely, and high) depending on the metric being calculated. This will be described in more detail later in this section. The input $\bar{r}_i$ identifies the reference system for each element in the system under consideration.

Since the factor calculation can use a chain of reference systems, a similar set of inputs is required for reference systems. Let,

$F^*_{i,p,r}$:	The complexity estimate for element *i*, in phase *p*, for reference system *r*.
$\bar{r}_{i,r} \in R$:	The reference system for element *i* of the reference system *r*.

These two inputs are similar to the previous set of inputs (i.e., $F^*_{i,p}$ and $\bar{r}_i$) except that these inputs provide information on the elements that are referenced by other systems. As with the previous inputs, it is assumed that a specific warhead option of interest has been selected for each reference system.

In order to calculate factors, a reference cost and conversion factor must be determined for each element and phase that is part of the system under consideration. As described in Subsection 2.4, the reference cost typically comes from one of two sources. In the case where the reference chain for a system ends at the base system, the reference cost from the base system is used. When this does not occur, the reference chain will end at the SME new scope reference system. In this case, the reference cost data for the new scope element will be used. The recursive equation below can be used to determine the reference cost for each element and phase. Equation (1) states that if the reference system of the current element and phase is either the base system or the new scope system then the reference system cost has been found. If this is not the case, then the next reference system in the chain is checked. The result of evaluating $\bar{C}(i,p,\bar{r}_i)$ is the reference cost for the system under consideration.

$$\bar{C}(i,p,r) = \begin{cases} C_{i,p,r}, & r = r^B \vee r = r^S \\ \bar{C}(i,p,\bar{r}_{i,r}), & otherwise \end{cases} \tag{1}$$

Similarly, Equation (2) below can be used to determine the conversion factor for each element and phase by calling $\bar{G}^*(i,p,\bar{r}_i)$. This equation takes the product of all intermediate estimates until the base system or the new scope reference system are reached in the reference chain. In cases where the reference system is the base system or the new scope reference system, the conversion factor is equal to 1.

$$\bar{G}^*(i,p,r) = \begin{cases} 1, & r = r^B \vee r = r^S \\ F^*_{i,p,r} * \bar{G}^*(i,p,\bar{r}_{i,r}), & otherwise \end{cases} \tag{2}$$

Given this, factors can be calculated using Equation (3), where A^*_j is the factor value associated with NWBS code j. The first term in this equation is equal to one divided by the total cost of the base system for each NWBS code. Quantity-based (e.g., production) phases are not currently included in factor calculations. Term one corresponds to the values in the denominators of all of the cases shown in Section 2. Since the denominator is the same for all terms in the factor calculation, it can be taken out of the summation. The second term is a summation over all elements and non-production phases. For each element and phase in the summation, the complexity of the system being analyzed is multiplied by the associated conversion factor and reference cost.

$$A^*_j = \frac{1}{\sum_{\substack{i,p|\alpha_{i,k_n}=j \\ \beta_p=0}} C_{i,p,r^B}} * \sum_{\substack{i,p|\alpha_{i,k_n}=j \\ \beta_p=0}} \overline{C}(i,p,\bar{r}_i) * \overline{G}^*(i,p,\bar{r}_i) * F^*_{i,p} \quad \forall j \in J(k_n) \tag{3}$$

During the SCORE process, SMEs provide low, most likely (ML), and high values for estimates. While Equation (3) above describes the approach for calculating factors, it does not describe how to calculate specific factor results such as the minimum, mean, or maximum. To date, two approaches have been adopted to make use of this information. The current and preferred approach assumes that estimates are characterized by a triangular distribution, as described in Subsection 3.1. Prior to SCORE Version 3.0, no assumption was made about the probability distribution and a weighted mean approach was used, as described in Subsection 3.2. It is worth noting that the user can choose to have SCORE Version 3.0 calculate scores or factors based on the triangular distribution and/or the weighted mean approach. Before looking at each of these approaches, it is useful to write Equation (3) in a more compact form, as shown in Equation (4) below. Using this representation, only the three terms with wildcards in the superscripts are shown. Equation (4) states that the factor depends on the estimate value and conversion factor that are used.

$$A^*_j = f(\overline{G}^*(i,p,\bar{r}_i), F^*_{i,p}) \tag{4}$$

3.1. Triangular Distribution

This subsection describes how factors are calculated when it is assumed that the uncertainty in estimates follows a triangular distribution. In this case, four types of factor metrics are calculated for each NWBS code – the minimum, mean, maximum, and percentiles. The first three metrics are calculated using exact analytic calculation.

Equations (5) and (6) below show how to calculate the minimum factor. When calculating the minimum factor value, the low estimate value for the system under consideration is used and the conversion factor is based on the low estimate values from the reference chain.

$$A^{Min}_j = f(\overline{G}^{Low}(i,p,\bar{r}_i), F^{Low}_{i,p}) \tag{5}$$

$$\bar{G}^{Low}(i,p,r) = \begin{cases} 1, & r = r^B \vee r = r^S \\ F^{Low}{}_{i,p,r} * \bar{G}^{Low}(i,p,\bar{r}_{i,r}), & otherwise \end{cases} \tag{6}$$

A similar approach can be used to calculate the maximum and mean factor values, as shown by Equations (7) and (8), respectively.

$$A^{Max}_{j} = f(\bar{G}^{High}(i,p,\bar{r}_i), F^{High}{}_{i,p}) \tag{7}$$

$$A^{Mean}_{j} = f(\bar{G}^{Mean}(i,p,\bar{r}_i), F^{Mean}{}_{i,p}) \tag{8}$$

Equations (9) and (10) show how to calculate the mean estimate value and conversion factor where estimates are assumed to follow a triangular distribution.

$$F^{Mean}{}_{i,p} = \frac{F^{Low}{}_{i,p} + F^{ML}{}_{i,p} + F^{High}{}_{i,p}}{3} \tag{9}$$

$$\bar{G}^{Mean}(i,p,r) = \frac{\bar{G}^{Low}(i,p,r) + \bar{G}^{ML}(i,p,r) + \bar{G}^{High}(i,p,r)}{3} \tag{10}$$

Finally, Monte Carlo sampling can be used to approximate uncertainty and probability distributions associated with factors. Equation (11) below shows how sample factors are calculated. When creating sample factor values, both the uncertainty in the conversation factor and the estimate of the system under consideration are considered. The basic process is to generate sample estimates for each element and phase of the system under consideration and for each element and phase in the reference chain. Refer to Law and Kelton for a discussion of how to generate samples from a triangular distribution [2]. These samples are then used to create a sample factor value. By repeating this many times, a histogram can be generated that approximates the true probability density function of the factors for a given NWBS code for the system under consideration. Using this result, summary statistics such as percentiles can be calculated.

$$A^{Sample}_{j} = f(\bar{G}^{Sample}(i,p,\bar{r}_i), F^{Sample}{}_{i,p}) \tag{11}$$

Figure 2 illustrates the key benefit of using sampling. The red line shows the actual probability distribution of a notional factor, and the blue bars show the approximation of this distribution that is obtained by sampling. The minimum and maximum SCORE factors that are obtained using the analytic calculation are also shown. While these values are the theoretical minimum and maximum values, they have very low probabilities of occurring. Note that the distribution in Figure 2 has long tails with low probabilities of occurrence. In practical applications, it is not reasonable to plan for these extreme values given their low likelihood of occurrence. A better strategy is to plan to a desired confidence level. For example, it may be sufficient to plan to the 90 percent confidence level. In this case the, the SCORE factor that corresponds to the 5th and 95th percentiles would be used for the calculation in Equation (11) instead of the minimum and maximum factor values. The histogram returned by the sampling process allows for these results to be produced. For the results shown in Figure 2, the bounds at the 90 percent confidence level would be 1.2 and 1.8 compared to 0.6 and 2.3 (when the minimum and maximum are used). Using this approach, planning factors can be generated that account for uncertainty and risk tolerance.

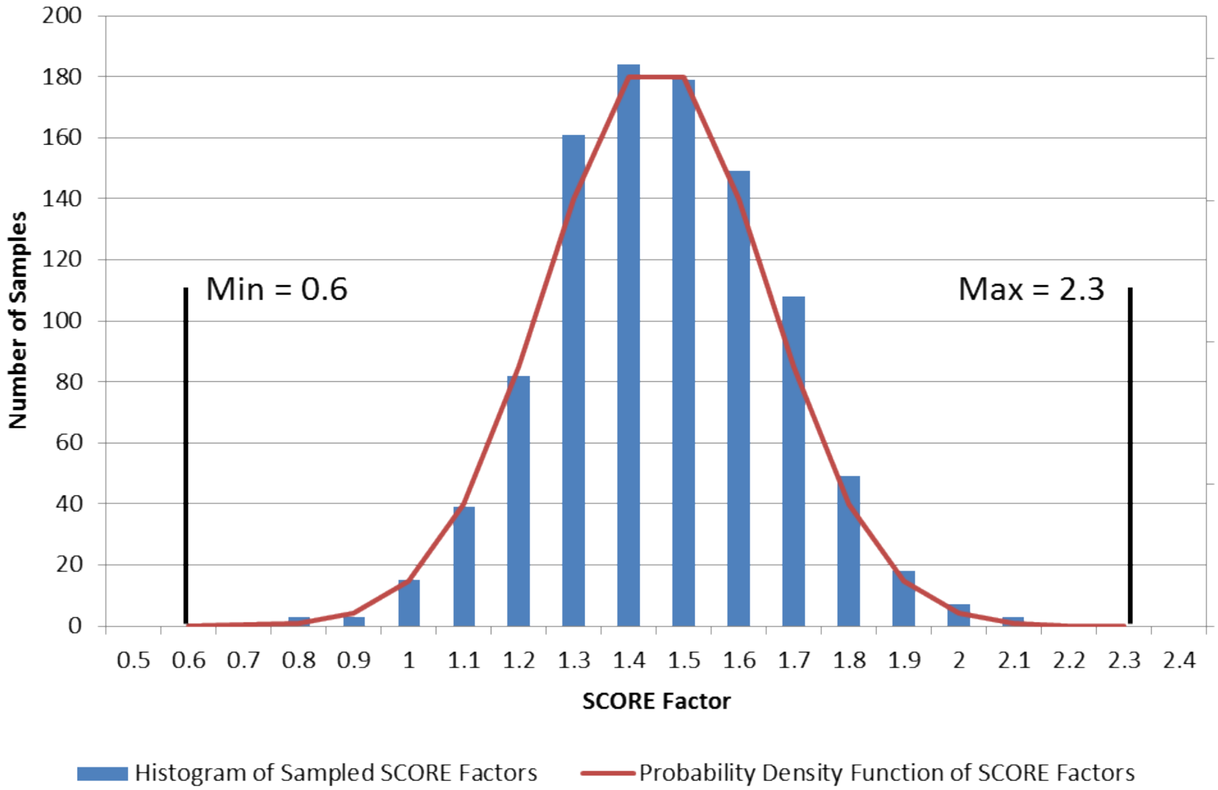

Figure 2. Notional example showing a histogram of SCORE factors that would be generated using sampling compared to probability density function.

3.2. Weighted Mean

An alternative approach for calculating factors relies on weighted means. This approach was used in previous versions of SCORE and can still be used if the user is so inclined. In this case, no underlying assumption is made about the shape of the distribution that estimates are drawn from. The only assumption that is made is that the mean of this estimate and conversion factor values are given by Equations (12) and (13) below, which puts more emphasis on the most likely (ML) value than the minimum or maximum, similar to a PERT 3-point approximation.

$$F^{Weighted}{}_{i,p} = \frac{F^{Low}{}_{i,p} + 4 \cdot F^{ML}{}_{i,p} + F^{High}{}_{i,p}}{6} \tag{12}$$

$$\overline{G}^{Weighted}(i,p,r) = \frac{\overline{G}^{Low}(i,p,r) + 4 \cdot \overline{G}^{ML}(i,p,r) + \overline{G}^{High}(i,p,r)}{6} \tag{13}$$

Since no distribution is assumed in this case, sampling cannot be used to estimate percentiles and only the minimum, maximum, and mean factor values are calculated. Equations (14), (15), and

(16) show how these values are calculated. In all three cases, the conversion factor is based on the weighted mean values from the reference systems. In the case of the minimum and maximum factor values, this reduces the impact of the low and high estimate values from the reference chain and narrows the gap between the minimum and maximum factors.

$$A_j^{Min} = f(\bar{G}^{Weighted}(i,p,\bar{r}_i),F^{Low}{}_{i,p}) \tag{14}$$

$$A_j^{Max} = f(\bar{G}^{Weighted}(i,p,\bar{r}_i),F^{High}{}_{i,p}) \tag{15}$$

$$A_j^{Mean} = f(\bar{G}^{Weighted}(i,p,\bar{r}_i),F^{Weighted}{}_{i,p}) \tag{16}$$

Again, the weighted mean approach for calculating factors is included since it is reflective of the previous approach used to generate factors before the sampling approach (Subsection 3.1) was utilized. Since it does not make full use of the probabilistic information available in the system under consideration or the reference systems, the weighted mean approach is not preferred for generating factors.

4. CONCLUSIONS

The factor calculation methodology extends the capability of SCORE, which is focused on comparing the relative complexity of proposed weapon system design variants of a single system to each other rather than many weapon system designs of the future to a base reference system. By establishing reference chains for each element and phase, full use can be made of the complexity estimate data provided by SMEs. The use of three point estimates of complexity allows for uncertainty to be captured in both the SME provided inputs and model outputs. The SCORE model and analysis process benefit the Nuclear Security Enterprise (NSE) by generating quantitative complexity outputs that capture uncertainty and can be traced back to clearly stated assumptions and input values.

5. REFERENCES

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