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Structural Dynamics Lunchtime Series - #1
The Power of the Modal Model

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U.S. DEPARTMENT OF ENERGY
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Introduction

- Re-Introduce the Modal Model using Real Mode Shapes in Time Domain
- Basic instrumentation insights from rigid body and elastic shapes
- Basic insights from FE models for response and force instrumentation
- The power of the rigid body mode shape in force reconstruction
- The power of the modal model for understanding the frequency domain
- The power of the modal force in multi-shaker simulation/control
- Boundary conditions unmasked - or - The power of the modal model for understanding test articles mounted on a fixture
- The power of the modal model to simulate our most typical random vibration nonlinearities

Re-introduce the modal model

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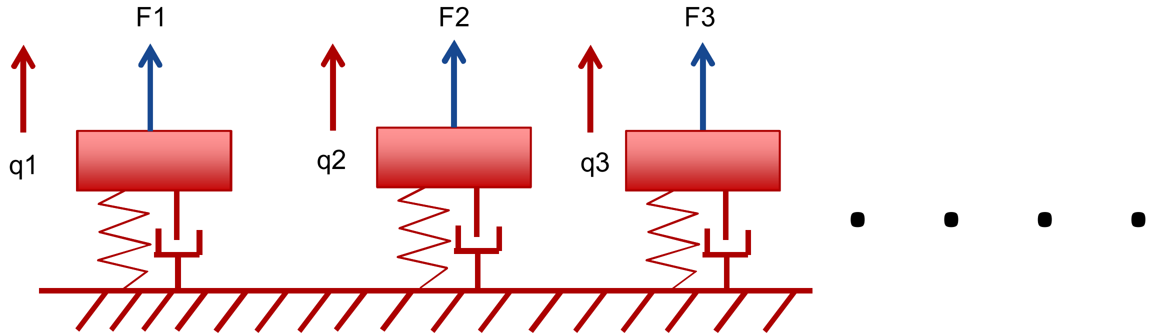


$$\mathbf{M}\ddot{\overline{\mathbf{x}}} + \mathbf{C}\dot{\overline{\mathbf{x}}} + \mathbf{K}\overline{\mathbf{x}} = \overline{\mathbf{f}}$$

$$\overline{\mathbf{x}} = \mathbf{\Phi}\overline{\mathbf{q}}$$

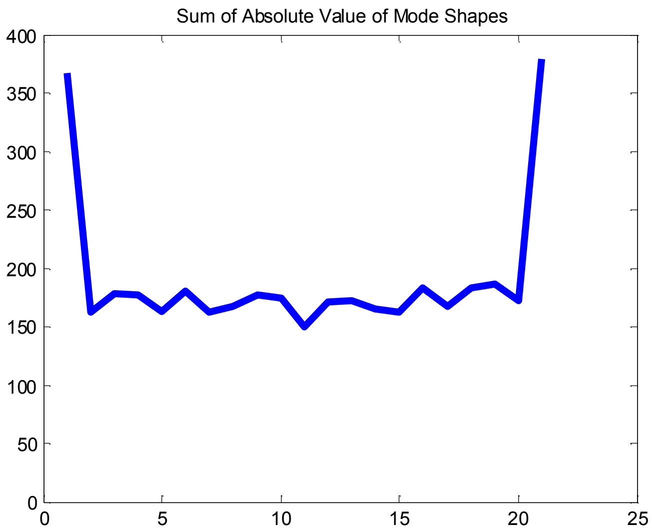
$$\mathbf{M}\mathbf{\Phi}\ddot{\overline{\mathbf{q}}} + \mathbf{C}\mathbf{\Phi}\dot{\overline{\mathbf{q}}} + \mathbf{K}\mathbf{\Phi}\overline{\mathbf{q}} = \overline{\mathbf{f}}$$

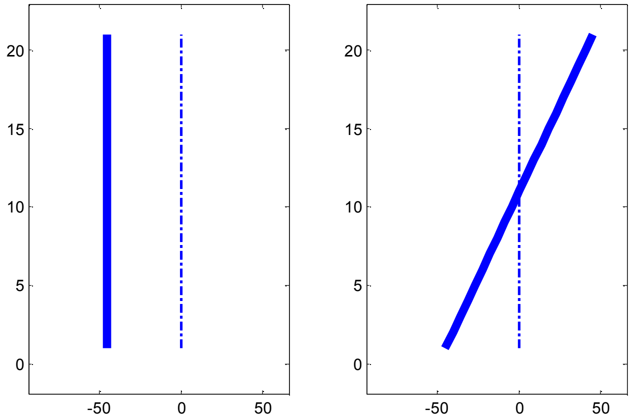
$$\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}\ddot{\overline{\mathbf{q}}} + \mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi}\dot{\overline{\mathbf{q}}} + \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}\overline{\mathbf{q}} = \mathbf{\Phi}^T\overline{\mathbf{f}}$$

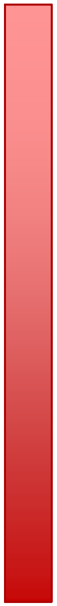
$$\mathbf{I}\ddot{\overline{\mathbf{q}}} + \left[{}^{\diagdown} 2\zeta_m\omega_m {}_{\diagdown}\right]\dot{\overline{\mathbf{q}}} + \left[{}^{\diagdown} \omega_m^2 {}_{\diagdown}\right]\overline{\mathbf{q}} = \overline{\mathbf{F}}$$

Rigid body modes – instrumentation insight

- Where would you instrument to observe the rigid body modes the best? (We will use bi-axial gages in plane of page)
- Where would you instrument to excite the rigid body modes the best?





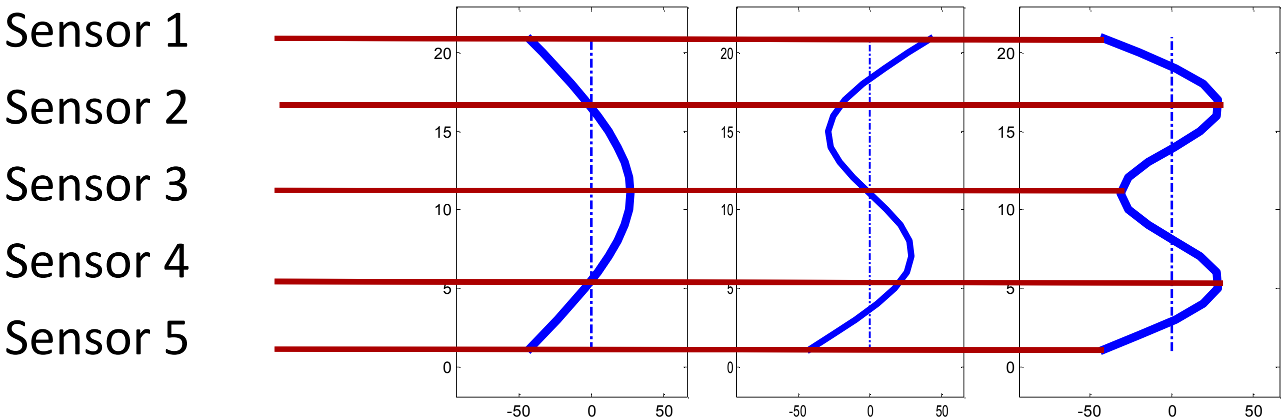


Elastic mode shapes – instrumentation insight

- Put instrumentation across every node
- Over instrument by a factor of 1.5 or better
- Across important joints
- Triaxes for visualization
  - If triaxes are not logistically possible, extend the mode shape to triaxes through:

$$\mathbf{\Phi}_{3d\_display} = \mathbf{\Phi}_{FE\_3d\_display}\mathbf{P}_{FE} = \mathbf{\Phi}_{FE\_3d\_display}\mathbf{\Phi}^{+}_{FE\_measured}\mathbf{\Phi}_{exp\_measured}$$

- Force inputs to excite every mode to at least 0.5 of max shape
- To separate closely space modes ⇨ force inputs across node line



Power of rigid body modes –force reconstruction

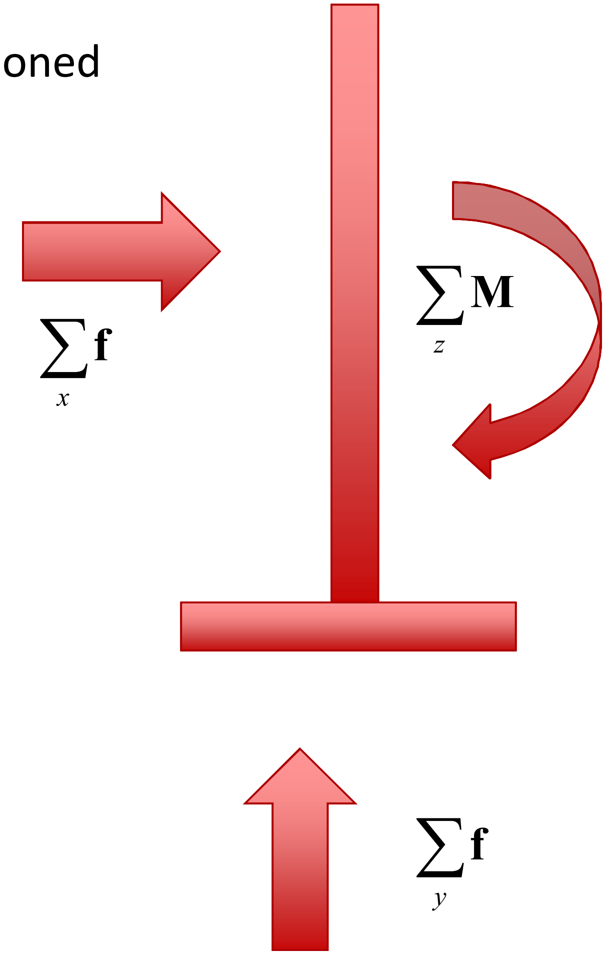
- Response measurements can give insight to force inputs
- Determining force inputs directly from FRFs is an ill-conditioned inverse problem
- One way to remove ill-conditioning uses rigid body modes

$$\mathbf{M}\boldsymbol{\Phi}\ddot{\overline{\mathbf{q}}} + \mathbf{C}\boldsymbol{\Phi}\dot{\overline{\mathbf{q}}} + \mathbf{K}\boldsymbol{\Phi}\overline{\mathbf{q}} = \overline{\mathbf{f}}$$

$$\boldsymbol{\Phi}_R^T\mathbf{M}\boldsymbol{\Phi}\ddot{\overline{\mathbf{q}}} + \boldsymbol{\Phi}_R^T\mathbf{C}\boldsymbol{\Phi}\dot{\overline{\mathbf{q}}} + \boldsymbol{\Phi}_R^T\mathbf{K}\boldsymbol{\Phi}\overline{\mathbf{q}} = \boldsymbol{\Phi}_R^T\overline{\mathbf{f}}$$

$$\begin{bmatrix} \backslash\, \mathbf{m}_r \,\backslash & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\overline{\mathbf{q}}}_R \\ \ddot{\overline{\mathbf{q}}}_E \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \end{bmatrix} \dot{\overline{\mathbf{q}}} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \end{bmatrix} \overline{\mathbf{q}} = \begin{Bmatrix} \sum_x \mathbf{f} \\ \sum_y \mathbf{f} \\ \sum_z \mathbf{M} \end{Bmatrix}$$

$$\boldsymbol{\Phi}_R^T\mathbf{M}\ddot{\overline{\mathbf{x}}}_{measured} = \begin{Bmatrix} \sum_x \mathbf{f} \\ \sum_y \mathbf{f} \\ \sum_z \mathbf{M} \end{Bmatrix}$$



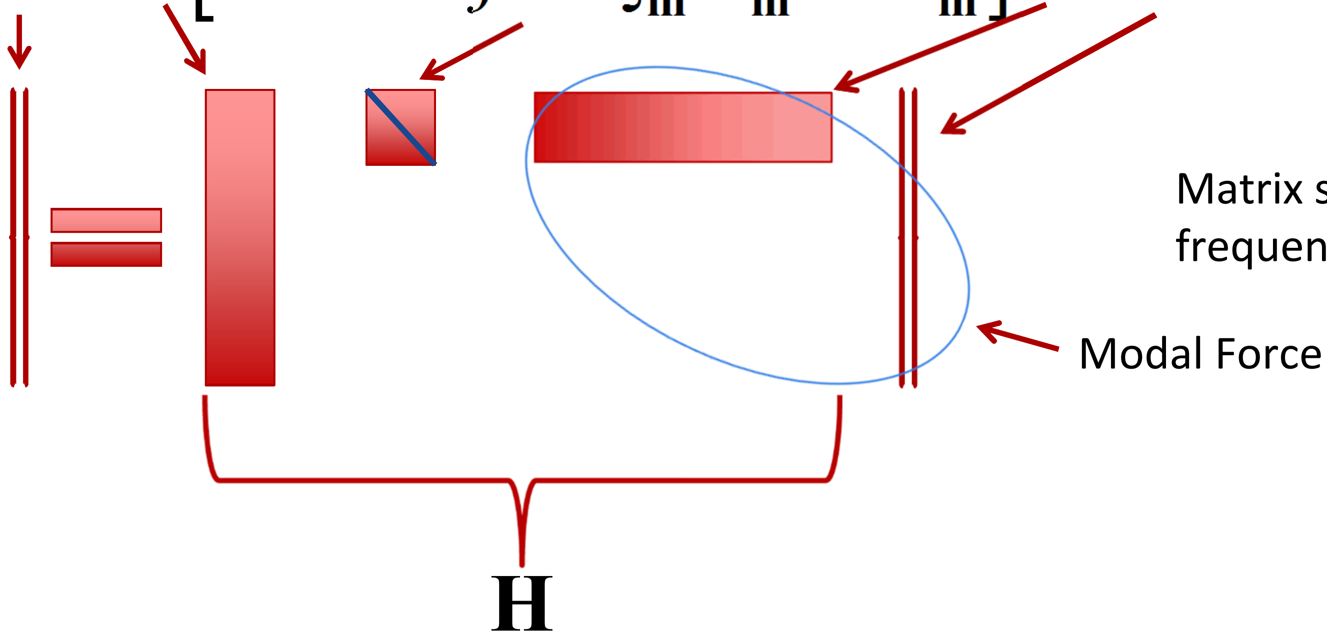
Modal Model in Frequency Domain

$$\mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi} \ddot{\bar{\mathbf{q}}} + \mathbf{\Phi}^T \mathbf{C} \mathbf{\Phi} \dot{\bar{\mathbf{q}}} + \mathbf{\Phi}^T \mathbf{K} \mathbf{\Phi} \bar{\mathbf{q}} = \mathbf{\Phi}^T \bar{\mathbf{f}}$$

$$\left[-\omega^2 \mathbf{I} + j\omega 2 \boldsymbol{\zeta}_\mathbf{m} \boldsymbol{\omega}_\mathbf{m} + \boldsymbol{\omega}_\mathbf{m}^2\right] \bar{\mathbf{q}} = \mathbf{\Phi}^T \bar{\mathbf{f}}$$

$$\bar{\mathbf{q}} = \left[-\omega^2 \mathbf{I} + j\omega 2 \boldsymbol{\zeta}_\mathbf{m} \boldsymbol{\omega}_\mathbf{m} + \boldsymbol{\omega}_\mathbf{m}^2\right]^{-1} \mathbf{\Phi}^T \bar{\mathbf{f}}$$

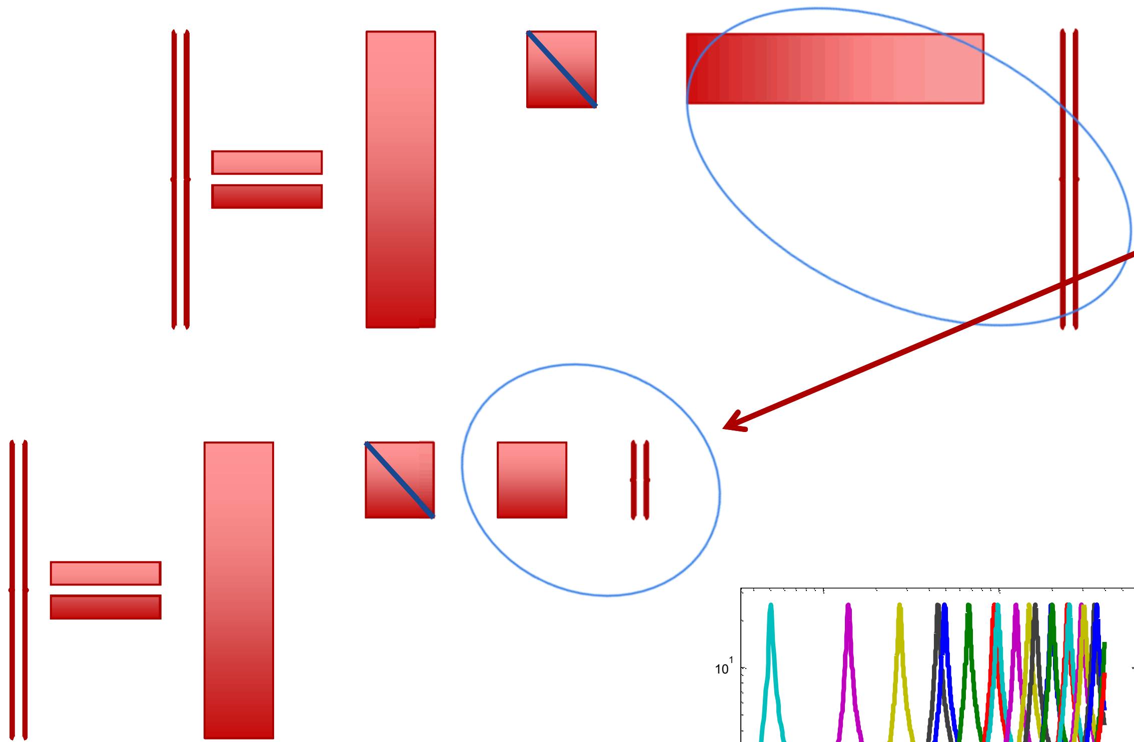
$$\bar{\mathbf{x}} = \mathbf{\Phi}\left[-\omega^2 \mathbf{I} + j\omega 2 \boldsymbol{\zeta}_\mathbf{m} \boldsymbol{\omega}_\mathbf{m} + \boldsymbol{\omega}_\mathbf{m}^2\right]^{-1} \mathbf{\Phi}^T \bar{\mathbf{f}} = \mathbf{H} \bar{\mathbf{f}}$$



Matrix shape at each frequency line

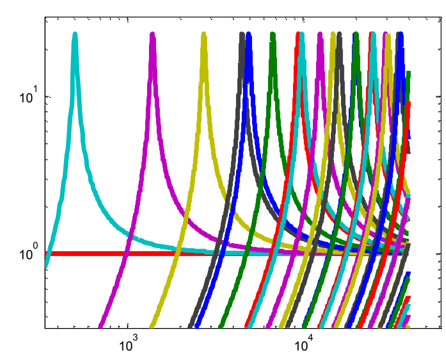
Power of modal model in multi-shaker simulation and control

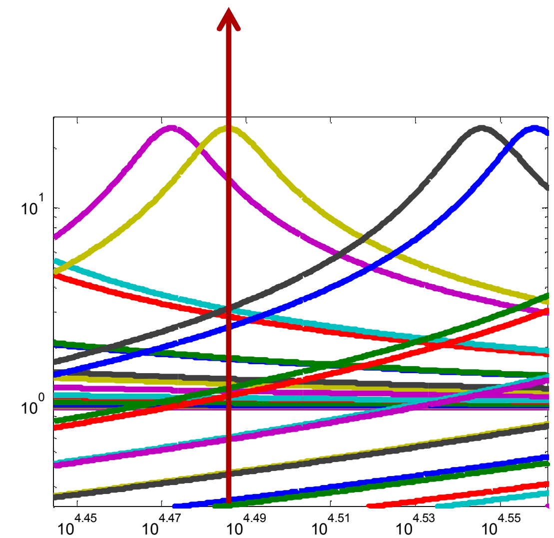
$$\bar{\mathbf{x}} = \mathbf{\Phi}\left[-\omega^2\mathbf{I} + j\omega 2\boldsymbol{\xi}_m\boldsymbol{\omega}_m + \boldsymbol{\omega}_m^2\right]^{-1}\mathbf{\Phi}^T\bar{\mathbf{f}} = \mathbf{H}\bar{\mathbf{f}}$$



Modal Force

30 kHz – 2.6/25

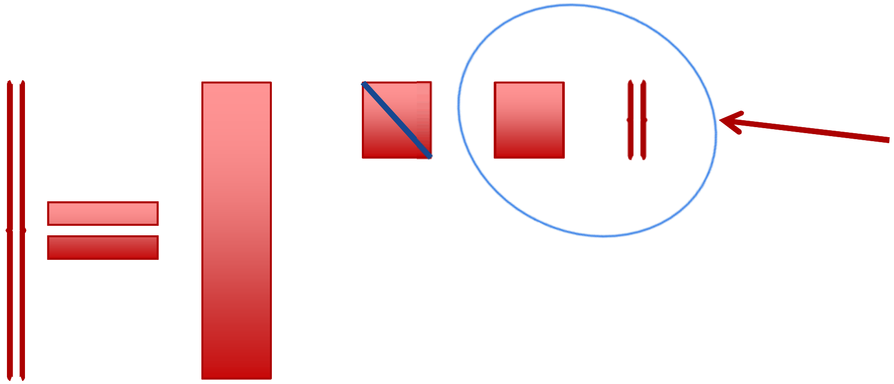




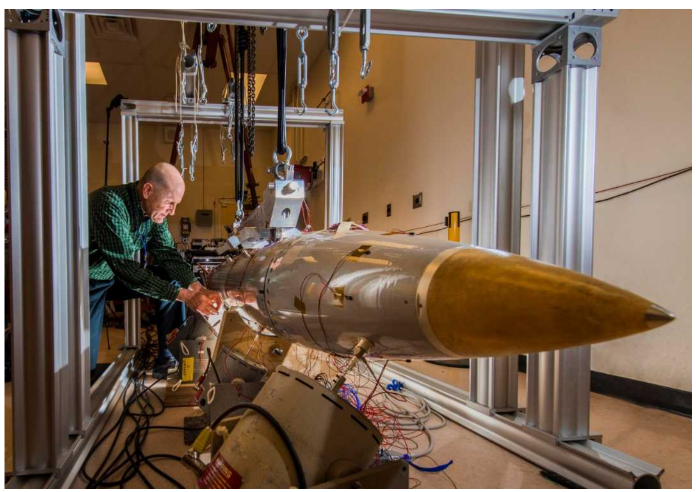
Consider just 6 modal responses at one frequency

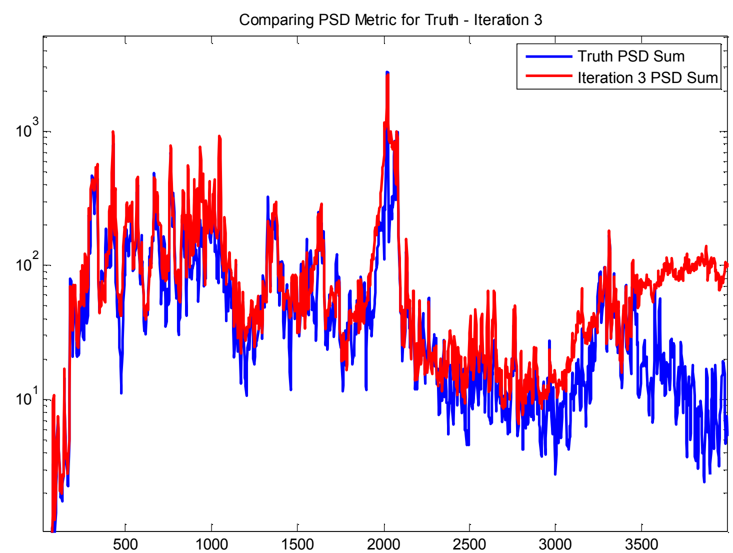
Power of modal model in multi-shaker simulation and control

$$\bar{\mathbf{x}} = \mathbf{\Phi}\left[-\omega^2\mathbf{I} + j\omega 2\boldsymbol{\xi}_m\boldsymbol{\omega}_m + \boldsymbol{\omega}_m^2\right]^{-1}\mathbf{\Phi}^T\bar{\mathbf{f}} = \mathbf{H}\bar{\mathbf{f}}$$



Modal Forces reduced down to 6 for system with hundreds of modes!

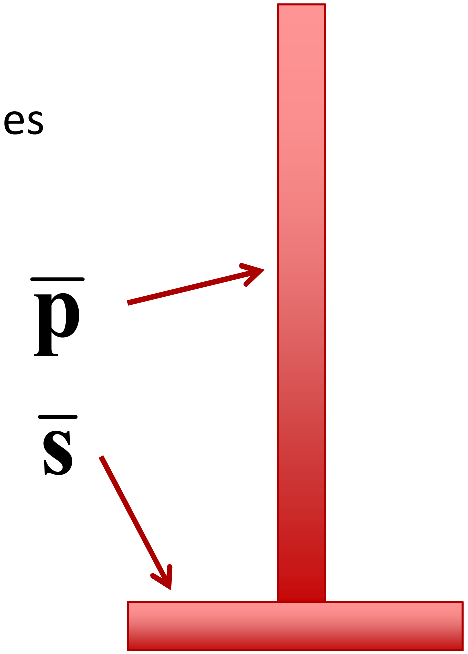




Boundary conditions unmasked! by the power of the modal model

- A modified modal model with the modal dof attached to the fixture was reported in 1972 (Wada, Bamford, Garba at JPL)
- It shows the relation between the free fixture rigid body modes (s dof) and the fixed based test article modes (p dof)

$$\begin{bmatrix} \mathbf{I} & \mathbf{M_{ps}} \\ \mathbf{M_{sp}} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \ddot{\bar{\mathbf{p}}} \\ \ddot{\bar{\mathbf{s}}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{\omega}_p^2 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \bar{\mathbf{p}} \\ \bar{\mathbf{s}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \bar{\mathbf{F}}_s \end{Bmatrix}$$



- Taking the first row and moving the fixture motion to the right hand side yields

$$\mathbf{I}\ddot{\bar{\mathbf{p}}} + \boldsymbol{\omega}_p^2 \bar{\mathbf{p}} = -\mathbf{M_{ps}} \ddot{\bar{\mathbf{s}}}$$

- Now the response p can be calculated directly from input s. For example the first p calculated in frequency domain is

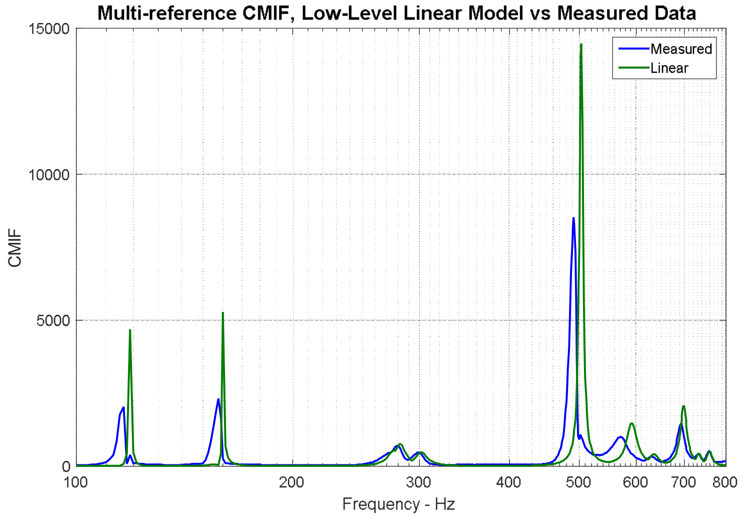
$$p_1(\omega) = \omega^2 (m_{11} s_1 + m_{12} s_2 + m_{13} s_3) / (\omega_1^2 - \omega^2)$$

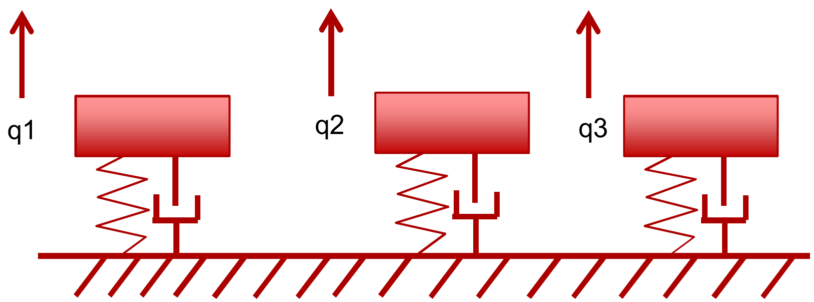
$$\bar{x} = \begin{bmatrix} \Phi_p & \Phi_s \end{bmatrix} \begin{Bmatrix} \bar{\mathbf{p}} \\ \bar{\mathbf{s}} \end{Bmatrix}$$

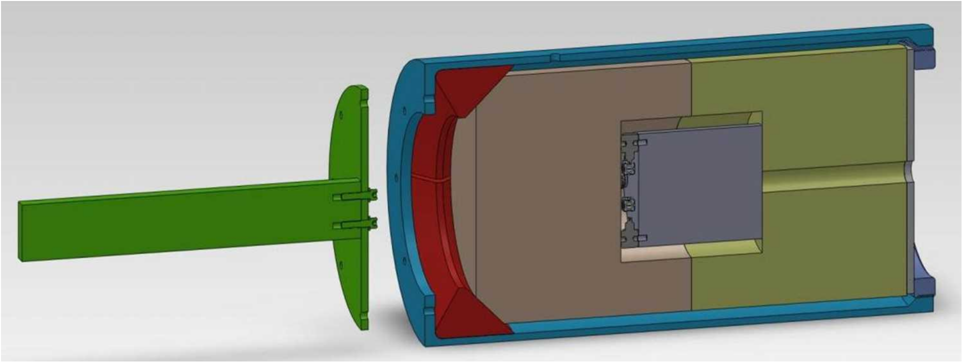
- The p generalized dof contain the damage potential to the test article since they contain all the energy absorbed by the test article. ***This appears directly applicable to energy methods.***

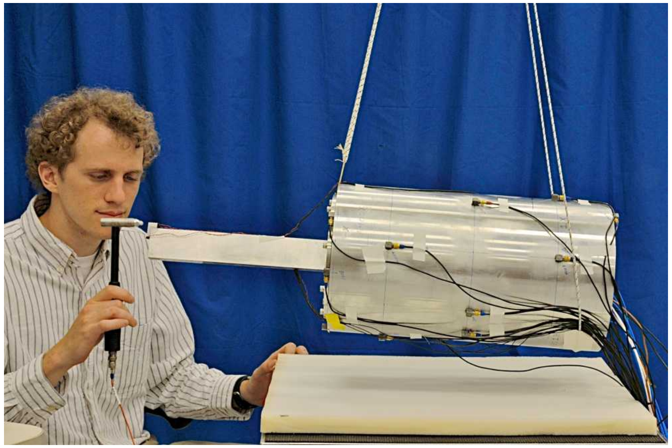
The power of the modal model to simulate our most typical random vibration nonlinearities

- The pictured hardware has a
  - Nonlinear bolted joint
  - Nonlinear foam supporting an instrumented internal component

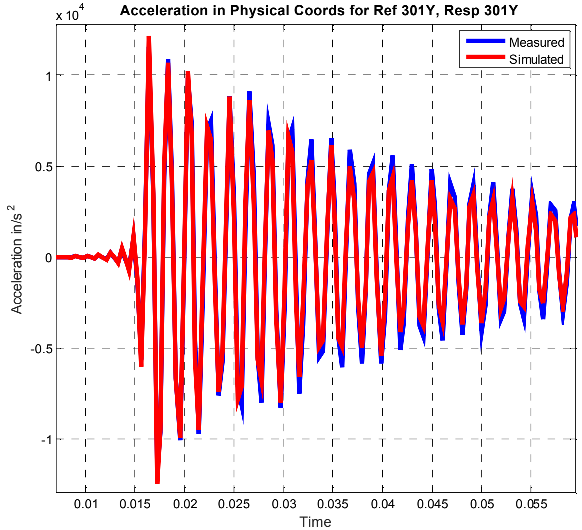


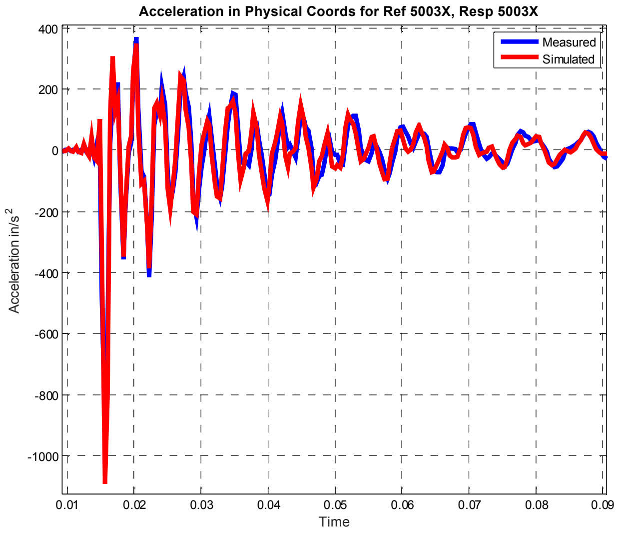






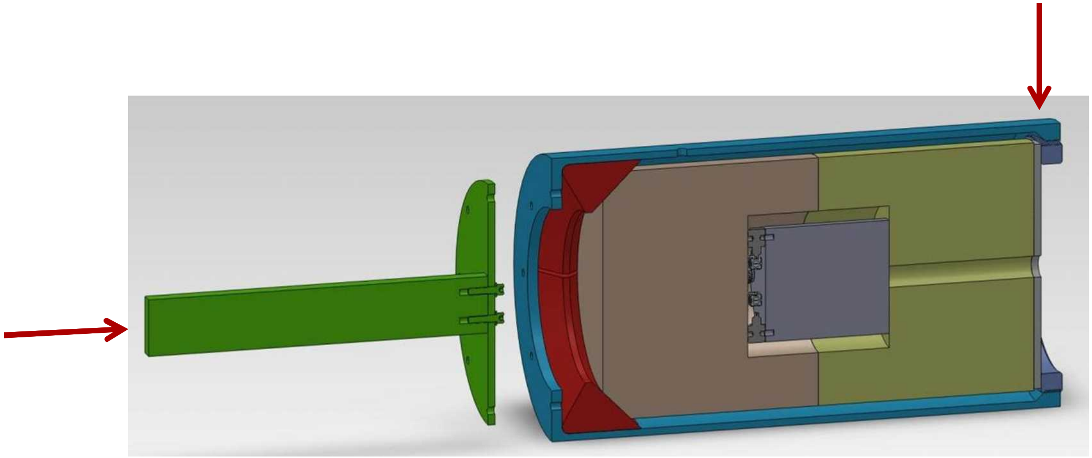
The power of the modal model to simulate our most typical nonlinearities





Axial Drive Point

Radial Drive Point



Conclusions – Our time has come

- Modal parameters have traditionally provided
  - Great physical insight
  - Great instrumentation insight
  - Great forcing insight
  - Great insight into the frequency domain
- Sandia forefathers developed robust force reconstruction approaches based on rigid body modes
- The power of the modal force is now at our disposal for multi-shaker simulation and control
- Slightly modified modal models provide tremendous understanding of boundary conditions, provide untapped insight into qualification testing, a hand-in-glove approach to energy methods
- We no longer have to shrug our shoulders at typical nonlinearity

- The time is upon us to advance our qualification and model validation approaches

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