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Gate Set Tomography on Two Qubits

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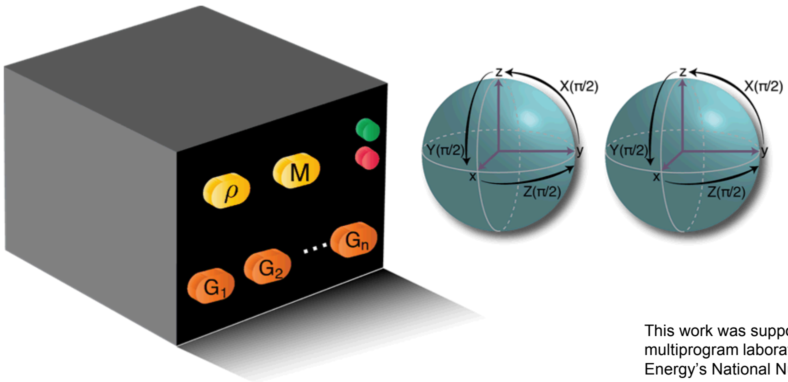

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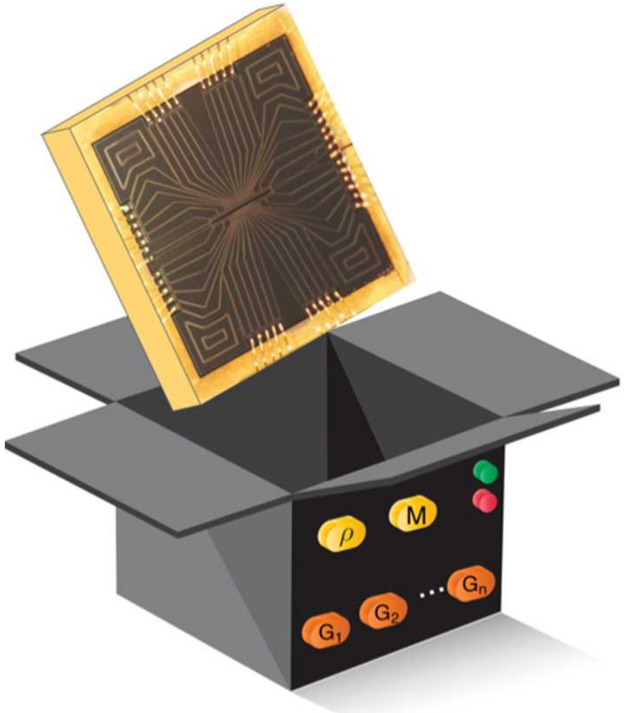
Gate Set Tomography

Experiment	Model ("gate set")



"gate sequence"

Press: (x 100) ρ | G_1 | G_2 | M → (green) / (red)

$= n$ (green), $N\text{-}n$ (red)

$p=n/N$ estimates p_0

ρ (prep) $= \begin{pmatrix} \rho_I \\ \rho_x \\ \rho_y \\ \rho_z \end{pmatrix}$

E (outcome) $= \begin{pmatrix} E_I & E_x & E_y & E_z \end{pmatrix}$

G_i (gate) $= \begin{pmatrix} 1 & 0 & 0 & 0 \\ \tau_x & \lambda_{xx} & \lambda_{xy} & \lambda_{xz} \\ \tau_y & \lambda_{yx} & \lambda_{yy} & \lambda_{yz} \\ \tau_z & \lambda_{zx} & \lambda_{zy} & \lambda_{zz} \end{pmatrix}$ $= p_0$

Matrix mult. (note order reversal):

$$\langle\langle E \,|\, G_2 \, G_1 \,|\, \rho \rangle\rangle = p_0$$

Long-sequence GST

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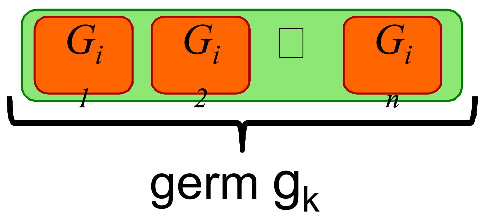
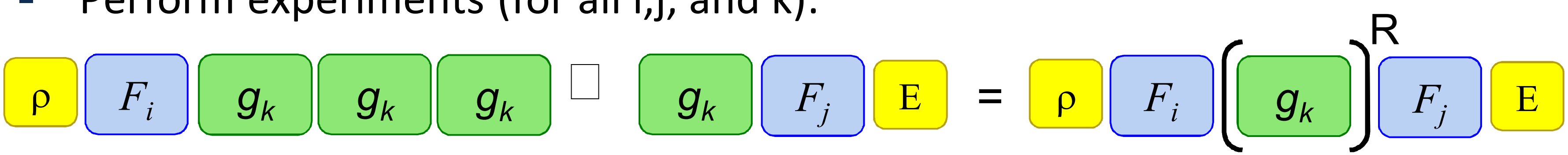
- Use error amplification to amplify **all** possible gate errors by repeating not just the gates themselves but a set of short gate sequences $\{g_k\}$ called "germs".



- Perform experiments (for all i,j, and k):



- Results in estimates ("frequencies") for $\langle\langle E | \mathrm{F}_j (g_k)^R \mathrm{F}_i | \rho \rangle\rangle$ which we compare with the probabilities predicted by the model using the log-likelihood or χ^2 statistic:

$$\log L = \sum_i N f_i \log(p_i) \qquad \chi^2 = \sum_i N \frac{(p_i - f_i)^2}{p_i}$$

  - N = #samples, f = frequency, p = probability, and i ranges over gate sequences *and* outcomes.
  - Maximizing the likelihood or minimizing χ^2 gives an estimate for the gate set.

Long-sequence GST: iterations

- To perform "long sequence GST" we do the following:
  - **Run LGST** to get an initial gate set estimate, $\mathcal{G}_0$
  - **Iteratively** maximize the log-likelihood to obtain sequentially better estimates:

Iteration	Gate Sequences used in log L	Starting gate set	Max. Likelihood Estimate
0	ρ F_i g_k F_j E	$\mathcal{G}_0$	$\mathcal{G}_1$
1	ρ F_i g_k g_k F_j E	$\mathcal{G}_1$	$\mathcal{G}_2$
2	ρ F_i g_k g_k g_k g_k F_j E	$\mathcal{G}_2$	$\mathcal{G}_3$
r	ρ F_i $[g_k]^{2^r}$ F_j E	$\mathcal{G}_{(r-1)}$	$\mathcal{G}_r$

Final GST Estimate

- Why iteratively? To avoid wrong "branch" of solution
- *Technical Point: actually use the χ^2 instead of log L for all but the final iter.

How is 2-qubit GST any different?

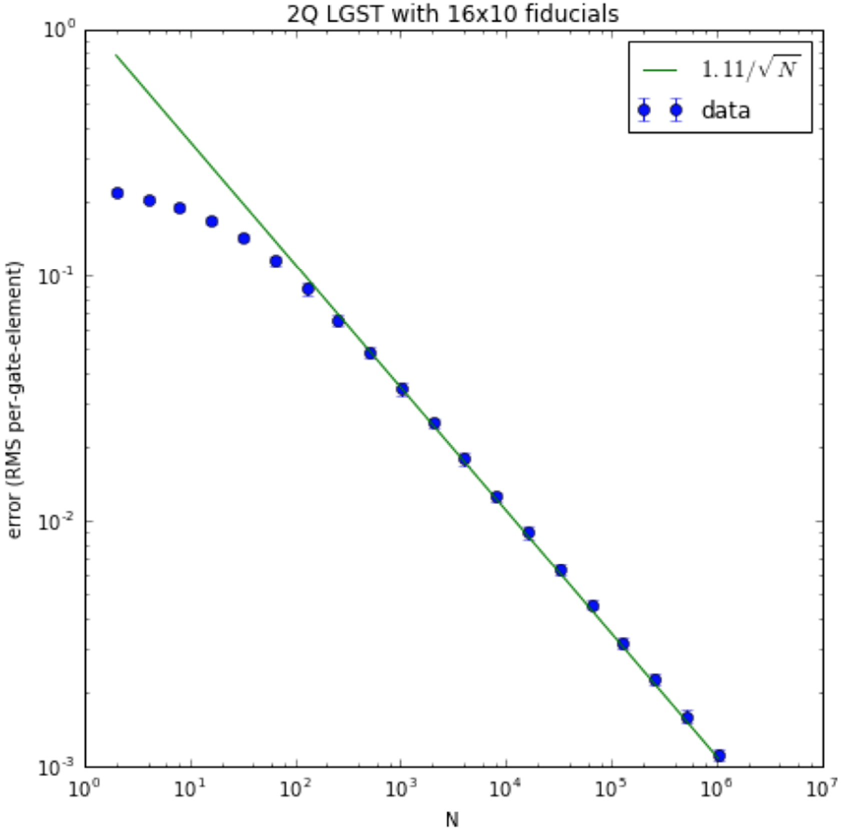
- It's not, in theory…
  - Gates & SPAM ops are just represented by larger matrices.
  - More gate sequences are required.

	min #germs	#fiducial pairs
1-qubit GST	11	16
2-qubit GST	71	160

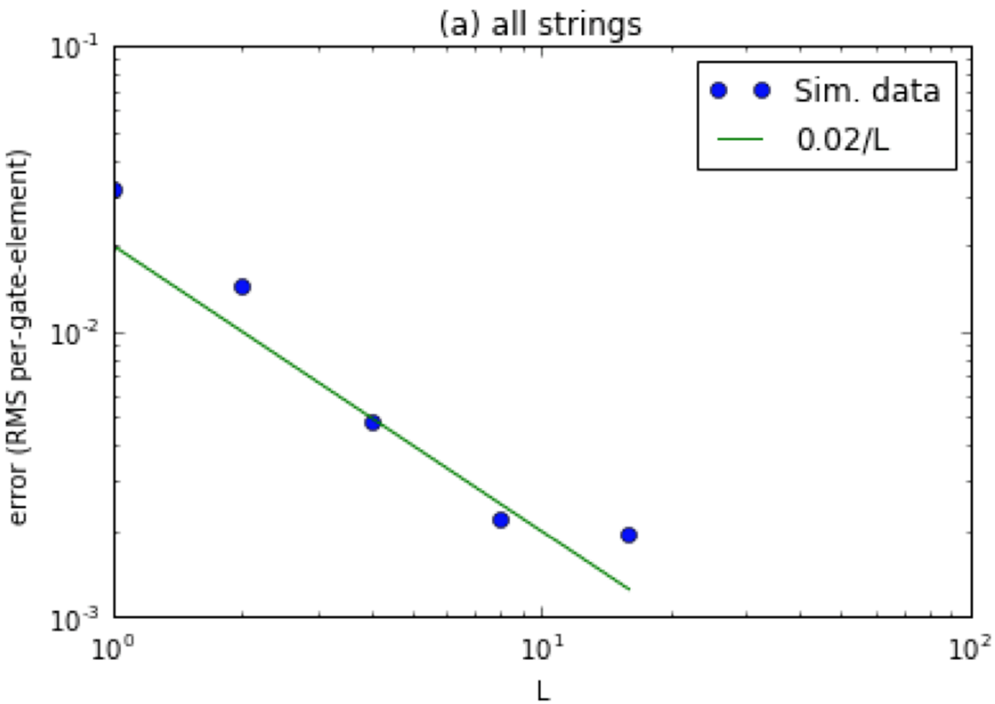
- But it is in practice…
  - The larger space requires **significantly more computational resources** to generate estimates (hours vs. minutes on laptop, tens of GB mem).
  - Effects due to a larger number of parameters became apparent at low numbers of experiment repetitions (LGST producing bad starting points)

Simulated 2-qubit GST

LGST only: 160 fiducial pairs, < 1 min. per point.



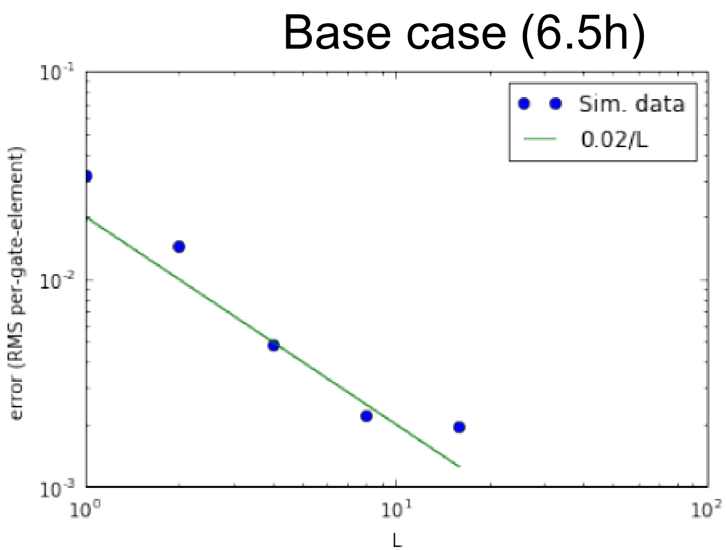
LS-GST: 71 germs, 160 fiducial pairs, ~1.5 hours per point.

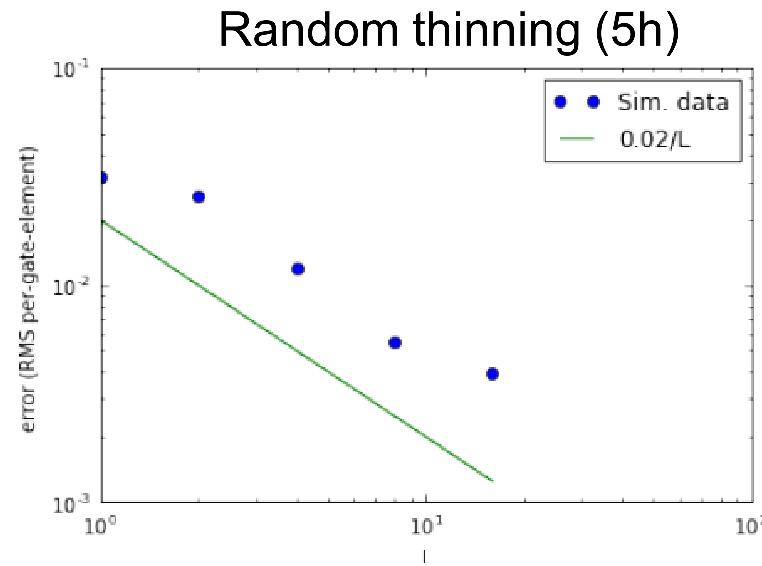


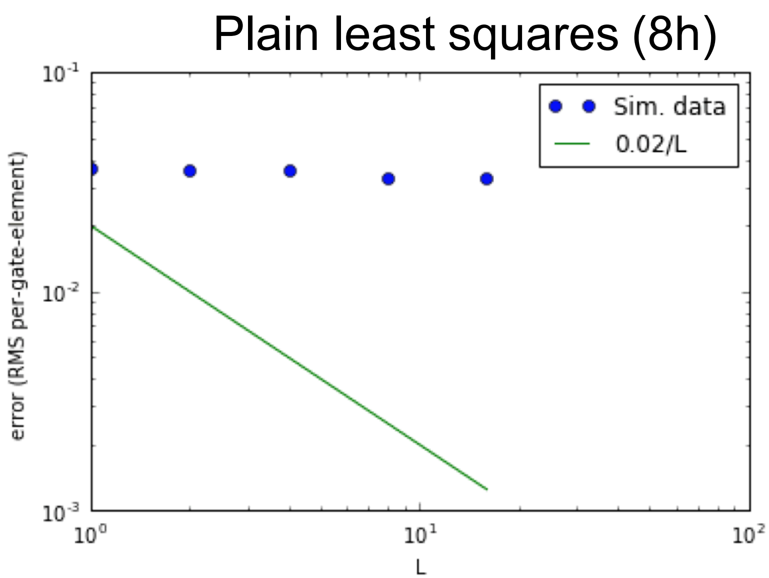
Speeding up GST

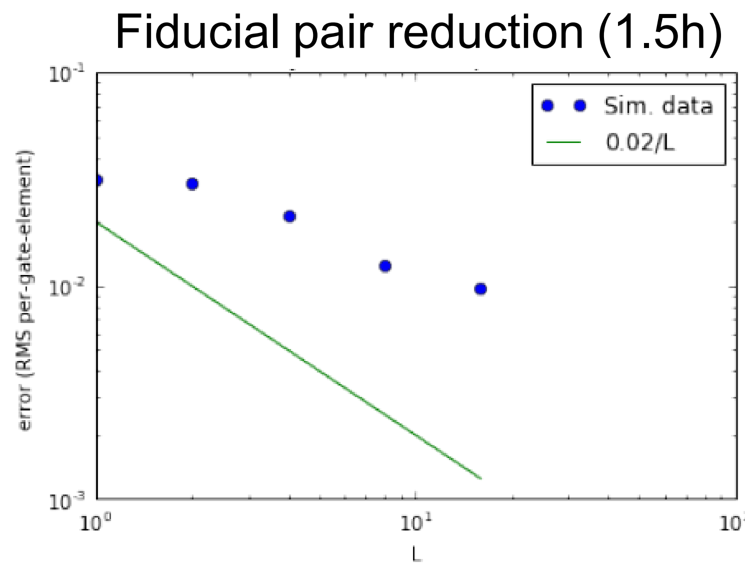
- What can be done to **decrease the time required** to perform GST (much more than time to take experiments!):
  - Reduce the **number of gate sequences** (objective fn looks smooth enough)
    - **Random thinning**
    - **Fiducial pair reduction**
    - Even more intelligent sequence pruning
  - Reduce the **number of parameters** in the gateset:
    - 1Q gates in a 2Q gateset (in progress)
  - Make the **function** we optimize ($\log L$ or χ^2) **smoother**
    - Use plain "least-squares" instead of the log-likelihood

Results


Base case (6.5h)
Sim. data
0.02/L
error (RMS per-gate-element)
L


Random thinning (5h)
Sim. data
0.02/L
error (RMS per-gate-element)


Plain least squares (8h)
Sim. data
0.02/L
error (RMS per-gate-element)
L


Fiducial pair reduction (1.5h)
Sim. data
0.02/L
error (RMS per-gate-element)
L

Initial experimental results

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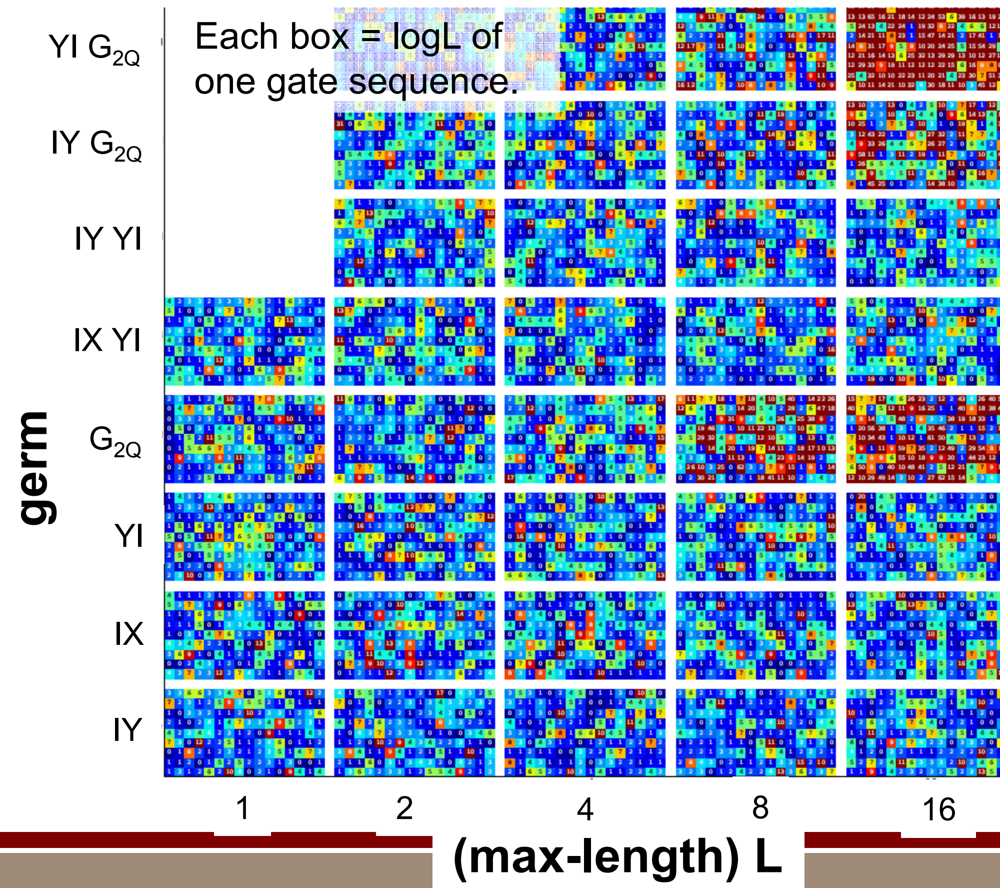
Gate	Process Infidelity	1/2 Trace Distance	1/2 ◊-Norm	Frobenius Distance
Gix	0.005268	0.063254	0.065685	0.148593
Giy	0.00578	0.064315	0.066165	0.157004
Gxi	0.006879	0.032782	0.03345	0.077547
Gyi	0.006534	0.041841	0.042923	0.100065
Gtq	0.045466	0.157726	0.159138	0.368332

- Two transmon qubits at BBN (fabricated by IBM)
- Native gate set:
  - IX, IY, XI, YI -- all $\pi/2$ rotations
  - G_{2Q} – a 2-qubit gate which rotates its second input by $\pm\pi/2$ based on the state of the first qubit.
- Performed **long-sequence GST** with maximum length L=16 (~12 hours on laptop)



Summary & a shameless ad…

- **Main Point:** 2-qubit GST works! (not just in theory)
  - theory is a straightforward extension of 1-qubit GST
  - time required to run it is manageable (and improving!)
  - 2Q-GST on simulated data shows expected Heisenberg scaling
  - Initial applications on real data is promising.

- Just-released open source GST software (doing GST is now *super easy!*)

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