

A Combinatorial Model of Dentate Gyrus Sparse Coding and Pattern Separation

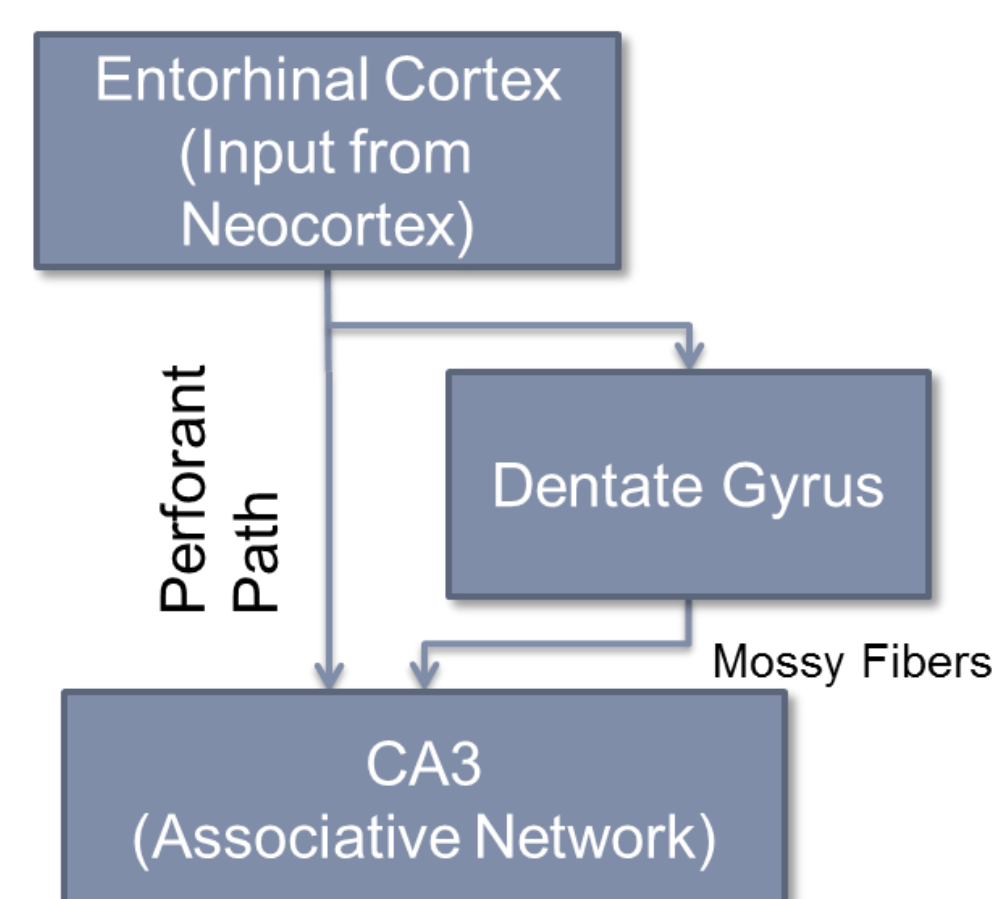
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Background

Dentate Gyrus (DG) Supplies Sparse Input to CA3

- Two distinct paths to CA3 (associative network)
 - Direct path is dense and weak—Used for recall
 - Indirect path via DG is sparse and strong—Used for training
- DG readies information for CA3:
 - Lower correlations
 - Lower activity
 - Pattern separation
 - Redundancy
- DG sparse coding differences:
 - Closed form
 - Thresholding
 - Controlled Fidelity



Formulation

Subsets Act as AND Filters

Mapping: $F: \{0,1\}^n \rightarrow \{0,1\}^k$

Collection of Subsets: $\Delta = \{\eta_i: \eta_i \subset \{1, \dots, n\}\}, k = |\Delta|$

Coding: $F = fA$ with $A = [a_{i,j}]$ where

$j \in \eta_i \Rightarrow a_{i,j} = 1/|\eta_i|$ or $a_{i,j} = 0$ otherwise, and f is the thresholding indicator function for $\{x \geq 1\}$

General Reduction: $\Delta = p - \text{sized subsets of } \{1, \dots, n\}$

Formal Results

- Noise filtering around 0; Lossless for $\{|x| \geq p\}$
- Coding non-trivially decreases normalized correlations
- High redundancy, distributed outputs: If $\|x\| = q$, then $\binom{q-1}{p-1}$ dimensions carry $F(x)$
- Action is never linear for non-zero inputs
- Outputs have assured minimum distance:

$$d(F(x), F(x')) = (q - r) \binom{q-1}{p-1} + (q' - r) \binom{q'-1}{p-1}$$

Grid Cell Refinement

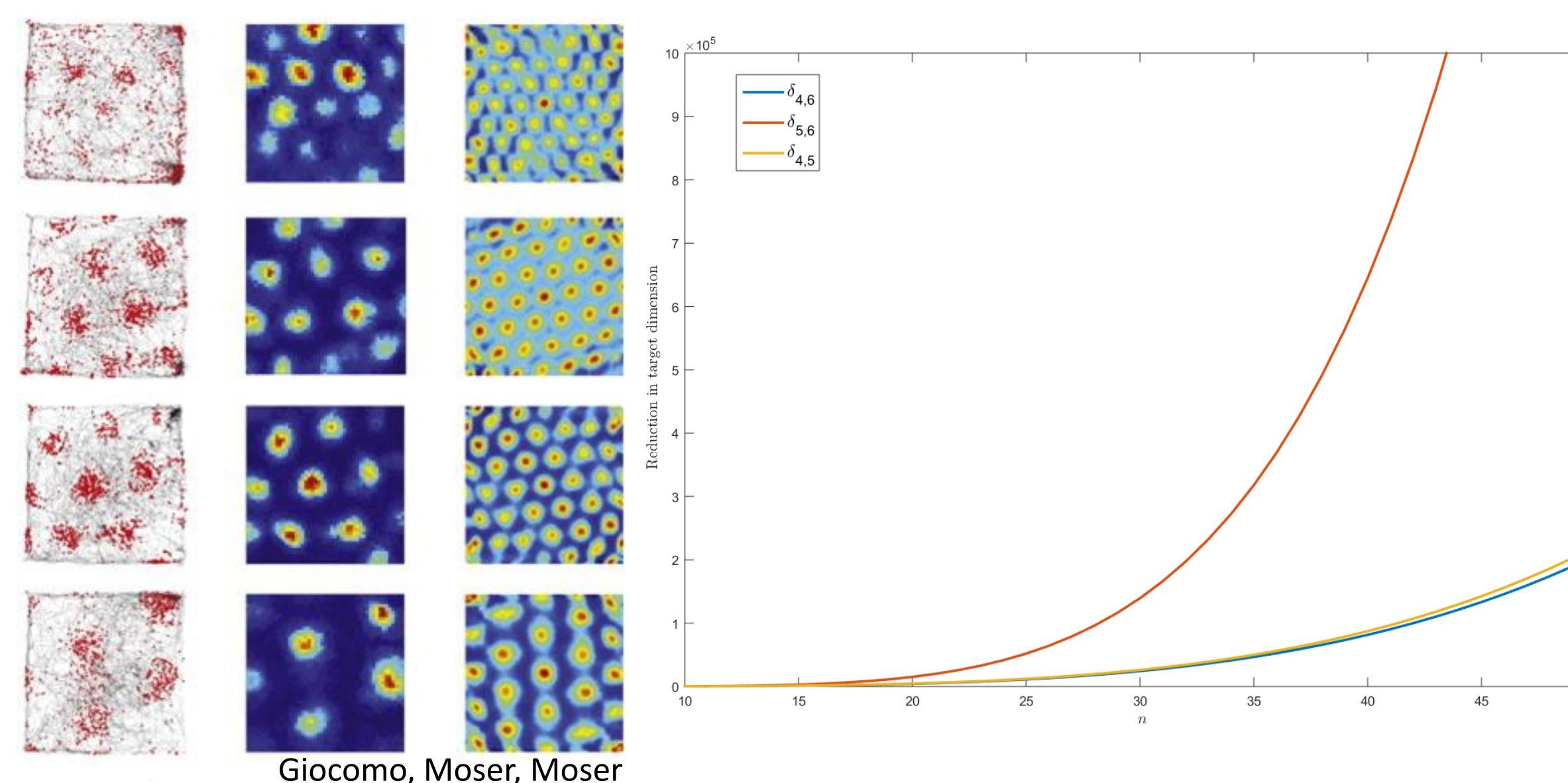
Input Distributions Curtails Target Dimension Growth

- Allowing for the entire space $\{0,1\}^n$ yields unrealistically large k
- Knowledge of input distribution, allows refining Δ to match

Biologically-inspired Grid Cell Refinement

- Grid cells located in the Entorhinal Cortex encode spatial information using a modular code

$$x \mapsto (x \bmod \lambda_i)_{i=1, \dots, T} \text{ for relatively prime } \lambda_i$$
- Action can be interpreted as toroidal dynamics, connected via fundamental group
- Construct input space X group isomorphic to grid cell input; remove η_i from Δ if η_i never occurs



- Resulting target dimension k is minimal without data loss, estimated by

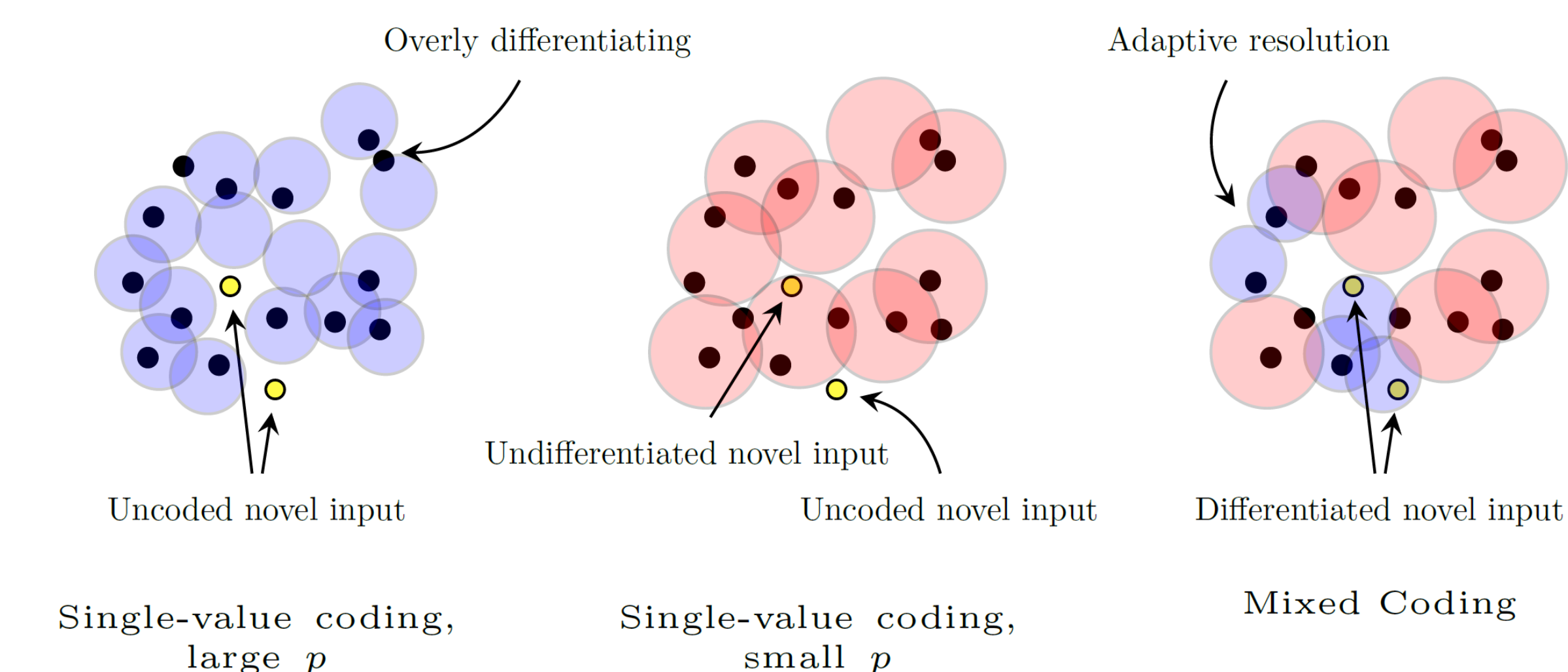
$$k < \lambda_{\max}^p \binom{T}{p} \leq \frac{n^p (T-1) \cdots (T-p+1)}{p! T^{p-1}}$$

- Using estimates for grid cell spacing in rats, model consistent with DG size
- Activity level satisfies requirement of Treves and Rolls

Mixed Coding

Adult Neurogenesis Improves Information Capacity

- Human DG develops new neurons throughout life
- Young neurons—Broadly tuned
- Old neurons—Tightly tuned
- Mixed coding allows for adaptive resolution, increased capacity



- Choose $\eta_i \in \Delta, p' < p$, expand

$$\Delta' = (\Delta \setminus \{\eta_i\}) \cup \{p' - \text{sized subsets of } \eta_i\}$$
- Conditions exists to guarantee sparsity and decorrelation

Conclusion

Simple Combinatorial Models → Meaningful Behavior

- Theoretically-robust combinatorial code
- Formal results satisfy desired DG properties
- Code is adaptable to biologically-inspired inputs
- Grid cell refinement results match literature
- Mixed coding extensions mimic adult neurogenesis

Further Directions

- Incorporate rate coding beyond binning
- Connect to larger Hippocampus model
- Biology imparting constraints onto formalism
- Neural models via input/output requirements
- Abstract neural coding methods