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Title:	Jezebel: Reconstructing a Critical Experiment from 60 Years Ago
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Jezebel: Reconstructing a Critical Experiment from 60 Years Ago

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Abstract

The Jezebel experiment of 1954-1955 was a very small, nearly-spherical, nearly-bare (unreflected), nearly-homogeneous assembly of plutonium alloyed with gallium. This experiment was used to determine the critical mass of spherical, bare, homogeneous Pu-alloy. In 1956, the critical mass of Pu-alloy was determined to be 16.45 ± 0.05 kg. The experiment was reevaluated in 1969 using logbooks from the 1950s and updated nuclear cross sections. The critical mass of Pu-alloy was determined to be 16.57 ± 0.10 kg.

In 2013, the ^{239}Pu Jezebel experiment was again reevaluated, this time using detailed geometry and materials models and modern nuclear cross sections in high-fidelity Monte Carlo neutron transport calculations. Documentation from the 1950s was often inconsistent or missing altogether, and assumptions had to be made. The critical mass of Pu-alloy was determined to be 16.624 ± 0.075 kg.

Historic documents were subsequently found that validated some of the 2013 assumptions and invalidated others. In 2016, the newly found information was used to once again reevaluate the ^{239}Pu Jezebel experiment. The critical mass of Pu-alloy was determined to be 16.624 ± 0.065 kg.

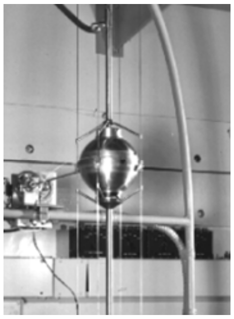
This talk will discuss each of these evaluations, focusing on the calculation of the uncertainty as well as the critical mass. We call attention to the ambiguity, consternation, despair, and euphoria involved in reconstructing the historic Jezebel experiment.

This talk is quite accessible for undergraduate students as well as non-majors.

Biography

Dr. Jeffrey Favorite received his Bachelor's, Master's, and Ph.D. in nuclear engineering from the Georgia Institute of Technology in 1993, 1994, and 1998, respectively, where he studied variational perturbation methods in nuclear reactor physics under Prof. W. M. Stacey. A paper based on his Master's thesis won the Mark Mills Award from the American Nuclear Society in 1995. In 1998, Dr. Favorite joined X-Division at Los Alamos National Laboratory, where he remains. His work and research are in the areas of neutron multiplication and criticality, neutron and photon shielding, and other neutron and photon simulations and analyses using the MCNP Monte Carlo code, the PARTISN discrete-ordinates code, and other transport codes. His particular interests are in perturbation and sensitivity methods as well as inverse and optimization methods for neutron and photon transport problems. Interface and boundary perturbations are of special interest. In Los Alamos, Dr. Favorite is active in the performing arts, youth programs, and the Episcopal church.

Photo



Jezebel: Reconstructing a Critical Experiment from 60 Years Ago

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with

Michael Zerkle (Bettis Atomic Power Laboratory)
Raymond L. Reed (URS Professional Solutions)
Roger Brewer (LANL XCP-3)

Purdue University
March 20, 2017

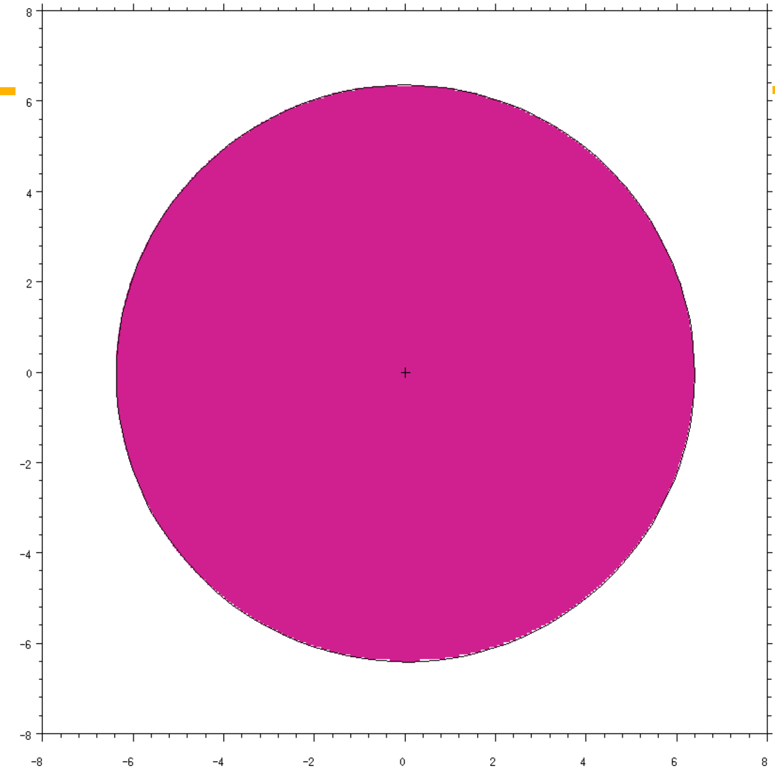
Jezebel

- Jezebel is a
  + one-dimensional spherical,
  + homogeneous,
  + bare,
  + plutonium,
  + critical benchmark.

  + Radius 6.3849 cm (5-1/32 inches diam.)

  + Density 15.61 g/cm^3

  + Mass 17,020 ± 100 g Pu alloy (± 0.6%)



- Material:

Nuclide	Atom Density (atoms/barn·cm)	Atom Fraction	Atom Fraction in Plutonium
Gallium	1.3752×10^{-3}	3.4132×10^{-2}	N/A
^{239}Pu	3.7047×10^{-2}	9.1951×10^{-1}	0.952
^{240}Pu	1.7512×10^{-3}	4.3465×10^{-2}	0.045
^{241}Pu	1.1674×10^{-4}	2.8975×10^{-3}	0.003

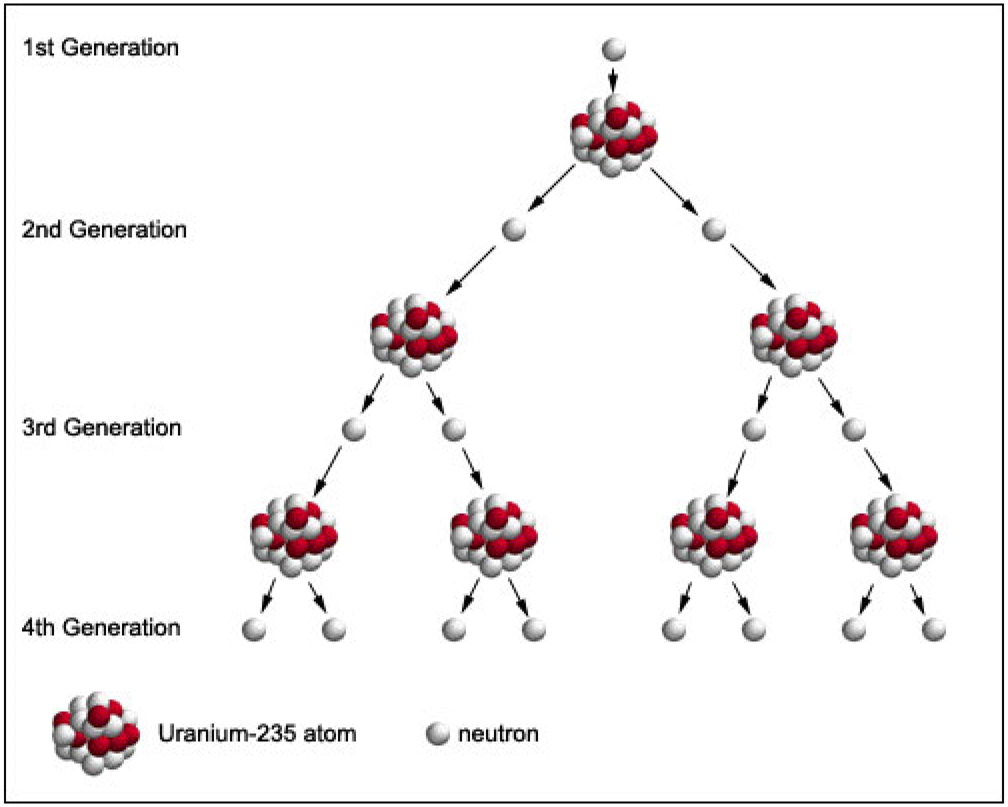
What is "Critical"?

- *Criticality* refers to the *neutron multiplication* of a fissioning system.

- We quantify criticality with a parameter called k_{eff}, the effective multiplication factor.

  + $k_{eff} < 1$, subcritical
  + $k_{eff} = 1$, critical
  + $k_{eff} > 1$, supercritical

- True criticality ($k_{eff} = 1$) is a balance: The neutron production rate is equal to the neutron loss rate (leakage, parasitic absorption, etc.)



http://www.atomicarchive.com/

- The *critical mass* is the minimum mass needed to sustain a chain reaction ($k_{eff} = 1$).

- The Boltzmann transport equation:

$$\hat{\mathbf{\Omega}}\cdot\vec{\nabla}\psi(\vec{\mathbf{r}},\hat{\mathbf{\Omega}},E)+\Sigma_t(\vec{\mathbf{r}},E)\psi(\vec{\mathbf{r}},\hat{\mathbf{\Omega}},E)-\int_{4\pi}d\hat{\mathbf{\Omega}}'\int_0^\infty dE'\,\Sigma_s(\vec{\mathbf{r}},\hat{\mathbf{\Omega}}'\to\hat{\mathbf{\Omega}},E'\to E)\psi(\vec{\mathbf{r}},\hat{\mathbf{\Omega}}',E')=$$
$$\frac{1}{k_{eff}}\int_{4\pi}d\hat{\mathbf{\Omega}}'\int_0^\infty dE'\,\chi(\vec{\mathbf{r}},E'\to E)\nu\Sigma_f(\vec{\mathbf{r}},E')\psi(\vec{\mathbf{r}},\hat{\mathbf{\Omega}}',E')$$

What is a “Benchmark”?

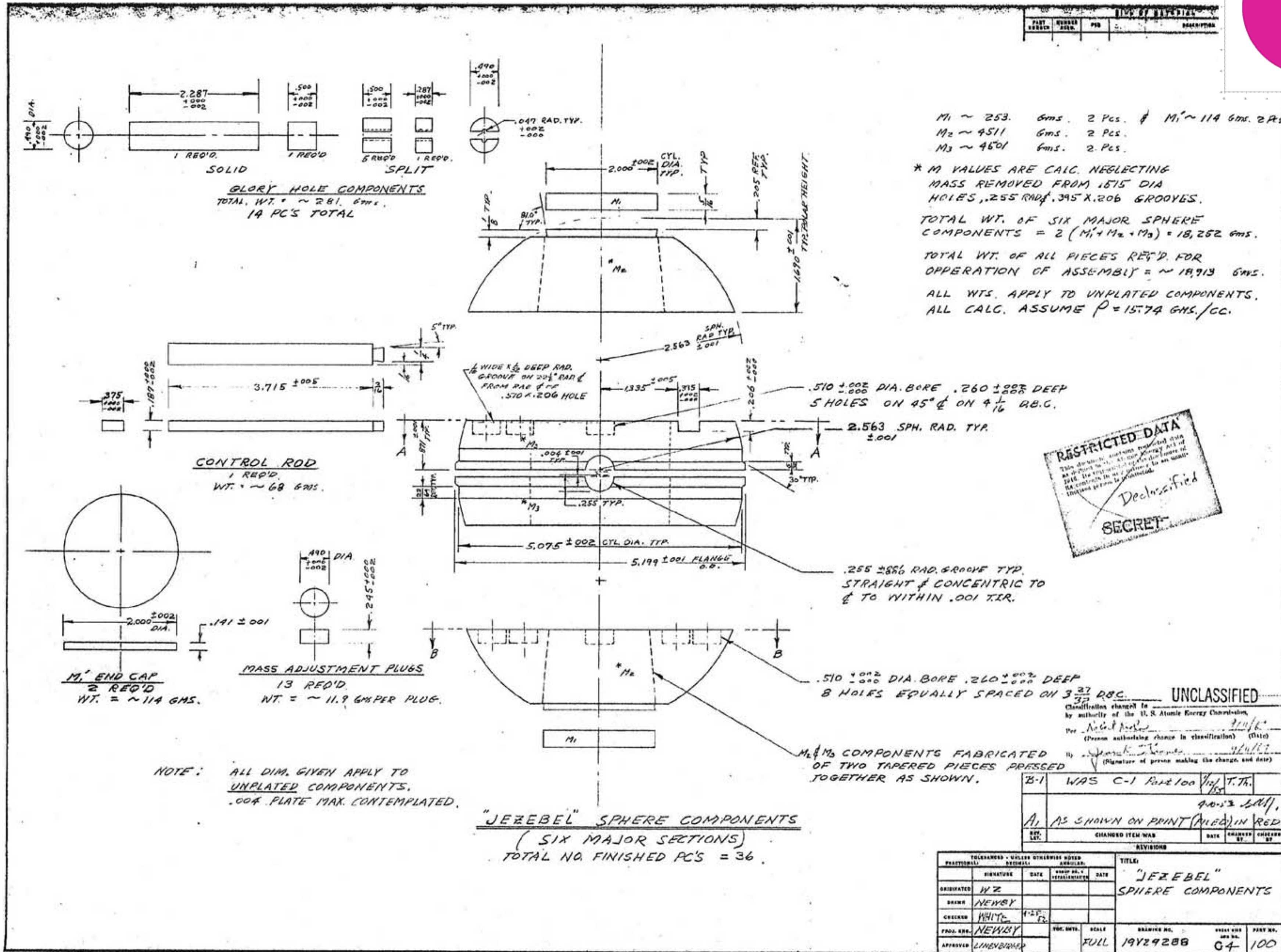
- Merriam-Webster, 2c: “a standardized problem or test that serves as a basis for evaluation or comparison”

- What are we evaluating?
  + Our neutron transport simulation codes *and* nuclear data

- What is the standardized test?
  + An experiment or measurement
  + that is of high quality
  + and is well documented

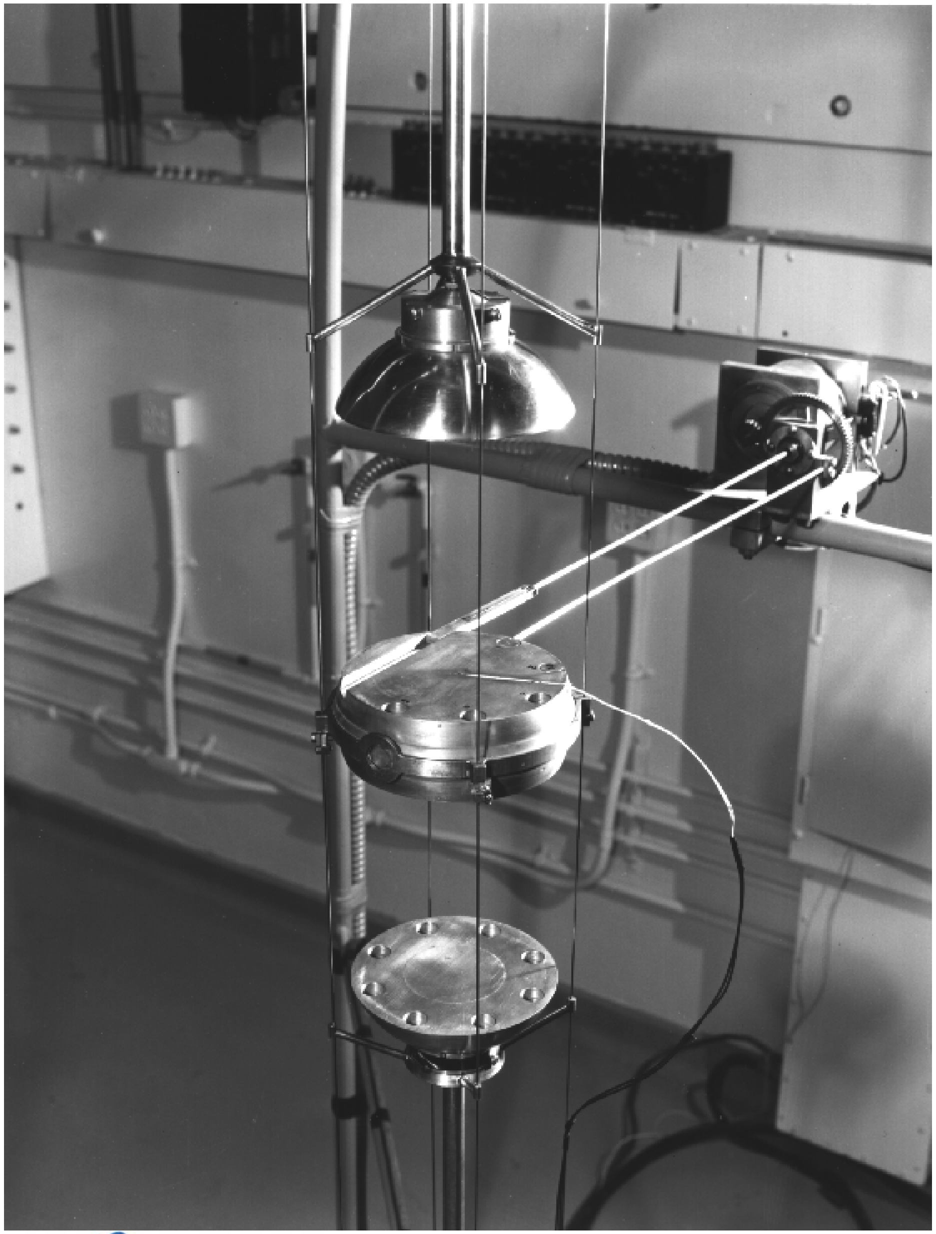
- A *critical benchmark* is an assembly that:
  + Is critical (or near critical)
  + Is of high quality and well documented
  + Has been evaluated

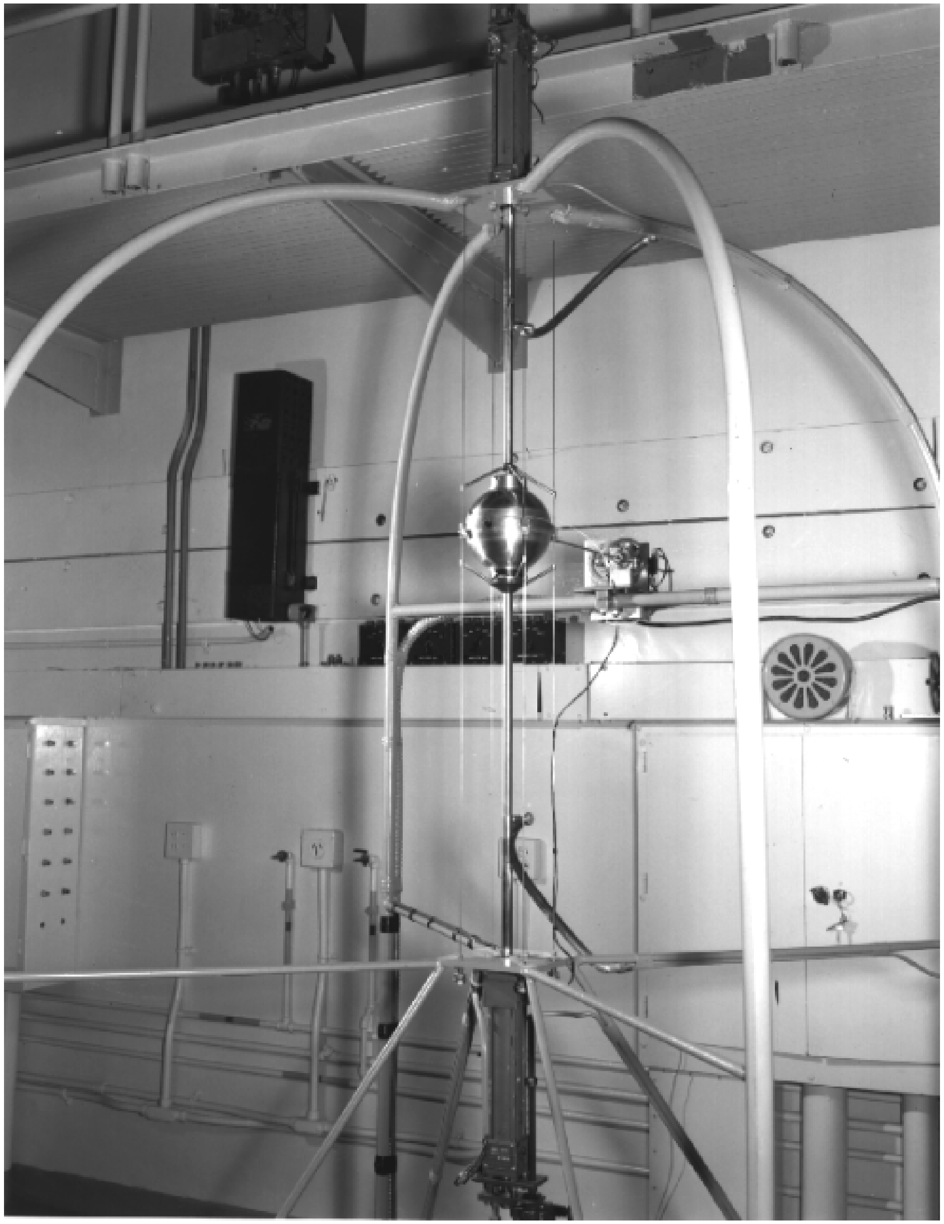
The actual Jezebel was more complicated…



Drawing 19Y29288 C4, April 1952

Photos – Jan. 24, 1955





Critical Mass Estimates in 1956, 1960, and 1969

- Los Alamos report LA-2044[a] (1956) gave the critical mass of "Jezebel Pu alloy" (1 wt.% gallium) as 16.45 ± 0.05 kg at a density of 15.82 g/cm^3.
  + Like everything else, this report was classified; it was declassified in 1965.

- A *Nuclear Science and Engineering* paper[b] (1960) gave the critical mass of "a solid, bare sphere of Pu (4½ [at.]% Pu^{240})" as 16.28 ± 0.05 kg at a density of 15.66 g/cm^3.
  + The NSE paper failed to mention the 1 wt.% gallium….

- Los Alamos report LA-4208[c] (1969) specified the full material and gave the critical mass as 17.02 kg Pu-alloy ± 0.6% at a density of 15.61 g/cm^3.
  + And 16.57 kg ± 0.6% at a density of 15.82 g/cm^3.

- The first official benchmarks (1974 through 2012) used the LA-4208 model.

---

[a] G. A. Jarvis, G. A. Linenberger, and H. C. Paxton, "Plutonium-Metal Critical Assemblies," Los Alamos Scientific Laboratory report LA-2044, May 1956.
[b] G. A. Jarvis, G. A. Linenberger, J. D. Orndoff, and H. C. Paxton, "Two Plutonium-Metal Critical Assemblies," *Nucl. Sci. Eng.*, **8**, *6*, 525-531, December 1960.
[c] G. E. Hansen and H. C. Paxton, "Reevaluated Critical Specifications of Some Los Alamos Fast-Neutron Systems," Los Alamos Scientific Laboratory report LA-4208, September 1969.

LA-4208 (1969)

- Described the development of a "reevaluated" one-dimensional model.

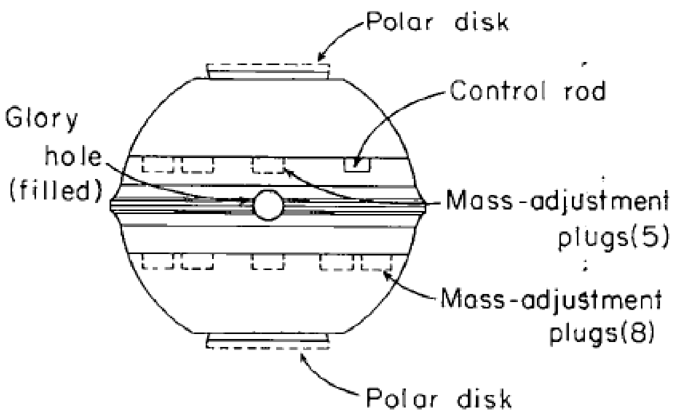


Fig. 6. Jezebel Pu (4.5% ^{240}Pu).
Configuration A, 16.751 kg alloy:

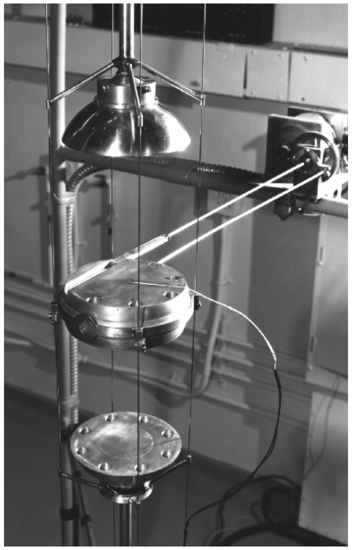
no polar disk; subcritical 0.43 lower mass-adjustment plug (or 10 g alloy at surface) with all mass-adjustment plugs in place and control rod fully inserted; critical mass is 16.761 kg alloy at average density 15.61 g/cm^3.

Configuration B, 16.909 kg alloy:

two polar disks; critical with 6 lower mass-adjustment plugs removed, and control rod retracted 1.375 in.; with all mass-adjustment plugs in place and control rod fully inserted, critical mass is 16.784 kg alloy at average density 15.60 g/cm^3.

Table I. JEZEBEL CORRECTIONS TO IDEALIZED SPHERES

	Pu(4.5% ^{240}Pu) Config. A	Config. B
Critical mass, kg[a]	16.761	16.784
(Density, g/cm^3)	(15.61)	(15.60)
Corrections, kg:		
Asphericity	-0.033	-0.047
Internal Ni and homogenization	0.047[b]	0.033[c]
Equatorial band	0.045	0.045
Polar supports	0.117	0.117
External Ni	0.074	0.074
Framework	0.002	0.002
Kiva reflection	0.010	0.010
Air reflection	0.004	0.004
Trace impurities[e]	-0.001	-0.001
Elevated temp.	-0.007	-0.007
Critical mass of homogeneous sphere, kg alloy	17.019	17.014
(Density, g alloy/cm^3)	(15.61)	~~(15.61)~~ 17.02±0.6% (15.61)



[a] Major cavities removed.

[b] Measured minus 144 g equivalent of 0.010-in.-thick Ni on one parting plane compares with calculated minus 142 g.

[c] Includes correction to ρ = 15.61 g/cm^3.

[d] Measured 75 g equivalent of upper polar support compares with calculated 78 g.

[e] Pu impurities are about 600 ppm (170 ppm C, 230 ppm 0, 115 ppm Fe); ^{233}U impurities are similar to those of Godiva.

- Corrections were estimated from a combination of measurements and calculations.

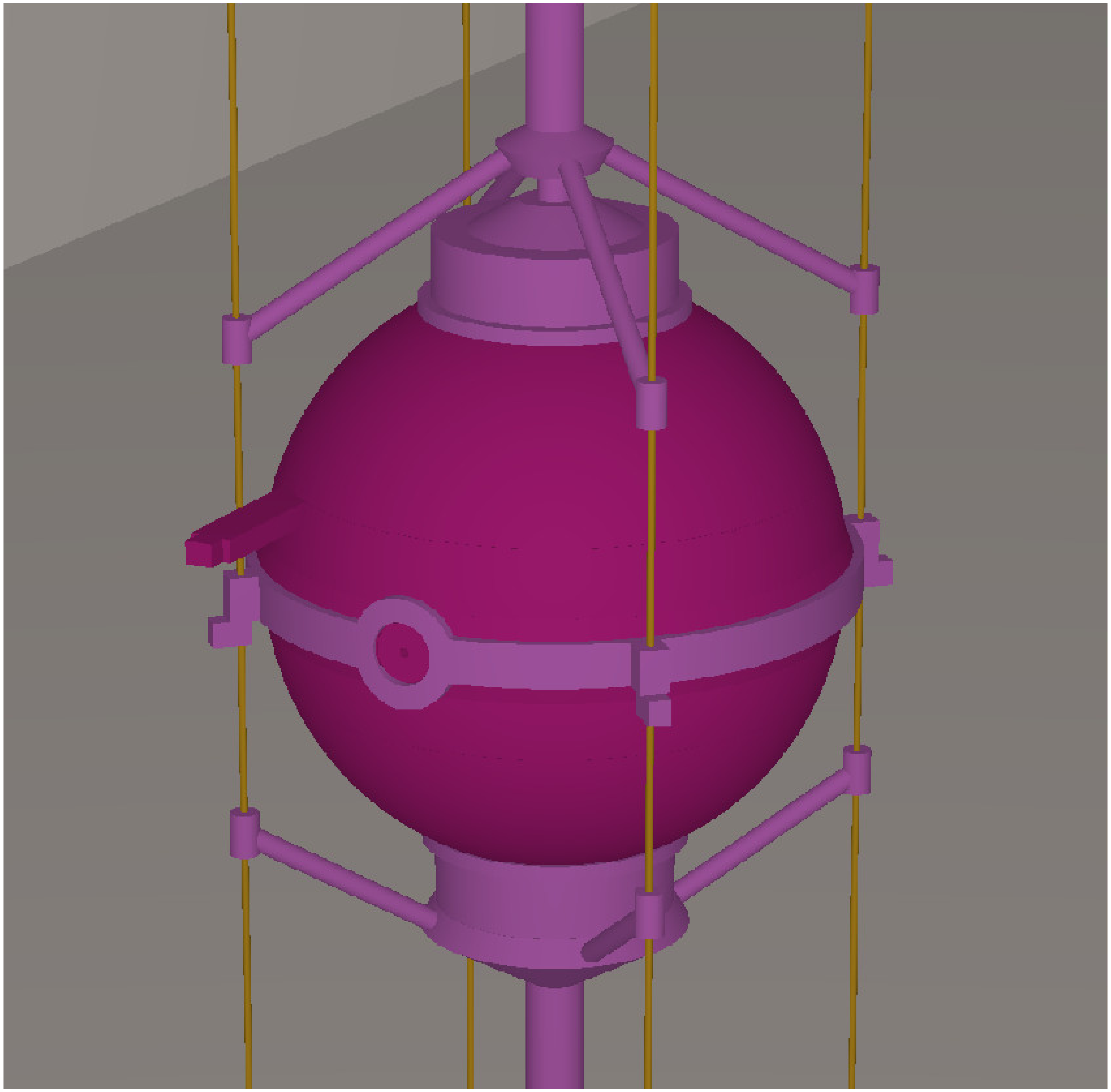
PU-MET-FAST-001 Rev. 4 (2016): Four Detailed Models

- Configurations A and B were described in LA-4208. B was found in the logbook. C and D are from the logbook.

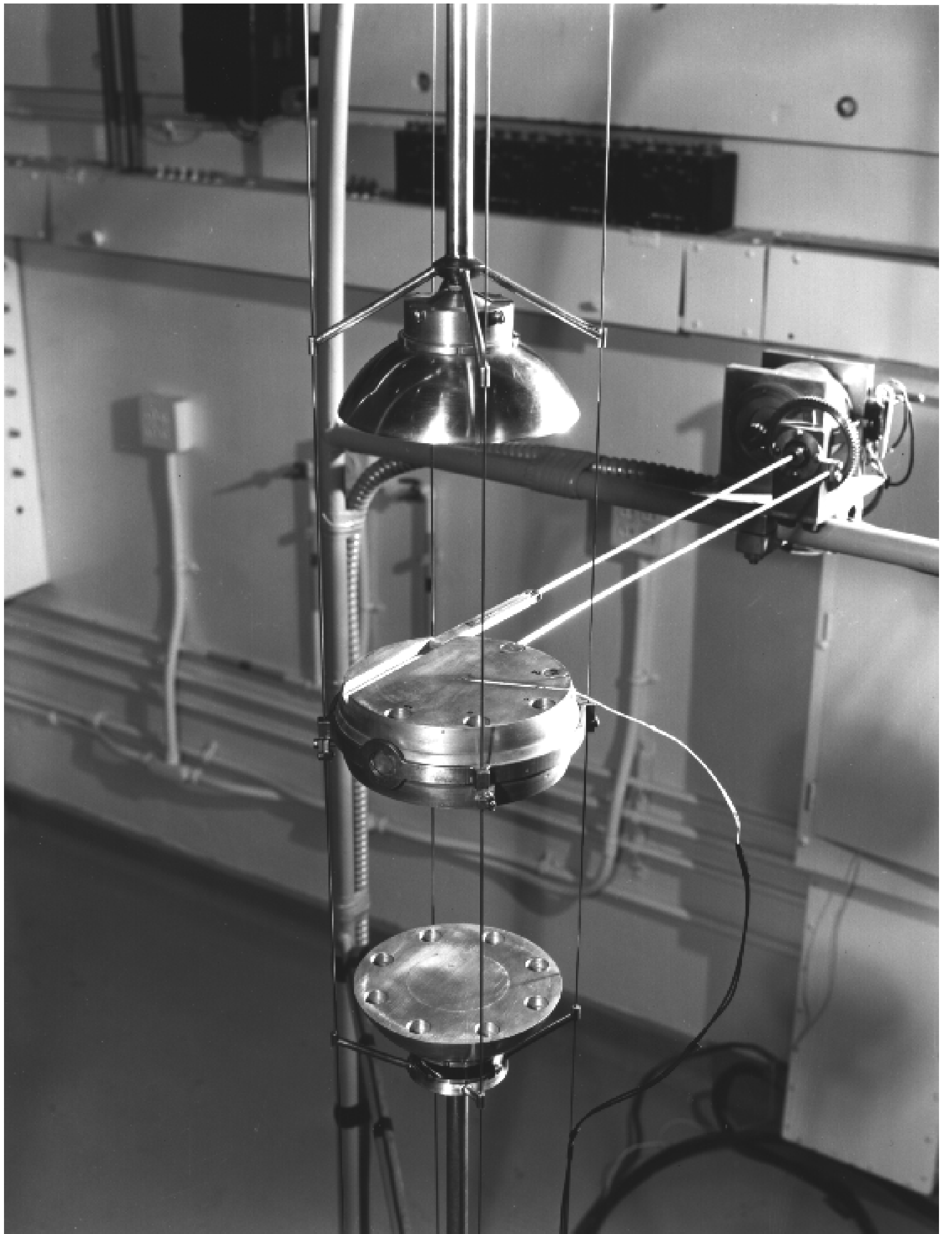
Configuration →	A	B	C	D
Experimental Assembly Mass (LA-4208) (kg Pu-alloy)	16.751	16.909	Not given	Not given
Model Assembly Mass (kg Pu-alloy)	16.751[a]	16.908	16.829	16.865
Average Pu-alloy Density (g/cm^3)	15.81[b]	15.81[b]	15.81[b]	15.81[b]
Control Rod Position	Fully inserted	Retracted 1.375 inches	Retracted 0.867 inch	Retracted 1.276 inches
Mass Adjustment Buttons in Upper Part M3	1, 2, 3, 4, 5	1, 2, 3, 4, 5	1, 2, 3, 4, 5	1, 2, 3, 4, 5
Mass Adjustment Buttons in Lower Part M2	6, 7, 8, 9, 10, 11, 12, 13	6, 7	6, 8, 10, 11, 13	6, 7, 8, 9, 10, 11, 12, 13
Glory Hole	Full	Full	Full	Full
Thin Polar End Caps (Upper and/or Lower M1′)	None	Upper and lower	Upper	Lower
Al Spacer Ring	Present	Present	Present	Present
Thick Polar End Caps (M1)	None	None	None	None

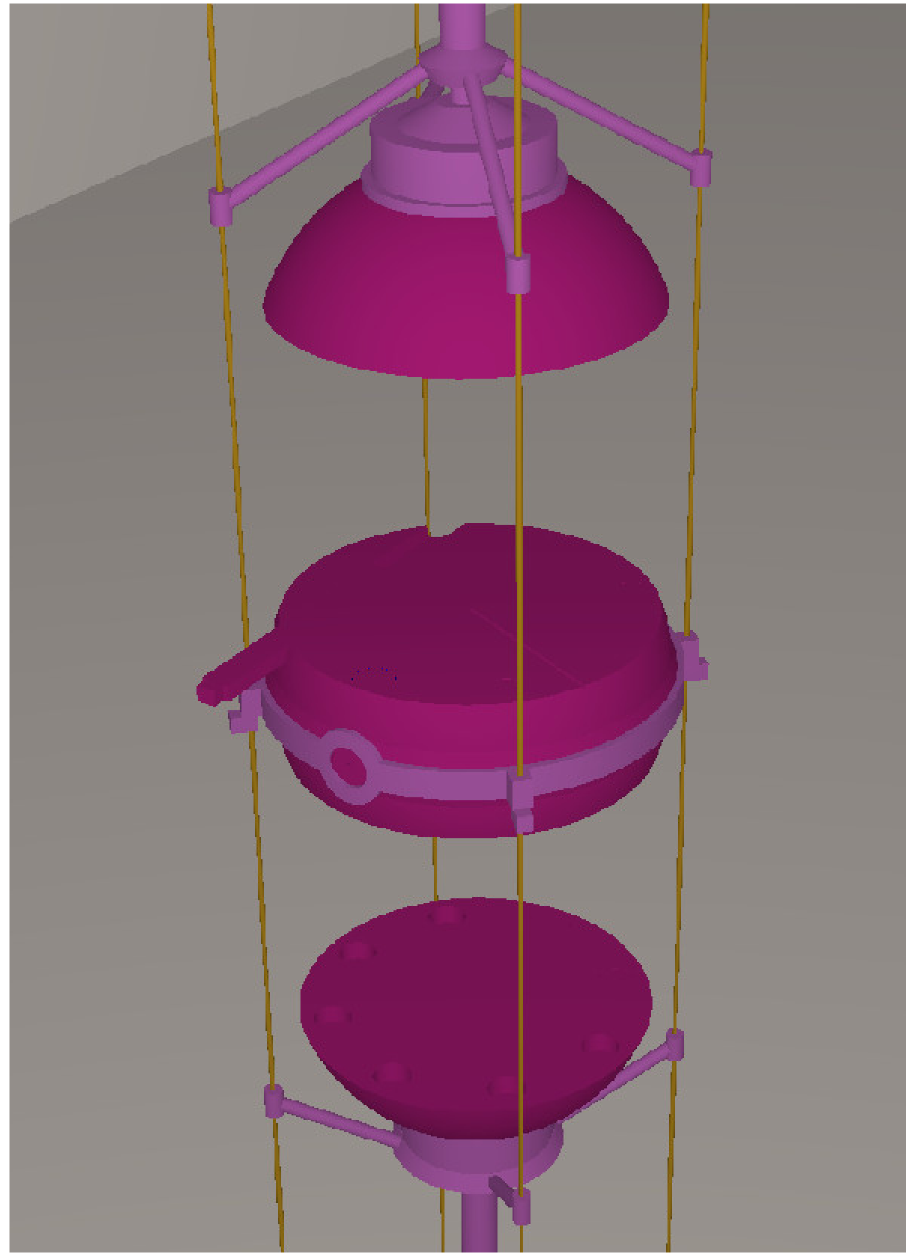
[a] Rev. 3 was 16.752 kg Pu-alloy.
[b] Rev. 3 was 15.78 g/cm^3.

MCNP Visual Editor Rendering of Configuration B (1 of 2)

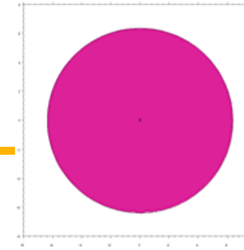


MCNP Visual Editor Rendering of Configuration B (2 of 2)



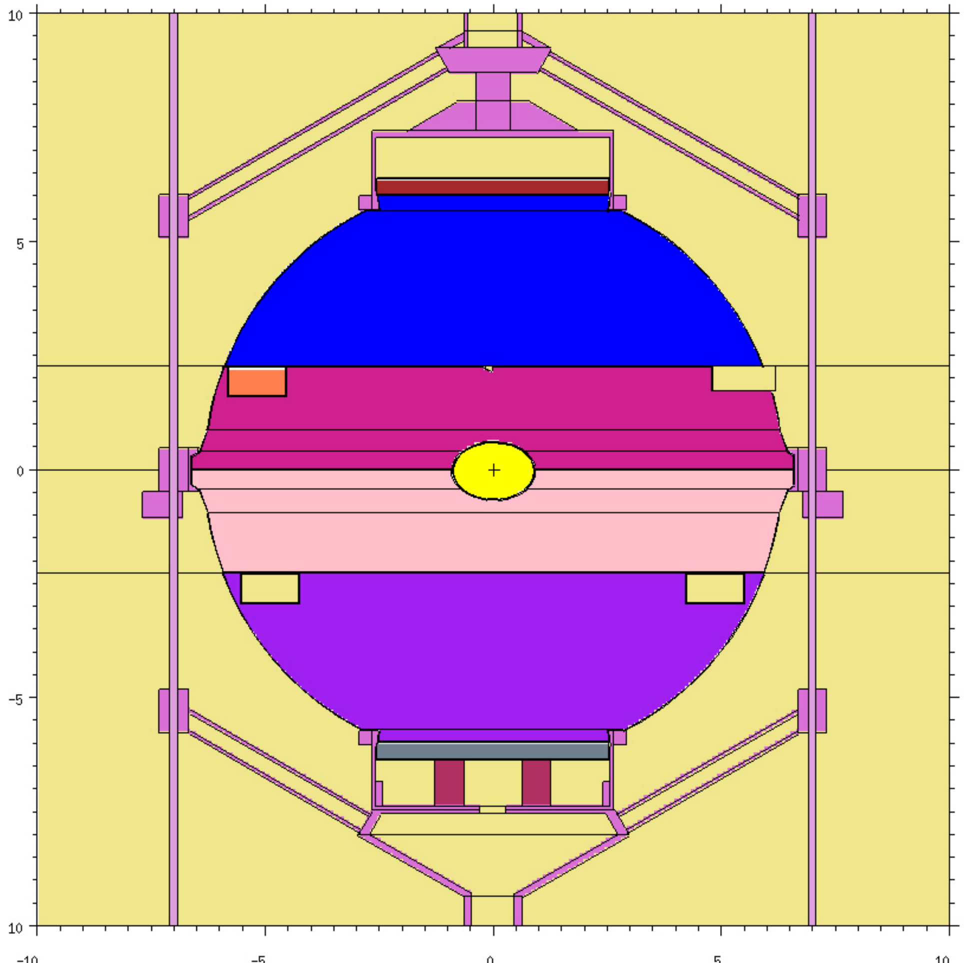


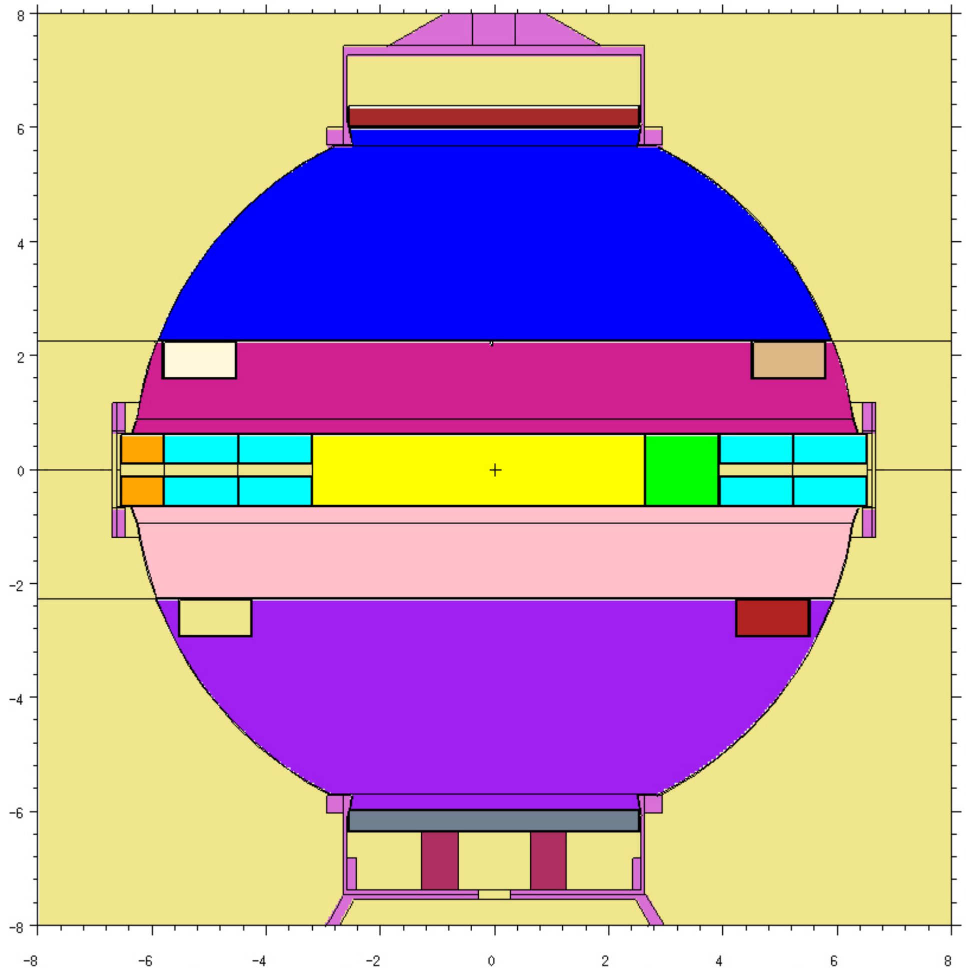
MCNP Renderings of Configuration B



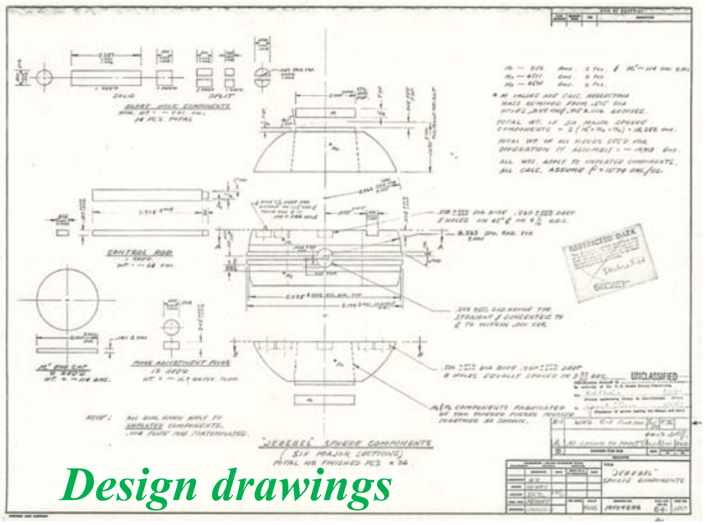
Spider assemblies, piano wire, belly band, wire lugs and clamps, control rod, mass adjustment buttons

Glory hole fill, mass adjustment buttons, external and internal nickel, thin polar end caps, aluminum shim

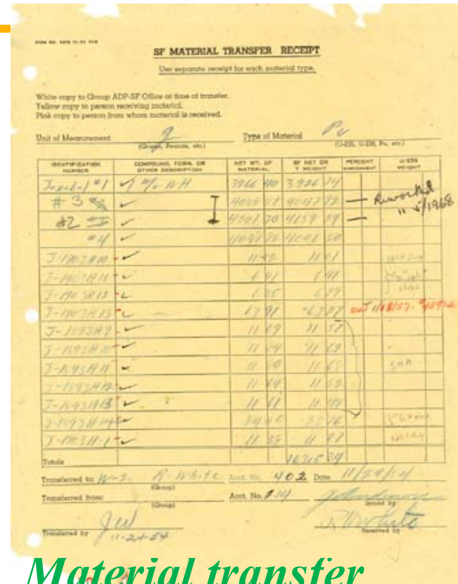




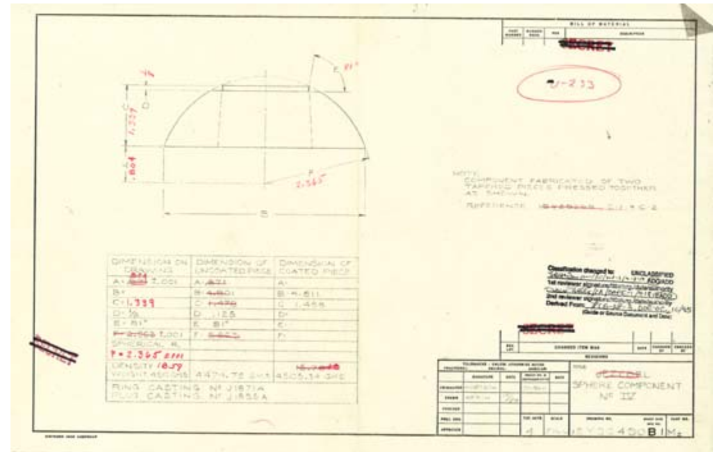
Sources for the Reevaluations



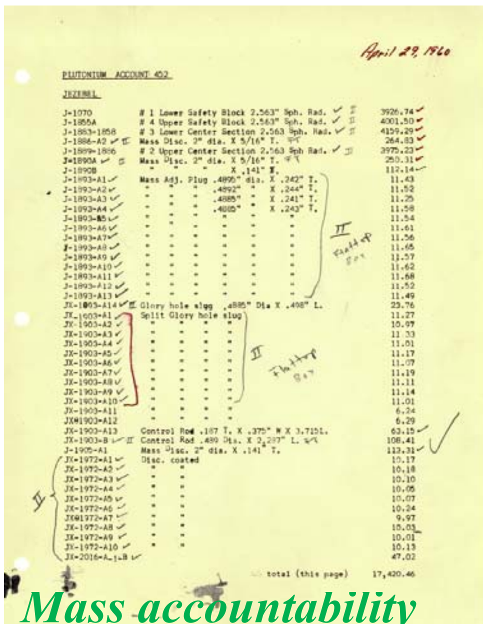
Design drawings



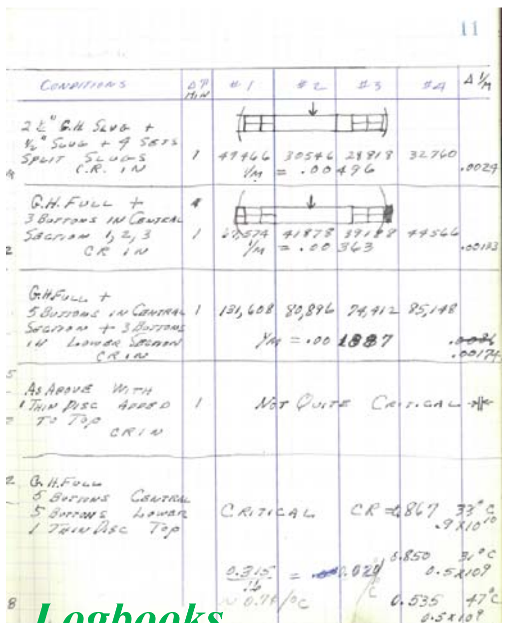
Material transfer receipt



As-built drawings



Mass accountability statements



Logbooks

Reevaluated Critical Specifications

of Some Los Alamos Fast-Neutron Systems

by

G. E. Hansen

H. C. Paxton

Reports (published and internal)

- The detailed model includes
  - 21-26 Pu-alloy parts (plus nickel plating for each)
  - ~32 structural parts
  - ~21 air gaps

Mass Accountability Statement

- This is p. 7 of a 22-page document.

April 29, 1960

PLUTONIUM ACCOUNT 452

JEZEBEL

J-1070	# 1 Lower Safety Block 2.563" Sph. Rad. II	3926.74
J-1855A	# 4 Upper Safety Block 2.563" Sph. Rad. II	4001.50
J-1883-1858	# 3 Lower Center Section 2.563 Sph. Rad. II	4159.29
J-1886-A2 II	Mass Disc. 2" dia. X 5/16" T. FT	264.83
J-1889-1886	# 2 Upper Center Section 2.563 Sph Rad. II	3975.23
J=1890A II	Mass Disc. 2" dia. X 5/16" T. FT	250.31
J-1890B	" " " X .141" T.	112.14
J-1893-A1	Mass Adj. Plug .4895" dia. X .242" T.	11.43
J-1893-A2	" " " .4892" " X .244" T.	11.52
J-1893-A3	" " " .4885" " X .241" T.	11.25
J-1893-A4	" " " .4885" " X .243" T.	11.58
J-1893-A5	" " " " " "	11.54
J-1893-A6	" " " " " "	11.61
J-1893-A7	" " " " " "	11.56
J-1893-A8	" " " " " "	11.65
J-1893-A9	" " " " " "	11.57
J-1893-A10	" " " " " "	11.62
J-1893-A11	" " " " " "	11.68
J-1893-A12	" " " " " "	11.52
J-1893-A13	" " " " " "	11.49
JX-1893-A14 II	Glory hole slug .4885" Dia X .498" L	23.76

II Flattop Box

Logbooks (Logbook II, 12/24/58, pp. 32-33)

32

Experiment Criticallity check
Date, Crew 12/24/58 Hoogterp, Wimett
Source Self
Monitor Grams -
Monitor Response -
Monit Grams -
Safety Notes -
Checking, etc. Jezebel center section re assembled after replating upper center section and Control rod because of leakage.

See next page for weight losses.

Placed 8 buttons in ~~center~~ lower section and 3 button in center section. [Two holes in center section will not take buttons]

Not critical with Control rod in

Glory hole parts would not fit in glory hole

1/5/58 - Loosened clamp and slipped 2½" long glory hole pc in E. End of Glory hole - J.C.H.

33

The lower center section and control rod of Jezebel had to be replated because of wear. Also, a number of mass adjustment plugs had pin holes in the nickel plating. The following changes in mass, due to the stripping and replating procedure are reported.

Piece No.	New Weight	Weight Change
JX-1889:1886	3975.23	- 72.69
JX-1893 A-1	11.43	- .05
A-2	11.52	- .06
A-3	11.25	- .22
A-4	11.58	- .03
A-5	11.54	- .06
A-7	11.56	- .04
JX-1903 A-2	10.97	- .03
A-4	11.01	- .32
A-6	11.07	- .02
A-11	6.24	- .17
		73.69

Ref. Receipts #61303 and #61304, dated 11/25/58 and 11/26/58, respectively.

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NATIONAL LABORATORY
EST.1943

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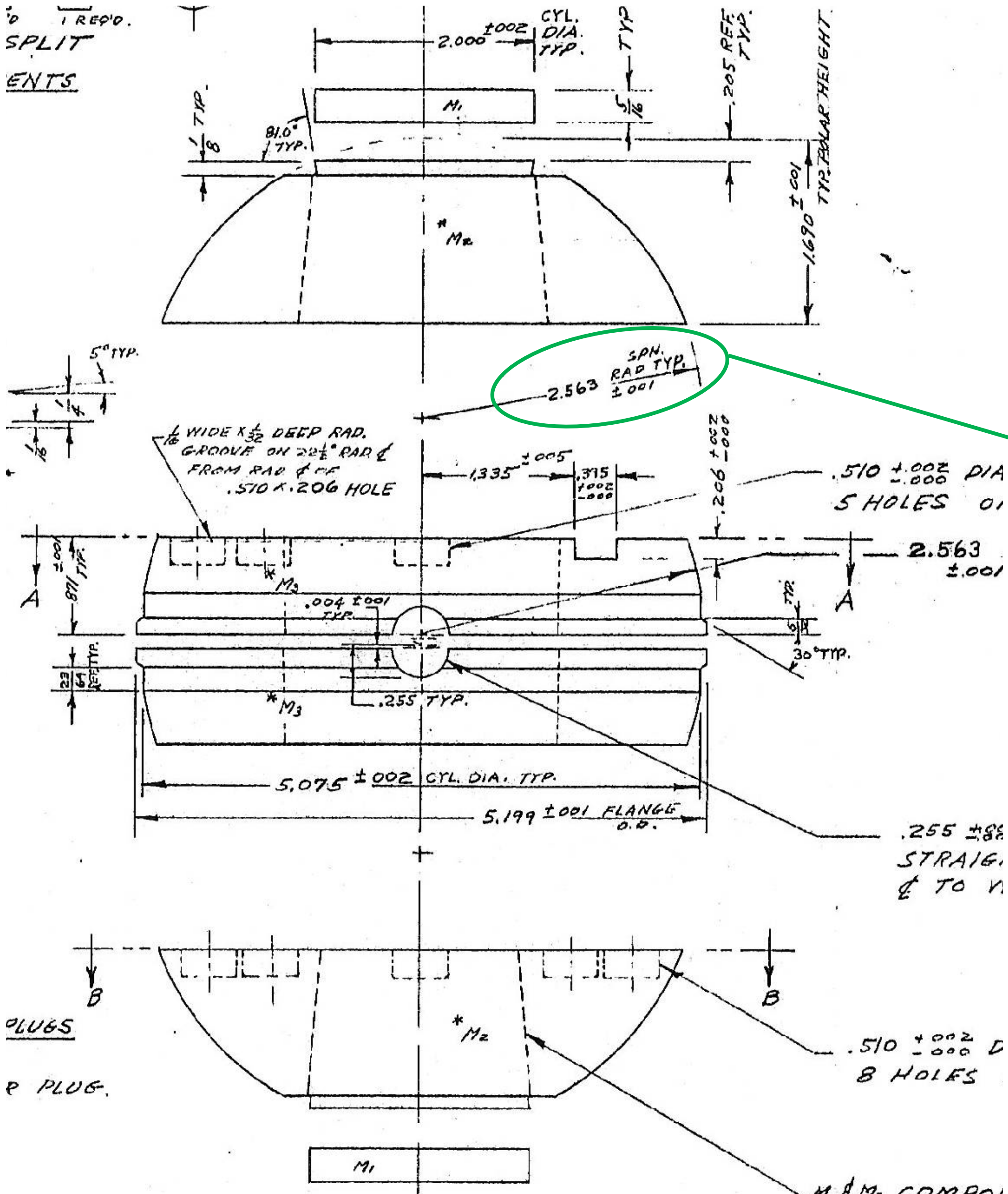
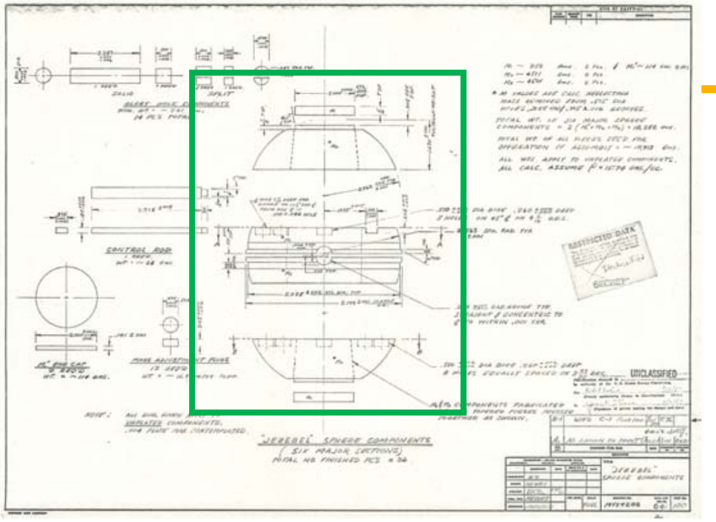
NNSA

Design Drawings (19Y29288 C4, April 1952)





Logbooks (Logbook I, Nov. 5, 1954, pp. 6-7)

6

Experiment JEZEBEL CRITICALITY MEASUREMENT
Date, Crew NOV. 5, 1954 - WHITE, GRUNDL, PAXTON
Source
Monitor Scrams O/K
Monitor Response
Hand Scrams
Safety Notes
Stacking, etc.

CONDITIONS	T	#1	#2	#3	#4
JEZEBEL UNMULTIPLIED	1 MIN	268	328	196	288
JEZEBEL ASS. NO DISKS C.R. OUT: $\frac{1}{M_{AV.}}$=.00689	1 MIN	53824 201 AV.=145	33214 101	30764 157	35048 122
SAME C.R. IN $\frac{1}{M}$=.00366	1 MIN.	102148 381 AV.=273	62444 190	57204 292	65762 228
JEZEBEL ASS., ½" SLUG FLUSH w/ O.D. OF SPHERE C.R. OUT: $\frac{1}{M}$=.00588	1 MIN.	62774 234 AV.=170	38856 118	36244 185	~~49~~ 40962 142
SAME - SLUG ½" INTO GLORY HOLE (OUTER END OF SLUG TO O.D.) $\frac{1}{M}$=.00518	1 MIN.	71462 267 AV.=193	44439 130	41014 209	46790 162
SAME - SLUG 1" $\frac{1}{M_{AV.}}$=.00493	1 MIN	76394 285 AV.=207	47536 145	44112 225	50000 174

7

CONDITIONS	T	#1	#2	#3	#4
SAME	1 MIN.	77623 290 AV.=211	48551 148	44773 228	51259 178
SAME - SLUG 1½" $\frac{1}{M}$=.00421	"	57946 ~~168359~~ 328 AV.=237	53950 164	50436 257	57420 199
SAME - SLUG 2¼" CENTER OF ASS: $\frac{1}{M}$=.00364	"	101388 378 AV.=274	62594 191	58058 296	66182 230
JEZEBEL ASS, 2¼" G.H. SLUG EXTENDING 1" INTO GLORY HOLE: $\frac{1}{M_{AV.}}$=.00392	" AV.=255	94390 352	58530 178	54096 276	61582 214
JEZEBEL ASS. 2¼" G.H. SLUG EXT. IN 1½" $\frac{1}{M}$=.00196	AV.=508	190184 709	116396 354	107188 546	122164 424
JEZEBEL ASS. C.R. OUT G.H. EMPTY 5 ADJ PLUGS IN D $\frac{1}{M}$=.00409	1 MIN. AV.=244	90088 336	55944 171	51408 262	58936 205
SAME SLUGS IN B $\frac{1}{M}$=.00404	1 MIN AV.=247	91156 91244 340	57230 57200 174	51920 52468 268	59299 59424 206
A+B+C+D only CR out $\frac{1}{M}$=.00653	AV.=153	56576 211	34800 106	32174 164	37182 129

Spherical surface remachined

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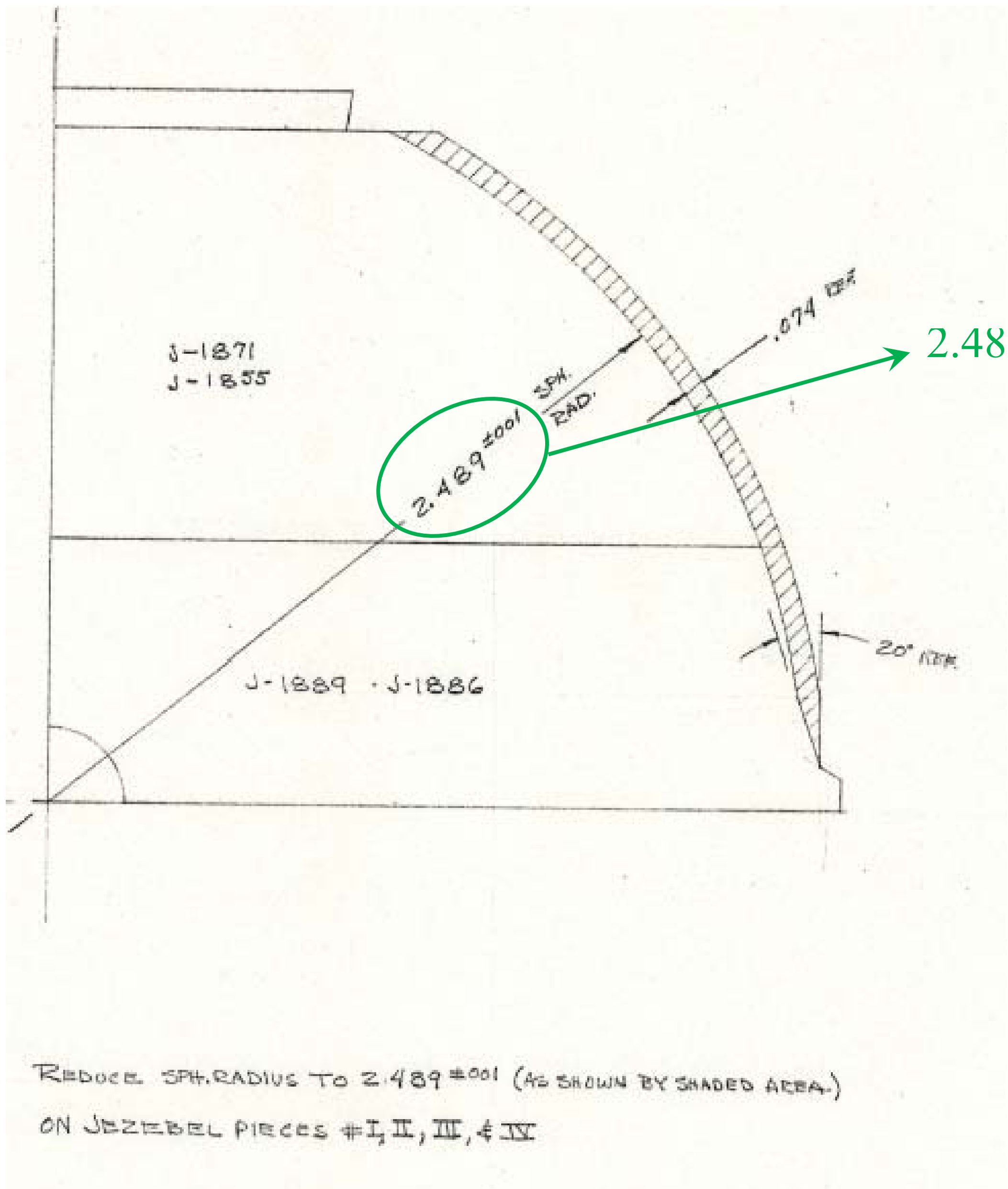
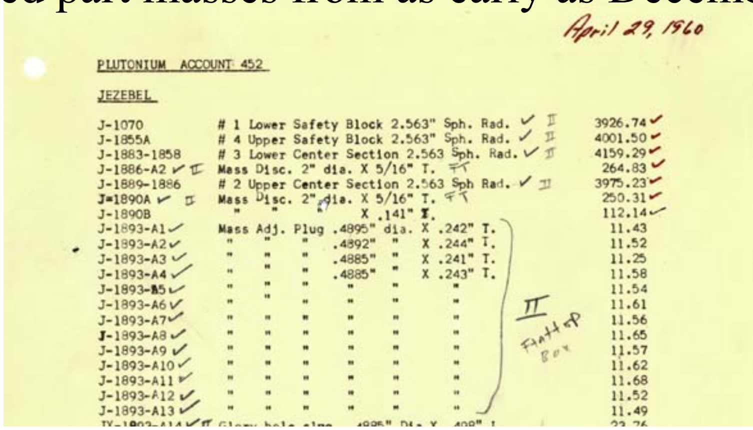
Design Drawings (19Y29288 C6, November 1954)



2.489 ± 0.001 inches

State of the Reevaluation, Spring 2013

- Measured part masses from as early as December 1958



April 29, 1960

PLUTONIUM ACCOUNT 452

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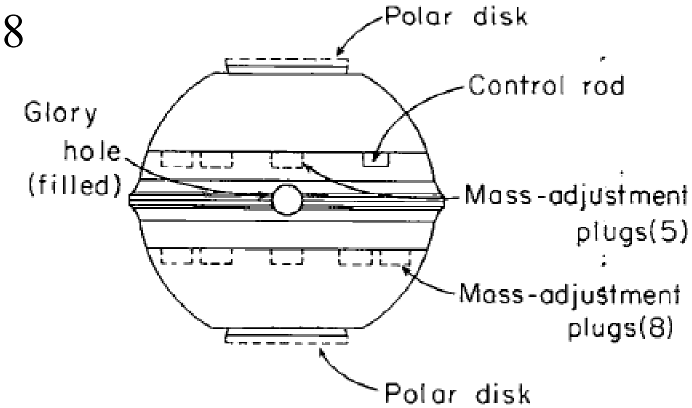
J-1893-A9	" " " " " "	11.57
J-1893-A10	" " " " " "	11.62
J-1893-A11	" " " " " "	11.68
J-1893-A12	" " " " " "	11.52
J-1893-A13	" " " " " "	11.49

II Flattop Box



- Design drawings (dimensions)

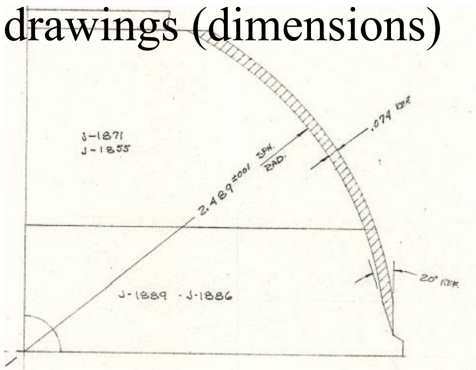


Fig. 6. Jezebel Pu (4.5% ^{240}Pu).
Configuration A, 16.751 kg alloy:

no polar disk; subcritical 0.43 lower mass-adjustment plug (or 10 g alloy at surface) with all mass-adjustment plugs in place and control rod fully inserted; critical mass is 16.761 kg alloy at average density 15.61 g/cm^3.

Configuration B, 16.909 kg alloy:

two polar disks; critical with 6 lower mass-adjustment plugs removed, and control rod retracted 1.375 in.; with all mass-adjustment plugs in place and control rod fully inserted, critical mass is 16.784 kg alloy at average density 15.60 g/cm^3.

- Calculated mass densities

- Two assembly masses (LA-4208) and detailed descriptions

Assumption: Assembly masses in LA-4208 are correct

- LA-4208 gave assembly masses for Configurations A and B.

- The earliest mass accountability statements (giving masses for individual parts) were from 1960.

- The logbooks describe an episode of nickel replating in Nov. 1958 in which one of the major parts lost 72.69 g.

- Adding the 1960 masses for Configurations A and B, and adding the mass lost in the nickel replating of Nov. 1958, the totals are ~169 g less than the LA-4208 masses.
  - + The control rod (plutonium) was replated in Nov. 1958 and "recoated" in Nov. 1957 but its new mass was not recorded either time.
    - ▫ Using the 1960 mass statements, the control rod density is 14.34 g/cm^3.
    - ▫ We added 5.58 g to the control rod to bring its density to 15.61 g/cm^3.
  - + We assumed some other undocumented process (perhaps nickel replating) in which the other three major parts lost a total of ~163 g.

- We distributed the remaining ~163 g equally among the three major parts that were not replated in Nov. 1958.

- What is the uncertainty associated with the uncertain mass distribution?

Plutonium Mass, Dimensions, and Density Uncertainties

- Linear dimensions were taken from drawings.

- Densities were not given for the individual parts (the average density was 15.82 g/cm^3).
  + LA-4208: the density of the "major parts [was] measured with a precision of ±0.2%."
  + During this period, mass could have been measured to less than a milligram. For many parts, mass is given to the nearest 0.01 gram.
  + Thus, the volume was measured to 0.2%.

- The relative uncertainty in k_{eff} due to correlated mass and volume uncertainties for each part independently is[d]

$$\left(\frac{\delta k_{eff}}{k_{eff}}\right)^2 = S_{k,\rho_d}^2\left[\left(\frac{u_{m_d}}{m_d}\right)^2+\left(\frac{u_{V_d}}{V_d}\right)^2\right]+\left(\frac{V_d}{k_{eff}}\frac{\partial k_{eff}}{\partial V_d}\right)_{\rho_d}^2\left(\frac{u_{V_d}}{V_d}\right)^2-2S_{k,\rho_d}\left(\frac{V_d}{k_{eff}}\frac{\partial k_{eff}}{\partial V_d}\right)_{\rho_d}\left(\frac{u_{V_d}}{V_d}\right)^2,$$

with $S_{k,\rho_d} \equiv \frac{\rho_d}{k_{eff}}\left(\frac{\partial k_{eff}}{\partial \rho_d}\right)_{V_d}$ and $\left(\frac{\partial k_{eff}}{\partial V_d}\right)_{\rho_d} = \frac{\sum_{n=1}^{N_d}\left(\partial k_{eff}/\partial r_n\right)_{\rho_d;\, r_m, m\neq n}}{\sum_{n=1}^{N_d}\left(\frac{1}{V_d}\partial V_d/\partial r_n\right)_{r_m, m\neq n}}$,

where N_d is the number of linear dimensions describing part d.

[d] J. A. Favorite, J. C. Armstrong, and T. Burr, "Uncertainty Analysis of Densities and Isotopics: Handling Correlations," *Proceedings of the International Conference on Mathematics and Computational Methods Applied to Nuclear Science and Engineering (M&C 2013)*, CD-ROM, Sun Valley, Idaho, May 5-9, 2013.

Plutonium Mass Distribution Correlations (Total Mass σ = ±2 g)

- The three large parts and the control rod, among which the "missing" 169 g was distributed, are correlated. The total $\delta k_{eff}/k_{eff}$ for the four parts is

$$\left(\frac{\delta k_{eff}}{k_{eff}}\right)^2 = \sum_{i=1}^{4} S_{k,\rho_i}^2 \left(\frac{u_{m_i}}{m_i}\right)^2 + 2\sum_{i=1}^{3}\sum_{j=i+1}^{4} S_{k,\rho_i} S_{k,\rho_j} \left(\frac{u_{m_i}}{m_i}\right)\left(\frac{u_{m_j}}{m_j}\right) r_{i,j},$$

where $r_{i,j}$ is the usual correlation coefficient, $r_{i,j} \equiv \mathrm{cov}(m_i, m_j) / \left(u_{m_i} u_{m_j}\right)$, and the covariance for M independent observations of m_i and m_j is $\mathrm{cov}(m_i, m_j) = \frac{1}{M-1}\sum_{l=1}^{M}(m_{i,l} - \overline{m}_i)(m_{j,l} - \overline{m}_j)$,

where $\overline{m}_d$ is the average mass of part d for the M observations.

- $M = 1 \times 10^6$ mass distributions were randomly generated.
  - \+ A mass to distribute was sampled from a Gaussian (169 ± 2 g);
  - \+ From 0 to 11.16 g was added to the control rod (random, uniform);
  - \+ The rest was distributed (randomly, uniformly) among the "big 3";
  - \+ Densities were not allowed to be less than 15.15 or greater than 16.41 g/cm^3.

Part	Base mass (g)	Mean (g)	Std. Dev. (g)	Std. Dev./Mean
Upper M2	4055.88	4055.5953	29.2222	0.7205%
Lower M2	3981.12	3980.8878	29.1966	0.7334%
Lower M3	4213.67	4213.4332	29.2049	0.6931%
Control rod	68.73	69.4841	1.6001	2.3028%

- The relative standard deviation is essentially unchanged if the total mass σ = ±10g.

k_{eff} Uncertainty Due to Pu Mass, Dims., and Densities (4 Parts)

- Results from 200 k_{eff} calculations for each case:

Total mass σ	Conf.	Base k_{eff}	Mean	Std. Dev	Difference Between Mean and Base k_{eff}
±2 g	A	1.00072	1.00080	0.00052	0.00008
	B	1.00115	1.00122	0.00049	0.00007
±10 g	A	1.00072	1.00070	0.00065	–0.00002
	B	1.00115	1.00113	0.00064	–0.00002

- The brute-force calculations did not include the volume uncertainty of 0.2%.

- Using $u_{V_d}/V_d = 0\%$ in the equation for $\delta k_{eff}/k_{eff}$ for Configuration B, and using only the four parts,
  - ± 2 g $\rightarrow \delta k_{eff}/k_{eff} = \pm 0.00047$
  - ± 10 g $\rightarrow \delta k_{eff}/k_{eff} = \pm 0.00067$

- CONCLUSION:
  - The uncertainty in the mass to distribute does not add much to the total uncertainty;
  - Or, the distribution of the mass is far more important than how much there is to distribute.

Total k_{eff} Uncertainty

- Due to Pu mass, dimensions, and densities (all parts):

Part	$\delta k_{eff}/k_{eff}$		
	Total mass σ ±2 g	Total mass σ ±10 g	No unc. due to mass distribution
Upper M2	±0.00127	±0.00128	±0.00021
Lower M2	±0.00128	±0.00129	±0.00021
Upper M3	±0.00035		
Lower M3	±0.00173	±0.00174	±0.00034
Upper M1′	±0.00000		
Lower M1′	±0.00000		
Control rod[(a)]	±0.00005	±0.00005	±0.00000
GH filler[(a)]	±0.00003		
Buttons[(a)]	±0.00000		
Cross terms	-5.82×10^{-6}	-5.68×10^{-6}	0.00
Total mass	+0.00002	+0.00003	0.00
Total	±0.00076	±0.00091	±0.00057

[(a)] Density uncertainty only.

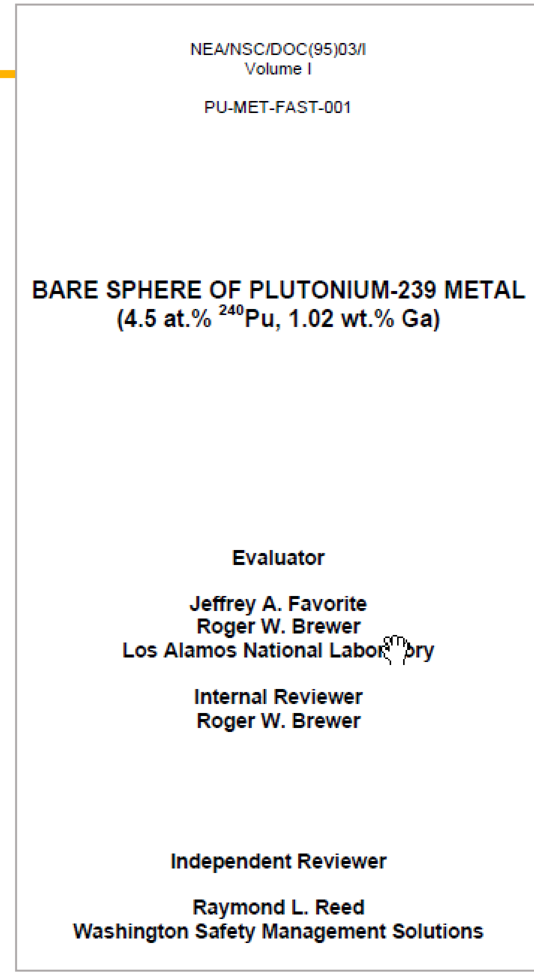
- The total uncertainty $\delta k_{eff}/k_{eff}$ was ±0.00129 (September 2013).[e]

[e] Jeffrey A. Favorite, Roger W. Brewer, and Raymond L. Reed, "Bare Sphere of Plutonium-239 Metal (4.5 at.% ^{240}Pu, 1.02 wt.% Ga)," *International Handbook of Evaluated Criticality Safety Benchmark Experiments*, PU-MET-FAST-001, Revision 3, Nuclear Energy Agency, Organization for Economic Co-Operation and Development (September 2013).

Critical Mass: PU-MET-FAST-001 Rev. 3 (2013)

- Benchmark results

	Experimental k_{eff}	Calculated k_{eff}	Calc./Exp.
Config. A	0.99999 ± 0.00129	1.00072 ± 0.00002	1.00073 ± 0.00129
Config. B	1.00016 ± 0.00129	1.00115 ± 0.00002	1.00099 ± 0.00129
Config. C	1.00020 ± 0.00129	1.00094 ± 0.00002	1.00074 ± 0.00129
Config. D	1.00128 ± 0.00129	1.00190 ± 0.00002	1.00062 ± 0.00129
Average	–	–	1.00077 ± 0.00016


NEA/NSC/DOC(95)03/I
Volume I

PU-MET-FAST-001

BARE SPHERE OF PLUTONIUM-239 METAL
(4.5 at.% ^{240}Pu, 1.02 wt.% Ga)

Evaluator

Jeffrey A. Favorite
Roger W. Brewer
Los Alamos National Laboratory

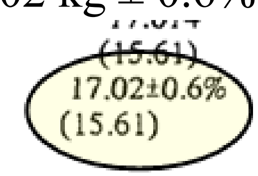
Internal Reviewer
Roger W. Brewer

Independent Reviewer

Raymond L. Reed
Washington Safety Management Solutions

- The benchmark one-dimensional model was redefined to be the one that gives k_{eff} = 1.00077 when ENDF-B/VII.1 cross sections are used.
  - + Mass = 17,073.2 ± 77 g Pu-alloy
  - + Density = 15.61 g/cm^3, same as previous benchmark (and the material is the same)
  - + Benchmark k_{eff} = 1.00000 ± 0.00129

- The reevaluated one-dimensional benchmark, 17.0732 kg ± 0.077 kg Pu-alloy, is statistically indistinguishable from the previous one-dimensional benchmark, 17.02 kg ± 0.6%.



- We did not know the mass distribution or the mass density of the Pu-alloy parts.

Major Discovery (November 2013): Material Transfer Receipt

FORM NO. 431B 10.50 50M

SF MATERIAL TRANSFER RECEIPT

Use separate receipt for each material type.

White copy to Group ADP-SF Office at time of transfer.
Yellow copy to person receiving material.
Pink copy to person from whom material is received.

Unit of Measurement g (Grams, Pounds, etc.) Type of Material Pu (U-235, U-238, Pu, etc.)

IDENTIFICATION NUMBER	COMPOUND, FORM, OR OTHER DESCRIPTION	NET WT. OF MATERIAL		SF NET OR T WEIGHT		PERCENT ENRICHMENT		U-235 WEIGHT	
Lower Part M2 Jezebel #1	✓ w/o WH	3966	40	3926	74			Reworked	
Upper Part M3 #3	✓	4088	81	4047	92	—		11/1968	
Lower Part M3 #2	✓	4201	30	4159	29	—		"	
Upper Part M2 #4	✓	4041	92	4001	50				
[illegible]	✓	11	12	11	01				(1/2 inch)
J-1903H-1	✓	11	38	11	[illegible]			split slug	(1/2 inch)
Totals				16315	34				

Transferred to: W-2 R-White (Group) Acct. No. 402 Date 11/24/54 11/24/54

Transferred from: (Group) Acct. No. 714 [illegible] Issued by

Transferred by JW 11-24-54 R White Received by

Recall: Mass Accountability Statement (and Logbook Entry)

April 29, 1960

PLUTONIUM ACCOUNT 452

JEZEBEL

J-1070	# 1 Lower Safety Block 2.563" Sph. Rad. ✓ II	3926.74 ✓
J-1855A	# 4 Upper Safety Block 2.563" Sph. Rad. ✓ II	4001.50 ✓
J-1883-1858	# 3 Lower Center Section 2.563 Sph. Rad. ✓ II	4159.29 ✓
J-1886-A2 ✓ II	Mass Disc. 2" dia. X 5/16" T. FT	264.83 ✓
J-1889-1886	# 2 Upper Center Section 2.563 Sph Rad. ✓ II	3975.23 ✓
J=1890A ✓ II	Mass Disc. 2" dia. X 5/16" T	[illegible]
J-1890B	" " " X .141" T	[illegible]
J-1893-A1 ✓	Mass Adj. Plug .4895" dia. X	[illegible]
J-1893-A2 ✓	" " " .4892" " X .244" T.	11.[illegible]
J-1893-A3 ✓	" " " .4885" " X .241" T.	11.[illegible]
J-1893-A4 ✓	" " " .4885" " X .243" T.	11[illegible]
J-1893-A5 ✓	" " " " " "	11[illegible]
J-1893-A6 ✓	" " " " " "	11[illegible]
J-1893-A7 ✓	" " " " " "	11[illegible]
J-1893-A8 ✓	" " " " " "	11[illegible]
J-1893-A9 ✓	" " " " " "	11[illegible]
J-1893-A10 ✓	" " " " " "	11[illegible]
J-1893-A11 ✓	" " " " " "	11.68
J-1893-A12 ✓	" " " " " "	11.52
J-1893-A13 ✓	" " " " " "	11.49
JX-1893-A14 ✓ II	Glory hole slug .4885" Dia X .498" T	23.76

II
Flatt op
Box

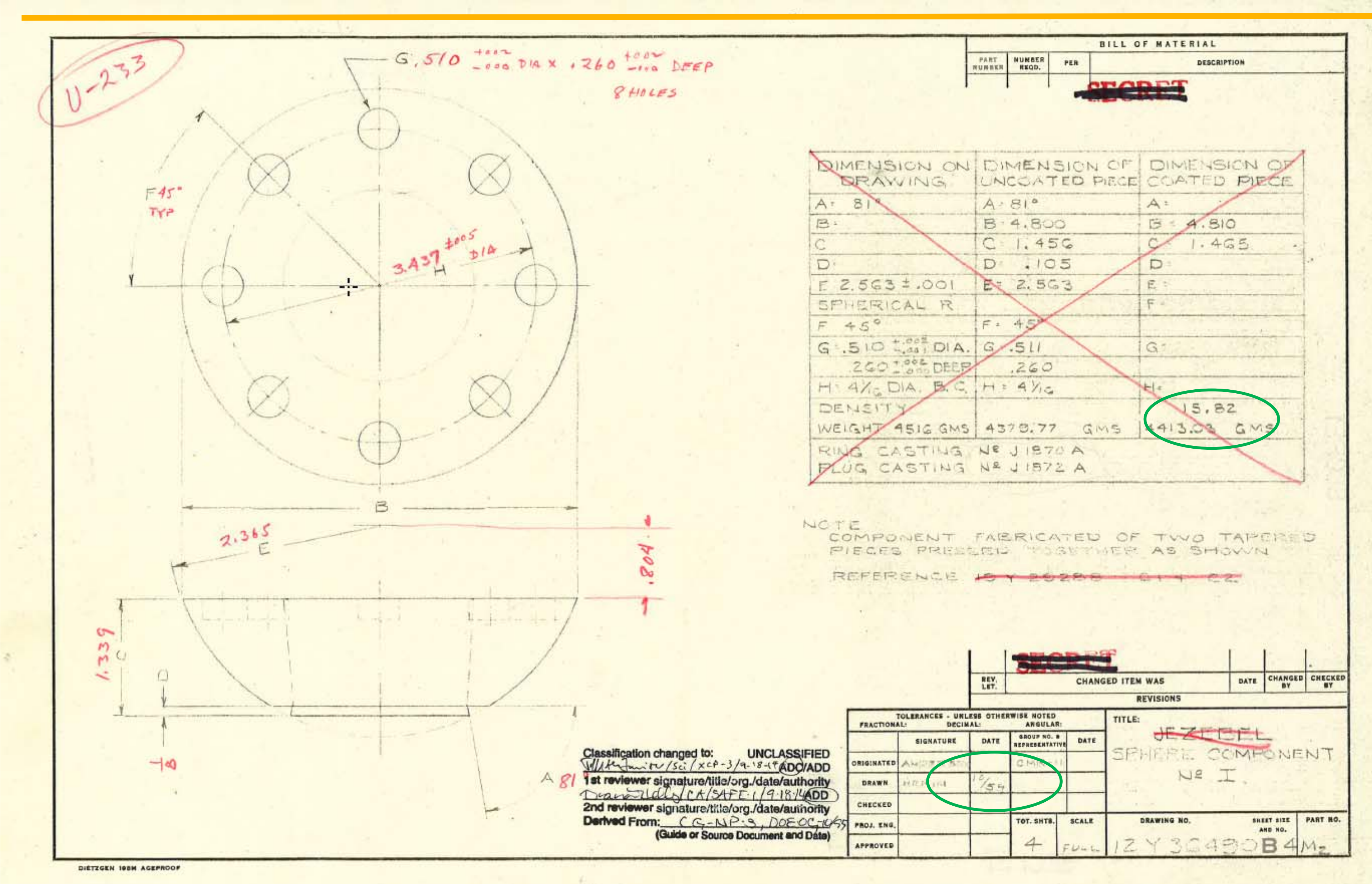
Piece No.	New Weight	Weight Change
JX-1889:1886	3975.23	- 72.69

Org. Weight 4047.92

NET WT. OF MATERIAL		SF NET OR T WEIGHT	
3966	40	3926	74
4088	81	4047	92
4201	30	4159	29

- *These masses include only the plutonium in the part, not the whole part!*

Major Discovery (Sept. 2014): As-Built Drawings (Example: Lower M2)



Interesting Discovery: Notes from 1969?

JEZEBEL

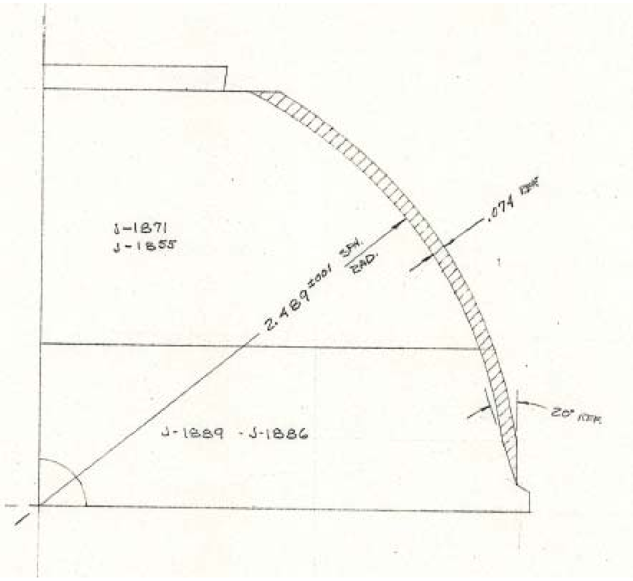
Part #'s + Description		wts 11/24/54	11/6/58	5/1/68	wts 9/21/69		Density
#1 J-1070 + J- ? Should be J-1870A and J-1872A Lower Safety Block Lower Part M2	Coated Alloy "T"	— 3966.40 3926.74			3966.40 3926.74		2 kilogram casting J-1070 12/19/50 was 15.751 Should be 15.82
#2 J-1889 + J-1886 Should be J-1889A and J-1886A Upper Center Upper Part M3	Coated Alloy "T"	— 4088.81 4049.92 Should be 4047.92 (see SF Material Transfer Receipt, Nov. 24, 1954)	4057.61 4015.38 3975.23	— 3995.00 3955.00	— 3995.00 3955.00		? Should be 15.84
#3 J-1883 + J-1858A Should be J-1883A (J-1858A is correct) Lower Center Lower Part M3	Coated Alloy "T"	4239.50 4201.30 4159.29		4180.00 4138.00	4180.00 4138.00		15.830
#4 J-1855A + J-1871A Upper Safety Upper Part M2	Coated Alloy "T"	4041.92 4001.50			4042.00 4002.00		15.782 Should be 15.78
	Total "T" wt.	16137.45 Should be 16135.45			16021.24 Should be 16021.74, assuming entries for parts are correct	Diff	116.21 Grams Should be 113.71

Incorporating the New Information into the Benchmark

- From the as-built drawings, we have major part densities and many dimensions from October 1954.

- From the Material Transfer Receipt, we have major part masses from late November 1954.

- In early November 1954, the four major parts were sent back to be remachined because they were too reactive.
  - "…1.2 kg of material was removed by decreasing the ball radius .075"."
  - We have a drawing for the remachining plan but no as-built dimensions.



- We have masses, densities, we have dimensions.
  - They are inconsistent.

- We accepted the masses and densities; we modified the dimensions to match.

Part	Rev. 4 Mass (g)	Rev. 3 Mass (g)	Difference Relative to Rev. 3	Rev. 4 Density (g/cm^3)	Rev. 3 Density (g/cm^3)	Difference Relative to Rev. 3
Upper M2	4041.92	4055.88	–0.344%	15.7800	15.5753	1.314%
Lower M2	3966.40	3981.12	–0.370%	15.8200	15.7082	0.712%
Upper M3	4088.81	4047.92	1.010%	15.8400	15.9045	–0.406%
Lower M3	4201.30	4213.67	–0.294%	15.8300	15.9797	–0.937%

Volume Change Needed to Match Benchmark Densities

- The spherical radius on the as-built drawings is 2.563 inches, but the radii were remachined to 2.489 inches.
  - We used the nominal remachined radius of 2.489 inches as the baseline spherical radius for all parts (same radius used in Rev. 3).
  - Otherwise, the dimensions from the as-built drawings were used for the nominal model, with only minor adjustments to correct ambiguities and obvious errors.

- The resulting calculated mass densities differed from the benchmark by far more than the drawing tolerances (typ. ±0.001 inch):

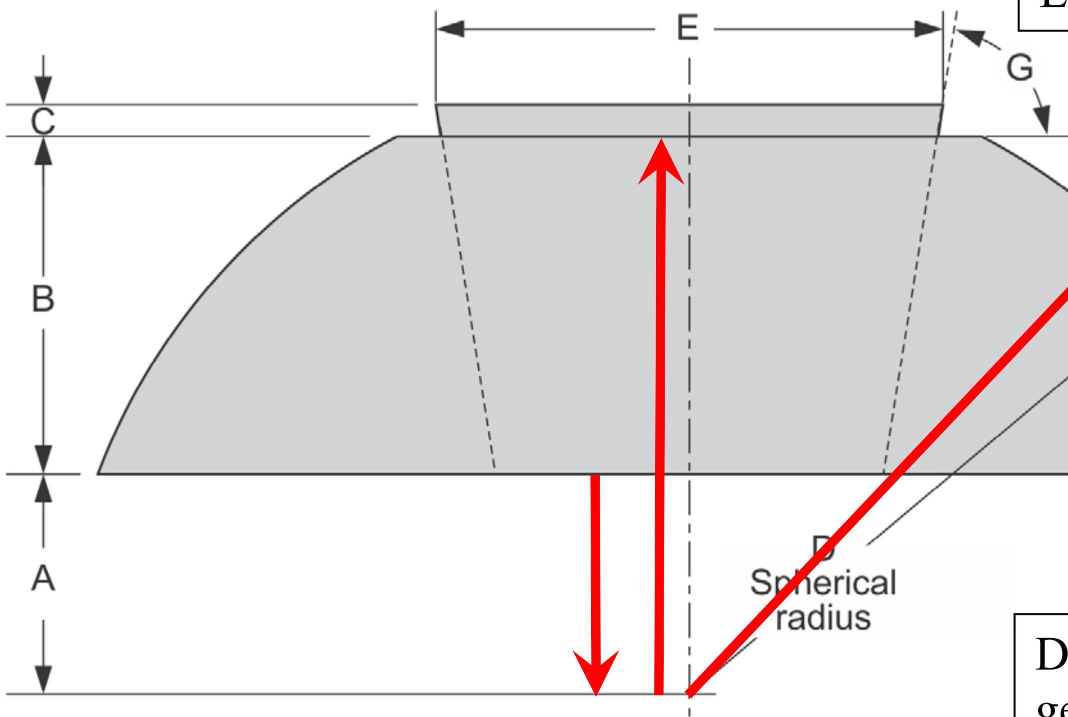
Part	Benchmark (g/cm^3)	Calculated (g/cm^3)	Volume Change Needed[a]
Upper M2	15.7800	15.5223	–1.633%
Lower M2	15.8200	15.7220	–0.619%
Upper M3	15.8400	15.9470	0.676%
Lower M3	15.8300	15.8178	–0.077%

[a] Relative to the volume calculated from the drawing dimensions.

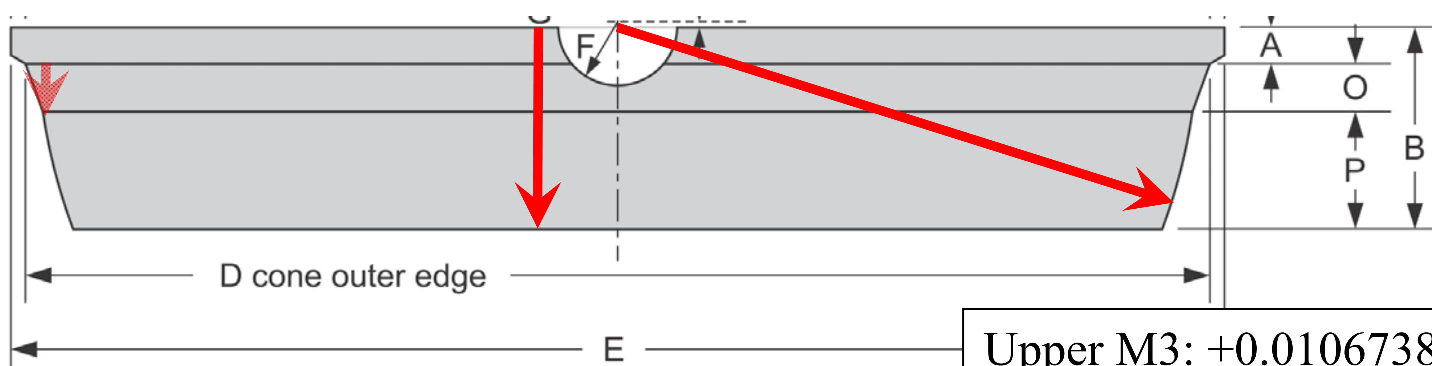
- We expanded or contracted each part nearly uniformly.

Modified Dimensions

Upper M2: –0.015707670 cm
Lower M2: –0.005754837 cm



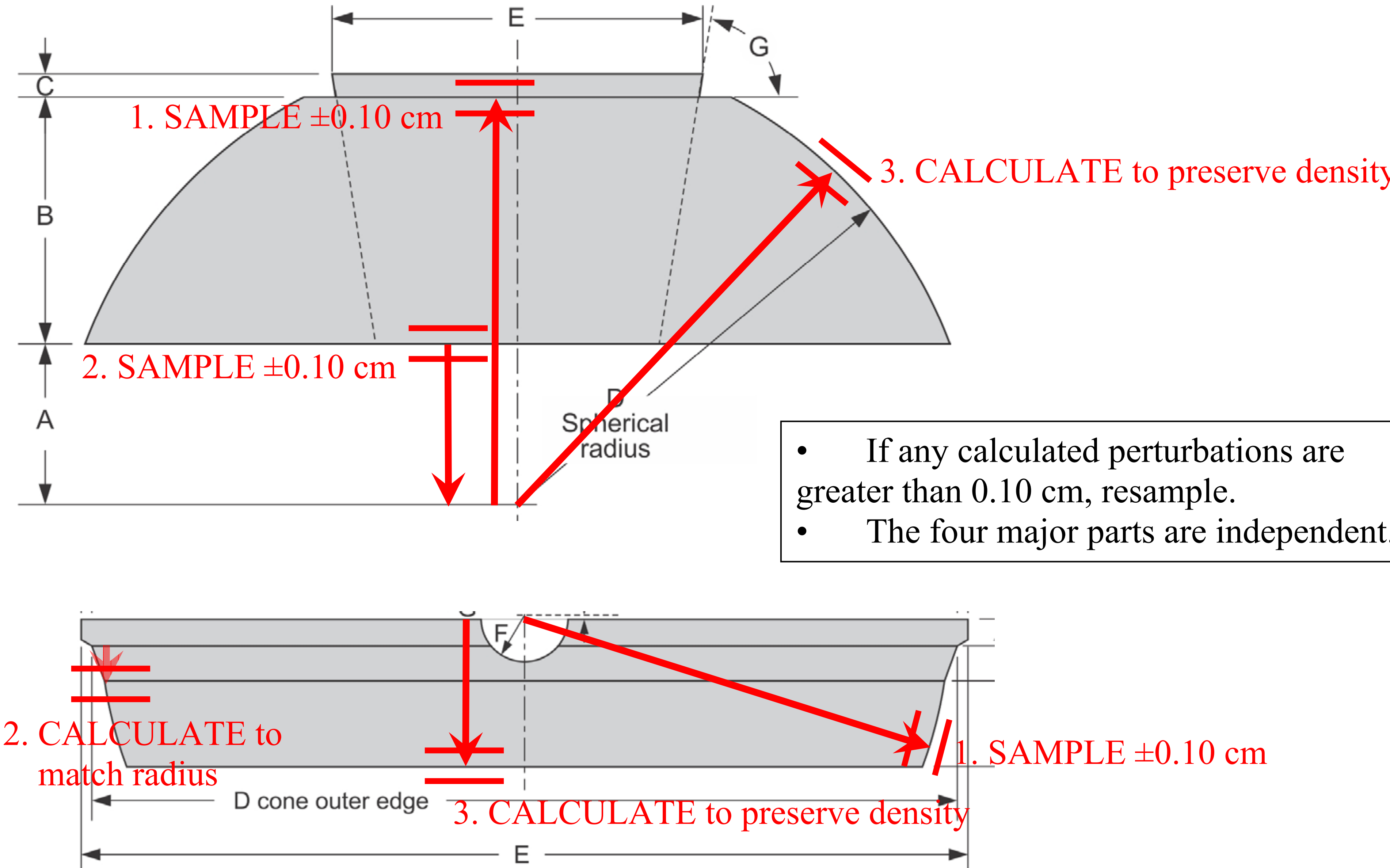
Dimension tolerances are generally 0.00254 cm



Upper M3: +0.010673814 cm
Lower M3: –0.001263601 cm

Estimating the Uncertainty: Brute-Force Sampling (1 of 2)

- Perturbations about the base case.



- If any calculated perturbations are greater than 0.10 cm, resample.
- The four major parts are independent.

Estimating the Uncertainty (2 of 2)

- 200 models were generated and k_{eff} was calculated for each configuration:

Configuration	Base k_{eff}	Mean k_{eff} of the 200	Std. Dev. of the 200	Difference Between Mean and Base k_{eff}
A	1.00067	1.00063	0.00005	–0.00004
B	1.00123	1.00121	0.00017	–0.00002
C	1.00092	1.00088	0.00013	–0.00004
D	1.00191	1.00187	0.00012	–0.00004

- Conclusion: Given fixed masses and densities of the major parts, the benchmark k_{eff} is only weakly dependent on the dimensions of the major parts.

- The largest standard deviation, ±0.00017, is used as the uncertainty in the benchmark k_{eff} due to uncertainty in the part dimensions.

Other Uncertainties

Source	$\delta k_{eff}/k_{eff}$	
	Rev. 3	**Rev. 4**
Dimensions of Major Pu-Alloy Parts (previous slides)	N/A	±0.00017
Correlated Pu-Alloy Mass, Dimensions, and Density (next slide)	±0.00094	+0.00064/ –0.00057
Lack of Planeness (Size of Gaps) Due to Nonuniform Nickel	±0.00056	±0.00052
Plutonium Isotopics	±0.00032	±0.00032
Nickel Plating Thickness and Density	±0.00053	±0.00047
Random Uncertainty (Deviation of the Samples About the Mean)	±0.00016	±0.00020
Control Rod Position	±0.00013	±0.00025
Aluminum Spacer	±0.00017	±0.00022
Other	±0.00011	±0.00019
Total	±0.00129	+0.00110/ –0.00107

Correlated Pu-Alloy Mass, Dimensions, and Density

- Densities were measured to 0.2% (not volumes!).
- The relative uncertainty in k_{eff} due to correlated mass and *density uncertainties* for each part independently is[f]

$$\left(\frac{\delta k_{eff}}{k_{eff}}\right)^2 = \left[\frac{V_d}{k_{eff}}\left(\frac{\partial k_{eff}}{\partial V_d}\right)_{\rho_d}\right]^2 \left[\left(\frac{u_{m_d}}{m_d}\right)^2 + \left(\frac{u_{\rho_d}}{\rho_d}\right)^2\right] + S^2_{k_{eff},\rho_d}\left(\frac{u_{\rho_d}}{\rho_d}\right)^2 - 2S_{k_{eff},\rho_d}\left[\frac{V_d}{k_{eff}}\left(\frac{\partial k_{eff}}{\partial V_d}\right)_{\rho_d}\right]\left(\frac{u_{\rho_d}}{\rho_d}\right)^2,$$

with $S_{k,\rho_d} \equiv \frac{\rho_d}{k_{eff}}\left(\frac{\partial k_{eff}}{\partial \rho_d}\right)_{V_d}$ and $\left(\frac{\partial k_{eff}}{\partial V_d}\right)_{\rho_d} = \frac{\sum_{n=1}^{N_d}\left(\partial k_{eff} / \partial r_n\right)_{\rho_d;r_m,m\neq n}}{\sum_{n=1}^{N_d}\left(\frac{1}{V_d}\partial V_d / \partial r_n\right)_{r_m,m\neq n}}$, where N_d is the number of linear dimensions describing part d.

- Results:

Part	u_m/m	u_ρ/ρ	$\delta k_{eff}/k_{eff}$
Upper M2	±0.025%	±0.2%	±0.00022
Lower M2	±0.025%	±0.2%	±0.00022
Upper M3	±0.025%	±0.2%	±0.00033
Lower M3	±0.025%	±0.2%	±0.00034
Upper M1′	±0.025%	+2%	+0.00001
Lower M1′	±0.025%	+2%	+0.00001
Control rod[(a)]	N/A	N/A	–0.00006
GH filler[(b)]	±1%	+2%	+0.00029
Buttons[(b)]	±0.025%	+2%	+0.00002
Total	N/A	N/A	+0.00064/–0.00057

(a) Evaluated separately.
(b) Density uncertainty only.

[f] J. A. Favorite and Z. Perkó, "The Uncertainty Due to Correlated Mass, Volume, and Density When Mass and Density are Measured," *Trans. Am. Nucl. Soc.*, **114**, 425-428 (June 2016).

Critical Mass: PU-MET-FAST-001 Rev. 4 (2016)

	Experimental k_{eff}	Calculated k_{eff}	Calc./Exp.
Config. A	0.99999 ± 0.00110	1.00067 ± 0.00002	1.00068 ± 0.00110
Config. B	1.00016 ± 0.00110	1.00123 ± 0.00002	1.00107 ± 0.00110
Config. C	1.00020 ± 0.00110	1.00092 ± 0.00002	1.00072 ± 0.00110
Config. D	1.00128 ± 0.00110	1.00191 ± 0.00002	1.00066 ± 0.00110
Average	–	–	1.00077 ± 0.00020

NEA/NSC/DOC(95)03/I
Volume I

PU-MET-FAST-001

BARE SPHERE OF PLUTONIUM-239 METAL
(4.5 at.% ^{240}Pu, 1.02 wt.% Ga)

Evaluator

Jeffrey A. Favorite
Los Alamos National Laboratory

Internal Reviewer

Roger W. Brewer
Los Alamos National Laboratory

Independent Reviewer

Michael Zerkle
Bettis Atomic Power Laboratory

- The Rev. 4 benchmark one-dimensional model was defined to be the one that gives k_{eff} = 1.00077 when ENDF-B/VII.1 cross sections are used.
  - + Mass = 17,073.2 ± 66 g Pu-alloy
  - + Density = 15.61 g/cm^3, same as previous benchmark (and the material is the same)
  - + Benchmark k_{eff} = 1.00000 ± 0.00110

- The Rev. 3 benchmark one-dimensional model was also the one that gave k_{eff} = 1.00077 when ENDF-B/VII.1 cross sections were used.
  - + Mass = 17,073.2 ± 77 g Pu-alloy
  - + Density = 15.61 g/cm^3, same as previous benchmark (and the material is the same)
  - + Benchmark k_{eff} = 1.00000 ± 0.00129

- These two reevaluated one-dimensional benchmarks are statistically indistinguishable from the previous one-dimensional benchmark, 17.02 kg ± 0.6%.

(15.61)
17.02±0.6%
(15.61)

One-Dimensional Model

- Jezebel is a one-dimensional bare sphere critical plutonium benchmark.
  - + Radius 6.3849 cm → 6.39157 cm (Revs. 3 and 4) (5-1/32 inches diam.) The difference in diameter is 0.005 inches (1/32 is 0.03125)
  - + Density 15.61 g/cm^3 → 15.61 g/cm^3 (Revs. 3 and 4)
  - + Mass 17,020 ± 100 g Pu alloy → 17,073.2 ± 77 g → 17,073.2 ± 66 g
  - + Material – gallium is separated into its isotopic constituents:

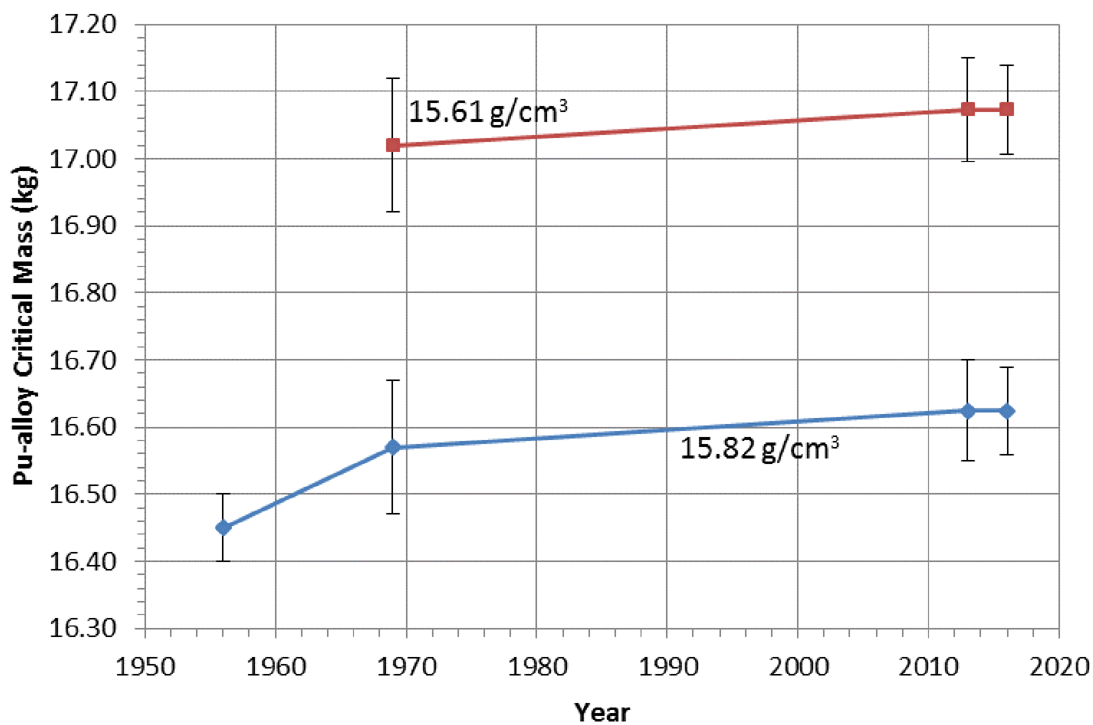
Nuclide	Atom Density (atoms/barn·cm)	Atom Fraction	Atom Fraction in Plutonium
^{69}Ga	8.2663×10^{-4}	2.0517×10^{-2}	N/A
^{71}Ga	5.4857×10^{-4}	1.3615×10^{-2}	N/A
^{239}Pu	3.7047×10^{-2}	9.1951×10^{-1}	0.952
^{240}Pu	1.7512×10^{-3}	4.3465×10^{-2}	0.045
^{241}Pu	1.1674×10^{-4}	2.8975×10^{-3}	0.003

- Benchmark k_{eff} 1.000 ± 0.002 → 1.00000 ± 0.00129 → 1.00000 ± 0.00110.
  - + ENDF/B-VII was tuned to the original one-dimensional Jezebel.
  - + The average C/E of the four detailed models, using ENDF/B-VII, is 1.00077 ± 0.00110.
  - + If the data were retuned to compute k_{eff} =1 for the new one-dimensional Jezebel, then it should compute C/E = 1 for the four detailed models.

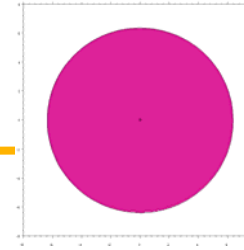
So What Is the Critical Mass of a Bare Sphere of Plutonium?

- The one-dimensional benchmark model uses 15.61 g/cm^3, determined in LA-4208.
- Using 15.82 g/cm^3, 1.02 wt.% Ga, Pu with 4.5 at.% ^{240}Pu:

Source	Year	Critical Mass of Pu-alloy (kg)
LA-2044	1956	16.45 ± 0.05
LA-4208	1969	16.57 ± 0.10
PU-MET-FAST-001 Rev. 3	2013	16.624 ± 0.075
PU-MET-FAST-001 Rev. 4	2016	16.624 ± 0.065

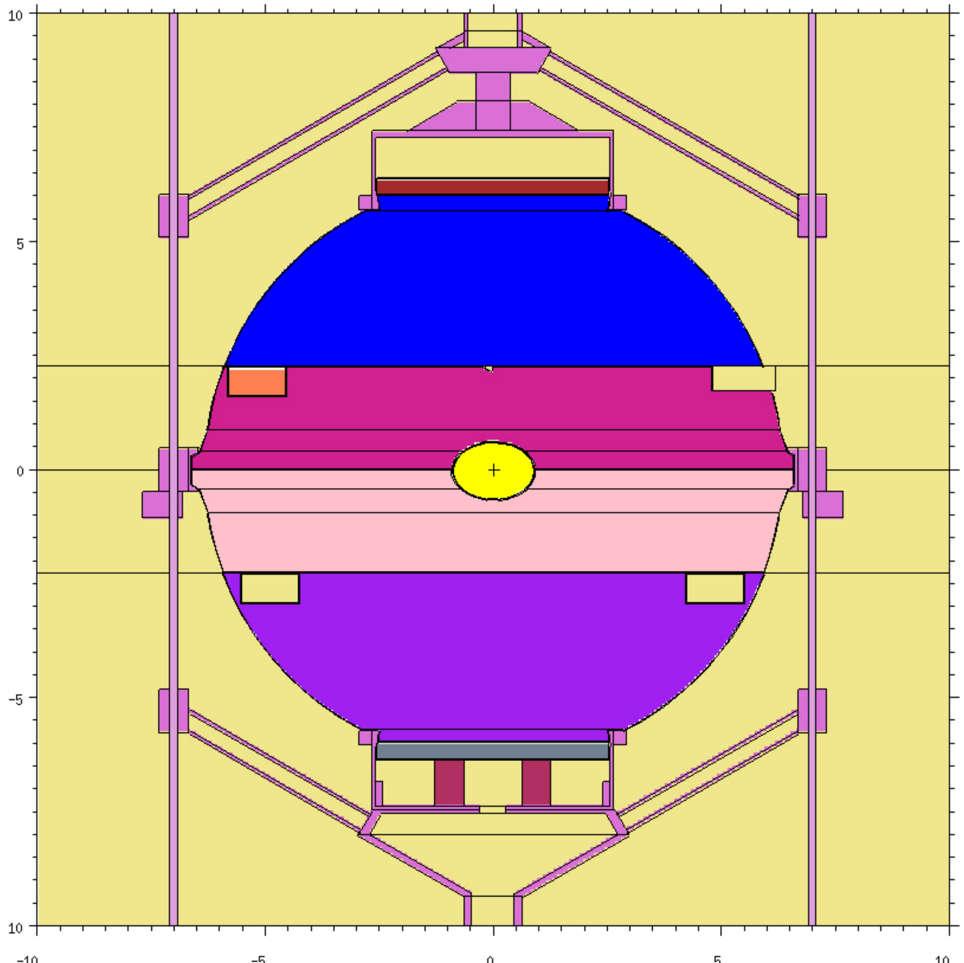


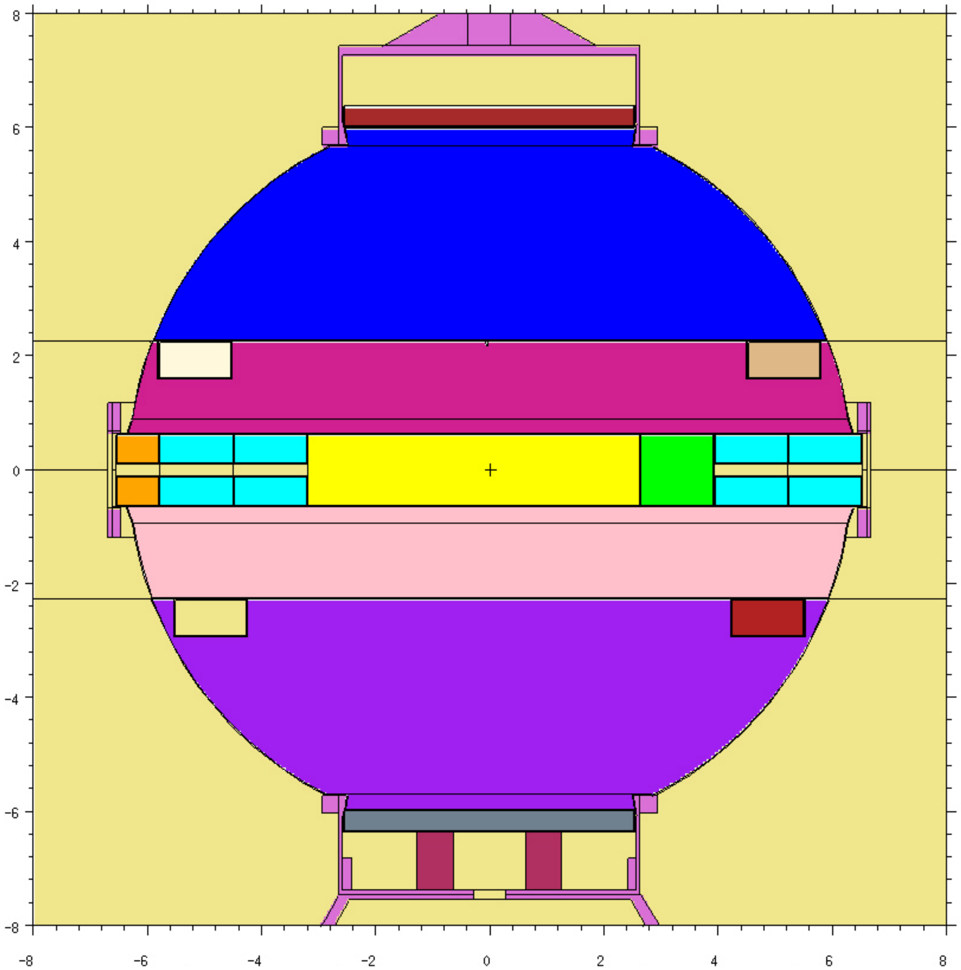
MCNP Renderings of Configuration B



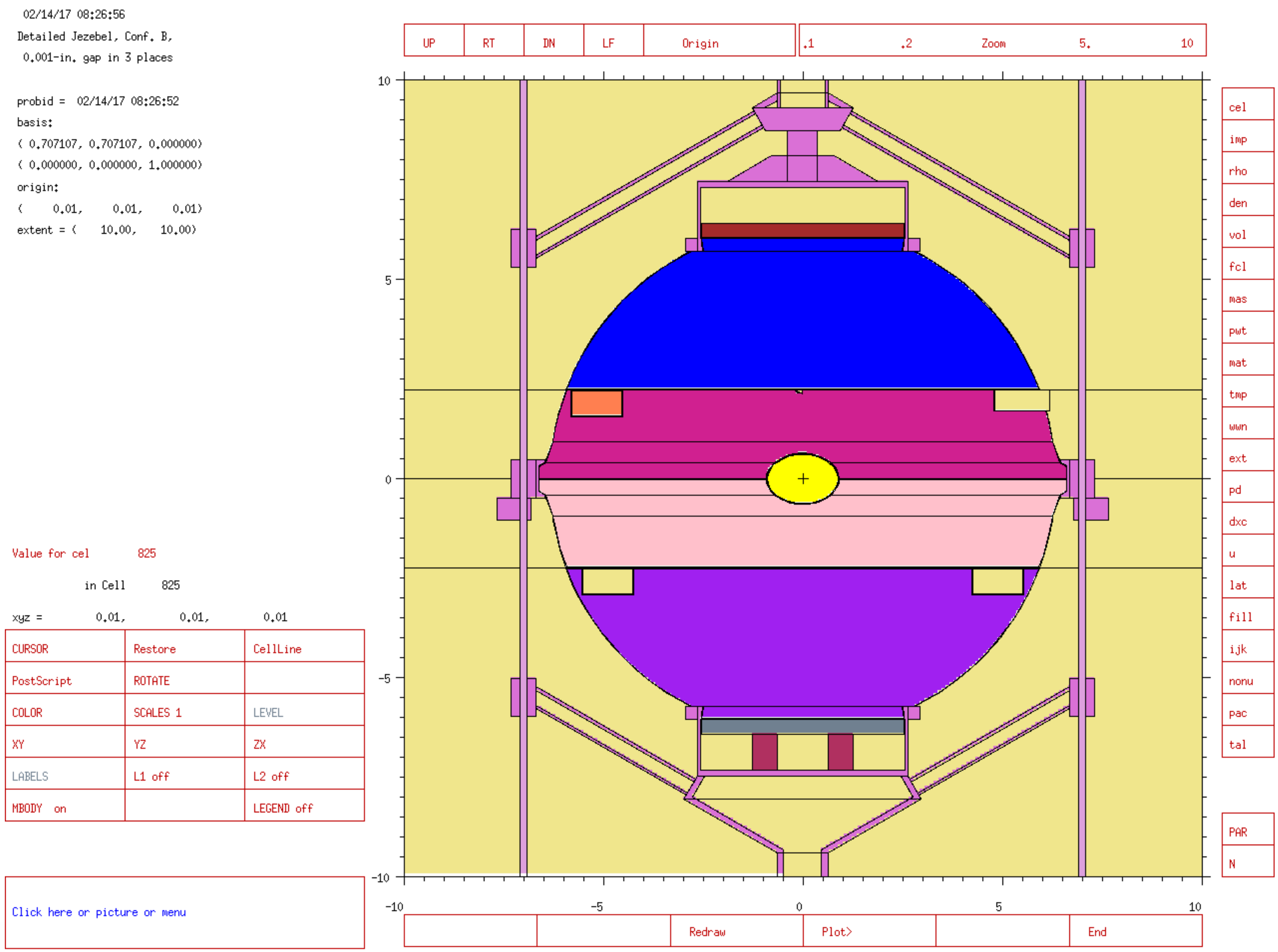
Spider assemblies, piano wire, belly band, wire lugs and clamps, control rod, mass adjustment buttons

Glory hole fill, mass adjustment buttons, external and internal nickel, thin polar end caps, aluminum shim

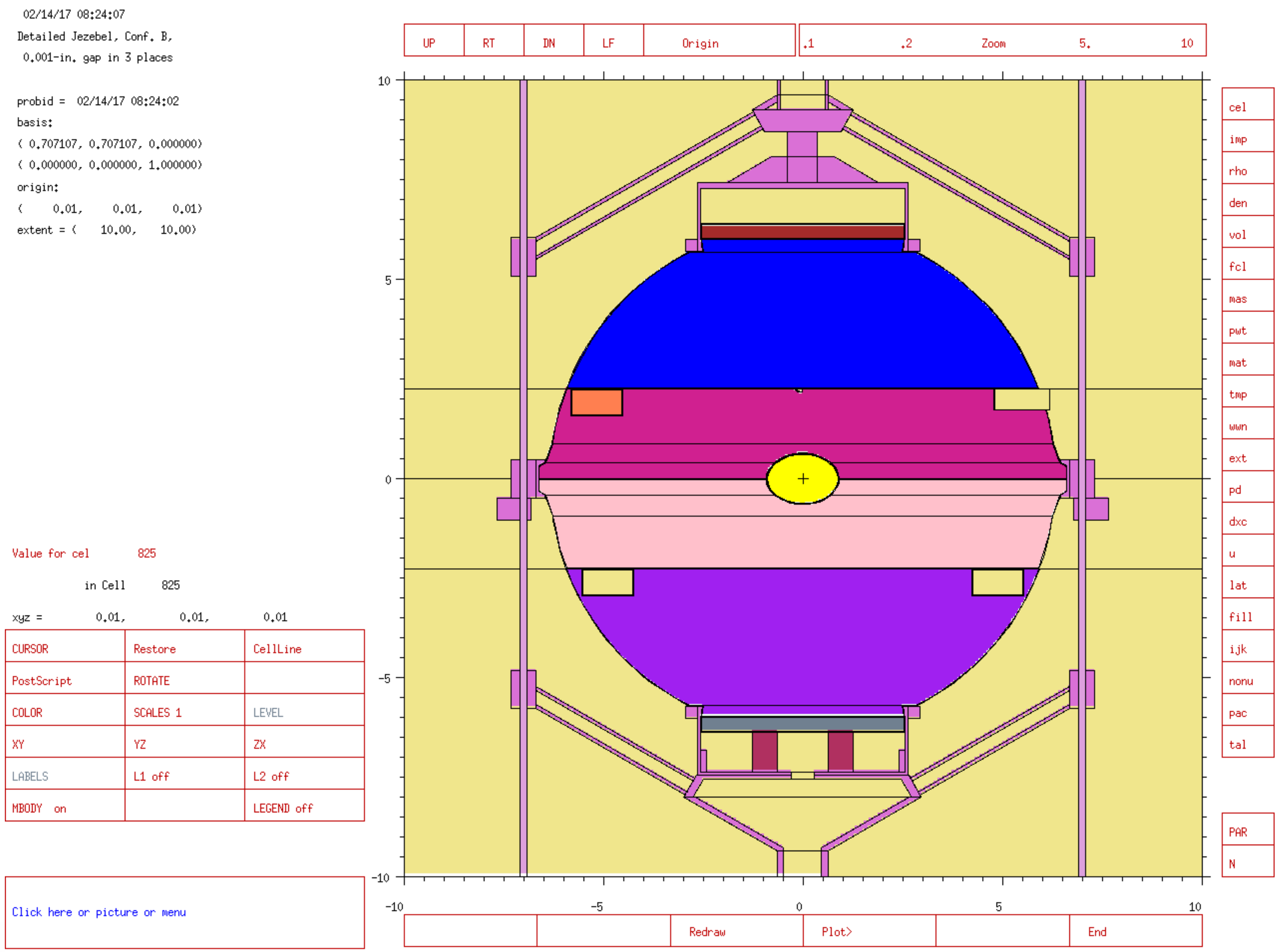




PU-MET-FAST-001 Rev. 3



PU-MET-FAST-001 Rev. 4



Is the Uncertainty Underestimated?

- The relative uncertainty in k_{eff} when the parts are treated independently:

Part	u_m/m	u_ρ/ρ	$\delta k_{eff}/k_{eff}$
Upper M2	±0.025%	±0.2%	±0.00022
Lower M2	±0.025%	±0.2%	±0.00022
Upper M3	±0.025%	±0.2%	±0.00033
Lower M3	±0.025%	±0.2%	±0.00034
Upper M1′	±0.025%	+2%	+0.00001
Lower M1′	±0.025%	+2%	+0.00001
Control rod[a]	N/A	N/A	–0.00006
GH filler[b]	±1%	+2%	+0.00029
Buttons[b]	±0.025%	+2%	+0.00002
Total	N/A	N/A	+0.00064/–0.00057

(a) Evaluated separately. (b) Density uncertainty only.

- What if the parts are correlated?

Assumed Correlation Coefficient	$\delta k_{eff}/k_{eff}$ due to Pu-Alloy Mass, Dimensions, and Density	Total Systematic $\delta k_{eff}/k_{eff}$	Total $\delta k_{eff}/k_{eff}$
0[a]	+0.00064 –0.00057	+0.00108 –0.00105	±0.00110
0.25	+0.00091 –0.00077	+0.00126 –0.00117	±0.00127
0.50	+0.00111 –0.00092	+0.00141 –0.00128	±0.00143
0.75	+0.00129 –0.00105	+0.00155 –0.00137	±0.00157
1	+0.00144 –0.00117	+0.00168 –0.00147	±0.00169

(a) This is the value assumed in the evaluation.

There is also evidence that $u_\rho/\rho = \pm 0.2\%$ is too small by half!

±0.00110 → ±0.00145;
±0.00169 → ±0.00250

Corrections in LA-4208 Compared with Three-Dimensional Calculations (Using Rev. 3)

- For Configuration B.
- Corrections are in kg Pu-alloy surface mass.

	Pu(4.5% ^{240}Pu) Config. A	Pu(4.5% ^{240}Pu) Config. B
Critical mass, kg[a]	16.761	16.784
(Density, g/cm^3)	(15.61)	(15.60)
Corrections, kg:		
Asphericity	-0.033	-0.047
Internal Ni and homogenization	0.047[b]	0.033[c]
Equatorial band	0.045	0.045
Polar supports	0.117	0.117
External Ni	0.074	0.074
Framework	0.002	0.002
Kiva reflection	0.010	0.010
Air reflection	0.004	0.004
Trace impurities[e]	-0.001	-0.001
Elevated temp.	-0.007	-0.007
Critical mass of homogeneous sphere, kg alloy	17.019	17.014
(Density, g alloy/cm^3)	(15.61)	(15.61)
		17.02±0.6%
		(15.61)

[a] Major cavities removed.

Effect	LA-4208	Calculated[(a)]
Asphericity	–0.047	–0.034 ± 0.002
Internal Ni & Homogenization	0.033	0.063 ± 0.002
Equatorial Band	0.045	0.040 ± 0.002
Polar Supports	0.117	0.118 ± 0.002
External Ni	0.074	0.074 ± 0.002
Framework	0.002	Not modeled
Building-Wall Reflection	0.010	0.008 ± 0.002
Air Reflection	0.004	0.005 ± 0.002
Trace Impurities	–0.001	0.006 ± 0.002
Elevated Temperatures	–0.007	–0.009 ± 0.002
Total	0.230	0.273[(b)] ± 0.006

(a) 1σ statistical uncertainties are given.
(b) Including 0.002 kg for the framework.

- Equivalent surface mass = $\frac{-\Delta k_{eff}}{\left(\partial k_{eff}/\partial m\right)_\rho}$, with $\left(\partial k_{eff}/\partial m\right)_\rho = 0.016825/\text{kg} \pm 0.117\%$.

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Summary and Conclusions

- The "original" (1969) Jezebel benchmark was a homogeneous bare sphere of Pu-alloy.

- In 2013, we reevaluated the classic Jezebel benchmark by modeling four actual experimental configurations as accurately as possible, with some assumptions.
  + The reevaluated critical mass was within the uncertainty of the original benchmark.

- Soon after, new data came to light establishing the part masses, mass densities, and some dimensions.
  + We found that we had made some wrong assumptions (and some right ones!).
  + In 2016, we reevaluated Jezebel again.
  + We assumed the part masses and mass densities are correct and we adjusted the dimensions to match.

- The average k_{eff} C/E for the four detailed configurations is 1.00077. The uncertainty $\delta k_{eff}/k_{eff}$ is ±0.00110.

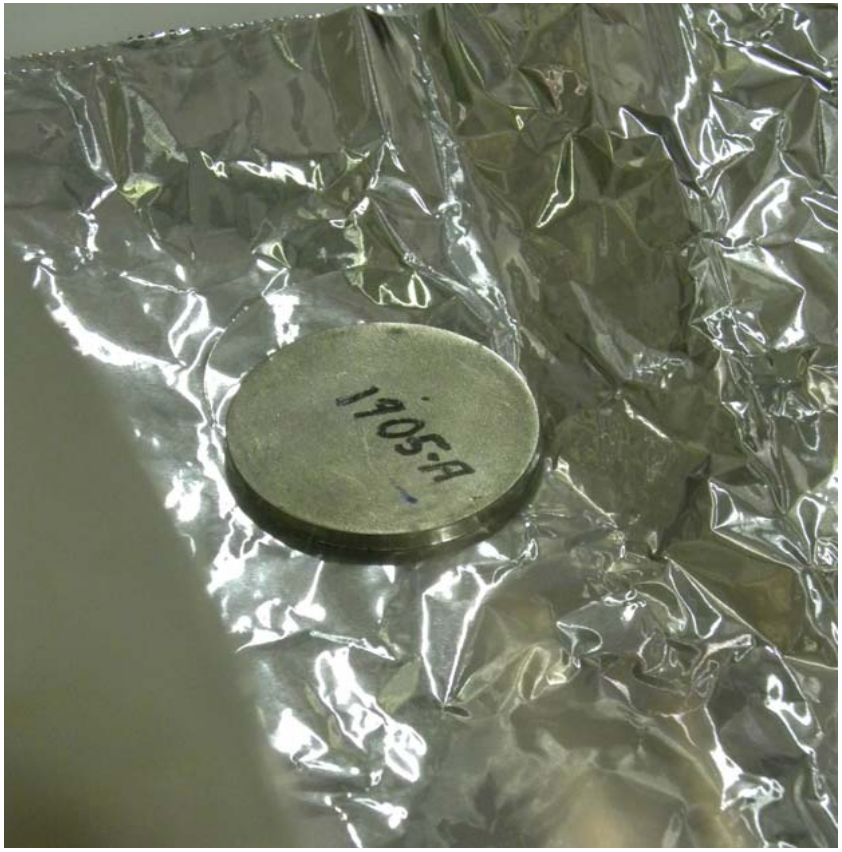
  + The average k_{eff} C/E for the four is the same as in 2013. The uncertainty is smaller (±0.00129 in 2013, ±0.00110 in 2016).

- The reevaluated one-dimensional simplification (17.0732 kg ± 0.066 kg Pu-alloy) is the same as in 2013 and is statistically indistinguishable from the "original" one (LA-4208; 17.02 kg ± 0.6%).

Lower M1′ Exists

- Has been at LANSCE (TA-53, LANL’s accelerator) since at least 2006, but nobody knew what it was.
- In ~2013, LANSCE decided to get rid of it.
- In December 2016 it was moved to the Nuclear Material Control & Accountability Group. (They now know of its historical significance.)

- Photos from when it was repackaged, Aug. 4, 2015:



2 inches diam. × 0.141 inches (~1/8+1/64), 114.47 g Pu-alloy