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Nuclear Targeting Terms for Engineers and Scientists

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Nuclear Targeting Terms for Engineers and Scientists

John W St. Ledger

Abstract

The Department of Defense has a methodology for targeting nuclear weapons, and a jargon that is used to communicate between the analysts, planners, aircrews, and missile crews. The typical engineer or scientist in the Department of Energy may not have been exposed to the nuclear weapons targeting terms and methods. This report provides an introduction to the terms and methodologies used for nuclear targeting. Its purpose is to prepare engineers and scientists to participate in wargames, exercises, and discussions with the Department of Defense. Terms such as Circular Error Probable, probability of hit and damage, damage expectancy, and the physical vulnerability system are discussed. Methods for compounding damage from multiple weapons applied to one target are presented.

Nuclear Targeting Terms for Engineers and Scientists

Introduction

So what do the terms CEP, probability of damage, damage expectancy, VNTK, weapon radius, R95, and damage sigma mean? These are a number of the terms used by nuclear planners to predict the damage to targets caused by nuclear weapons. This paper will introduce these terms to the reader, and give examples of how they are used. It will first define the probability of hit, and how it is calculated. This lays the ground work for explaining the Circular Error Probable (CEP) and Spherical Error Probable (SEP), and how they are used. Next, the paper discusses weapon impact or aiming error when the error is not circular. This is followed by defining the probability of damage using a cookie-cutter damage function. Then the term damage expectancy is explained. This leads to a discussion of how to compound damage when more than one weapon is used against a target. Finally, a review is given of the physical vulnerability system, and the Probability of Damage Calculator (PDCALC) computer code.

In the discussions below a number of equations are given to explain how calculations are actually performed. Engineers and physicists like to see the details of how things are calculated, but for many readers the important details are to understand the terms, the assumptions, and how they are used in nuclear targeting.

Probability of Hit

The probability of hit is simply the probability that a weapon will land on or within a target given an aimpoint that may be on the target or offset from the target. One fundamental assumption underlies most probability of hit calculations. That is that the distribution of bomb hits around an aimpoint are circular normally distributed. This means that the error in the x, and the y directions is normally (Gaussian) distributed, with the standard deviation of the errors being the same in both directions. The probability of hit equations are given below, starting with one dimensional targets, and moving up to 3 dimensions.

NOTE: There are actually four assumptions made when describing the weapon impact error: (ST1)

1. The errors in the x, y (and z) directions are statistically independent.
2. The distribution of the errors in each direction is normally (Gaussian) distributed.
3. The variance (or standard deviation) of the error in each direction is the same.
4. The mean point of impact is at the aimpoint.

When these assumptions are valid, then the terms CEP and SEP, which will be defined below, are correctly used. In most targeting calculations, the CEP or SEP is used as a measure of the impact error, it is a value given to the targeteer, and it is *assumed* to be valid.

Probability of Hit on a Line

Assuming a one-dimensional line target along the x axis, and that the probability of a bomb landing at a point x, near the aimpoint x_0, is normally distributed with a standard deviation of σ, then the probability of hit distribution is given by:

$$f(x; x_0, \sigma)dx = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-x_0}{\sigma}\right)^2} dx \qquad (1)$$

which is the normal, or Gaussian distribution.

Figure 1 shows the one-dimensional distribution function plotted against the x axis. Assume a line target starts at x_1 and ends at x_2. With the aimpoint at x_0, the probability of hitting the line target can be calculated by integrating the distribution function from x_1 to x_2. This is shown in equation 2.

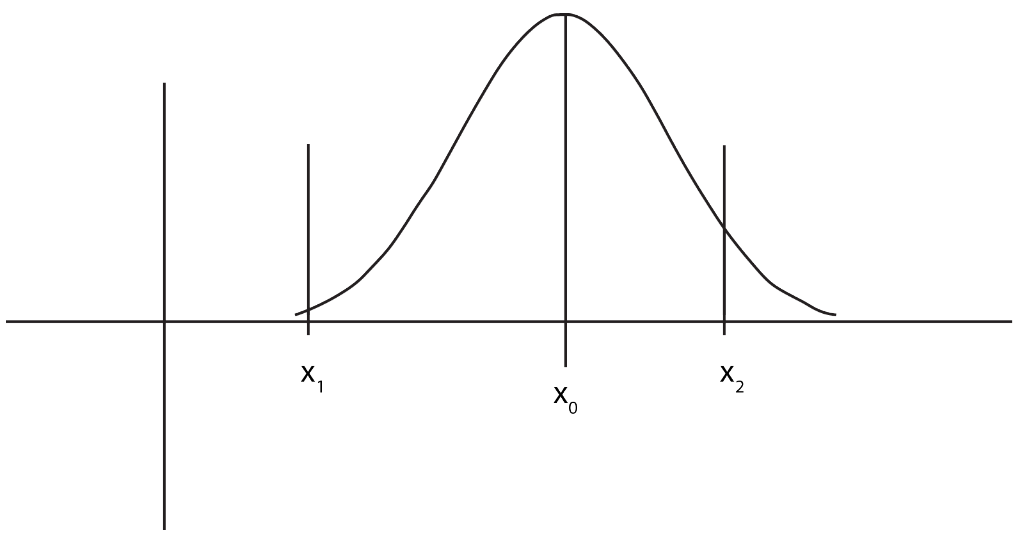

Figure 1—1D Probability of Hit Distribution

$$P(x_1, x_2) = \int_{x_1}^{x_2} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-x_0}{\sigma}\right)^2} dx \qquad (2)$$

The integral in equation 2 cannot be calculated analytically, but a number of approximations to the integral are available in the literature. (AB1)

Probability of Hit on a Rectangle

Figure 2 shows a rectangular target with a weapon aimpoint at (x_0, y_0). The rectangle extends from x_1 to x_2 in the x direction, and y_1 to y_2 in the y direction. The probability of the weapon hitting within the rectangle is given by equation 3. It is simply the product of the probability of hitting along the line x_1 to x_2, and the probability of hitting along the line y_1 to y_2. If σ_x equals σ_y, the hitpoint distribution is circular normally distributed. If σ_x and σ_y are not equal, then the hitpoint distribution is elliptically, as opposed to circularly, distributed. Once again, the underlying assumption is that the bomb impact errors are normally distributed about the aimpoint.

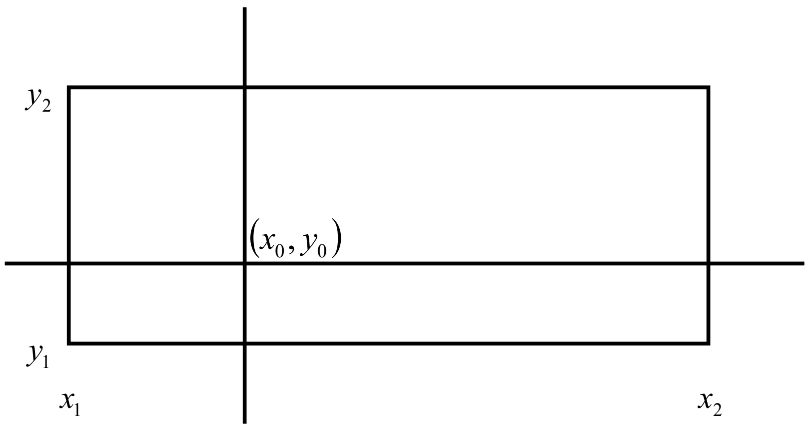

Figure 2—Aiming at a Rectangular Target

$$P(x_1, x_2, y_1, y_2) = \int_{x_1}^{x_2} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{1}{2}\left(\frac{x-x_0}{\sigma_x}\right)^2} dx \int_{y_1}^{y_2} \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{1}{2}\left(\frac{y-y_0}{\sigma_y}\right)^2} dy \quad (3)$$

Probability of Hit on a Circle

Figure 3 shows a circular target with a radius of R. Assume that σ_x equals σ_y, and that the aimpoint is at the center of the circle, which is at the origin. Then the hitpoint or weapon impact distribution is given in equation 4.

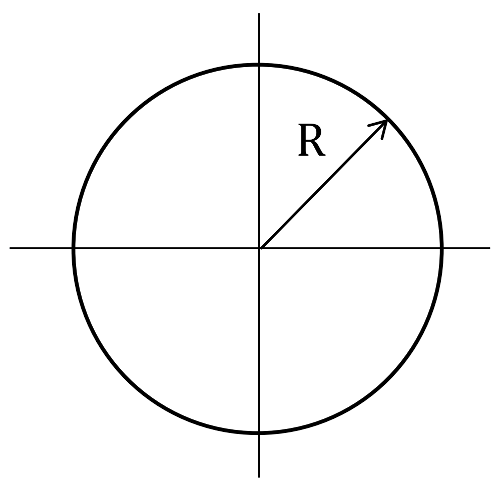

Figure 3—Circular Target

$$f(x, y, \sigma)dxdy = \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}\left(\frac{x^2+y^2}{\sigma^2}\right)} dxdy \quad (4)$$

In polar coordinates:

$$dxdy = rdrd\theta = 2\pi rdr \tag{5}$$

and

$$x^2 + y^2 = r^2 \tag{6}$$

Substituting equations 5 and 6 into equation 4, and integrating the distribution function across the circle yields the probability of the weapon hitting within the circle in equation 7.

$$P(0 \leq r \leq R) = \int_0^R \frac{r}{\sigma^2} e^{-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2} dr = 1 - e^{-\frac{R^2}{2\sigma^2}} \tag{7}$$

Circular Error Probable

The Circular Error Probable (CEP) is defined as the radius of a circle within which 50 percent of the bombs will hit. Operationally, test weapon drops are performed, and the along-track and cross-track standard deviations of the weapon impacts from the aimpoint are calculated to determine the CEP. One underlying assumption is that the hitpoint distribution is circular normally distributed. However, if the along-track and cross-track standard deviations are different, several formulas have been used to calculate the CEP as if the two standard deviations were equal. Some of these approximations will be presented below.

Using equation 7, and the definition of the CEP as the radius of a circle within which 50 percent of the weapons will fall, leads to:

$$0.50 = 1 - e^{-\frac{CEP^2}{2\sigma^2}} \tag{8}$$

Solving for the CEP gives:

$$CEP = \sqrt{ln(4)}\sigma \tag{9}$$

Using equation 9, equation 7 can be written as:

$$P(0 \leq r \leq R) = 1 - e^{-\ln(2)\frac{R^2}{CEP^2}} \tag{9a}$$

where R is the target radius.

The CEP is a single number that communicates to the analysts, crews and mission planners what the accuracy of a weapon is. A particular weapon will typically have different CEPs for different delivery conditions, fuze settings, or weather conditions. In nuclear war planning USSTRATCOM is the final arbiter of what CEP is correct for a given set of circumstances. For many analyses, such as setting CEP requirements for a new weapon, one CEP will often be used to estimate the effectiveness of a particular weapon design under all delivery conditions. Using equation 9a it is easy to show that 94% of the weapons will land within 2 CEP of the aimpoint, and about 100% of the weapons will land within 3 CEP. Only about 0.2% of the weapons will land outside of 3 CEP.

Offset Circle Probability of Hit

A common targeting problem is to calculate the probability of hit when the aimpoint is offset from the center of a circular target. There are at least two reasons why an aimpoint might be offset from the center of a target. Sometimes there are multiple targets in the target database which are located close to each other. An example would be an airfield with a fuel storage depot, a command post, and a weapon storage area. Rather than using 3 weapons to target the airfield, one weapon could be used with an aimpoint in the "center" of the targets, and then the probability of hitting each target could be calculated. Another example could be a target near a facility that we don't want to damage. For instance, an electric power distribution center could be a target near the electric power generators that we don't want to damage. We might be able to offset the aimpoint away from the generators to have a very low probability of damaging them, but still have a probability of damaging the distribution center. Figure 4 shows the offset aimpoint geometry. Unless the CEP is zero, there is a probability that the weapon will land within the distance R of the target.

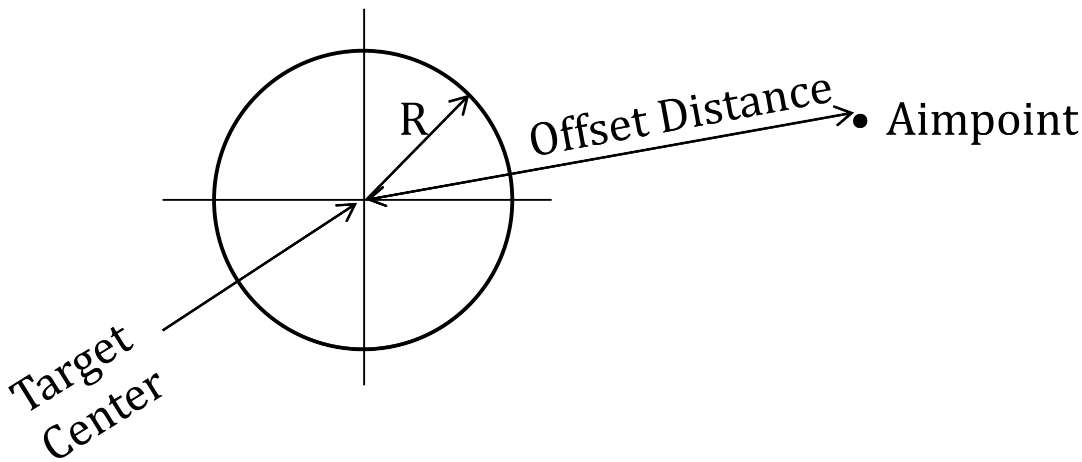

Figure 4—Offset Aimpoint Geometry

The offset circle problem cannot be solved analytically, but there are several methods that use an approximate solution to calculate the probability of hit for circular normal hitpoint distributions. Williams presents several approximations for calculating the probability of hit with different run times and accuracies. (WI1) The Germond-Wegner approximation is very accurate in the tails of the distribution. It is:

$$P(R, D, \sigma) = 1 - \phi\left(\frac{D}{\sigma} - \sqrt{\left(\frac{R}{\sigma}\right)^2 - 1}\right) \qquad (10)$$

where:
$P(R, D, \sigma)$ is the probability of hitting within the circle of radius R, with an aimpoint offset a distance D, a hitpoint standard deviation of σ, and
ϕ is the cumulative normal function.

This can be used when $R/\sigma > 5$. For other cases, a series approximation by Vitalis can provide answers accurate to better than 6E-07:

$$P(x,y) = e^{-y} \sum_{n=0}^{T} \frac{y^n}{n!} \left[1 - e^{-x} \sum_{m=0}^{n} \frac{x^m}{m!}\right] \tag{11}$$

where:
R is the target radius,
D is the offset aimpoint distance,

$x = \frac{1}{2}\left(\frac{R}{\sigma}\right)^2$

$y = \frac{1}{2}\left(\frac{D}{\sigma}\right)^2$

$T = 5 + 6\frac{D}{\sigma} + 0.31\left(\frac{D}{\sigma}\right)^2$

Equations 10 and 11 can be used to calculate the probability of hit for circular normal hitpoint distributions. For an elliptical hitpoint distribution, DiDonato has developed a numerical solution which is accurate to 8 significant figures or better. (DI1) The use of these methods will be illustrated in the probability of hit examples below.

Probability of Hit for a Polygon

DiDonato has developed a method to numerically calculate the probability of a weapon with an elliptical hitpoint distribution, and an offset aimpoint, hitting within any arbitrary polygon. The polygon may be simple or complex, concave or convex, with any winding. The method is much too complicated to reproduce here, but DiDonato was able to solve the problem by calculating the probability that the weapon would land outside of the polygon, and subtracting that value from 1. (DI2)

Offset Sphere Probability of Hit

Imagine a target at the center of a sphere of radius R, with an offset aimpoint a distance D from the target. The weapon fired at the aimpoint has a Spherical Error Probable. That is $\sigma_x = \sigma_y = \sigma_z = \sigma$., and 50 percent of the weapons detonate within 1 SEP of the aimpoint. Guenther shows that the probability of the weapon hitting within the sphere is: (GU1)

$$P(r,d) = \phi(d+r) - \phi(d-r) - \frac{1}{d\sqrt{2\pi}}\left[e^{-\frac{(d-r)^2}{2}} - e^{-\frac{(d+r)^2}{2}}\right] \tag{12}$$

where:
P (r, d) is the probability the weapon will land within the sphere of radius R, with the aimpoint offset a distance D where
r is R/σ,
d is D/σ, and
ϕ is the cumulative normal function.

Equation 12 is useful, except that targeting documents give weapon accuracy in terms of the SEP, not in terms of the sigma. When $\sigma_x = \sigma_y = \sigma_z = \sigma$, the probability of hitting within a sphere of radius r when aiming at the center is: (GU1)

$$P(r) = \sqrt{\frac{2}{\pi}} \int_0^r \rho^2 e^{-\frac{\rho^2}{2}} d\rho \qquad (13)$$

where:
P(r) is the probability the weapon detonates within the sphere of radius R, with $r = R/\sigma$,
$\rho = \frac{\sqrt{x^2 + y^2 + z^2}}{\sigma}$

By definition, when r is the SEP the probability is 0.50. Setting the integral in equation 13 to 0.50 when r is the SEP, and integrating by parts, gives:

$$\operatorname{erf}\left(\frac{SEP}{\sqrt{2}}\right) - \frac{SEP}{\sqrt{2}}\left(\frac{2}{\sqrt{\pi}}\right) e^{-\left(\frac{SEP}{\sqrt{2}}\right)^2} = 0.50 \qquad (14)$$

where erf is the error function. Solving for the SEP gives:

$$SEP \approx 1.5382\sigma \qquad (15)$$

Converting Sigmas to a CEP or an SEP

Sometimes the standard deviations of the hitpoint error may be given in a test document. They will probably be unequal, but the CEP or SEP may be needed. Stolle gives a formula for estimating the CEP when $\sigma_x \neq \sigma_y$ and $\sigma_x > \sigma_y$: (ST1)

$$CEP \approx 0.6125\sigma_y + 0.5640\sigma_x \quad for\ \frac{\sigma_y}{\sigma_x} \geq 0.3 \qquad (16)$$

$$CEP \approx 0.6745\sigma_x + 0.8200\frac{\sigma_y^2}{\sigma_x} - 0.0070\sigma_y \quad for\ \frac{\sigma_y}{\sigma_x} < 0.3 \qquad (17)$$

These equations give the CEP accurate to about 0.5% or better.

Childs gives a similar set of equations that can be used to estimate the SEP when $\sigma_x \neq \sigma_y \neq \sigma_z$ and $\sigma_x > \sigma_y > \sigma_z$: (CH1)

$$SEP \approx 0.670\sigma_x - 0.015\sigma_y - 0.066\sigma_z + 0.888\frac{\sigma_y^2}{\sigma_x} + 1.11\frac{\sigma_z^2}{\sigma_x}$$
$$\text{when } \frac{\sigma_y}{\sigma_x} \leq 0.3\ and\ \frac{\sigma_z}{\sigma_x} \leq 0.3 \qquad (18)$$

$$SEP \approx 0.558\sigma_x + 0.622\sigma_y + 0.283\sigma_z - 1.65\frac{\sigma_y\sigma_z}{\sigma_x}$$
$$\text{when } \frac{\sigma_y}{\sigma_x} \geq 0.3\ and\ \frac{\sigma_z}{\sigma_x} \leq 0.3 \qquad (19)$$

$$SEP \approx 0.462\sigma_x + 0.622\sigma_y + 0.621\sigma_z - 0.165\frac{\sigma_y\sigma_z}{\sigma_x}$$
$$\text{when } \frac{\sigma_y}{\sigma_x} \geq 0.3 \ and \ \frac{\sigma_z}{\sigma_x} \geq 0.3 \qquad (20)$$

These equations give the SEP accurate to within 2 percent. Of course, equations 9 and 15 should be used when the standard deviations are equal.

Probability of Hit Examples

The Los Alamos National Laboratory (LANL) Nuclear Weapon Analysis Tools has 13 separate applications for conducting simulations, calculating nuclear effects, and calculating probabilities of hit. (ST2)

The Nuclear Weapon Requirements Tools have a tab for calculating the probability of hit on circles, rectangles, and polygons. The application can also calculate optimum aimpoints and headings to maximize the probability of hit. (ST3) Figure 5 shows the top part of the probability of hit tab where the aiming error is set. The error can be either circular normal with a CEP, or elliptical normal with the standard deviations in the along track and cross track directions. The along track error is in the direction of the delivery heading. The probability of hit, or an optimum aimpoint and probability of hit may be calculated.

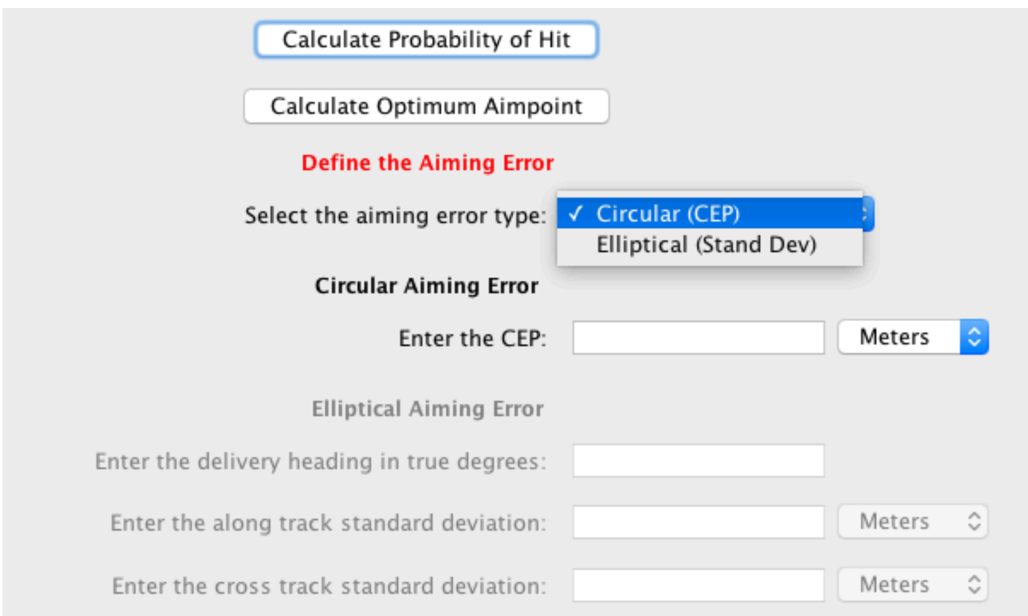

Figure 5—Setting the Hitpoint Distribution

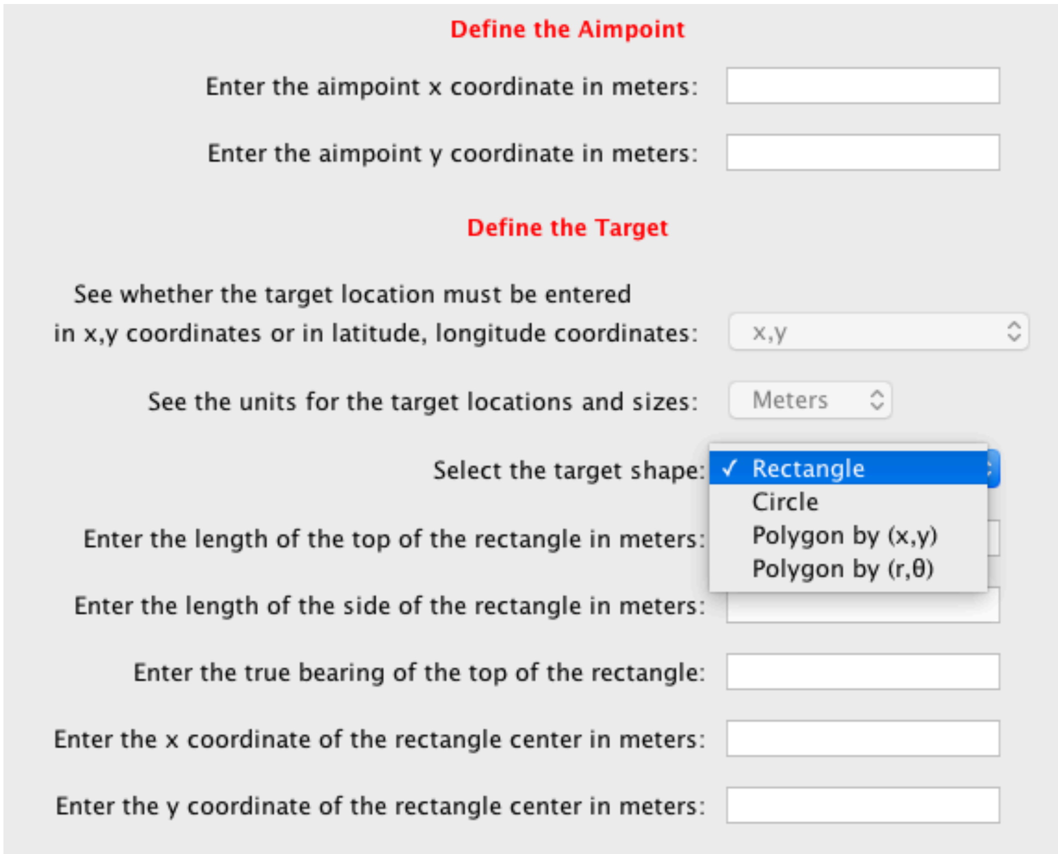

Figure 6—Setting the Aimpoint and Target

Figure 6 shows the bottom part of the probability of hit tab where the aimpoint and target are defined. The aimpoint may be defined with Cartesian coordinates or the latitude and longitude. Rectangular and circular targets are defined with their dimensions and the target center coordinates. Polygons are defined in a text file, with the coordinates of each vertex being on one line. The polygon vertices may be defined in Cartesian (x, y) coordinates, or in (r, θ) with each point being defined by a distance and bearing from some reference point.

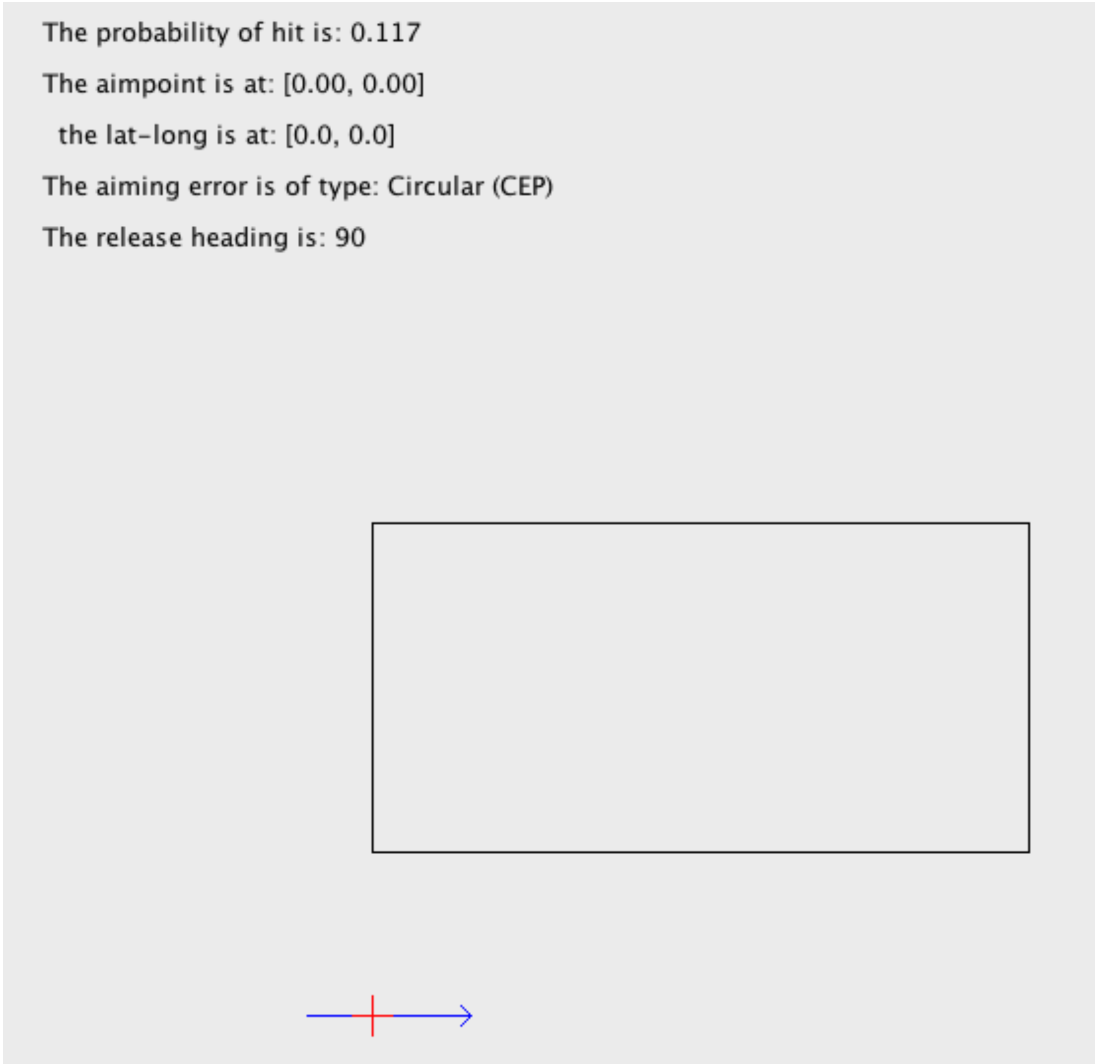

Figure 7—Offset Aimpoint on a Rectangle, Circular Error

Figure 7 shows the results of an offset aimpoint with a rectangular target. The rectangle is 10 meters long by 5 meters wide, and the rectangle center is at (5, 5). The hitpoint error is circular normal with a 5 meter CEP. The aimpoint is at (0, 0), and the aircraft heading is 090 degrees. The probability that a bomb will land within the rectangle is 0.117.

Figure 8 shows the results for an optimum aimpoint calculation on the same target as in Figure 7. The bomb now has an elliptical hitpoint distribution. The along track standard deviation is 10 meters, and the cross track standard deviation is 5 meters. The optimizer uses a genetic algorithm to find the optimum aimpoint and heading. The optimum aimpoint is at (5, 5), with a heading of either 090 or 270 degrees. The optimizer gets within 1 degree of the optimum heading, and within a few centimeters of the optimum aimpoint. The optimum probability of hit is actually 0.147, which matches the calculated value in Figure 8.

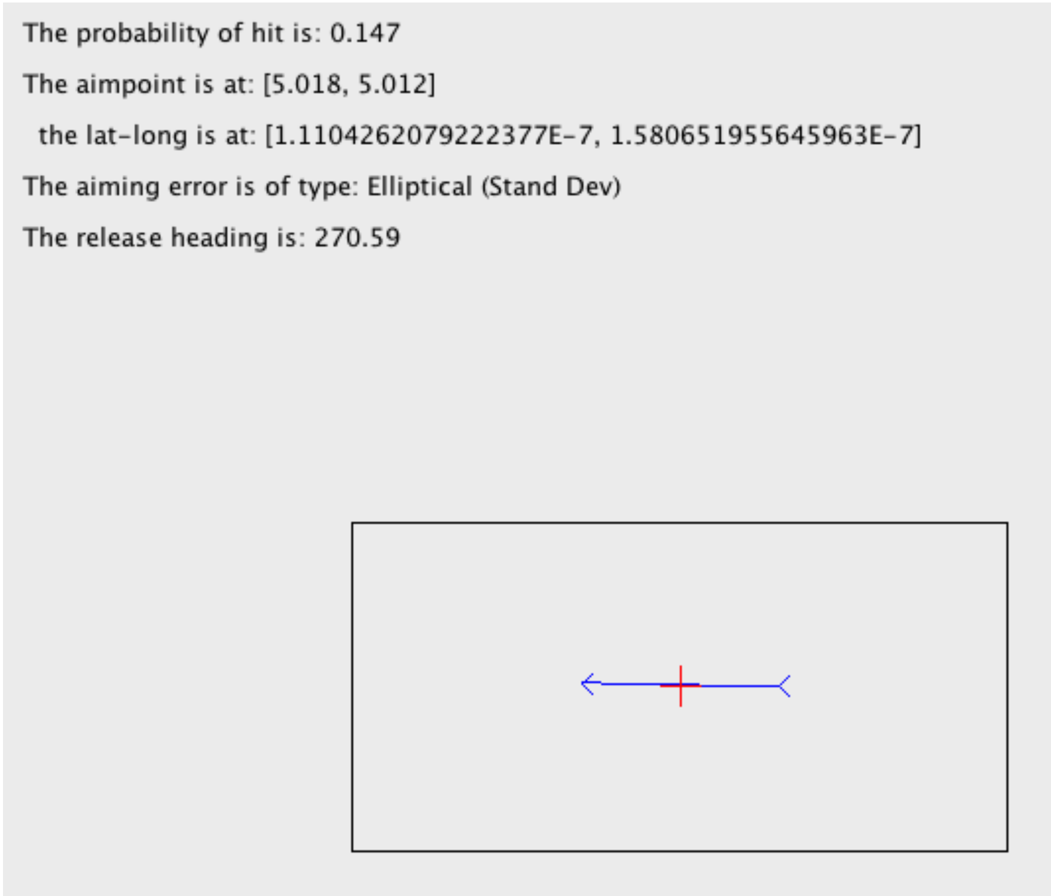

Figure 8—Optimum Aimpoint and Heading on a Rectangle, Elliptical Error

Figure 9 shows an optimum aimpoint calculation on a polygon, in this case a triangle with vertices at (0, 0), (5, 5), and (-5, 27). The calculated optimum aimpoint, heading, and probability of hit are shown.

Figure 10 shows the 3-dimensional probability of a weapon detonating ("hitting") within a sphere, with an offset aimpoint, and with the weapon having an SEP. This graph was generated using equation 12. The offset distance, and SEP have been normalized by dividing them by the sphere radius. Normalization like this is a common technique to use to make one graph or table that can be applied to many weapons, targets, and release conditions.

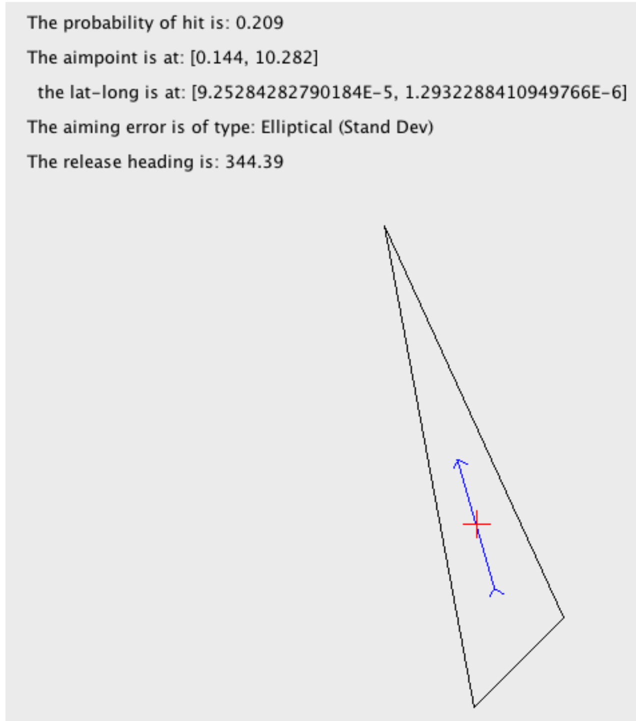

Figure 9—Optimum Aimpoint and Heading on a Polygon, Elliptical Error

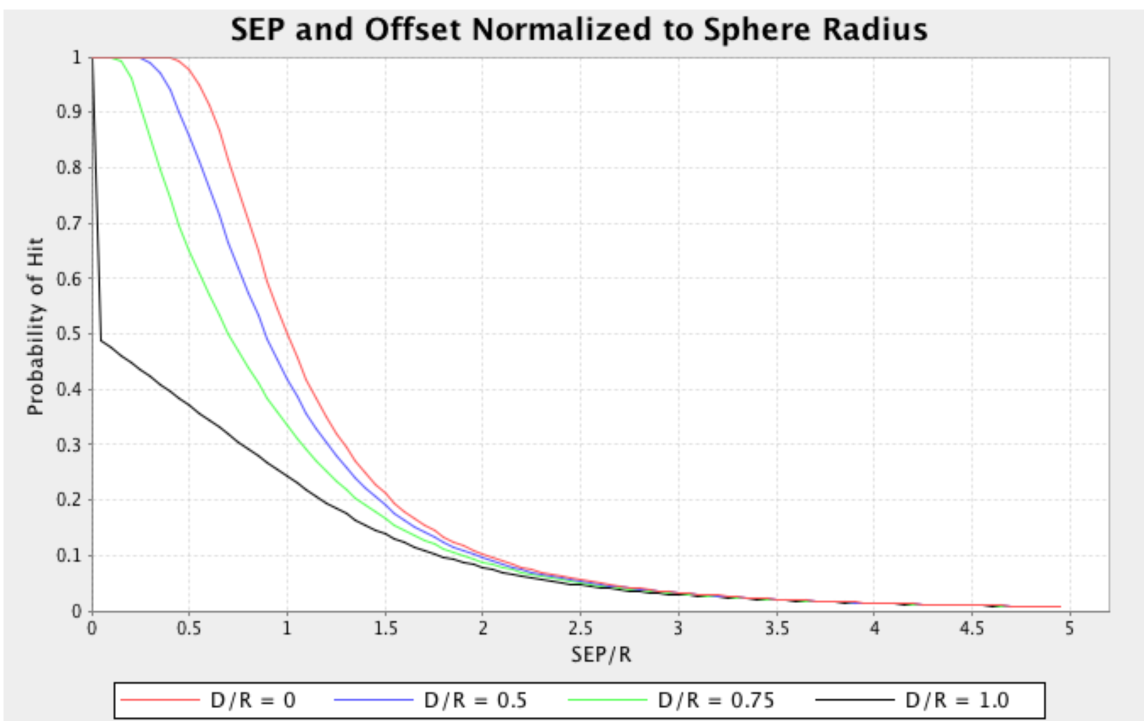

Figure 10—Probability of Hit for Offset Sphere

Probability of Damage

The probability of hit and probability of damage can be calculated using many of the same equations or approximations, but they are very different terms. As defined above, the probability of hit is defined as the probability that a weapon will land within some circle or polygon. But this definition says nothing about whether or not the target is damaged, or what type of damage is inflicted. For now, just assume that a hit within a circle or polygon implies some sort of damage to the target. The damage levels and probability of damage will be explored in the discussions below. However the probability of damage is defined, the probability of survival is just 1 minus the probability of damage.

Damage Levels

The probability of damage in nuclear targeting doesn't have meaning, unless the level of damage is also given. For instance, a probability of damage of 0.7 is ambiguous at best, and incorrect within the targeting community. However, a probability of damage of 0.7 for severe damage is correctly stated. This means there is a 70 percent chance that the target will suffer severe damage, *or worse*. For nuclear targeting, there are typically three damage levels: severe, moderate, and light. Severe damage means that the target suffers a degree of damage such that it cannot be repaired or reused. It must be replaced or rebuilt. For a building, severe damage implies structural collapse. For an aircraft inflight, it implies that the aircraft crashes. It is not flyable and cannot land. Moderate damage means that the target cannot be used for its intended purpose until major repairs are made. A building might have no windows, interior partitions may have been knocked down, elevators may not be working, and the electrical and plumbing systems may have to be replaced or undergo major repairs. An aircraft will not be able to continue its mission, and will have to perform an emergency landing as soon as possible. Light damage means that the target can continue to be used. Any repairs needed are minor. Buildings might need new windows, an aircraft in flight might have charred paint, but any needed maintenance can be delayed until after the mission is completed. Once again, the damage level is the minimum damage inflicted. A probability of damage of 0.7 for light damage says that there is a 70 percent chance that light damage *or worse* will occur.

In survivability studies, the terms sure kill, mission completion, and sure safe are sometimes used. For an aircraft, sure kill means that the aircraft will be destroyed. It is essentially the same as severe damage. Sure safe means that the aircraft is operational, it does not need repairs, and it can experience multiple nuclear detonations at the sure safe level and below, with no impact on the aircraft or its mission. Ionizing radiation damage is assumed to be negligible with this definition. Mission completion means that the aircraft is damaged *from one detonation*, it will take an experienced crew to complete the mission, and some aircraft capabilities are degraded. Mission completion damage levels sound rather benign, but a description from an Air Force Weapons Laboratory report gives some insight into the definition.

> "It should be emphasized that the aircraft, after exposure to the mission completion environments presented here, is no longer bright and shiny. After exposure to the 1 Mt detonation two or three miles away from the aircraft, the aircraft configuration is a structural and aerodynamic malaise. The paint is charred except where shadowed, thin skins have started to melt, secondary structure is deformed and debonded, control surfaces are marginally effective, and pitot-static instruments have been disrupted during shock passage. What has not been discussed is crew response to this emergency. Will their corrective actions be proper to keep the aircraft flying?... It is vital that the crew are aware of how bad things can get and still complete the mission." (AF1)

In a targeting analysis, the terms defensive conservative and offensive conservative may be used. Defensive conservative means that from the point of view of the defense, that the probability of *survival* from an enemy attack will be no lower than the estimate. For instance, let's say that the Air Force is building a new bomber. The aircraft is very early in the design stage, so the "exact" hardness of the aircraft to nuclear blast damage cannot be given. However, the aircraft engineers assure you that the sure safe blast level for the aircraft will be between 2 and 3 psi. The analysis may assume that the aircraft is sure safe below 2 psi, and sure kill at 2 psi and above. This is defensive conservative, in that it ensures that the calculated probability of survival for the defense will be no lower than the calculated value. It is conservative in that it gives credit to the offense for any uncertainty in the damage levels. If our analysis shows that 70 percent of the new bombers can survive a particular nuclear attack scenario, then conservatively *at least* 70 percent of our force is expected to survive. The actual survival rate will most likely be higher by some unknown amount.

The term offensive conservative may be used when we are assessing the capability of a weapon to damage a target. Offensive conservative means that from the point of view of the offense, that the probability of *damage* to a target will be no lower than the estimate. Let's say that the Air Force is building a new long range missile with a nuclear warhead that will be used to attack another country's nuclear missile silo. The Air Force intelligence officers tell you that we're not exactly sure how hard the missile silo is, but that it will be severely damaged somewhere between 10,000 and 15,000 psi of static overpressure. When we assess the capability of our new missile, we might assume that we need to deliver 15,000 psi against the target silo to cause severe damage. This is offensive conservative, in that we will calculate a conservative estimate of our ability to severely damage the missile silo. If our probability of severe damage is 0.90, we know that the probability of severe damage of 0.90 is a conservative estimate. We will actually do somewhat better with the new missile.

Cookie-Cutter Damage Function

A cookie-cutter damage function is like the circular target radius R in Figure 4. Figure 11 shows the cookie-cutter damage function. The P_d is the probability of damage to a given level. It is 1.0 anytime a weapon lands within a distance R of the target, and 0.0 everywhere else. In Figure 4, it is the radius R of the circle.

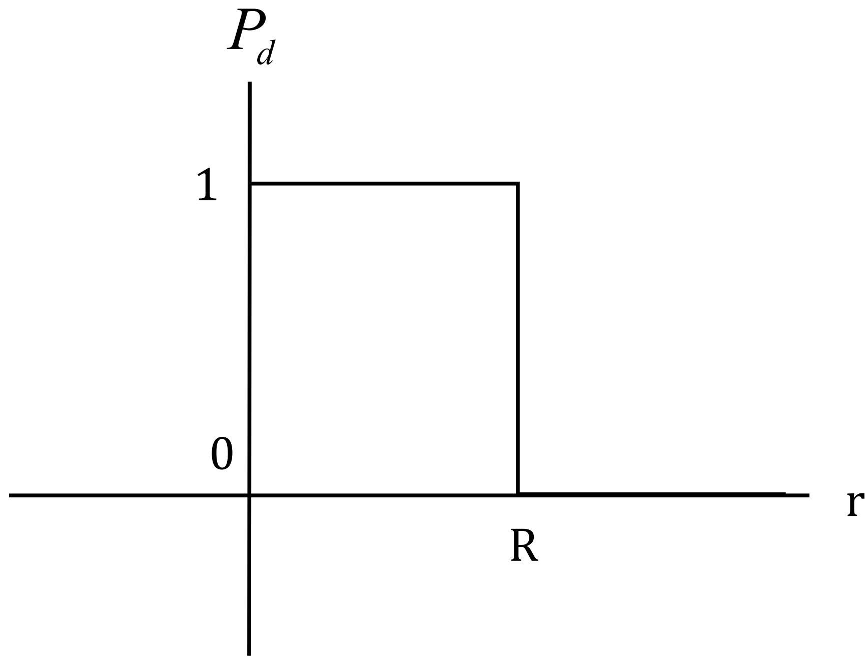

Figure 11—Cookie-Cutter Damage Function

The cookie-cutter damage function may either be specified as a distance or an environment level. If a distance is given for a damage level, then the probability of hit equations can be used to calculate the probability of damage. The probabilities will be the same. If the cookie-cutter damage function is given as an environment level, such as 2 psi of static overpressure, or 10 calories per square centimeter of thermal energy, then the environment must be converted to a range. The Simple Nuclear Effects Calculator (SNEC) in the LANL Nuclear Weapon Analysis Tools (ST4) can calculate a range for an environment level.

Figure 12 shows a nuclear detonation occurring above the ground. The HOB, or height of burst, is the distance from the detonation to the ground. The SR is the slant range distance from the detonation to the target. GR is the ground range. It is the distance along the ground from the point directly below the detonation to the target. If the ground range from the detonation to the target is less than R in Figure 11, then the target is damaged. If it is greater than R, then the target is undamaged. The ground range is the range used to calculate the probability of damage when the cookie cutter is given as a distance. In SNEC, the GR is called the ground range, and also the RTE, or range to effect.

Note: For targets on the ground, which is the most usual case, the cookie-cutter damage radius is always measured along the ground. For targets above the ground, the slant range may be used as the cookie-cutter damage radius. The analyst should explicitly say what the cookie-cutter damage radius is based upon.

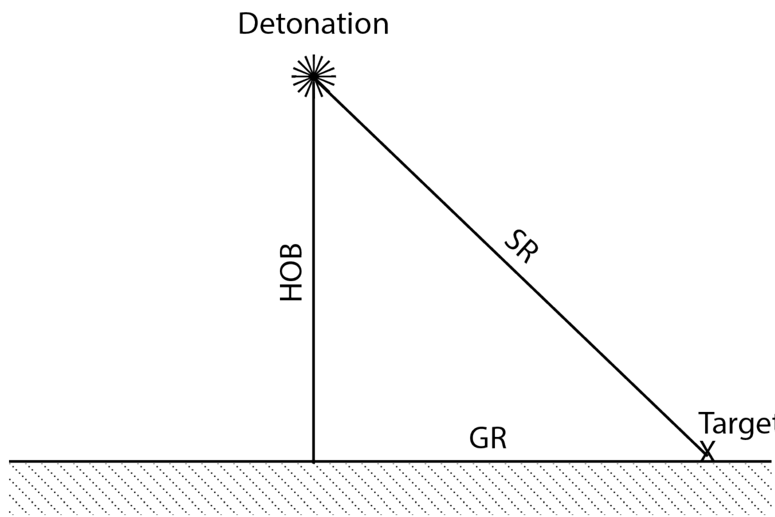

Figure 12—Detonation Geometry

Damage Expectancy

The probability of damage discussed so far is the predicted damage from one weapon detonation being applied against a target. In conducting targeting calculations, a term called the damage expectancy that accounts for several other probabilities is often used. For instance, a nuclear war plan will report the damage expectancy of a given option or scenario, rather than the probability of damage. The damage expectancy is usually defined as:

$$DE = PLS * PA * WSR * PD \tag{20}$$

where:
DE is the damage expectancy,
PLS is the probability of launch survival,
PA is the probability of arrival,
WSR is the weapon system reliability, and
PD is the probability of damage.

The probability of launch survival is the probability that a weapon system will survive an attack and be able to launch. For instance, an intercontinental ballistic missile will have a probability of surviving an attack and being able to launch from its silo. The PLS will have different values depending upon the scenario. If the scenario calls for the missile force to ride out an attack, the PLS will be lower than if the scenario has the missiles launching before the attacking force arrives. A bomber on ground alert will have a probability of surviving an attack on the alert airfield. The PLS will vary depending upon the alert status of the bombers, and upon the tactics used by the attacking force. The PLS value is calculated by USSTRATCOM.

The probability of arrival is the probability that a nuclear weapon that survives launch will survive to arrive at the target. For bombers or cruise missiles, this is the probability that the bomber or missile will survive the enemy air defenses enroute to the target. The PA value is calculated by USSTRATCOM.

The weapon system reliability is made up of two parts: the reliability of the weapon carrier, and the warhead reliability. The carrier reliability is the probability that the missile or aircraft will successfully fly to the target when the launch order is given. The warhead reliability is the probability that the weapon will detonate when a detonation is desired. The reliability of the weapon carrier is provided by USSTRATCOM, and the nuclear warhead reliability is provided by the Department of Energy.

Depending upon the weapon system, other probabilities may be included in the damage expectancy. For instance, the probability of surviving penetration into the ground could be included for an earth penetrating weapon.

The probability of damage may be calculated using some of the probability of hit and damage techniques discussed above. In many analyses, the probability of damage will be calculated by the Probability of Damage Calculator (PDCALC). This is a computer code sponsored by USSTRATCOM, and maintained by the Defense Threat Reduction Agency (DTRA). It is the official accredited method for calculating the probability of damage for nuclear war planning. A short description of the PDCALC methodology is given in the description of the Physical Vulnerability System below.

In some cases, intelligence estimates of foreign weapon systems will include a value for the weapon system reliability. An analysis may include the WSR in the probability of damage calculation, and call the combined value the probability of damage. This is not technically correct, but this approach is often used. The analyst should explicitly describe how the probability of damage was calculated.

Compounding of Damage

In the discussion to this point, the probability of damage, or the damage expectancy, has been described for one weapon used against one target. In targeting calculations if the probability of damage is not high enough from one detonation, then multiple weapons will be employed. Two methods for compounding the probability of damage from multiple weapons, compounding for damage, and compounding for reliability, are presented below. The more commonly used names are independent and dependent compounding. These two methods have been used in past analyses.

Independent Compounding (Compounding for Damage)

The term compounding for damage has been used for some nuclear analysis, but it is not a generally recognized term within the targeting community. It is more generally called independent compounding. This type of compounding is used when the results of a weapon detonation are independent of the results of all other detonations. To illustrate the compounding rules, first assume that we have 100 identical targets, and we want to achieve a probability of damage of 0.8 against this target set. Further assume that the probability of damage for one weapon used against one target is 0.50. If we fire 100 weapons at the target set, then on average, 50 targets will have been damaged. If we fire another 100 weapons at the target set, then 75 targets will have been damaged, and if we fire an additional 100 weapons, then on average 87.5 targets will have been damaged. It takes 3 weapons applied against each target to achieve a 0.80 probability of damage against the target set. The independently compounded probability of damage can be calculated from:

$$PD(n) = 1 - PS^n = 1 - (1 - PD)^n \qquad (21)$$

where:
PD(n) is the compounded probability of damage after n weapons are used on a target,
PS is the probability of survival after 1 weapon is used,
PD is the probability of damage after 1 weapon is used, and
n is the number of weapons used.

There are a couple of assumptions that are inherent to this method. One is that the targets are identical, in that they have the same probability of damage for one weapon being used. Another is that the targets don't change after a detonation occurs. The probability of damage does not change after one or a series of detonations occurs. The probability of damage for a single weapon is independent of the number of weapons used. The target hardness is independent of the number of weapons used. Compounding for damage means to apply more and more weapons until we have achieved our probability of damage goal.

Dependent Compounding (Compounding for Reliability)

Dependent compounding, or compounding for reliability means that we need to have at least one weapon function to achieve our damage goal. To illustrate this case, assume we are attacking a missile silo. Assume that our damage goal is 0.90, but that our weapon system reliability is 0.85, and our probability of damage is 0.95, *if our weapon is reliable*.

Let's say that we use a shoot-look-shoot tactic. That is, we attack with the first weapon. If reconnaissance shows that the target is not destroyed, then we attack with a second weapon, and so on until the target is destroyed. Since the attacks are independent, we can use equation 21 to calculate the probability of damage after n weapons are used.

Now let's assume that we don't want to use a look-shoot-look attack. If we wait on our reconnaissance assets, the enemy will have time to launch any surviving missiles against us. Therefore, we will launch our weapons so that they all arrive at about the same time. In this case, if the first weapon detonates, we assume that the following weapons are destroyed by the first detonation. The detonations are now dependent upon each other. What we need to calculate is the probability that at least weapon will detonate if n weapons are employed. To calculate the dependent probability, let's first modify equation 21 to include the weapon system reliability:

$$PD(n) = 1 - PS^n = 1 - (1 - R \cdot PD)^n \qquad (22)$$

where all of the terms are as defined in equation 21, and R is the WSR, or weapon system reliability. Given the WSR, and employing n weapons, the probability that at least one detonation will occur is:

$$R_n = 1 - (1 - WSR)^n \qquad (23)$$

and equation 22 can be rewritten as:

$$PD(n) = 1 - PS_n = 1 - (1 - R_n \cdot PD) \qquad (23)$$

where
PD(n) is the dependent probability of damage for n weapons used, or the probability of damage for one weapon times the probability that at least one weapon will detonate,
PS_n is the probability of survival if n weapons are employed, and
R_n is the adjusted weapon reliability from equation 23.

With the assumed values from above: the WSR equals 0.85, the damage goal is 0.90, and the single weapon PD is 0.95, the probability of damage using one weapon is 0.81, and using 2 weapons it is 0.93. It takes two weapons to meet our damage goal.

Physical Vulnerability System

The methods of calculating the probability of damage discussed above are often used in the analysis and vulnerability of weapons systems, and the effectiveness of weapons. These types of analyses are performed when new scenarios are introduced into war games, or when the employment of a weapon system or the threat to a weapon system changes. These kinds of techniques are also used to help set requirements for new weapon systems early in the DoD acquisition process. However, nuclear targeting and war plan development is done using the physical vulnerability system, sometimes called the VNTK system.

The physical vulnerability system has been in use since the early 1950s, and continues to be modified as new requirements emerge for nuclear targeting. Chapter 15 of Bridgman give a concise overview of the history of the development of the physical vulnerability system. (BR1)

One of the important features of the physical vulnerability system is that the damage functions are not cookie cutters or step functions, but they are distributions. This introduces some complexity to the math of calculating the probability of damage. The physical vulnerability system and its terms will be introduced below. The results will be presented without derivation. The interested reader can read Dorsch to see the mathematical details of the system. (DO1)

VNTK

VNTK (pronounced one letter at a time) stands for Vulnerability Number, T factor, and K factor. The vulnerability number for a target can be entered into a formula, and the vulnerability of the target to a nuclear environment can be calculated. The T factor determines the slope of the damage function, or it says how "fuzzy" the damage function is. The K factor describes how sensitive the target vulnerability is to changes in the weapon yield.

There are three basic target types usually covered in an introduction to the VNTK system: P type, Q type, and Z type. P type targets are vulnerable to the static over pressure of a blast wave. The static over pressure is the increase in air pressure behind the blast front. Q type targets are vulnerable to the dynamic pressure of the blast wave. The dynamic pressure is the force of the wind behind the blast front. Z type targets are generally shallow buried targets that are vulnerable to cratering effects. Several other target types are used in the physical vulnerability system, and they are discussed below.

Damage Function

When the VNTK system was being developed, an attempt was made to define the probability of damage as a function of a blast effect (over pressure, dynamic pressure, or crater radius). The physical vulnerability system uses a cumulative log-normal function to calculate the probability of damage as a function of the over pressure, dynamic pressure, or crater radius. The data does not exactly fit a log normal, but of all of the functions tried the log normal function most nearly matched the data from Hiroshima, and Nagasaki. (BI1)

Figure 13 shows the probability of damage for a 12P0 target as a function of static overpressure. A target with a VN of 12 has a probability of damage of 0.50 at 10 psi. Figure 13 shows that the damage function is "fuzzy", in that it is not a cookie cutter with a very sharp edge. The T factor in the VNTK is related to the log of the standard deviation of the overpressure. For a P type target, the T factor can be one of the letters L, M, N, O, P. The letter defines how steep or flat the probability of damage curve is. Figure 14 shows how the probability of damage curve changes with the T factor. The curves get flatter or steeper, depending upon the T factor. Notice that the steepness of the curves is not in alphabetical order. This is for historical reasons. The Hiroshima and Nagasaki T factor for P type targets was a P. When the other letters were added, the P value was left unchanged.

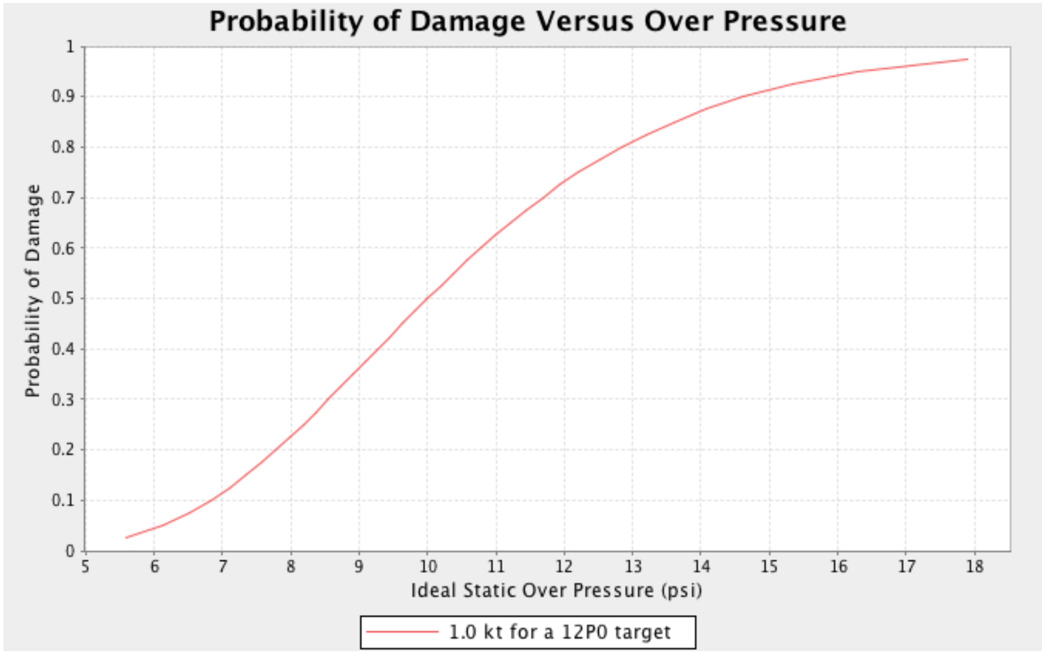

Figure 13—Probability of Damage versus Overpressure

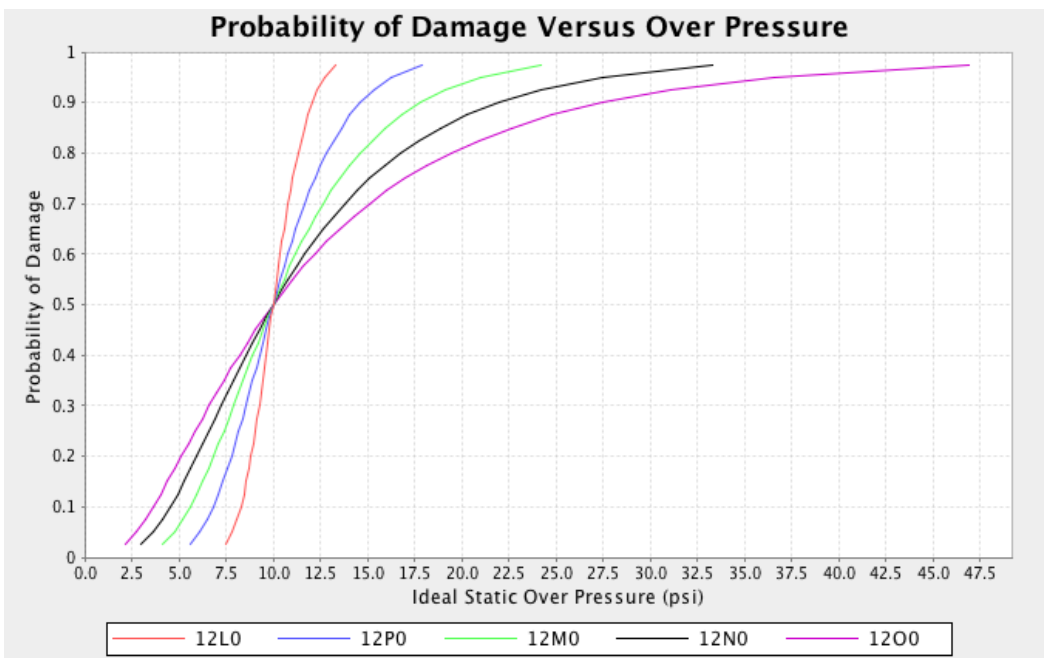

Figure 14—Probability of Damage Variation with T Factor

The T factor for a Q type target can be any of the letters Q, R, S, T, U. The T factor for a Z type target can be any of the letters V, W, X, Y, Z. As Figure 14 shows, the “slope” or shape of the curve changes with the T factor, but the over pressure for a 50 percent probability of damage does not change.

The K factor is an integer that determines how sensitive the target vulnerability is to changes in the yield. For instance, the VN in the VNTK is valid for a yield of 20 kt. If the K factor is zero, it means that the target is damaged only by the peak value of the over pressure wave. If the K factor is greater than 0, it means that the target is sensitive to the duration, or impulse, of the blast wave and not just the peak value. The greater the K factor, the more sensitive the target is to the pressure impulse. As the K factor increases, the probability of damage occurs at lower peak over pressures for a given yield. Figure 15 shows the probability of damage as a function of yield with a constant K factor. Figure 15 shows that for a constant K factor greater than zero, the probability of damage occurs at lower peak overpressures as the yield increases. Also, with the x axis as a log axis, the effect of changing the yield is to shift the probability of damage curve is to the right or left.

Figure 16 shows the probability of damage for a constant yield and a changing K factor. With the x axis as a log axis, the effect of changing the K factor is to shift the probability of damage curve to the right or the left.

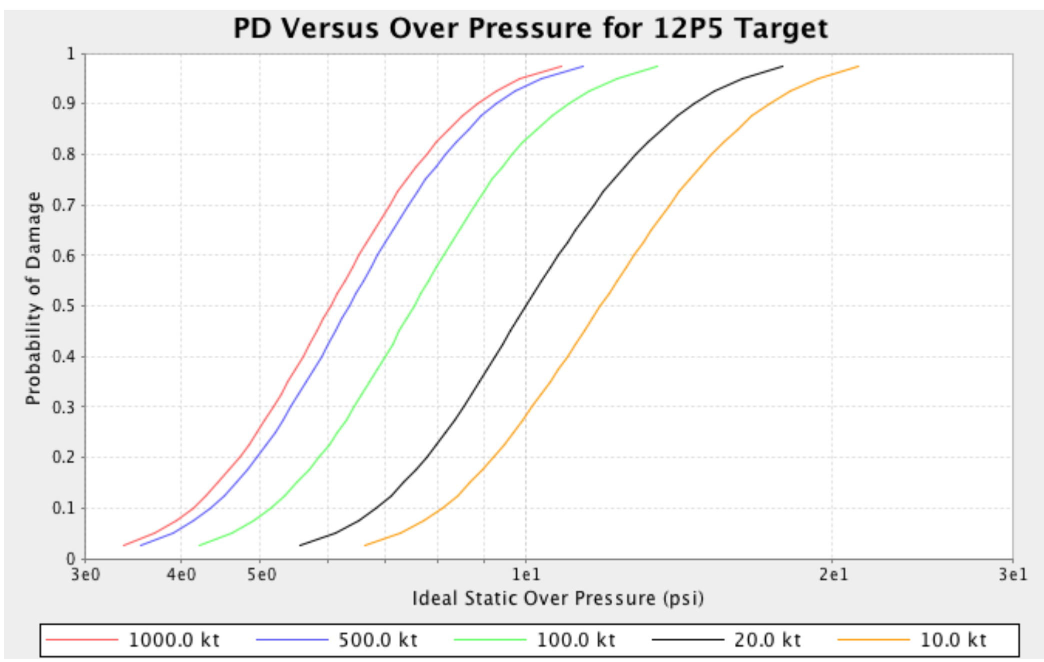

Figure 15—Probability of Damage Variation with Constant K Factor

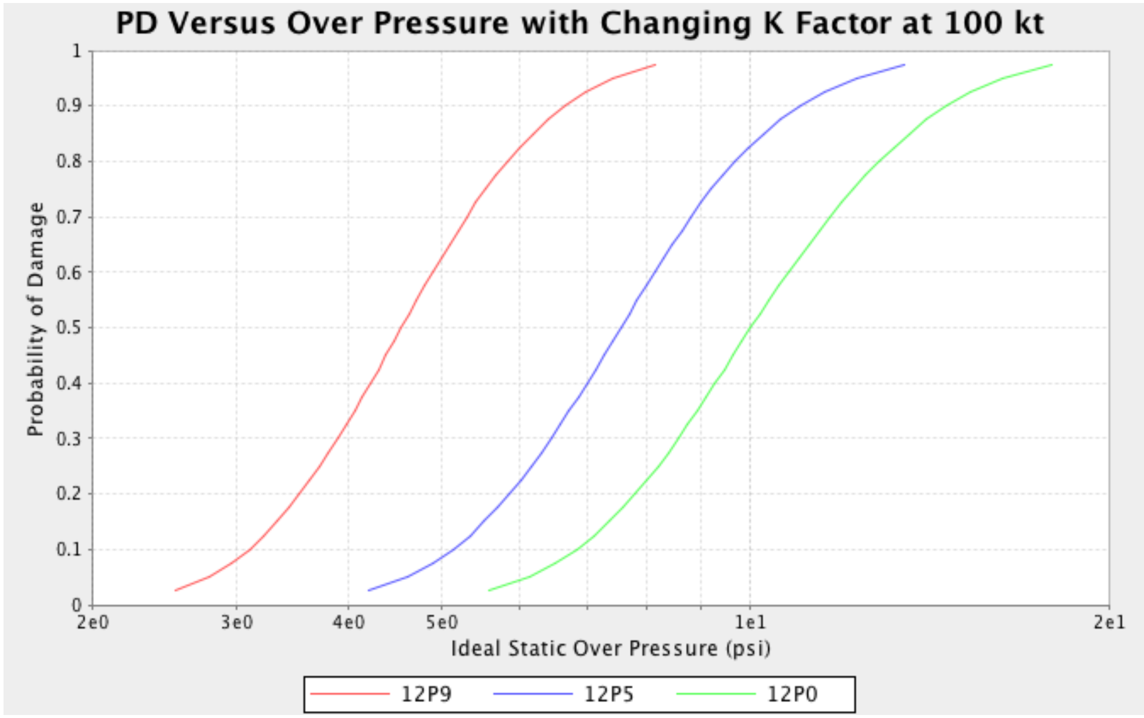

Figure 16—Probability of Damage Variation with Variable K Factor

Distance Damage Function

Although the Hiroshima and Nagasaki data was fit to a log normal distribution in air blast space, the most common way of calculating the probability of damage is to use a probability distribution in range space. This is probably because it is easier to use the concepts of CEP and offset aimpoint with the probability of damage as a function of range. The probability of damage range function is known as the distance damage function. The distance damage *distribution* is log normal in range space:

$$p(r;\alpha,\beta)dr = \frac{1}{\sqrt{2\pi}\beta r} e^{-\frac{1}{2}\left[\frac{ln(\alpha)-ln(r)}{\beta}\right]^2} dr \qquad (24)$$

where:
α is the median of the distribution,
r is the distance of the detonation from the target, and
β is the is the standard deviation of the ln(*r*).

The probability of damage at a distance R from the detonation is:

$$P(R) = 1 - \int_0^R p(r;\alpha,\beta)dr \qquad (25)$$

Equation 25 is the distance damage function. Notice that the probability is the complementary cumulative log normal function, because we require the P(R) to be 1 at zero range, and to be monotonically decreasing as the range increases.

Weapon Radius and Distance Damage Sigma

Rather than using α and β to describe the distance damage function, the terms weapon radius (WR) and distance damage sigma (σ_d) are used. The weapon radius is defined as the second moment about the origin of the distribution function in equation 24. Inside the weapon radius there are just as many targets that are undamaged, as there are targets damaged outside of the weapon radius. This means that the cookie-cutter damage radius in Figure 11 is a weapon radius, because there are zero targets left undamaged inside of R, and there are zero targets damaged outside of R. The σ_d is the variance of the distance damage function made dimensionless by dividing it by the weapon radius. A small σ_d means that the distance damage function has a rapid fall off of the probability of damage as the range increases. It can be shown that: (DO1)

$$\sigma_d^2 = 1 - e^{-\beta^2} \tag{26}$$

and

$$WR = \alpha e^{-\beta^2} \tag{27}$$

The distance damage sigma, σ_d, has a value between 0 and 1. As a matter of practice in the physical vulnerability system, the distance damage sigma has values of 0.1 to 0.5 as shown in Table 1. If the σ_d were 0, this would result in a cookie-cutter damage function as shown in Figure 11.

Table 1—Distance Damage Sigmas for P, Q, and Z Type Targets

Over Pressure (P Type)		Dynamic Pressure (Q Type)		Crater (Z Type)	
T Factor	σ_d	T Factor	σ_d	T Factor	σ_d
L	0.10	Q	0.30	V	0.50
M	0.30	R	0.10	W	0.40
N	0.40	S	0.20	X	0.20
O	0.50	T	0.40	Y	0.10
P	0.20	U	0.50	Z	0.30

From Dorsch (DO1)

The physical vulnerability system uses the distance damage function, rather than a cookie-cutter damage function, to calculate the probability of damage. Although the approximations are different, in principle the calculations are very similar to the probability of hit and damage methods presented above. See Dorsch for a detailed description of the calculations in the VNTK system. (DO1)

Point versus Area Target

The probability of hit calculations above used a target with an area, and calculated the probability that a weapon would land within the target area. The probability of damage calculations used a cookie-cutter damage function, and inherently assumed that the targets were point targets. The point target is damaged to the required level if the weapon lands within the cookie-cutter damage radius. Many targets in the target database can be considered as point targets. Their dimensions are very small compared to the damage radius from a nuclear detonation.

Area targets may be either circular normally distributed, or uniformly distributed, and the target area is assumed to be a circle. The term R95 is used to describe the radius of area targets. For a circular normally distributed target area, the R95 is the radius that contains 95% of the target elements. For a uniformly distributed target, the R95 is the target radius.

For a point target, the probability of damage calculated is the probability of the target being damaged to the specified level.

> *For an area target, the probability of damage is the fraction of the target that is expected to be damaged to the specified level.*

Target Types Other Than P, Q, and Z

There are several other target types described in the physical vulnerability system. Equivalent Target Area (ETA) targets are long and narrow targets, such as runways, dams, and bridges. An ETA VNTK is used to describe the vulnerability of these targets. Personnel targets have VNTKs that can combine the damage from several effects, such as air blast, thermal radiation, and ionizing radiation. Personnel VNTKs are identified with a T Factor of I, and there are three different methods for specifying the VNTK based on historical usage. Deeply buried targets use a Ground shock Vulnerability Number (GVN), which is a 10 character VNTK which describes the target vulnerability based on the depth to a given shock level, instead of the ground range to a given air blast level. A physical vulnerability data sheet (PVDS) target is an historical VNTK method for calculating the damage to a deeply buried target. This has been "replaced" by the GVN methodology, but it is still part of the physical vulnerability system. The 2VNTK is a new way of specifying the vulnerability levels of some types of urban-industrial structures. The 2VNTK is a nine character VNTK. The first 4 characters specify the target vulnerability for a target on the ground. It is the same value as for the traditional VNTK. The last four characters specify the target vulnerability for the roof of the target, when the detonation is directly above the structure. The 5th character describes how to make the transition in range from the traditional to the roof. The 2VNTK can only be used for targets with a T factor of P or Q. (WR1)

PDCALC

The DIA has the responsibility for developing, maintaining, and improving the physical vulnerability system. They are also responsible for assigning the VNTK to the targets in the target database. PDCALC stands for the Probability of Damage Calculator. It implements the physical vulnerability system, and it is the officially accredited tool for calculating the probability of damage to a target from a nuclear detonation.

PDCALC is a Fortran code which was first developed by the Strategic Air Command, and which is now maintained by DTRA. The code is a Fortran subroutine, which is used in multiple planning tools within the DoD. The calling arguments to the subroutine do not change with revisions to PDCALC, so that the new subroutine can be plugged directly into the legacy planning tools. The legacy tools will continue to work without any other modification. Some of the personnel VNTK calculations are classified Confidential, so there are two versions of the PDCALC subroutine. One is classified Confidential, and can calculate the probability of damage for all of the personnel VNTKs. The second version is unclassified, and can calculate the probability of damage for only the unclassified personnel VNTKs

To calculate the probability of damage using PDCALC, the user must input the VNTK, yield, height of burst, CEP, R95, offset distance, and depth for deeply buried targets. PDCALC may be obtained from DTRA. There is also an unclassified version with a GUI interface in the LANL Nuclear Weapon Analysis Tools. (ST5)

Other Sources of Error

In the discussion to this point, only one type of error has been used: the weapon impact distribution about the aimpoint, or the CEP. In the real world, there are a number of other error types that are considered. Target location error is the error in the latitude and longitude of the target. The target location may be determined by several different methods, and all of the methods have an error associated with them. There is also error in the estimate of the target altitude above sea level, and different methods for calculating sea level. Map error is the error in the coordinates of a symbol on a map. The map legend for an aeronautical map will have the year of the data that the map is based upon, and the expected error of the map as a distance. The usual approach to handling these many different error types is to include them in the estimate of the CEP. That way the standard methods for calculating the probability of damage can be used. If the assumption can be made that the different error types are independent of each other, then the adjusted CEP is often calculated by using the root mean square of the different errors. But sometimes the errors are dependent.

Figure 17 shows a target with the flight path of a weapon. The aimpoint is the DGZ, or the Designated Ground Zero. The figure shows the flight path of the weapon with the solid or dashed line. Assume that the solid line is the planned flight path of the weapon. If the fuze functions at the correct height of burst, then the detonation will occur over the target. This is the blue detonation on the solid line. If the fuze functions too high, say at the yellow burst, then the fuze error and the "impact" error are not independent. The two errors are linked. Now assume that the dashed line is an actual flight path of a test weapon. If the fuze functions at the correct HOB, then the "impact" location will be in error. If the height of burst fuze fails, and a backup contact fuze functions, (the yellow detonation on the dashed line) then the detonation will occur off target. In both cases the flight path error and fuze error are linked. If the errors in the fuze and the flight path were given as separate values, it would not be appropriate to use the root mean square of the two errors. The best way to determine the CEP would be to use test data. That way the interdependence of the two errors would be included in the measured errors.

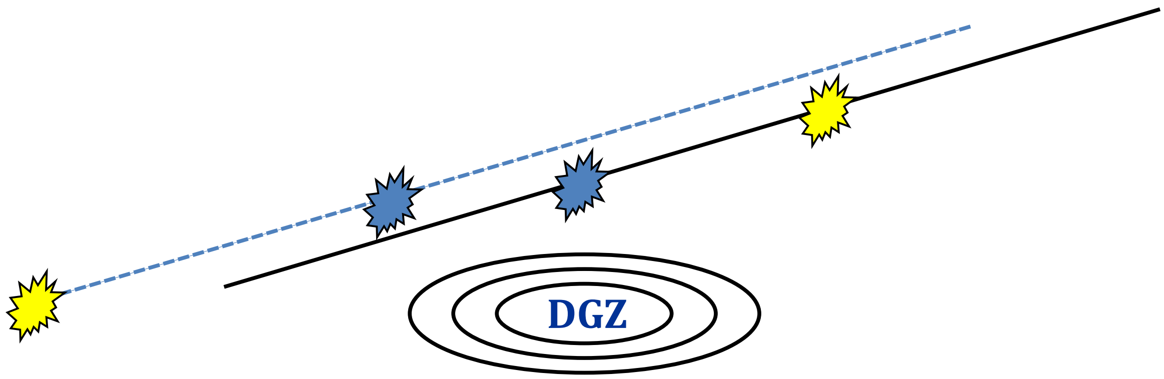

Figure 17—Height of Burst Errors

Summary

There are a number of terms used in nuclear targeting. The probability of hit is the probability that a weapon will land within a designated area. The probability of damage is the probability that a weapon will cause a given level of damage to a target. The CEP is the radius of a circle within which 50 percent of the weapons will land, and SEP is the radius of a sphere within which 50 percent of the weapons will detonate. There are a number of assumptions inherent in the CEP and SEP terms. If the weapon impact error is not circular or spherical normal, it is termed as being elliptical with unequal standard deviations of the impact error. Many vulnerability or survivability analyses use a cookie-cutter damage radius. Sure kill is an environment level which ensures that the target is destroyed. A target may experience many encounters at or below the sure safe level with no damage to the target. Mission completion is the environment level, which after being experienced just one time, damages the target, but the target is just able to complete its mission. The R95 describes the radius of an area target. The physical vulnerability system uses VNTKs to describe the vulnerability of targets to nuclear effects. The system uses a distance damage function which is log normal in range, rather than being a cookie cutter. PDCALC is the officially recognized and accredited nuclear targeting tool for calculating the probability of damage. Knowing these terms will not make the reader an expert in probability of damage calculations, but it will give the reader a basic understanding of how nuclear targeting and survivability analyses are performed.

There is one last foot stomper. This paper only applies to nuclear targeting. *The targeting process for conventional weapons is completely different, and has a completely different set of terms and methodologies.*

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Glossary of Terms

CEP	Circular Error Probable
DE	Damage Expectancy
DGZ	Designated Ground Zero
DoD	Department of Defense
DOE	Department of Energy
DTRA	Defense Threat Reduction Agency
ETA	Equivalent Target Area
GR	Ground Range
GVN	Ground Shock Vulnerability Number
HOB	Height of Burst
ICBM	Intercontinental Ballistic Missile
kt	kilotons
LANL	Los Alamos National Laboratory
P type	Vulnerable to Over Pressure
PA	Probability of Arrival
Pd, PD	Probability of Damage
PDCALC	Probability of Damage Calculator
PLS	Probability of Launch Survival
PS	Probability of Survival
psi	pounds per square inch
PVDS	Physical Vulnerability Data Sheet
Q type	Vulnerable to Dynamic Pressure
R95	Target radius within which 95% of a circular normal target is contained, or the radius of a uniformly distributed target
RTE	Range to Effect

SEP	Spherical Error Probable
SNEC	Simple Nuclear Effects Calculator
SR	Slant Range
VNTK	Vulnerability Number, T factor, K factor
WR	Weapon Radius
WSR	Weapon System Reliability
Z type	Vulnerable to Cratering
A	Ground Range to 50% Probability of Damage
σ_d	Distance Damage Sigma

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