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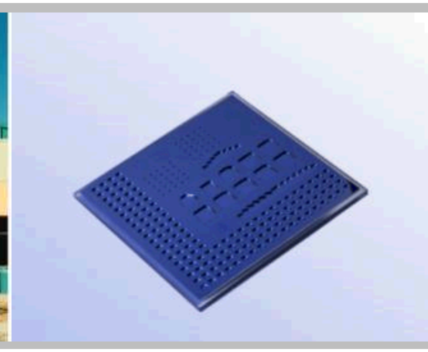

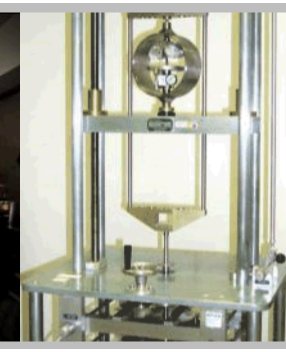

Topics in Dimensional Metrology

Eric C. Forrest, PhD; Hy D. Tran, PhD, PE

Feb. 2, 2016

NCSLI Tech Training Exchange, Jacksonville, FL

U.S. DEPARTMENT OF ENERGY
NNSA National Nuclear Security Administration

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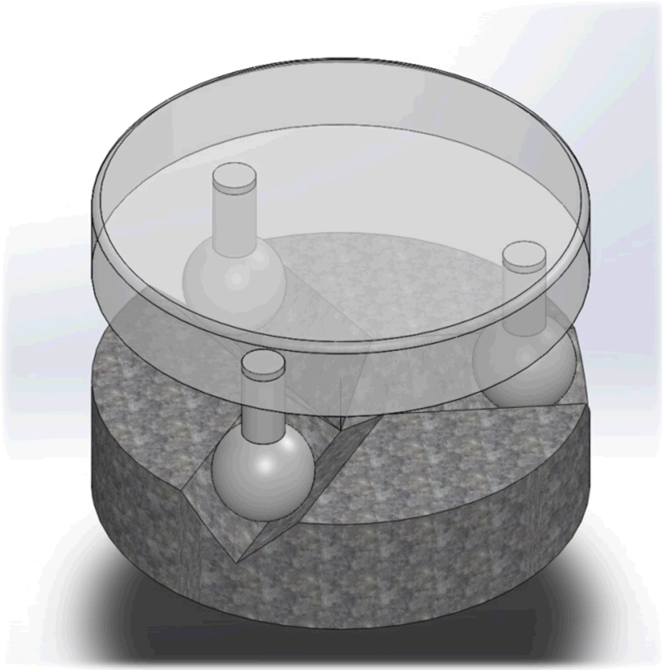
Desired outcomes

- Sources of errors in precision dimensional instruments
  - Patterns in instrument design:
  - Repeatability
  - Kinematics
  - Isolation, metrology loop
  - Abbe
  - Stiffness
  - Thermal effects
- Displacement transducers



Type II Kelvin Clamp Illustrating 2-2-2 Constraint.

PRINCIPLES OF KINEMATIC FIXTURING

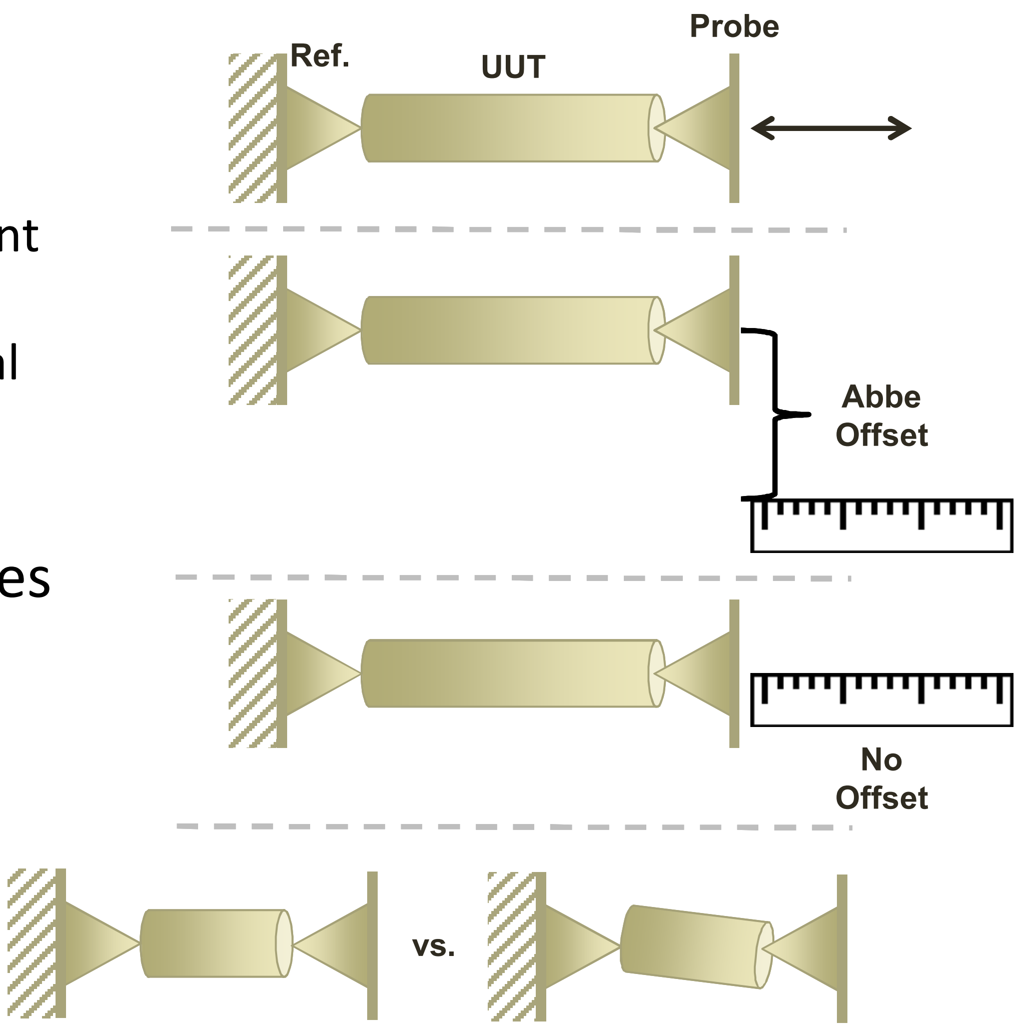
Material adapted from MIT Precision Engineering Research Group, © Alex Slocum.

Outline

- Motivation
- Fundamentals of Kinematic Constraint
- Overview of Coupling Methods
- Hertz Contact
- Types
  - Standard
  - Canoe (high load)
  - Compliant
  - Three tooth
- Examples
- Elastic Averaging

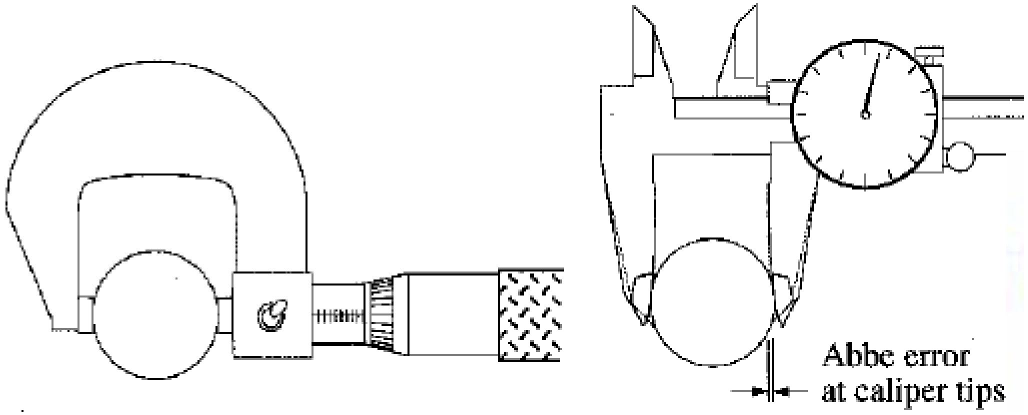
Motivation

- Probe measures displacement.
  - Modern length measurement machines use various transducers, typically optical scales or laser interferometer.
- Offset of ‘ruler’ from probes may introduce error in reading (Abbe offset).
- ***Reproducibility of UUT positioning critical to achieving precise measurements.***



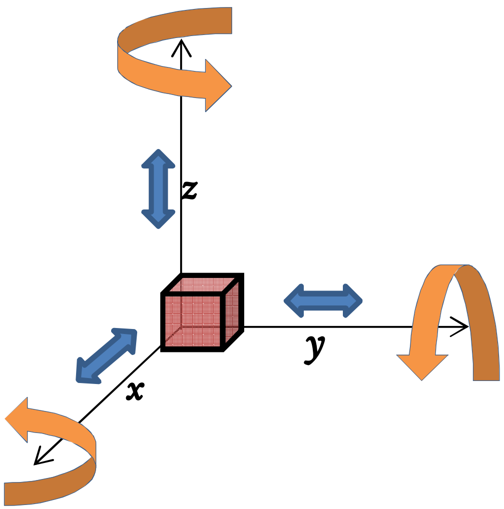
Abbe's Principle- Locating Components

- Geometric: Angular errors are amplified by the distance from the source.
  - Measure near the source, and move the bearings and actuator near the work.
- Thermal: Temperatures are harder to measure further from the source.
  - Measure near the source!

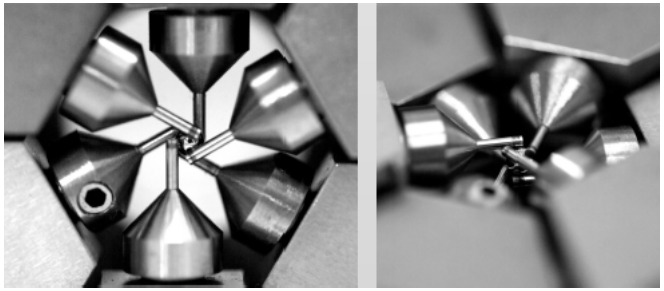


Fundamentals of Kinematic Constraint

- Unconstrained rigid body has six degrees of freedom.
  - Three in translation, three in rotation.
- Properly constrained body will only have point contact restraint for each DoF.
- Consider the three vs. four-legged stool.
- Same principles used to position D-T fusion fuel spheres at NIF.
- Practical considerations may require deviation from kinematic principles (elastic deflection, manufacturing considerations, etc.) .

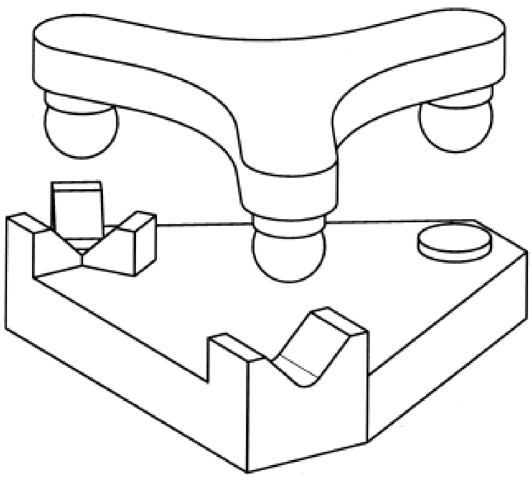


Degrees of Freedom in Cartesian Coordinates.

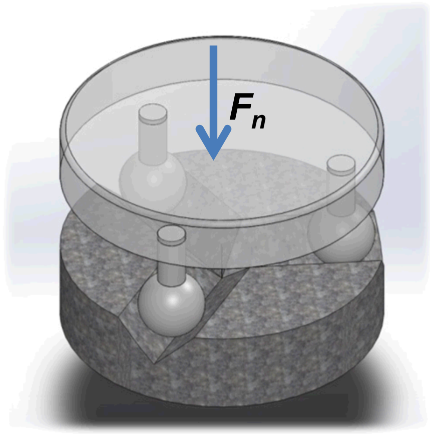

Manipulation Apparatus for D-T Fuel Spheres.
Source: Doyle and Fontana, CAARI 2014.

Fundamentals of Kinematic Constraint

- When body constrained by number of points equal to degrees of freedom, said to be exactly constrained.
  - Number contact points = degrees of freedom constrained.
- Low to medium force applications.
- Do not enable sealing contact.
- Moderate stiffness.
- Moderate cost.
- Excellent repeatability.
  - 0.25 micron typical.
  - On order of surface finish.

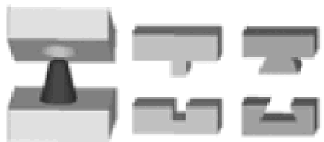


Type I Kelvin Clamp Illustrating 3-2-1 Constraint. *Source: Hale and Slocum, Precision Engineering (2001).*

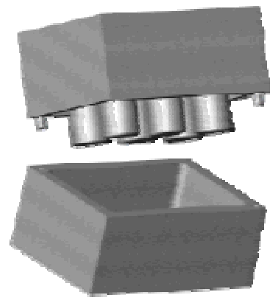


Type II Kelvin Clamp Illustrating 2-2-2 Constraint.

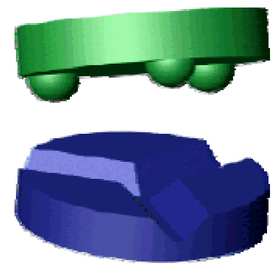
Overview of Coupling Methods



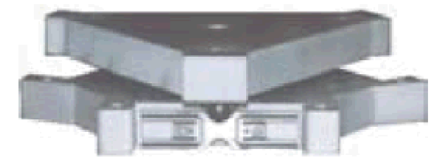
Elastic Averaging
Non-Deterministic



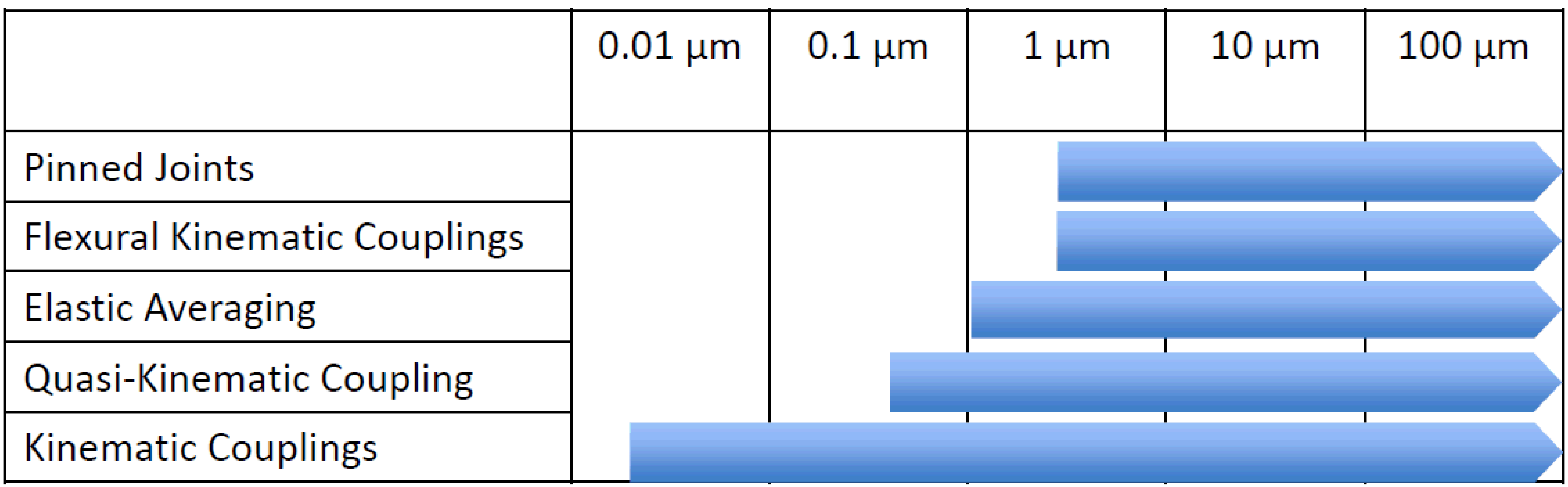
Pinned Joints
No Unique Position



Kinematic Couplings
Kinematic Constraint



Flexural Kin. Couplings
Kinematic Constraint



Hertz Contact

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- <u>Equivalent Modulus:</u>

$$E_{eqivalent} = \frac{1}{\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}}$$

- <u>Equivalent Radius:</u>

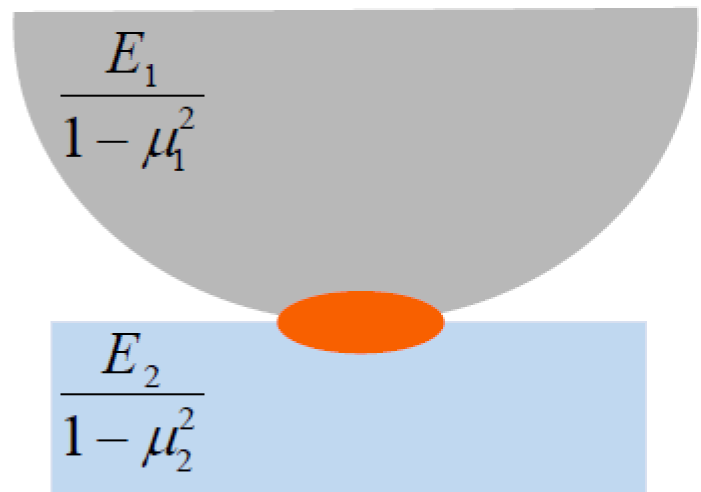
$$R_{eqivalent} = \frac{1}{\frac{1}{R_{1,mjr}} + \frac{1}{R_{1,mnr}} + \frac{1}{R_{2,mjr}} + \frac{1}{R_{2,mnr}}}$$

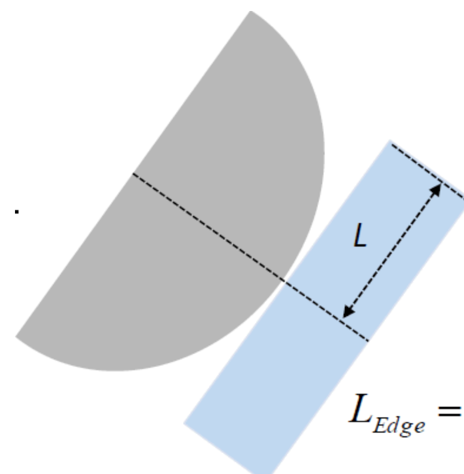
- <u>Contact Radius and Stress:</u>

$$a_{contact} = \left(\frac{3 \cdot F \cdot R_e}{2 \cdot E_e} \right)^{1/3} \qquad q_{contact} = \frac{a \cdot E_e}{\pi \cdot R_e}$$

$$q_{HertzMax} = \frac{2 \cdot \sigma_{allowable}}{1 - 2 \cdot \mu}$$

* Keep Contact Stress Below Elastic Limit



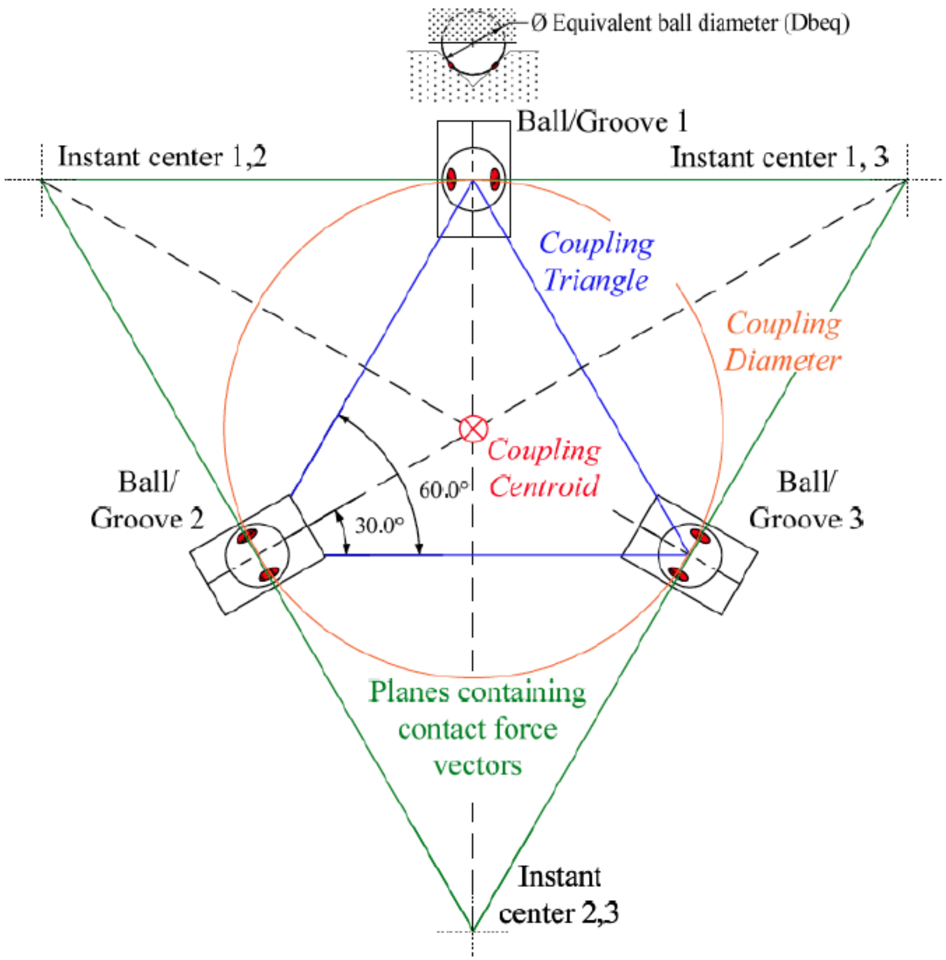


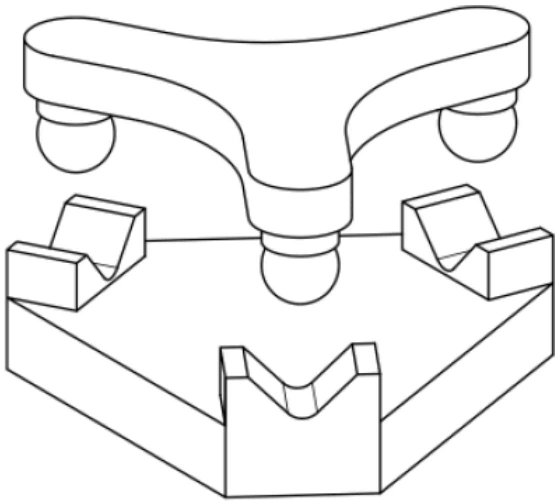
$$L_{Edge} = 10 \cdot a_{contact}$$

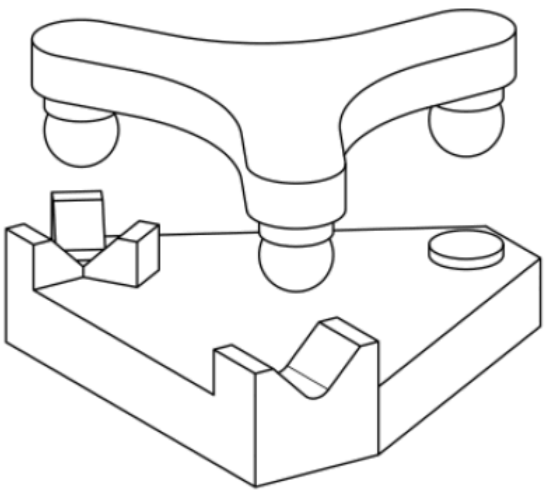
Hertz Relations

- Contact pressure proportional to:
  - Force to the 1/3rd power
  - Radius to the (-2/3)rd power
  - Modulus to the 2/3rd power
- Deflection is proportional to:
  - Force to the 2/3rd power
  - Radius to the (-1/3)rd power
  - Modulus to the (-2/3)rd power
- Contact ellipse diameter is proportional to:
  - Force to the 1/3rd power
  - Radius to the 1/3rd power
  - Modulus to the (-1/3)rd power
- Do not allow contact ellipse within one diameter of edge of surface!

Standard Kinematic Coupling

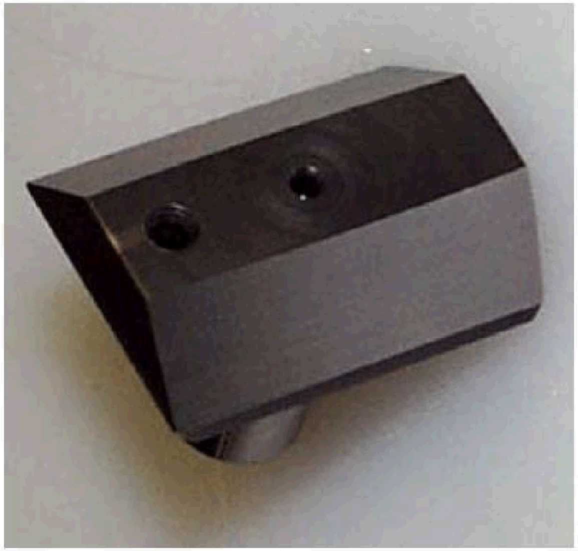


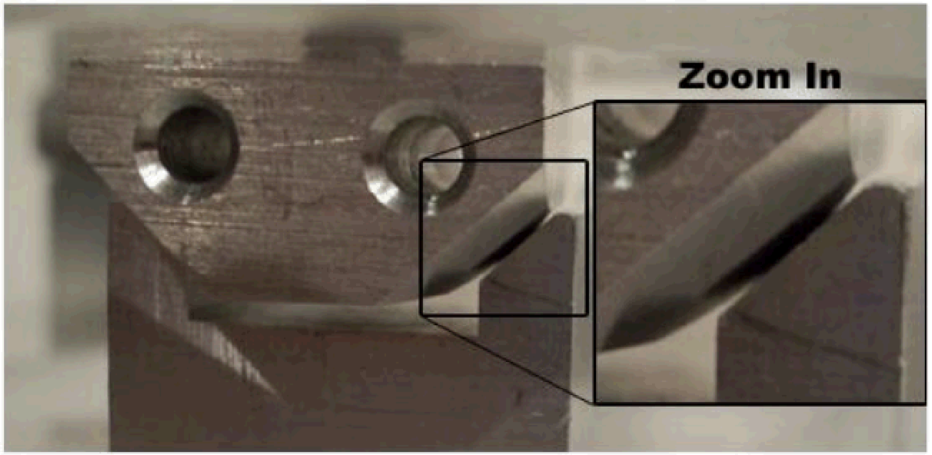




Canoe-Ball Kinematic Interface (High Load)

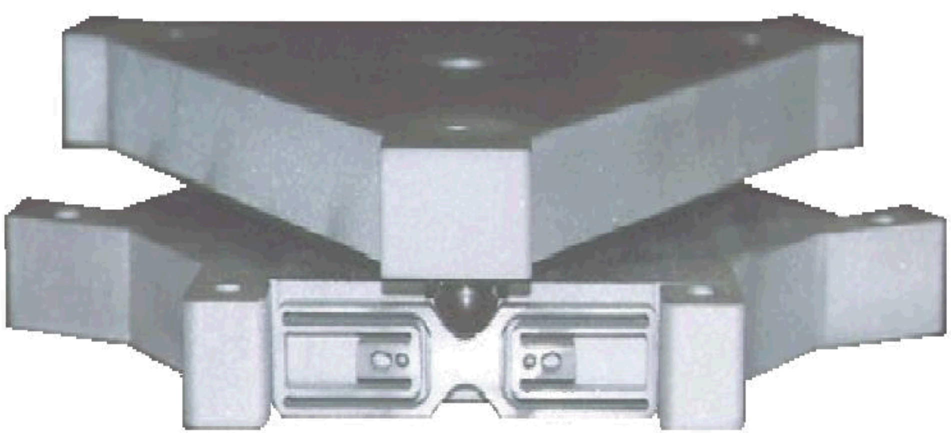
- "Canoe Ball" shape acts lime a 1m diameter ball.
- 100 times the stiffness and load of normal ball.
- 10x higher load capacity than crowned cone.
- Large, shallow Hertzian zone (improves repeatability).
- US Patent 5,711,647.

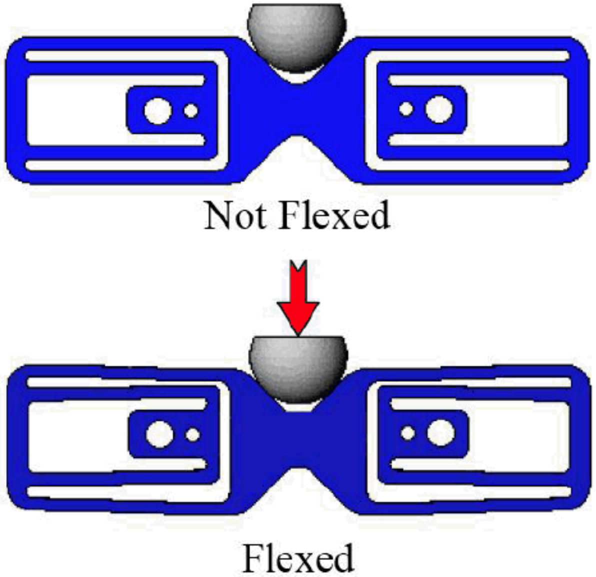




Compliant Kinematic Couplings

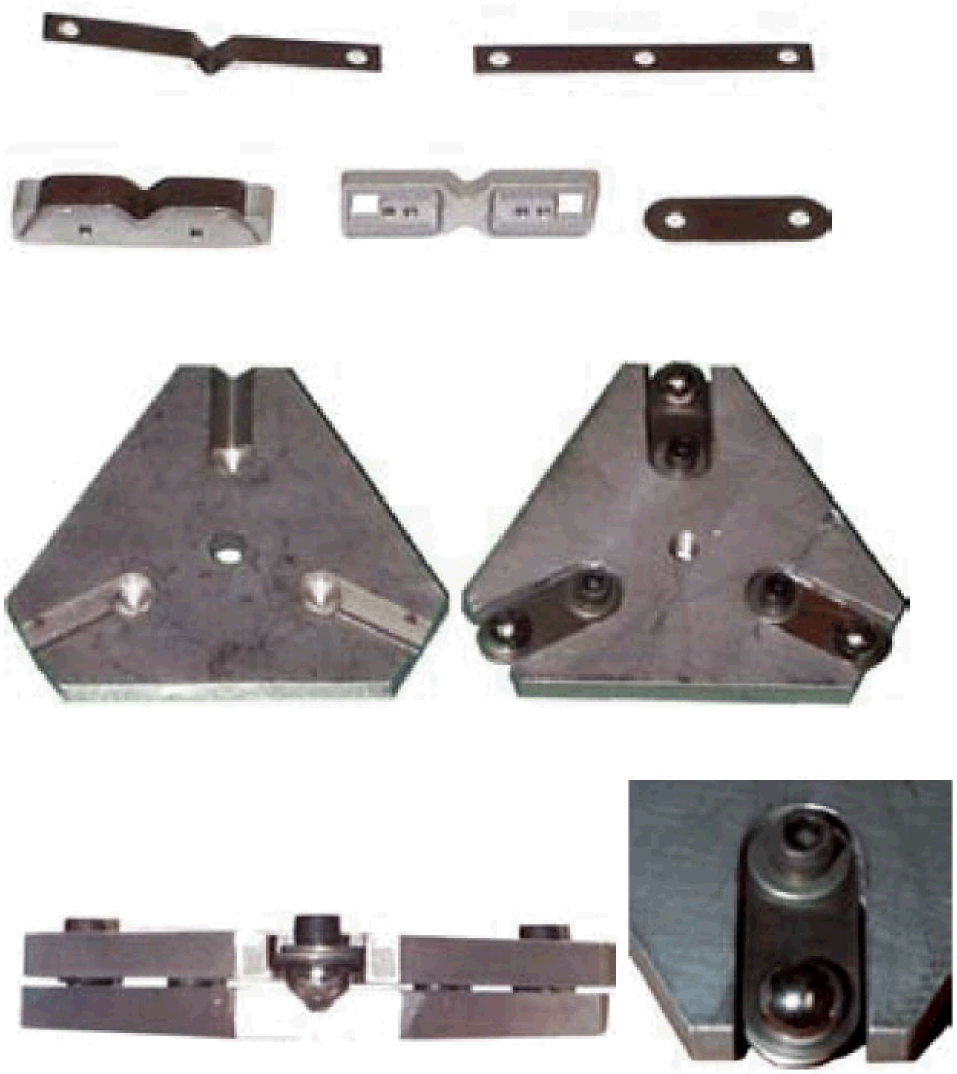
- Clamping load/friction/mated surface.
- Joint location.
- Compliant members.
- Kinematic interface.





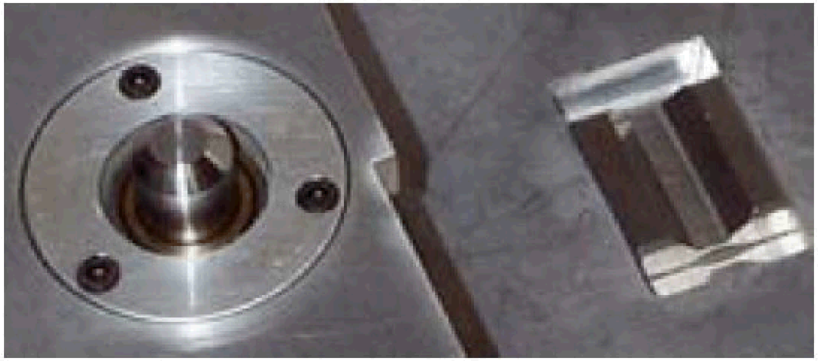
Compliant Kinematic Couplings

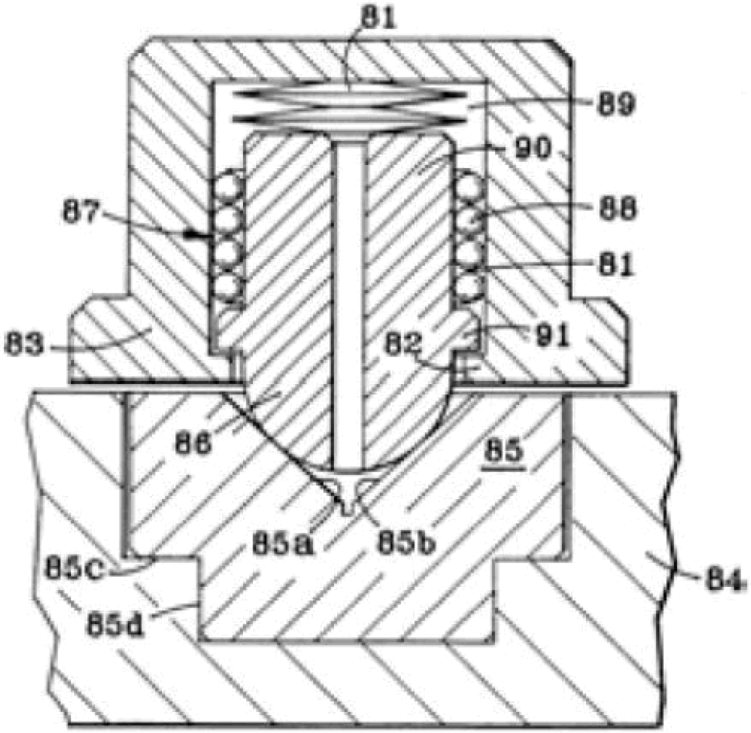
- Characteristics:
  - Low cost
  - Small to medium stroke
  - 5-10 µm precision
- Applications/Processes:
  - Assembly lines
  - Stamping, forging, forming equipment for die alignment
  - Semiconductor mfg.
  - Casting dies
- Cost:
  - ~$10-$200



Compliant Kinematic Couplings

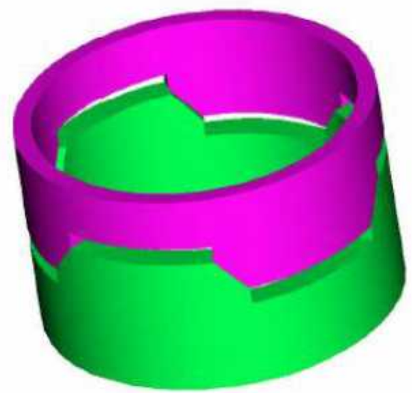
- Characteristics:
  - Medium cost
  - Long stroke
  - 2.5 µm precision
- Applications/Processes:
  - Assembly lines
  - Casting
  - Fixtures
- Cost:
  - ~$2000

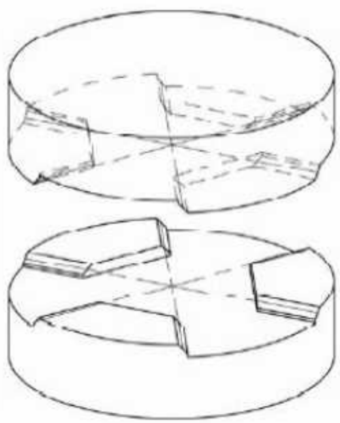




Three Tooth

- Semi-kinematic
- Six-point mating
- Layton Hale (LLNL)added crowns to one set of teeth yielding 2.5 µm repeatability.





United States Patent [19]
Hale

[11] **Patent Number:** 6,065,898
[45] **Date of Patent:** *May 23, 2000

[54] THREE TOOTH KINEMATIC COUPLING

[75] Inventor: **Layton C. Hale**, Livermore, Calif.

[73] Assignee: **The Regents of the University of California**, Oakland, Calif.

[*] Notice: Under 35 U.S.C. 154(b), the term of this patent shall be extended for 634 days.

[21] Appl. No.: **08/511,980**

[22] Filed: **Aug. 7, 1995**

[51] **Int. Cl.**^{7} **F16D 1/00**
[52] **U.S. Cl.** **403/364**; 403/190; 403/340; 403/381; 464/157; 192/69.83
[58] **Field of Search** ..... 403/190, 291, 403/364, 311, 340, 381; 192/114 T, 69.81, 69.82, 69.83; 464/149, 157

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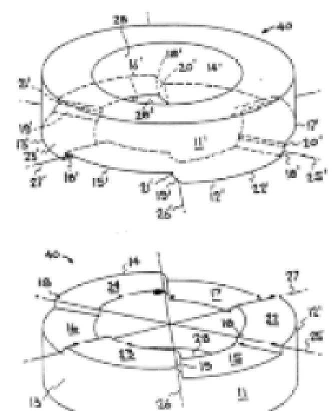
Primary Examiner—Daniel P. Stodola
Assistant Examiner—Bruce A. Lev
Attorney, Agent, or Firm—Alan H. Thompson; L. E. Carnahan

[57] **ABSTRACT**

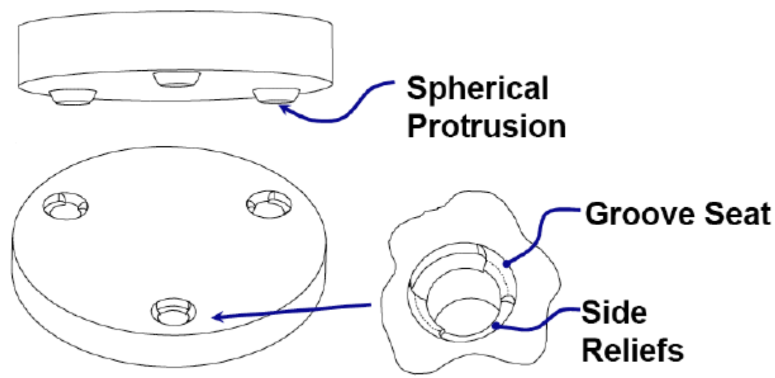
A three tooth kinematic coupling based on having three theoretical line contacts formed by mating teeth rather than six theoretical point contacts. The geometry requires one coupling half to have curved teeth and the other coupling half to have flat teeth. Each coupling half has a relieved center portion which does not effect the kinematics, but in the limit as the face width approaches zero, three line contacts become six point contacts. As a result of having line contact, a three tooth coupling has greater load capacity and stiffness. The kinematic coupling has application for use in precision fixturing for tools or workpieces, and as a registration device for a work or tool changer or for optics in various products.

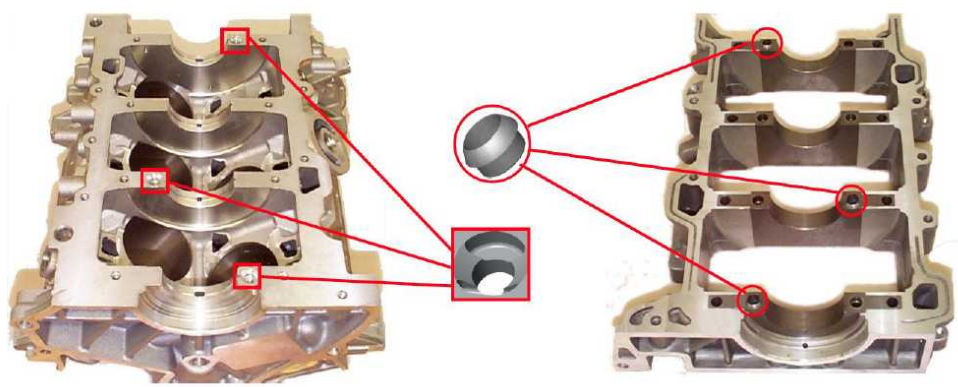
15 Claims, 2 Drawing Sheets



Quasi Kinematic Coupling

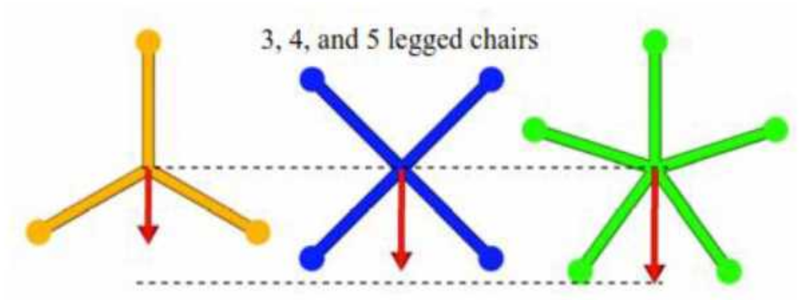
- Attain sub-micron repeatability.
- Use less restrictive tolerances.
- More flexibility in assigning precision tolerances.
- Better precision at lower cost.
- Extension of practical HVM precision
- Eliminate precision pinned joints.
- Reduce number of parts.

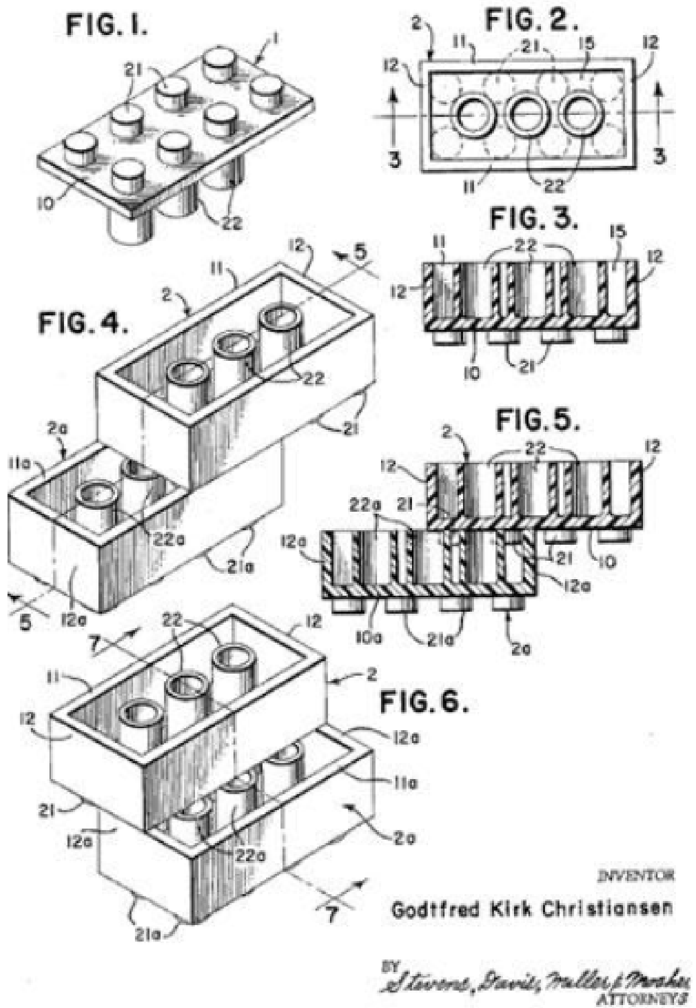




Elastic Averaging

- Multiple compliant contact points.
- Members elastically deform.
- High overall stiffness.
- Application from toys, to office chairs, to couplings





Schlieren photograph of a bullet passing through a candle flame. Edgerton and Vandiver, 1973.

INTRODUCTION TO HEAT TRANSFER

It is the fire that warms the cold, the cold that tempers the heat…

-Don Quixote, 1615

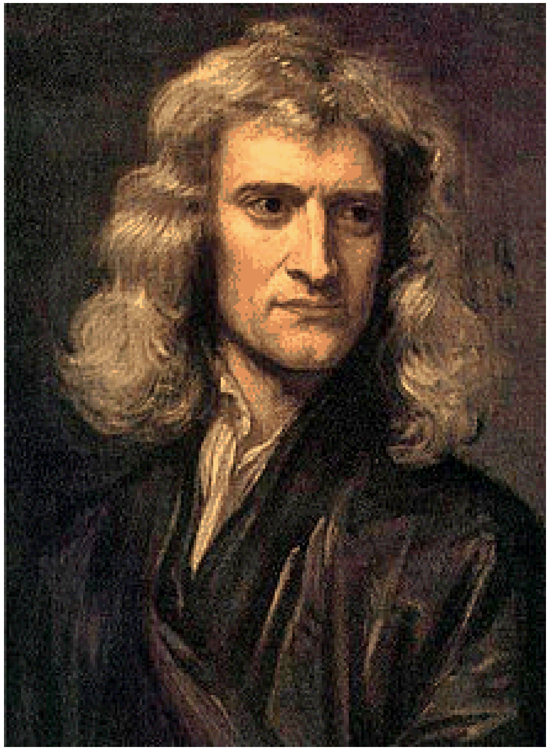
Historical Perspective

Isaac Newton

- *Scala Gradum Caloris* (1701)

  "The heat which hot iron, in a determinate time, communicates to cold bodies near it, that is, the heat which the iron loses in a certain time is as the whole heat of the iron; and therefore, if equal time of cooling be taken, the degrees of heat will be in geometrical proportion."

- Newton understood a time-temperature change relationship existed for objects hotter than surroundings.
- However, modern concept of heat transfer **did not exist at the time and were unknown to Newton.**



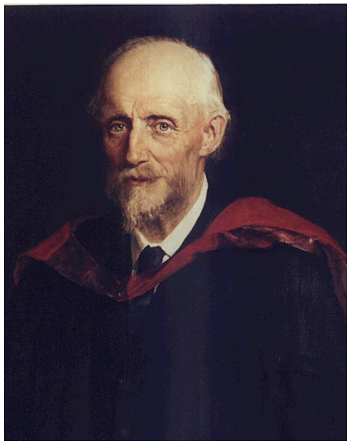
Sir Isaac Newton, circa 1689

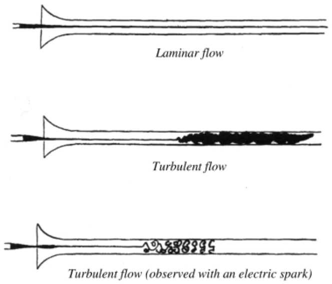
Jean-Baptiste (Joseph) Fourier

- Founder of modern heat transfer.
- *Théorie Analytique de la Chaleur* (1822)
  - Introduces Fourier's Law of Conduction
  - Introduced what most incorrectly refer to as 'Newton's Law of Cooling'

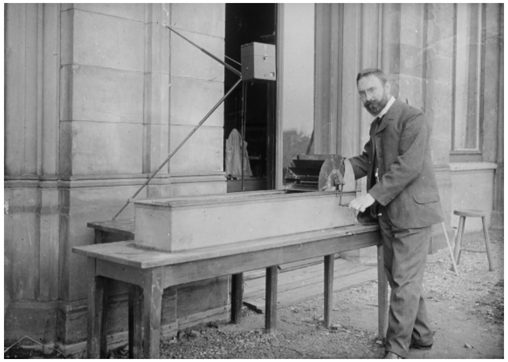
Reynolds, Nusselt, and Prandtl

- Late 1800's and early 1900's saw significant advances in our understanding of heat transfer.
- Laminar and turbulent flow regimes (Reynolds).
- Dimensionless heat transfer analysis (Nusselt).
- Boundary layer theory (Prandtl).
  - In addition to importance for heat transfer, formed basis for modern aerodynamic sciences (flight would not be possible without it).





Osbourne Reynolds and his drawings of flow regimes.



Wilhelm Nusselt (left) and Ludwig Prandtl (right).

Definitions and Units

General Definitions

- **Temperature**, *T* (Kelvin, K, or degrees Celsius, ^{o}C)
  - Relates the degree of thermal vibrational motion of atoms in matter. At *T*=0 K (absolute zero), there is no thermal motion.
- **Thermal Energy**, *Q* (Joules)
  - For this lecture, refers to energy associated with thermal processes.
- **Power** or **Heat Rate**, $\dot{Q} = \frac{Q}{t}$ (Watts, W)
- **Heat Flux**, $q'' = \frac{\dot{Q}}{A_{surf}}$ (W/m^2)
  - Most heat transfer problems surface area dependent so a heat *flux* will be used for calculations.
- **Heat Transfer Coefficient**, *h* (W/m^2-K)
  - Relates temperature change to heat flux for convective processes. Will discuss more later.

Material Property Definitions

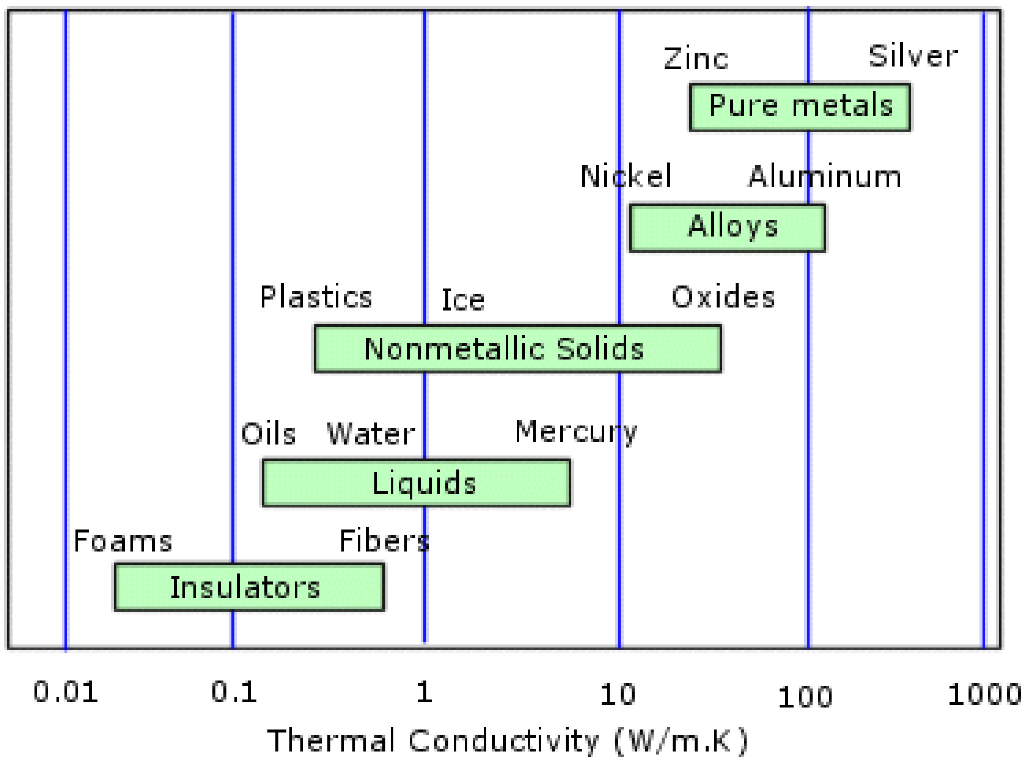
- **Density**, ρ (kg/m^3)
- **Thermal Conductivity**, k (W/m-K)
  - Measure of how well a material transfers heat.
- **Specific Heat Capacity**, c_p (J/kg-K)
  - Measure of how much thermal energy a material can store per unit mass.
- **Thermal Diffusivity**, $\alpha = \frac{k}{\rho c_p}$ (m^2/sec)
  - Measure of relative ability for material to transfer heat vs. store heat.
- **Coefficient of Thermal Expansion**, $\beta = \frac{1}{L_0} \times \frac{(L_f - L_0)}{(T_f - T_0)}$ (ppm/K)
  - Fractional change of material's dimension per change in temperature. Positive value for most materials. Typically reported as linear CTE, but volumetric CTE also used (not necessarily equivalent).
- **Dynamic Viscosity**, μ (Pa-sec ≡ kg/m-sec)
  - Measure of resistance to flow of a fluid.

Unit Conversions

- Many older handbooks will contain an amalgam of information in English Standard, CGS, and SI units.
  - → For heat transfer problems, easiest to convert everything to SI and work in that!
- Temperature: For *intervals* (temperature differences) or *properties*, K and °C interchangeable. Otherwise see table.
- Energy: 1 cal = 4.184 Joules, 1 BTU = 1055.06 Joules

	From Kelvin	To Kelvin
Celsius	[°C] = [K] − 273.15	[K] = [°C] + 273.15
Fahrenheit	[°F] = [K] × $\frac{9}{5}$ − 459.67	[K] = ([°F] + 459.67) × $\frac{5}{9}$
Rankine	[°R] = [K] × $\frac{9}{5}$	[K] = [°R] × $\frac{5}{9}$
For temperature *intervals* rather than specific temperatures, 1 K = 1 °C = $\frac{9}{5}$ °F = $\frac{9}{5}$ °R		

Thermal Conductivity Comparison of Material Types



Thermophysical Properties of Common Solid Materials

Material	Density, ρ	Thermal Conductivity, k	Specific Heat Capacity, c_p	Thermal Diffusivity, α	Linear Coefficient of Thermal Expansion, β
Metals					
Plain Carbon Steel (AISI 1030 Steel)	7850 kg/m^3	51.9 W/m-K	486 J/kg-K	1.36 x10^{-5} m^2/sec	11.7 ppm/°C
Copper (C10100)	8940 kg/m^3	391 W/m-K	385 J/kg-K	1.14 x10^{-4} m^2/sec	17.0 ppm/°C
6061-T6 Aluminum	2700 kg/m^3	167 W/m-K	896 J/kg-K	6.90 x10^{-5} m^2/sec	23.6 ppm/°C
304L Stainless Steel	7900 kg/m^3	15 W/m-K	475 J/kg-K	4.0 x10^{-6} m^2/sec	16.5 ppm/°C
Non-Metals					
Tungsten Carbide	15,700 kg/m^3	110 W/m-K	945 J/kg-K	7.41 x10^{-6} m^2/sec	5.9 ppm/°C
Granite	2630 kg/m^3	2.79 W/m-K	775 J/kg-K	1.37 x10^{-6} m^2/sec	3.70 to 11.0 ppm/°C
Wood (hardwood, e.g. oak)	720 kg/m^3	0.16 W/m-K	1255 J/kg-K	1.77 x10^{-7} m^2/sec	N/A
Acetal (Delrin)	1420 kg/m^3	0.289 W/m-K	1480 J/kg-K	1.38 x10^{-7} m^2/sec	102 ppm/°C
PTFE (Teflon)	2200 kg/m^3	0.25 W/m-K	1300 J/kg-K	8.74 x10^{-8} m^2/sec	125 ppm/°C
Glass (Soda-Lime)	2530 kg/m^3	0.94 W/m-K	880 J/kg-K	4.22 x10^{-7} m^2/sec	9.5 ppm/°C

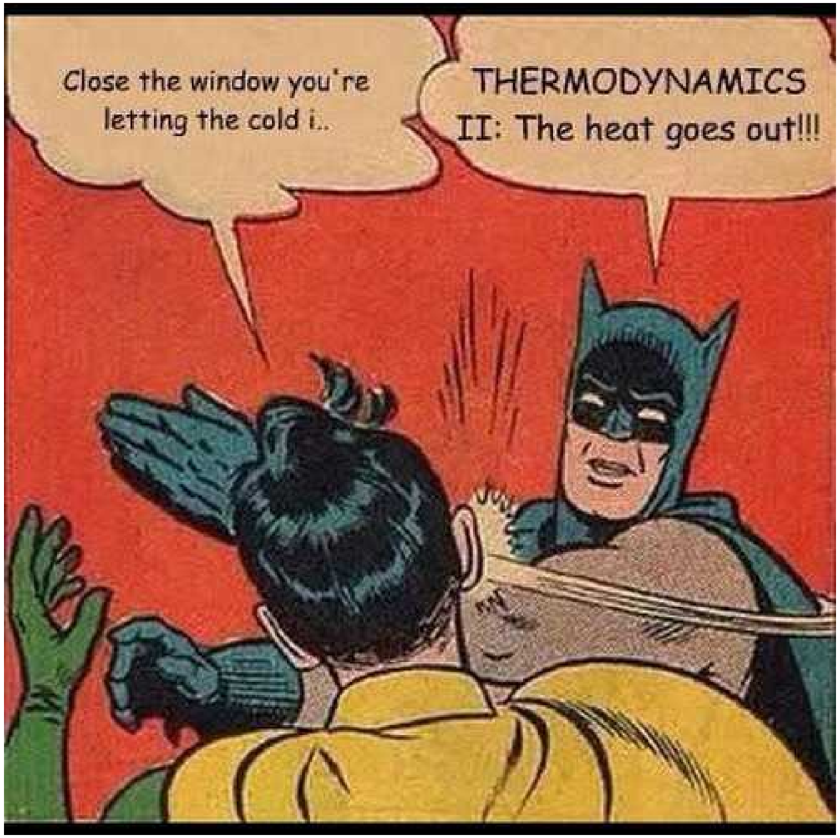
*Typical properties at ambient conditions.

Thermophysical Properties of Air and Water

	Density, ρ	Thermal Conductivity, k	Specific Heat Capacity, c_p	Thermal Diffusivity, α	Volumetric Coefficient of Thermal Expansion, β_V	Viscosity, μ
Fluids						
Air	1.20 kg/m^3	0.0262 W/m-K	1012 J/kg-K	2.3x10^{-5} m^2/sec	3430 ppm/°C	18.6x10^{-6} Pa-sec
Water	997 kg/m^3	0.609 W/m-K	4181 J/kg-K	1.43x10^{-5} m^2/sec	207 ppm/°C	894x10^{-6} Pa-sec

*Typical properties at ambient conditions.

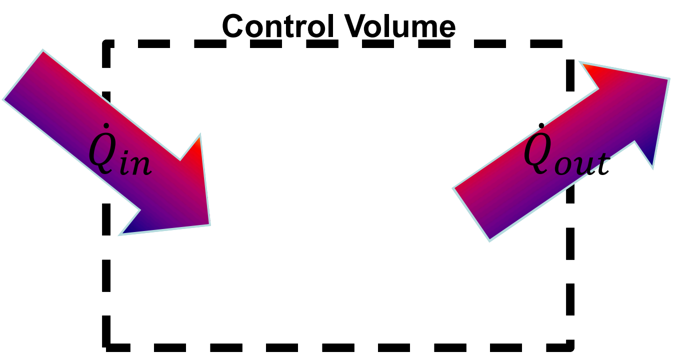
Thermodynamics



Thermodynamics I

- First Law of Thermodynamics
  - Derived from conservation of energy.
  - *"In a thermodynamic process involving a closed system, the increment in the internal energy is equal to the difference between the heat accumulated by the system and the work done by it."*

$$\frac{\Delta E_{tot}}{\Delta t} = \dot{Q} + \dot{W}$$



→ **For a closed system at steady-state (equilibrium) with no work input, heat in must equal heat out!**

Practical note- Friction dissipates energy, and so the energy dissipated goes into the material, which turns to heat & raises temperatures.

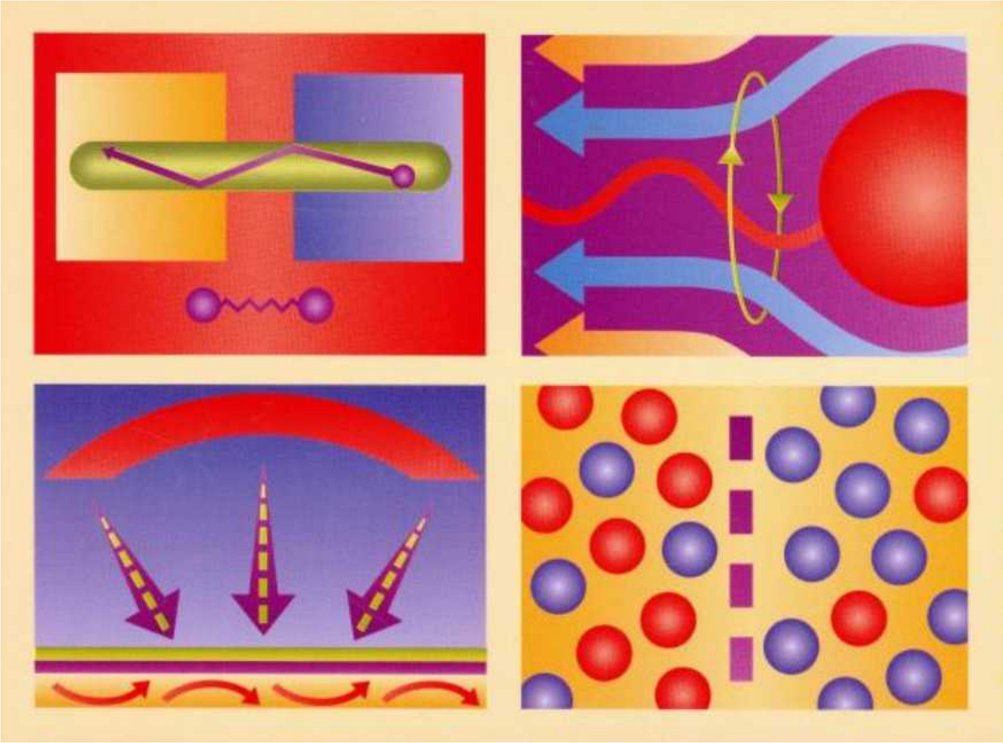
Thermodynamics II

- Second Law of Thermodynamics
  - Entropy always increases, except in an ideal process where it does not change. Cannot "un-create" entropy.
  - *"Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time."*

→ **In absence of work, thermal energy must flow from objects of higher temperature to objects of lower temperature.**

Practical note- The only means to alter heat flow is through some work input, such as with a heat pump (e.g. air conditioner) or thermoelectric device, etc.

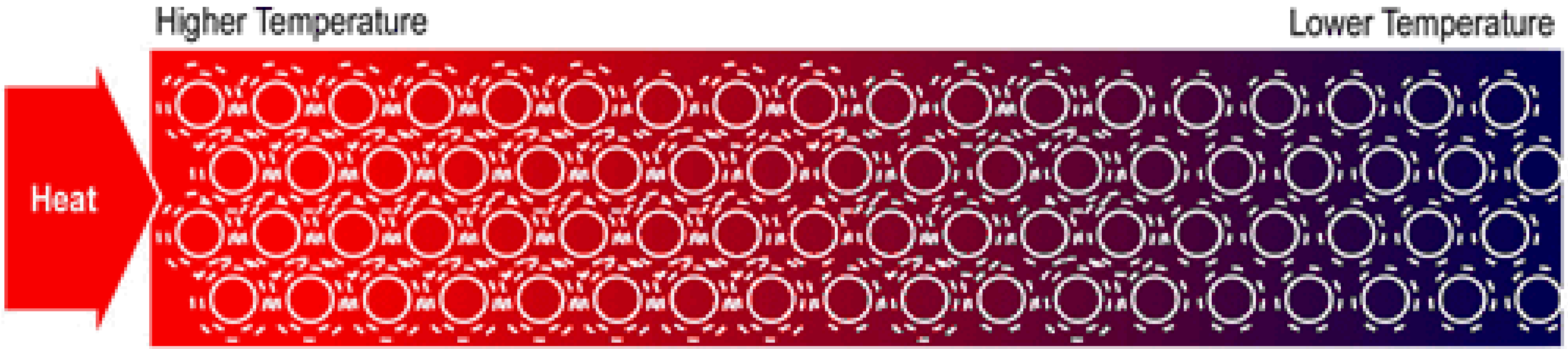
2nd Practical note- Thermoelectric devices themselves are good thermal conductors, so it actually takes a *lot* of energy to use them to create temperature differences.



Introduction to Conduction, Convection, and Radiation

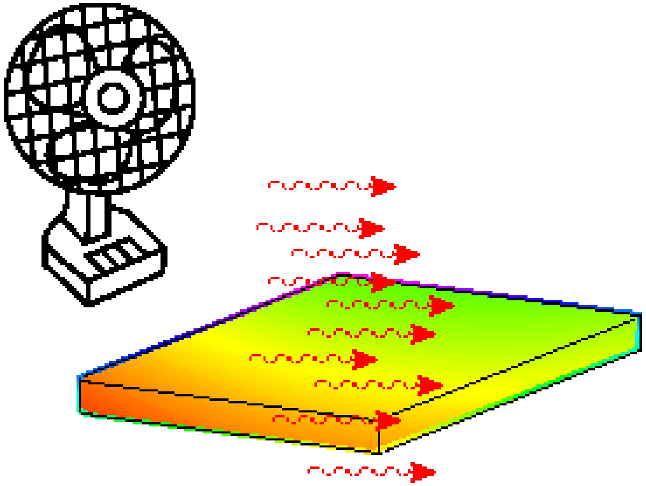
Conduction Heat Transfer

- Direct kinetic energy transfer between adjacent atoms or molecules.
- Bodies must be connected or in intimate contact for conduction heat transfer to occur.



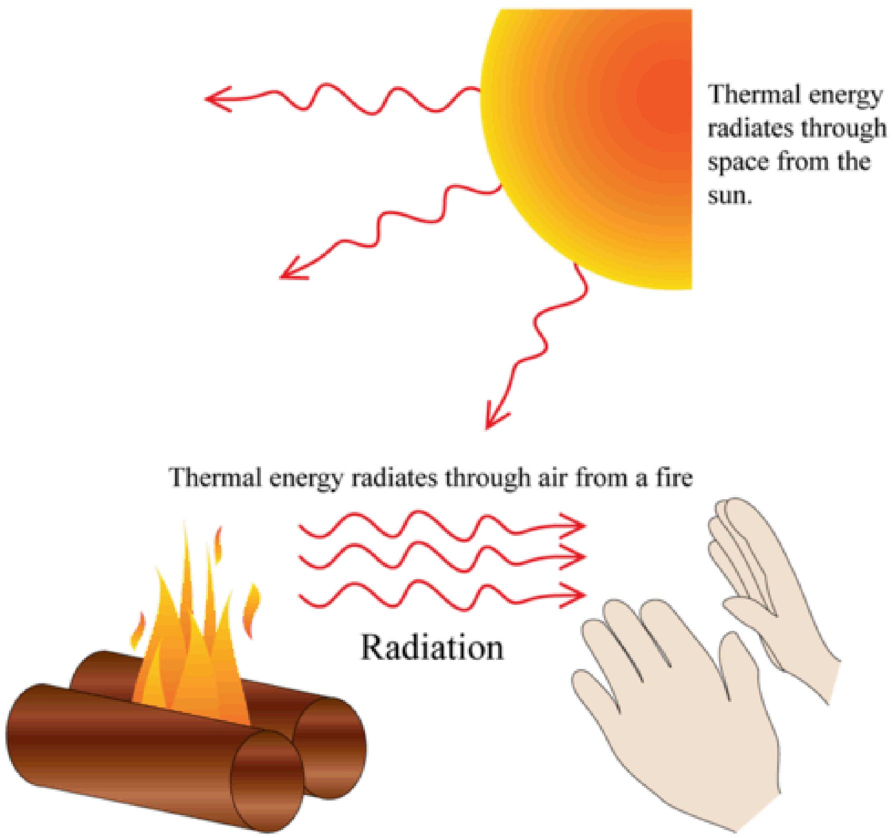
Convection Heat Transfer

- Thermal energy transfer due to bulk (macroscopic) motion of a fluid.
- Convection heat transfer only occurs in fluid mediums (liquid or gas, e.g., air).
- In a fluid, conduction occurs as well, but convective heat transfer typically dominates.



Radiation Heat Transfer

- **All matter at a temperature greater than absolute zero emits thermal radiation.**
  - Thermal energy carried by electromagnetic waves (visible light, infrared, etc.).
- Important at very high temp. (>800 °C).
  - Non-negligible at room temperature.
- **Surface condition effects radiation heat transfer.**
  - Dull, black surfaces absorb more thermal radiation.
  - Shiny, polished surfaces reflect more. **Can be shielded** with using thin, opaque material (radiation baffle).



Fourier's Law of Heat Conduction

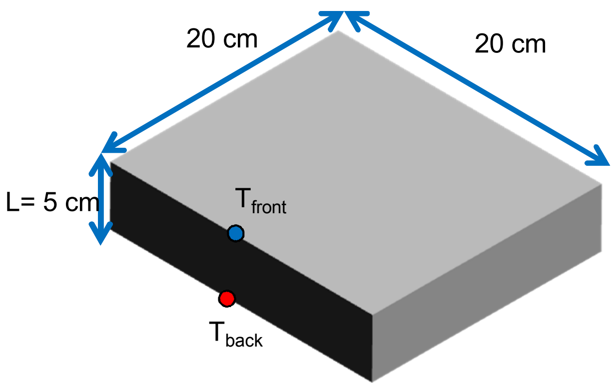
- Can relate surface temperature difference to heat flow if thermal conductivity and thickness of material is known.
- The one-dimensional, steady-state case:

$$q'' = k\frac{\Delta T}{L}$$

- **Remember that the heat flow is always from high temperature to low temperature!**

Heat Conduction Example

- Let's take a plate of **carbon steel**, 5 cm thick, with dimensions of 20 cm x 20 cm.
- Assume heat transfer occurs in one dimension only (across thickness).
- If the back side of the steel plate is in direct contact with a hot plate at 250 ^{o}C that applies 40 W of heat uniformly to the plate, what is the temperature on the front side of the steel plate?



Heat Conduction Example (cont.)

- Let's set up the equation:

$$q'' = k_{steel} \frac{(T_{back} - T_{front})}{L}$$

- We are given the heat rate from the hot plate. We need to calculate a heat flux:

$$q'' = \frac{\dot{Q}}{A_{surf}} = \frac{40\ \text{W}}{0.2\ \text{m}\ \times 0.2\ \text{m}} = 1000\ \text{W/m}^2$$

- We can now solve for T_{front}

$$T_{front} = T_{back} - \frac{q'' L}{k_{steel}} = 250\ ^\circ\text{C} - \frac{1000\text{W/m}^2 \times 0.05\text{m}}{51.9\ \text{W/(m}\ ^\circ\text{C)}}$$

$=249.0\ ^\circ\text{C}$

Thermophysical Properties of Common Solid Materials

Material	Density, ρ	Thermal Conductivity, k	Specific Heat Capacity, c_p	Thermal Diffusivity, α	Linear Coefficient of Thermal Expansion, β
Metals					
Plain Carbon Steel (AISI 1030 Steel)	7850 kg/m^3	51.9 W/m-K	486 J/kg-K	1.36 $x10^{-5}$ m^2/sec	11.7 ppm/°C
Copper (C10100)	8940 kg/m^3	391 W/m-K	385 J/kg-K	1.14 $x10^{-4}$ m^2/sec	17.0 ppm/°C
6061-T6 Aluminum	2700 kg/m^3	167 W/m-K	896 J/kg-K	6.90 $x10^{-5}$ m^2/sec	23.6 ppm/°C
304L Stainless Steel	7900 kg/m^3	15 W/m-K	475 J/kg-K	4.0 $x10^{-6}$ m^2/sec	16.5 ppm/°C
Non-Metals					
Tungsten Carbide	15,700 kg/m^3	110 W/m-K	945 J/kg-K	7.41 $x10^{-6}$ m^2/sec	5.9 ppm/°C
Granite	2630 kg/m^3	2.79 W/m-K	775 J/kg-K	1.37 $x10^{-6}$ m^2/sec	3.70 to 11.0 ppm/°C
Wood (hardwood, e.g. oak)	720 kg/m^3	0.16 W/m-K	1255 J/kg-K	1.77 $x10^{-7}$ m^2/sec	N/A
Acetal (Delrin)	1420 kg/m^3	0.289 W/m-K	1480 J/kg-K	1.38 $x10^{-7}$ m^2/sec	102 ppm/°C
PTFE (Teflon)	2200 kg/m^3	0.25 W/m-K	1300 J/kg-K	8.74 $x10^{-8}$ m^2/sec	125 ppm/°C
Glass (Soda-Lime)	2530 kg/m^3	0.94 W/m-K	880 J/kg-K	4.22 $x10^{-7}$ m^2/sec	9.5 ppm/°C

*Typical properties at ambient conditions.

Heat Conduction Example (cont.)

- What if we replace the steel plate with one made of **Teflon** with the same dimensions?

$$T_{front} = T_{back} - \frac{q''L}{k_{Teflon}} = 250\ ^{\circ}\mathrm{C} - \frac{1000\mathrm{W/m^2} \times 0.05\mathrm{m}}{0.25\ \mathrm{W/(m\ ^{\circ}C)}}$$

$=50.0\ ^{o}$C

→ The much lower thermal conductivity of Teflon *increases* the temperature drop, resulting in a cooler surface. Teflon and plastics in general serve as good **thermal insulators**.

Thermophysical Properties of Common Solid Materials

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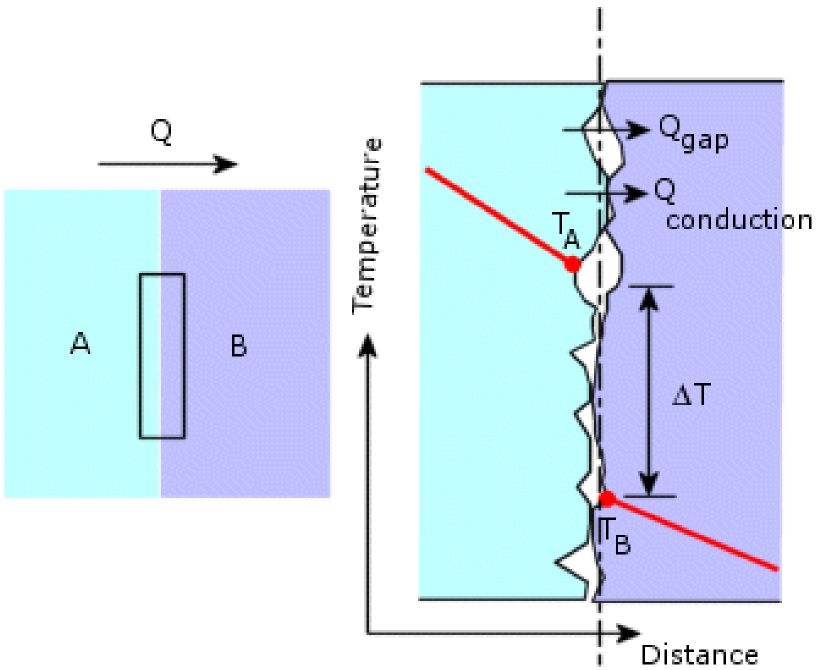
*Typical properties at ambient conditions.

Thermal Contact Resistance

- In previous example, we assumed the steel (or Teflon) plate was in perfect contact with the hot plate.
- However, conduction is a microscopic process, and if we looked at the interface under a microscope:



- **Large temperature change** occurs since solid-to-solid **conduction only possible where peaks touch**. Heat conduction also occurs through air in gap, but air has a very low thermal conductivity.

Thermal Contact Resistance (cont.)

- We can define a thermal contact resistance per unit area, R_{tc}, as:

$$R_{tc} = \frac{1}{h_c} = \frac{\Delta T_{int}}{q''} \qquad \text{(m}^2\text{-K/W)}$$

  where ΔT_{int} is the temperature drop across the interface and h_c is the thermal contact conductance.
- The thermal contact resistance is difficult to predict analytically- depends on **contact pressure**, **surface texture**, **material**, hardness, and **other factors**. Usually it must be determined empirically.
- Typical values of contact conductance, h_c provided in the next Table.

Practical note- Thermal greases or thin foils of soft metals (gold, lead, copper) can be used to reduce thermal contact resistance at interfaces.

Thermal Contact Resistance (cont.)

Situation	h_c (W/m^{2}K)
Iron/aluminum (70 atm pressure)	45,000
Copper/copper	10,000 – 25,000
Aluminum/aluminum	2,200 – 12,000
Graphite/metals	3,000 – 6,000
Ceramic/metals	1,500 – 8,500
Stainless steel/stainless steel	2,000 – 3,700
Ceramic/ceramic	500 – 3,000
Stainless steel/stainless steel (evacuated interstices)	200 – 1,100
Aluminum/aluminum (low pressure and evacuated interstices)	100 – 400

- In our previous example, what would the temperature drop between the hot plate and steel plate be if we assume a contact resistance of 5.0x10^{-3} m^2-K/W (h_c=200 W/m^2-K)?

$$\Delta T_{int} = q'' \times R_{tc} = 1000\,\frac{\mathrm{W}}{\mathrm{m}^2} \times 5.0 \times \frac{10^{-3}\mathrm{m}^2\mathrm{K}}{\mathrm{W}}$$

=5 °C

Convective Law of Cooling (Newton's Law of Cooling)

- Can relate temperature difference between a solid surface and ambient fluid (e.g., air in room) using a **heat transfer coefficient**, *h:*

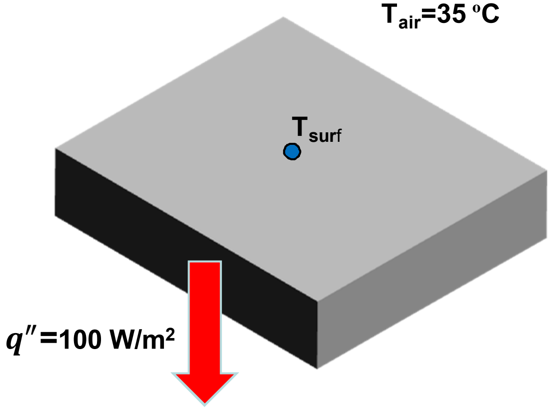
$$q'' = h\left(T_{surf} - T_{air}\right) \text{ for } T_{surf} > T_{air}$$

$$q'' = h\left(T_{air} - T_{surf}\right) \text{ for } T_{surf} < T_{air}$$

- **Remember that the heat flow is always from high temperature to low temperature!**

Convection Heat Transfer Example

- Let's take the same plate of **carbon steel** from the conduction example.
- The HVAC system in our room is out, and the air temperature is at a sweltering 35 ºC (T_{air}=35 ºC).
- If we connect a chiller to cool the backside of the plate that removes q''=100 W/m^2 , and we know the convective heat transfer coefficient in air from natural circulation is ~10 W/m^2-K, what is the temperature on the front of the plate?



Convection Heat Transfer Example (cont.)

- The heat is flowing from the air, through the plate, and out the back of the plate to the chiller. Even though we don't know the backside temperature, if we assume steady-state conditions the heat into the top surface must equal the heat to the chiller (First Law).

$$q'' = h\left(T_{air} - T_{surf}\right)$$

$$100\ \mathrm{W/m^2} = 10\mathrm{W/m^2K} \times \left(35^{\circ}\mathrm{C} - T_{surf}\right)$$

$$\rightarrow T_{surf} = 25\ ^{\circ}\mathrm{C}$$

Typical Values of the Convection Heat Transfer Coefficient

- **Free convection** refers to the motion of a fluid without external agitation.
  - Temperature gradients (due to a warm object) cause density change in fluid. Fluid motion from buoyancy.

- **Forced Convection** result of external agitation (e.g. fan, blower, pump, etc.).

Process	Heat Transfer Coefficient, *h* (W/m^2-K)
Free Convection	
Gases (e.g., air)	2 to 25
Liquids	50 to 1000
Forced Convection	
Gases (e.g., air)	25 to 250
Liquids	100 to 20,000
Convection with Phase Change	
Boiling or condensation	2500 to 100,000

Natural Convection Heat Transfer Horizontal Plate Correlations

- Heat transfer from the upper surface of a *hot* plate in air:

$$h = 0.54\left(\frac{k_{air}}{L_c}\right)\left[\frac{g\beta_{v,air}\left(T_{surf} - T_{air}\right){L_c}^3}{\left({}^{\mu_{air}}/{}_{\rho_{air}}\right)\alpha_{air}}\right]^{1/4}$$

- Heat transfer to upper surface of a *cold* plate:

$$h = 0.27\left(\frac{k_{air}}{L_c}\right)\left[\frac{g\beta_{v,air}\left(T_{air} - T_{surf}\right){L_c}^3}{\left({}^{\mu_{air}}/{}_{\rho_{air}}\right)\alpha_{air}}\right]^{1/4}$$

Where g is the acceleration due to gravity (9.8 m/sec^2), and L_c is the characteristic length in meters, defined by the ratio of the plate surface area to perimeter: $L_c \equiv \frac{A_{surf}}{P}$

Forced Convection Heat Transfer

Plate Correlations for Laminar and Turbulent Flow

- Heat transfer from plate under laminar flow, uniform heat flux conditions:

$$h = 0.680\left(\frac{k_{air}}{L}\right)\left[\frac{\rho_{air}\mathrm{v}_{air}L}{\mu_{air}}\right]^{1/2}\left[\frac{\mu_{air}c_{p,air}}{k_{air}}\right]^{1/3}$$

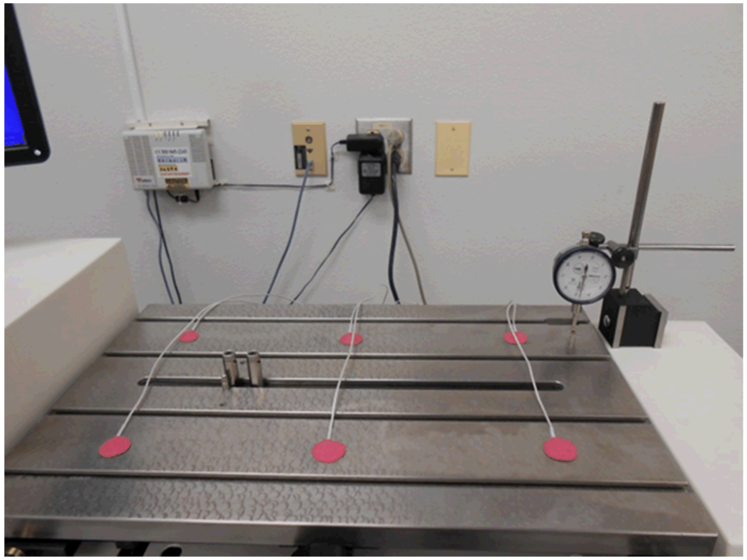
- Heat transfer from plate under turbulent flow, uniform heat flux conditions:

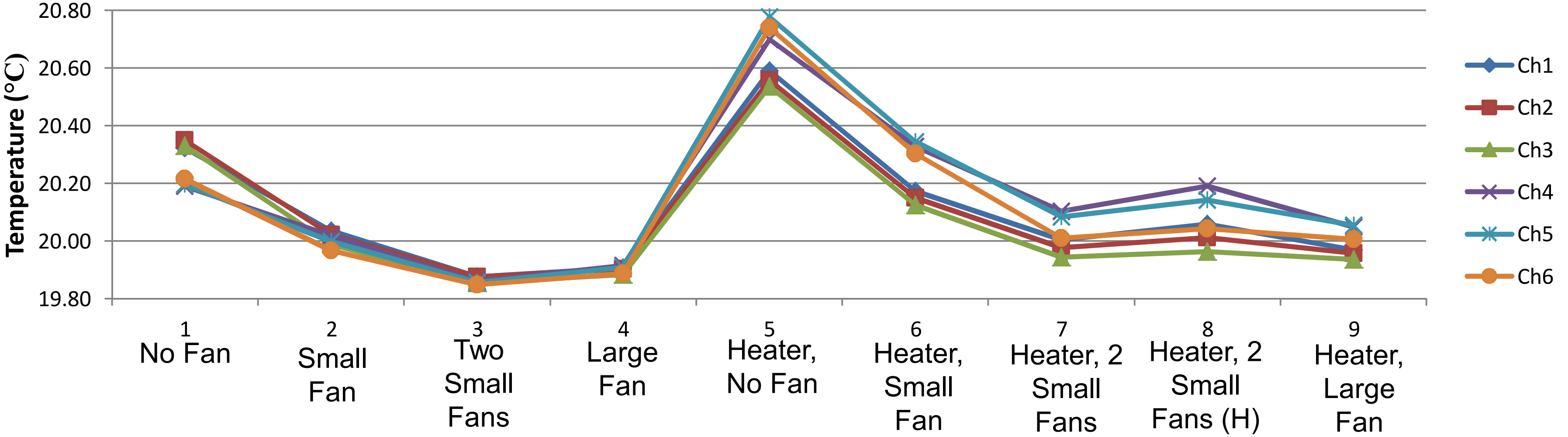
$$h = 0.037\left(\frac{k_{air}}{L}\right)\left[\frac{\rho_{air}\mathrm{v}_{air}L}{\mu_{air}}\right]^{4/5}\left[\frac{\mu_{air}c_{p,air}}{k_{air}}\right]^{1/3}$$

Where *L* is the length of the plate in meters, and v_{air} is the flow velocity of the air parallel to the plate.

A Practical Example- Temperature Uniformity of LMU Table

- Each channel represents location on table.
- Use of fans improves temperature uniformity, stability of table.





The Transient Problem

- Thus far, we have assumed steady-state conditions (thermal equilibrium).
- However, for dimensional measurements, one may need to know ***how long*** it takes for a system to reach thermal equilibrium.
- Useful parameter is the **Biot Number, *Bi*** (dimensionless):

$$Bi = \frac{hL_c}{k_{solid}}$$

- For systems where $Bi < 0.1$, we can use a *lumped capacitance analysis* to solve the transient problem.

The Transient Problem (cont.)

- The temperature of some solid object in a fluid at ambient temperature (e.g., air) after a certain amount of time is given by:

$$T_{solid}(t) = T_{amb} + \left[T_{i,solid} - T_{amb}\right] \times exp\left[-\left(\frac{hA_{surf}}{\rho_{solid} V c_{p,solid}}\right) t\right]$$

where $T_{solid}(t)$ is the solid object temperature at time t (t in seconds), $T_{i,solid}$ is the initial temperature of the solid object, A_{surf} is the surface area of the solid, V is the volume of the solid object, and h is the heat transfer coefficient of the fluid (e.g., air).

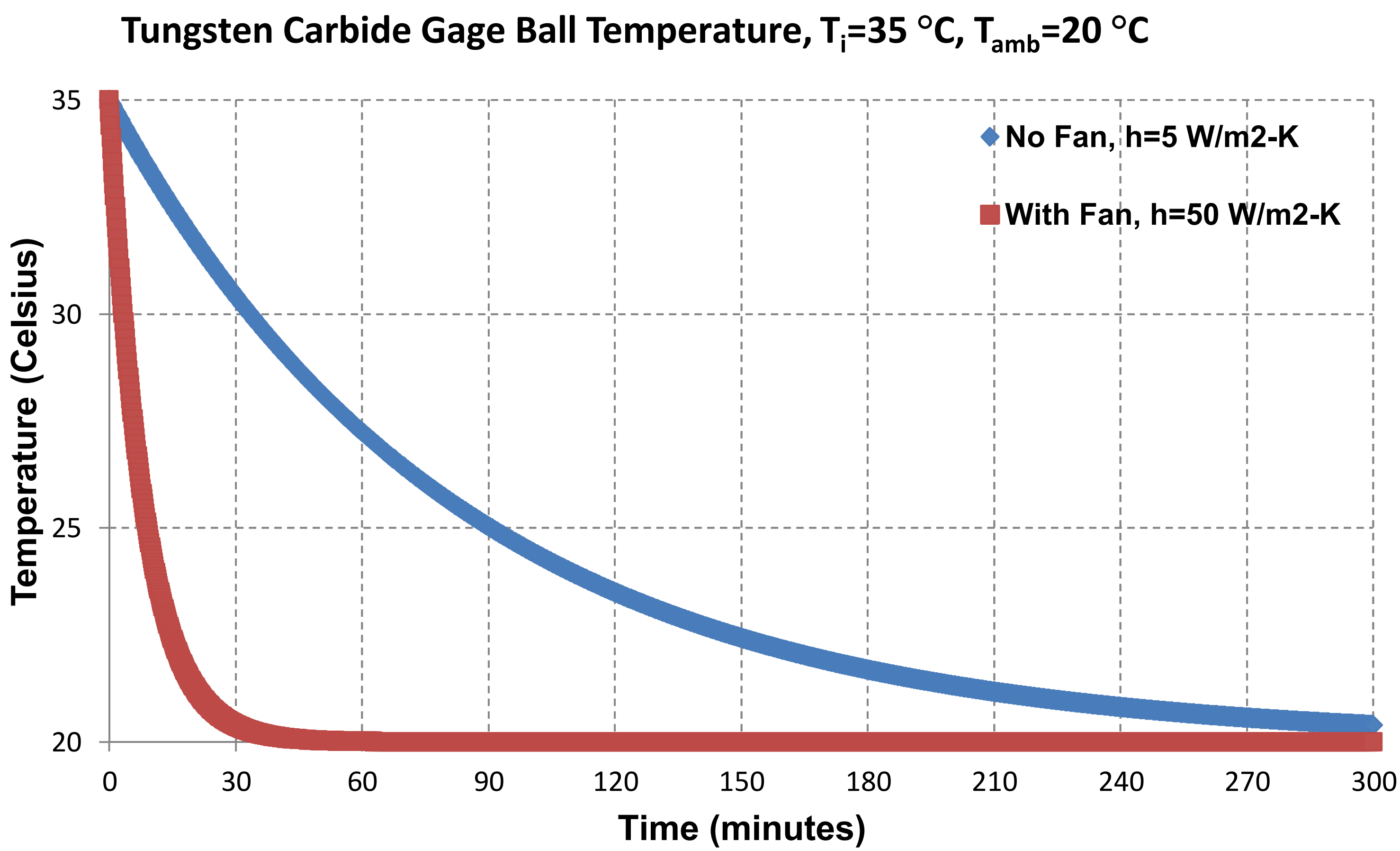
Transient Problem with Gage Ball

- A metrologist carries around a **10mm tungsten carbide** gage ball in his closed hand, bringing the temperature to **35 ^{o}C**.
- After realizing his mistake, he sets the gage ball down on a well-insulated tray open to air, with the ambient temperature at T_{amb}**=20 ^{o}C**.
- If the natural convection heat transfer coefficient is ***h*=5 W/m^2-K**, how long will it take the gage ball to cool to room temperature?
- What if the metrologist realizes that a small portable fan could **increase *h* to 50 W/m^2-K**? How long will it take to cool with the fan?

Transient Problem with Gage Ball (cont.)

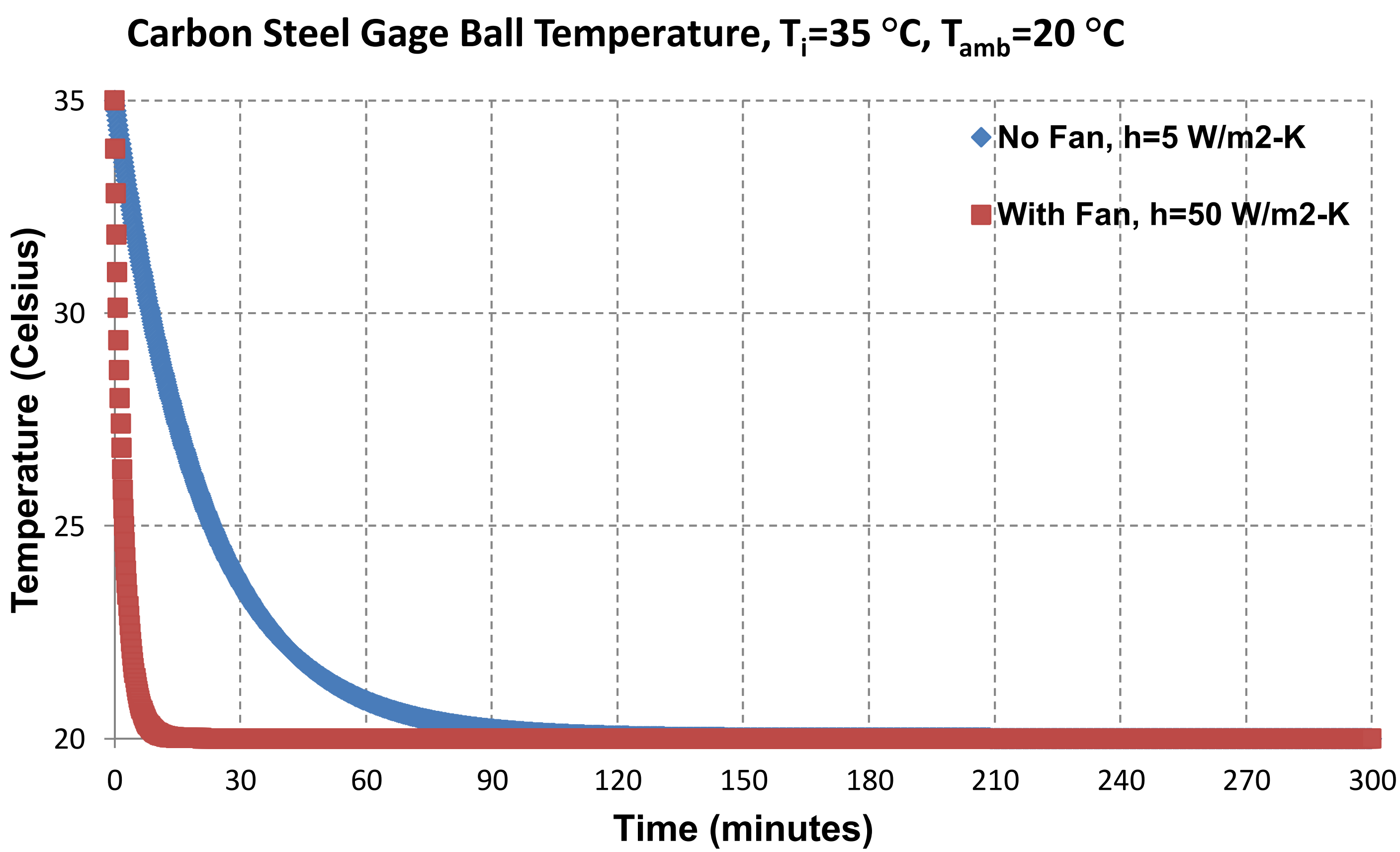
- First, we need to calculate the Biot number. For the case with and without the fan, *Bi*= 0.002 and 0.0002, respectively. Therefore a lumped capacitance approach is valid.
- Treat the characteristic length of a sphere as half the diameter, so L_c=0.005 m.
- Tungsten carbide properties can be found in the table.

Transient Problem with Gage Ball (cont.)

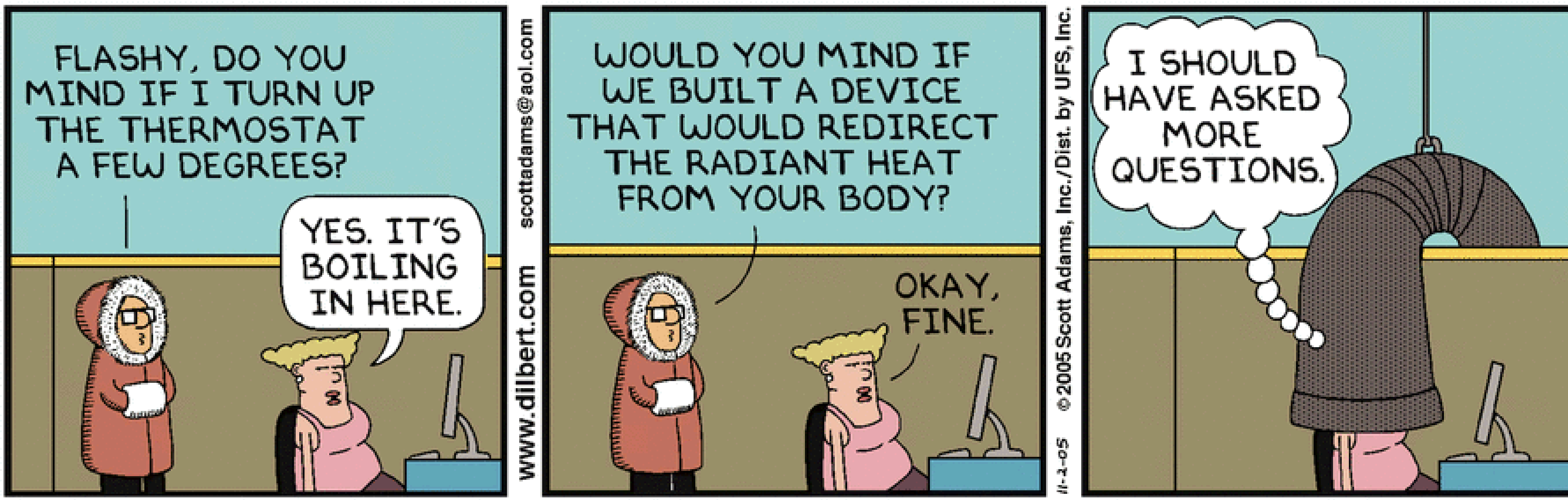


Transient Problem with Gage Ball (cont.)

- What if the gage ball were made from carbon steel?



Sandia National Laboratories



Bioheat Transfer

The Human (Heating) Element

- Average person dissipates ≈100 W of heat under non-strenuous activity.

Removing the Human (Heating) Element

- A person dissipates heat via heat transfer modes already discussed.
  - Conduction: Picking up an object (e.g., gage block) at ambient will increase its temperature due to conduction between your fingers and the object. Use **Teflon-tipped tweezers** instead!
  - Convection: The human body will induce additional convection currents in the vicinity, affecting measurements. Use of an **≈100 W radiant heater** (e.g., a boot tray warmer) can be used to simulate a person to establish equilibrium convection currents prior to measurements.
  - Radiation: Exposed skin radiates more heat than clothing. Use of **labcoats** during measurements reduces radiant heat transfer to units being measured.

A Few Notes on Temperature Measurement

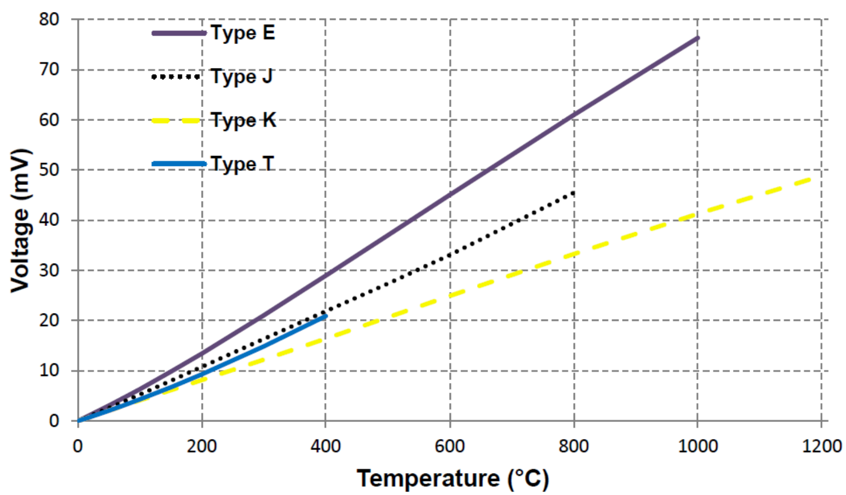
Thermocouples

- When two dissimilar metals are joined electrically, a small voltage is produced.
- Amplitude of voltage is proportional to temperature at the junction.
- Properties of common thermocouple elements listed in table.

Practical note- Sharp bends in thermocouple sheath can cause grounding, resulting in erroneous readings!

	Type E	Type J	Type K	Type T
(+) Junction Material	Chromel (90%-10% Ni-Cr)	Iron	Chromel (90%-10% Ni-Cr)	Copper
(-) Junction Material	Constantan (55%-45% Cu-Ni)	Constantan (55%-45% Cu-Ni)	Alumel (94%-3%-2%-1% Ni-Mn-Al-Si)	Constantan (55%-45% Cu-Ni)
Operational Range	-253 °C to 1000 °C	-253 °C to 760 °C	-253 °C to 1370 °C	-253 °C to 400 °C
Recommended Range* **(protected element)**	-243 °C to 650 °C	-18 °C to 590 °C	300 °C to 1090 °C	-185 °C to 370 °C
Seebeck Coefficient	58.5 µV/K at 0 °C	50.2 µV/K at 0 °C	39.4 µV/K at 0 °C	38.0 µV/K at 0 °C
Other Considerations	Non-magnetic; Annealed wire recommended	Magnetic	Magnetic; Curie point at 152.5 °C	Non-magnetic; High thermal conductivity

*Upper temperature limit may also depend on wire gauge.



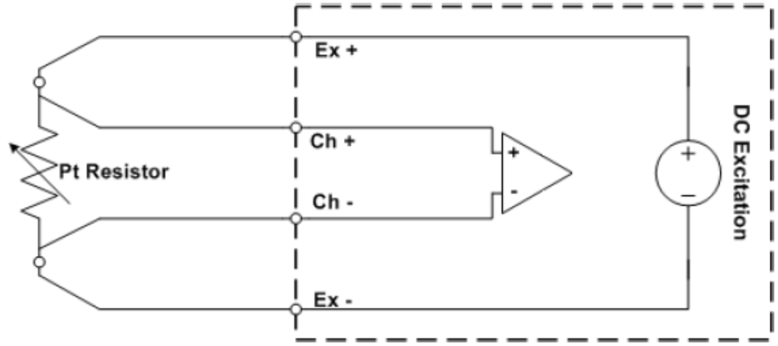
Thermistors

- "Negative Temperature Coefficient" (NTC) Thermistors
- Nonlinear behavior of electrical resistance to temperature
- Highly sensitive, e.g. lots of mV for small change in T (thermistor systems can achieve ~ 10 mK uncertainty in the 20 C range & 1 mK readability)
- Inexpensive, repeatable

- Somewhat fragile
- Cannot handle large swings in temperature
- As long as used only within std dimensional range, typically the preferred temperature transducer

Resistive Temperature Devices (RTDs)

- Platinum resistance thermometers (PRTs)
- Thermistors
- Typically higher accuracy available, but more expensive and fragile.
- Four-wire configurations most accurate; eliminate lead wire resistance effects.



Measurement Uncertainty

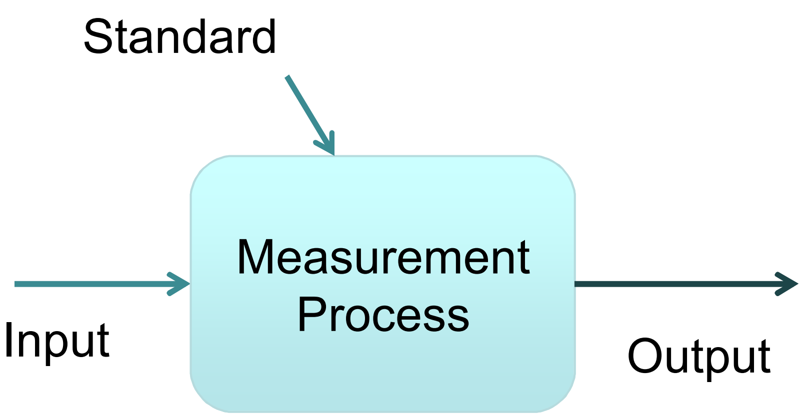
- A statement of measurement is ***incomplete*** without a statement about its uncertainty
- The best uncertainty statement ever published:

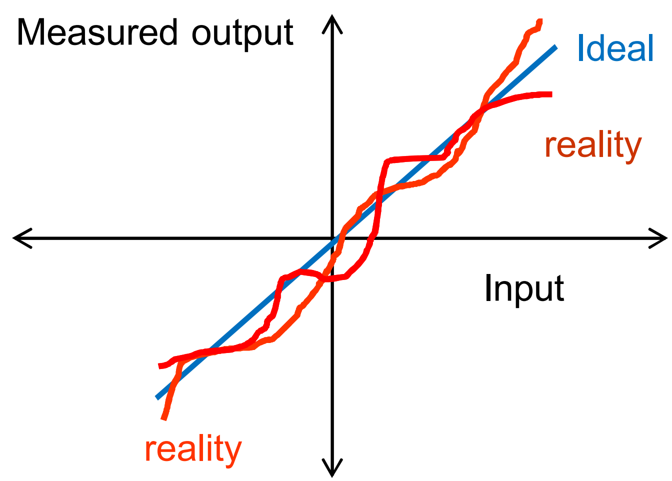
> "We think our reported value is good to 1 part in 10000. We are willing to bet our own money at even odds that it is correct to 2 parts in 10000. Furthermore, if by any chance our value is shown to be in error by more than 1 part in 1000, we are prepared to eat the apparatus and drink the ammonia."
>
> —Anecdotal publication from NBS (now NIST), on measuring heat capacity of ammonia (Meyers or Myers; 1930's)

- Traceability: Comparisons (including uncertainties) that trace to the SI through national or international standards (NMI's), or intrinsic standards

Where does measurement uncertainty come from?

- Measurements are fundamentally comparisons
  - Uncertainty in the unknown
  - Uncertainty in the standard for comparison
  - Uncertainty in the process of making the measurement
- Even “direct reading” instruments make a comparison—you just don’t see the comparison (the comparison is hidden)



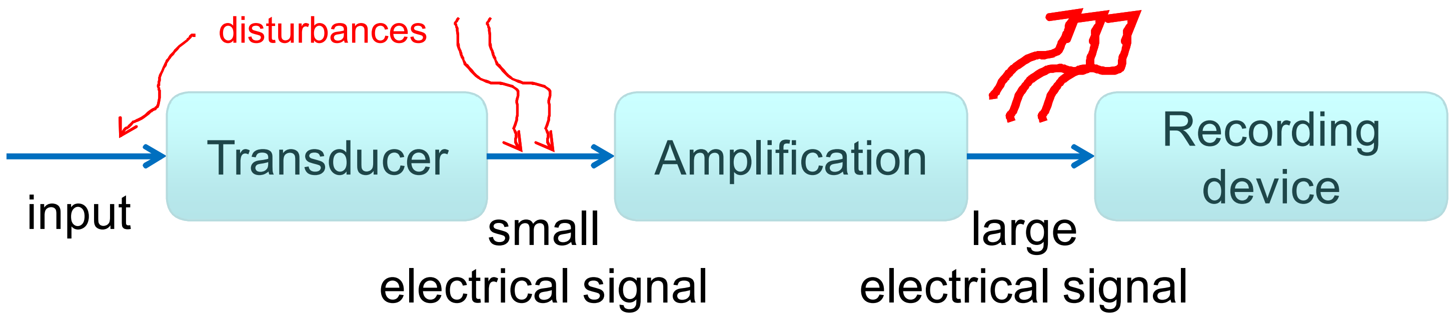


GUM uncertainty

- Imperfect process/standard/input leads to “doubt about the validity of the result”
- Formal method for expressing “uncertainty of result of measurement”—Estimate presented as a range and a confidence level (risk of having to eat equipment!)
  - Uses a model for the measurement
  - Divides into statistical analysis (type A evaluation) & non-statistical analysis (type B evaluation)
  - Part of the model: Statistical & non-statistical evaluations are ***independent*** (i.e. not correlated), so you combine the result by adding the variances
  - Part of the model: Influence factors (example: combining mass & dimensional measurements to measure density—some measurements have a greater influence, or “weight” on the result).

Noise—the model

- What we commonly call "noise" is more properly a combination of noise + disturbances
- We typically deal with noise by ***repeating*** the measurement several times, and doing a statistical evaluation (average of measurement)
- What assumptions are we making? (are there better ways of dealing with noise than repeating measurements?)
- Modeling a mechanical displacement measurement: Each step adds to the uncertainty

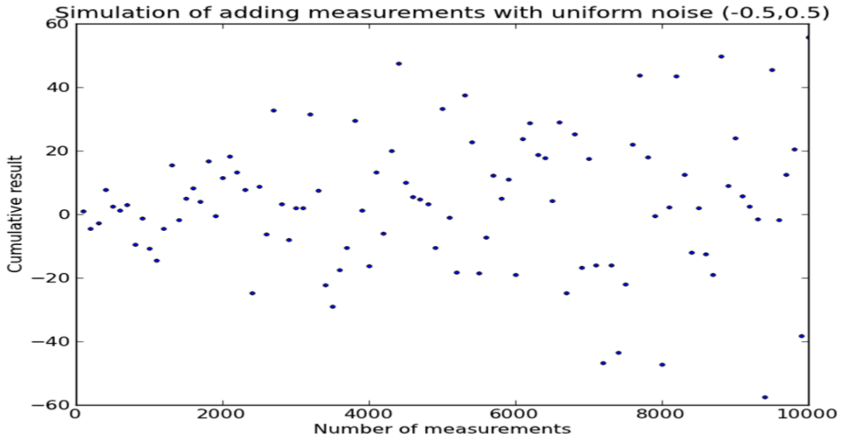


More on noise:

- Lowell Howard's law: "Sensors that make no noise are probably not plugged in" (Lowell attributes this to *anonymous*, 1996)
- If I have "uniformly" distributed noise, can I take multiple measurements?
  - Be careful: Let's say I have a measurement with an expected value of zero, and I'm adding a random number (to simulate noise) between (0.5,-0.5)
  - Let's take multiple measurements and add them. I should get a value near zero, because the mean of my random number is zero?

Simulation: add random numbers (mean=0) ***n*** times (100 to 10000)

Expect as ***n*** goes up, result goes to:



Enough digression…

- Let’s move on to sensors/transducers—purpose of digression was to make you aware of some possible “gotchas”. Details are too long for this short talk
- When is it easier to have a 10 nm repeatability than a 10 mm repeatability?
- When your span of measurement is 1 μm, vs 1 km!
  - 1 part in 10^2 vs 1 part in 10^5 !
- That’s why most high end M&TE is used as comparative equipment, as opposed to direct-reading equipment

Linear ***V***ariable ***D***ifferential ***T***ransformer

The LVDT measures displacement. Many of the ideas in the principle of LVDT operation and signal conditioning can be used with other precision sensors

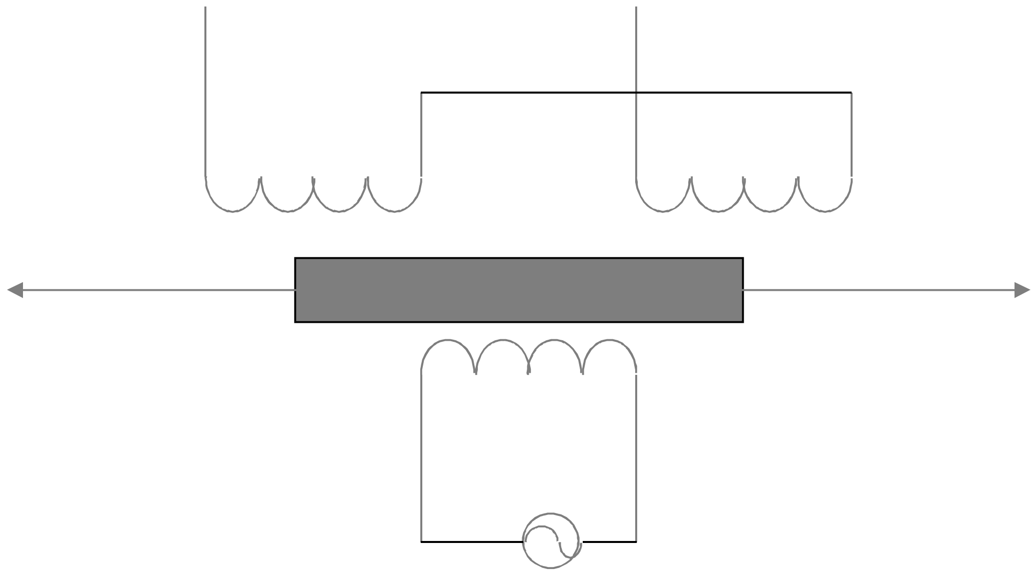
LVDT’s are used in:

- CMM probe head (3 LVDT’s)
- Gage block comparators (~ 1 nm repeatability!)
- Stylus-based surface finish machine
- Roundness machine (0.1 nm repeatability!)

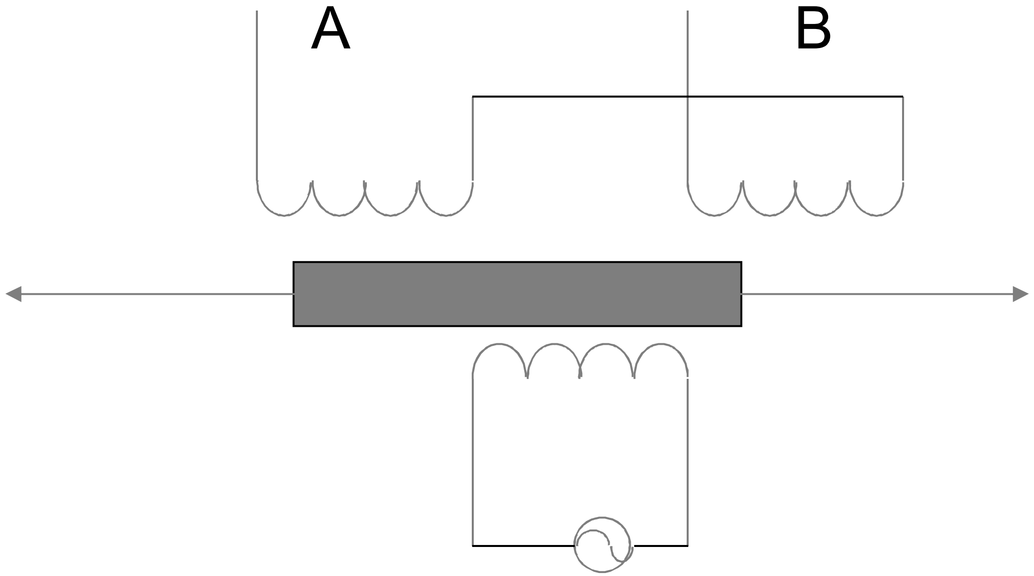
Scale-based systems are beginning to approach capability at the low end; they typically are used where large ranges of motion is desired.

Photo of Lucas-Schaevitz LVDT

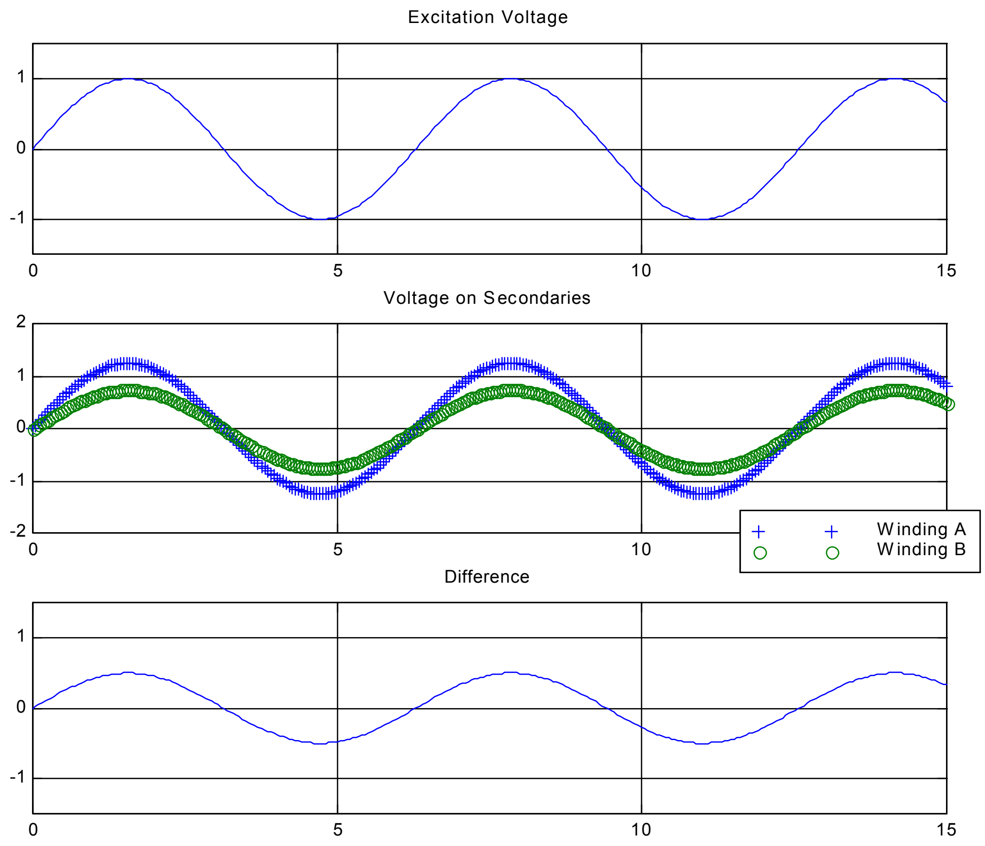
Basic LVDT Theory of Operation



Basic LVDT Operation (continued)

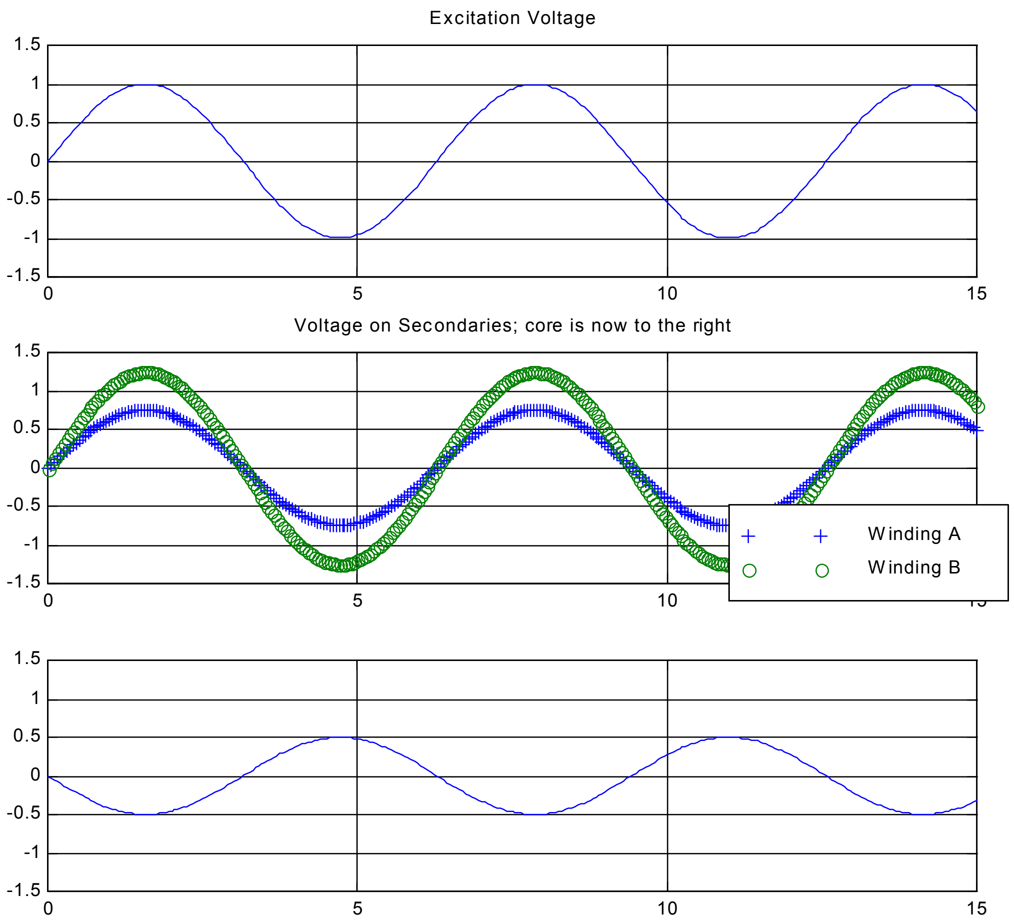


Waveforms



Excitation Voltage
1
0
-1
0
5
10
15
Voltage on Secondaries
2
1
0
-1
-2
0
5
10
Winding A
Winding B
Difference
1
0
-1
0
5
10
15

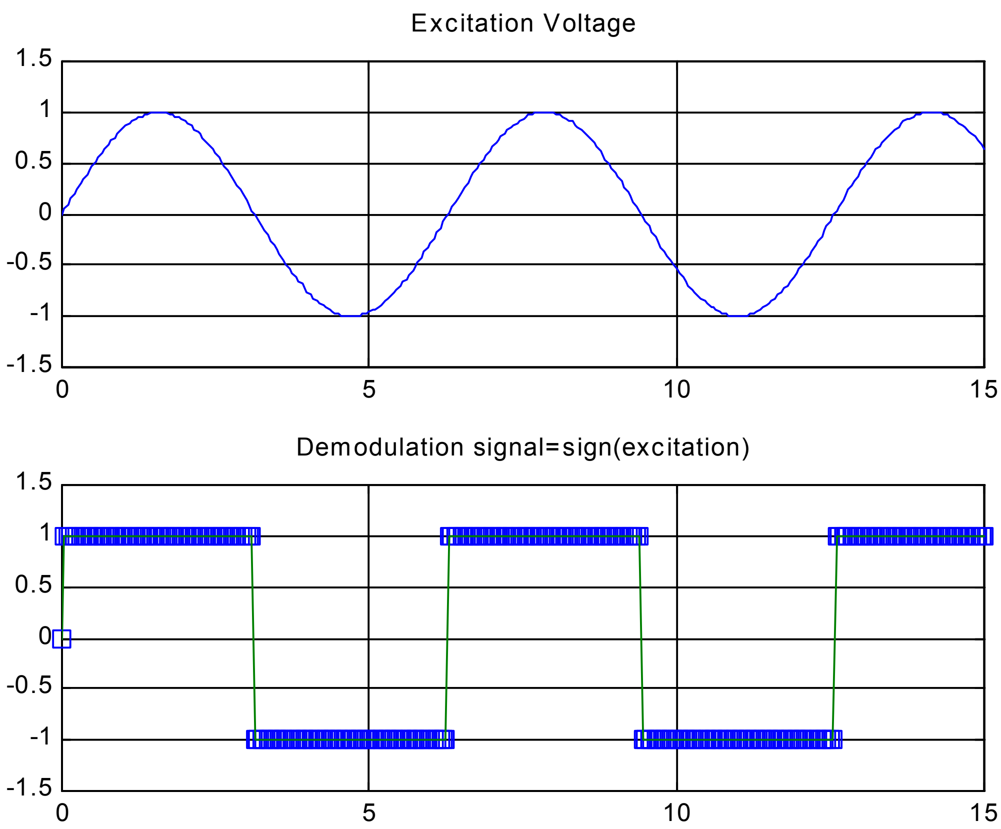
Move core to the right: Waveforms



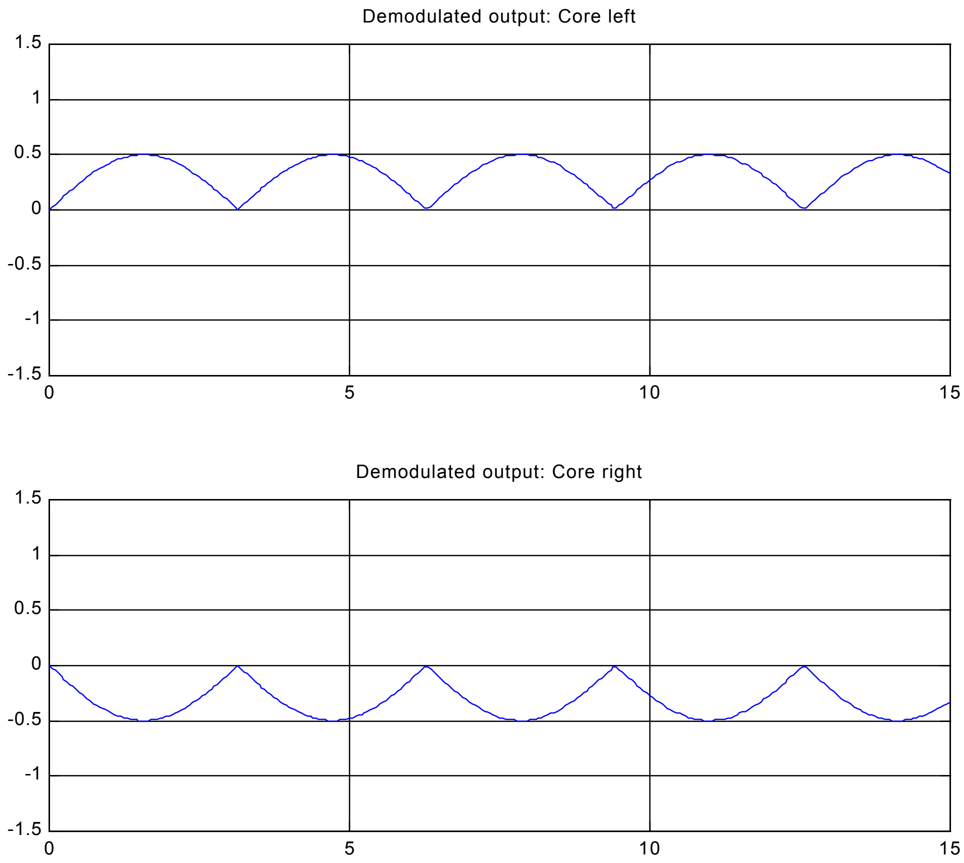
Synchronous Demodulation

- Use synchronous demodulation (e.g. “intelligent” rectification) to extract direction information
- How to do synchronous demodulation: Multiply signal to be demodulated by the demodulating signal. The demodulating signal is at the same frequency and constant phase offset with the excitation.

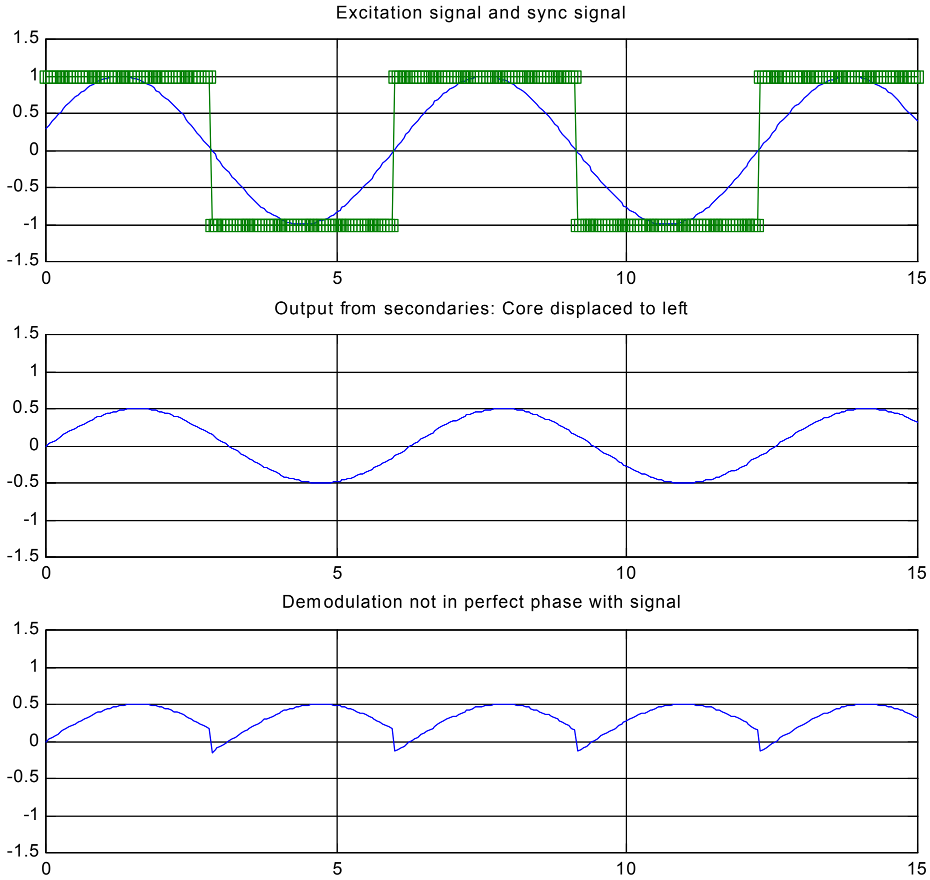
Sync. Demod. waveforms



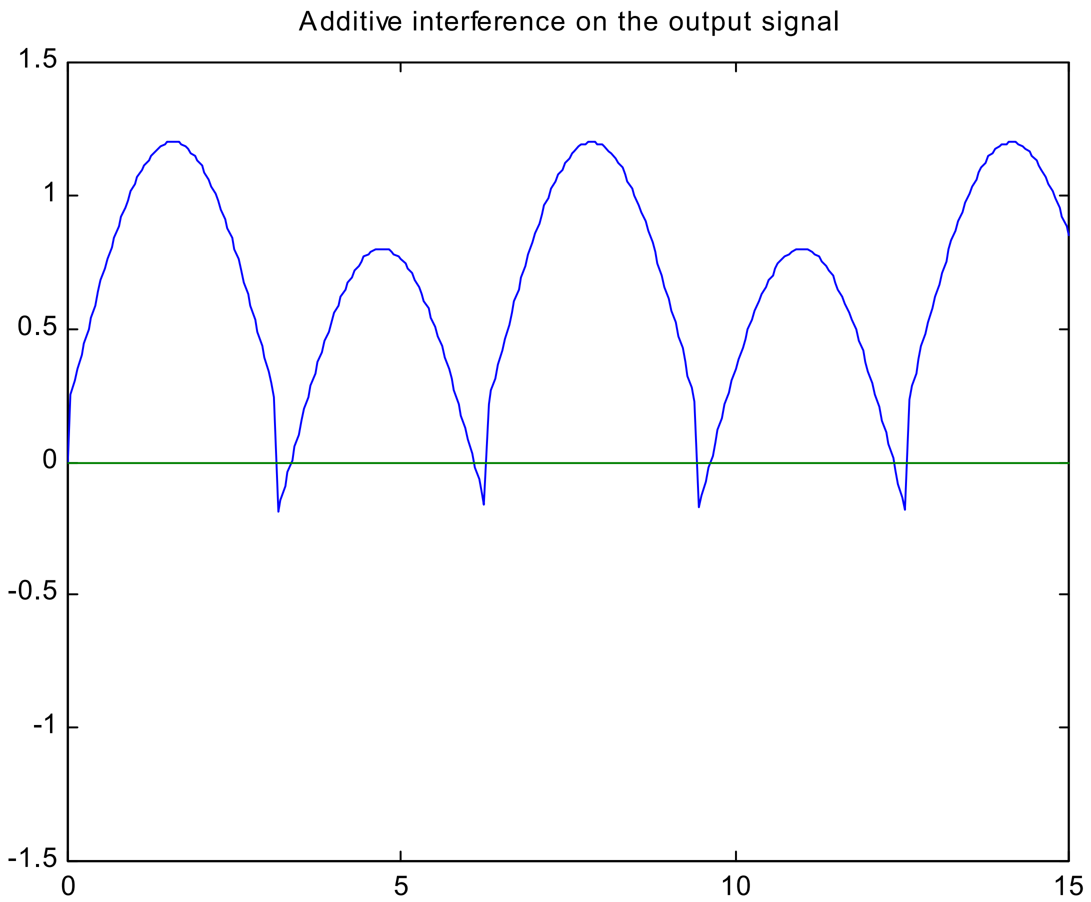
Demodulated Output



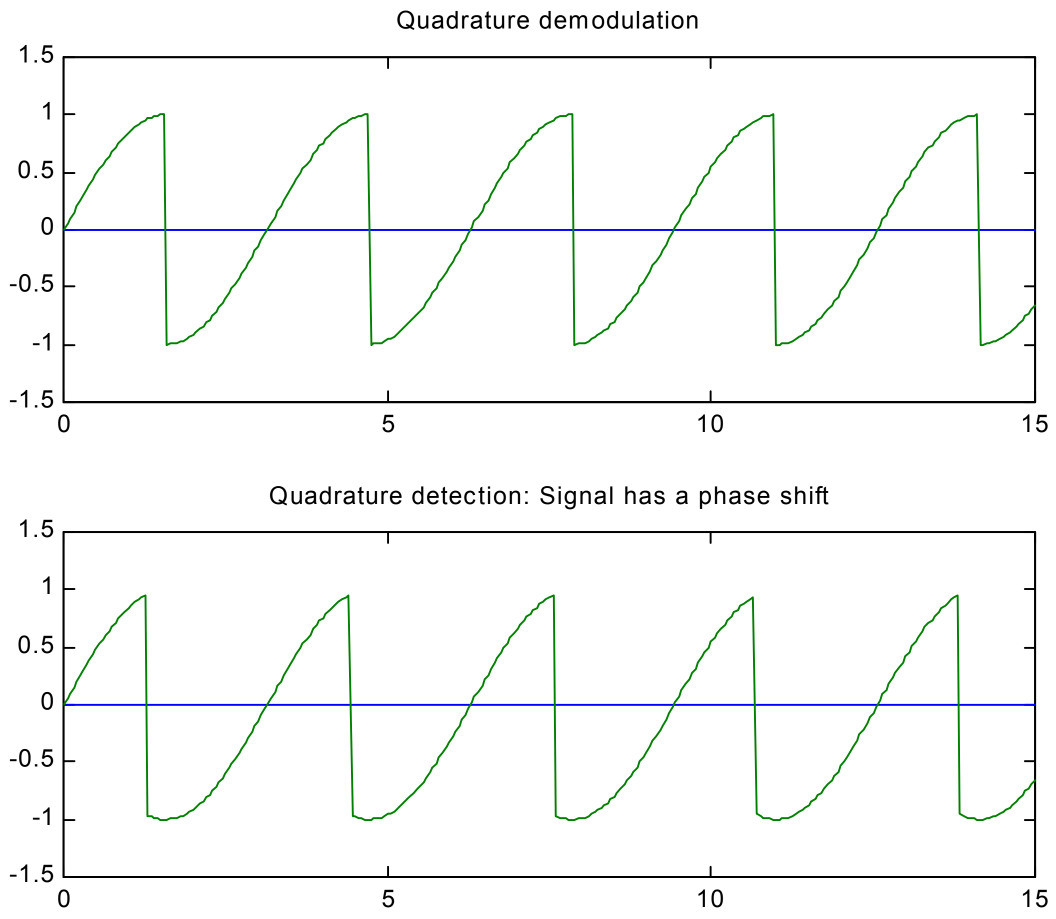
Effect of phase error



Effect of added interference



Quadrature detection



Signal conditioning technique is common

- The technique I presented is commonly used—For example, the electronics used to drive the load cells in the Force lab
- Insensitive to 60 Hz (if you've done it right)
- Relatively insensitive to other types of electronic interference

- Key is that it moves your operating range outside of where you expect noise/interference to be
- More signal, less noise!

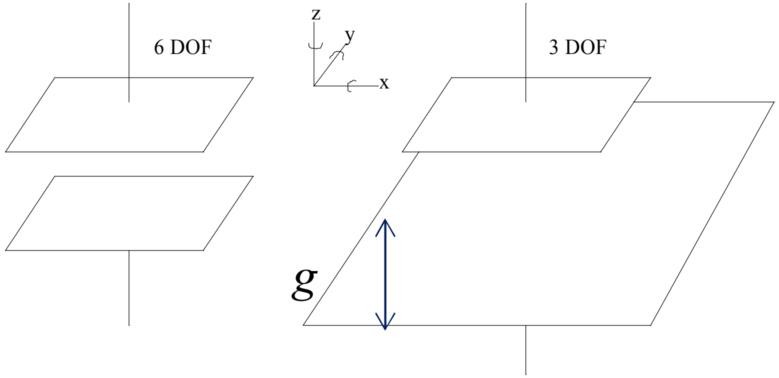
Capacitive sensors

- Not in LMF, but used in many high resolution transducers (diaphragm pressure transducers, for example)
- Non-contact, relatively inexpensive, can get ***very*** high resolution (attometer resolution!), fast operation!
- But—they have small capacitances (typically pF range)—susceptible to interference; depending on environment, susceptible to contamination (oil/surface contamination is a major issue)

Principle of operations

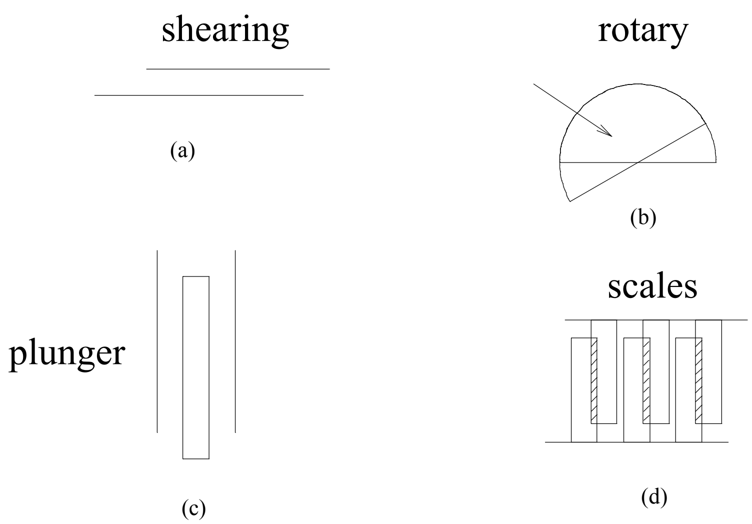
Parallel Plates

- $C = \frac{\varepsilon A}{g}$
- >1 DOF can cause problems



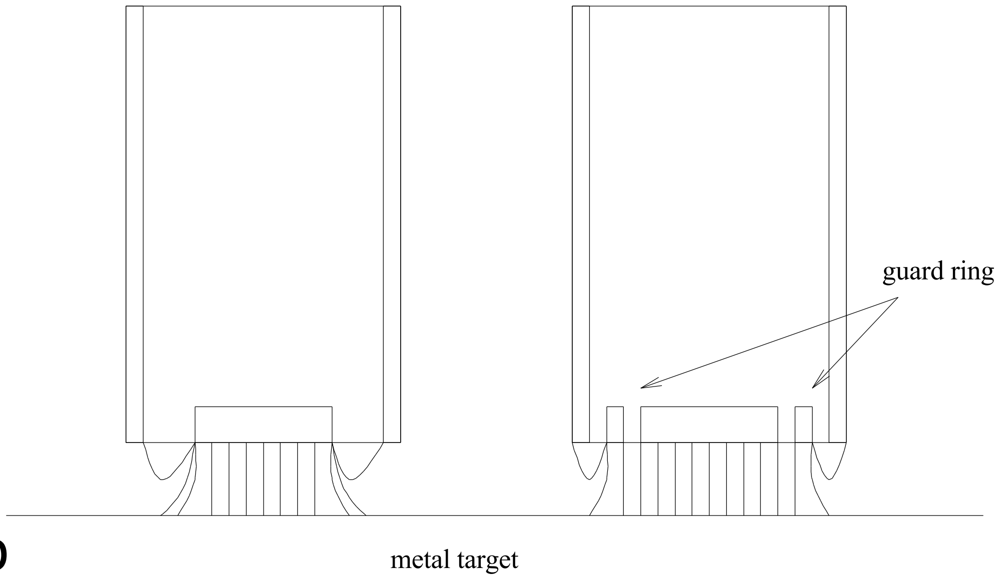
in vacuum, $\varepsilon \equiv \frac{1}{\mu_0 c^2}$ ~8.85×10^{-12} F/m

Measurement Geometries

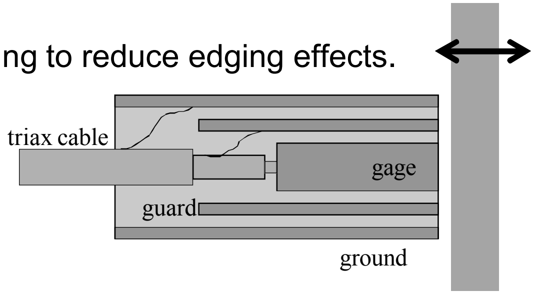


Some details requiring careful attention

- Edge effects
- Tilts
- Environmental effects—humidity, surface films
- Mechanical stability of electrode
- Dimensional stability of gap
- Stability of reference capacitor
- Triboelectric effects of cables
- The electronics too!



Use guard ring to reduce edging effects.



Optical short-range sensors

Sandia National Laboratories

- Typically, non-contact, reasonably standoff distance (order of mm)
- Robust to electromagnetic interference
- Fast
- But—what is the true mechanical surface? (do you care?)
- Atmospheric turbulence, beam bending?
- If you operate too slowly, gotchas with 1/f noise
- Susceptible to surface contamination

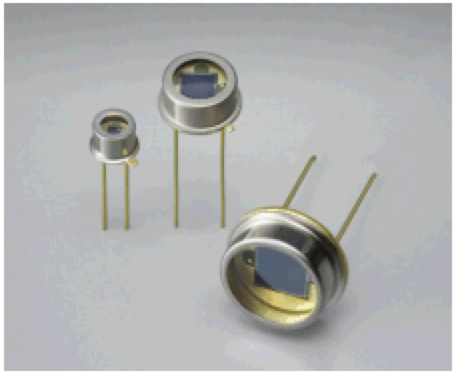
- Used as safety lockout in CMM
- Used as measurement mechanism in mass comparators
- Used in Talysurf CLI surface profile

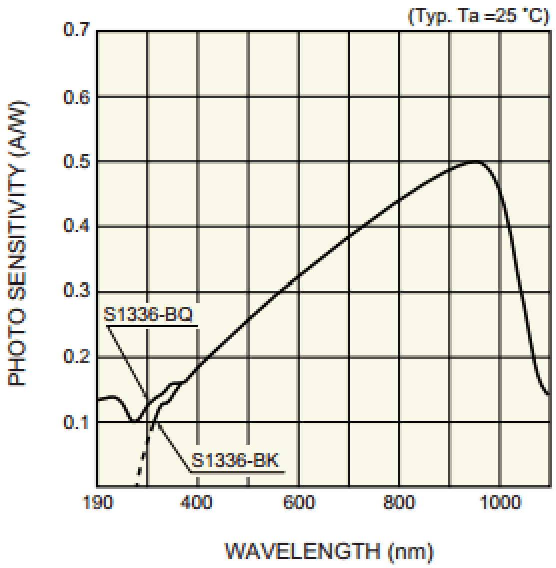
Optical sensors have:

- A light source
- Some sort of mechanism for converting displacement to affect the light
  - Shadow
  - Optical lever
  - Triangulation
  - Focus change
- A light detector

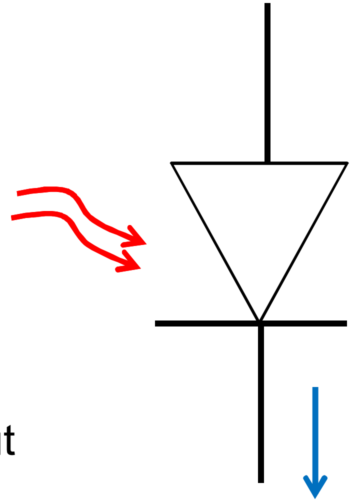
Let's start at the detector:

- Regular photodiodes, position-sensing-detectors (aka lateral effect photodiode), and split-cell photodiodes (bi-cell, quad-cell etc.)
- The basic photodiode:





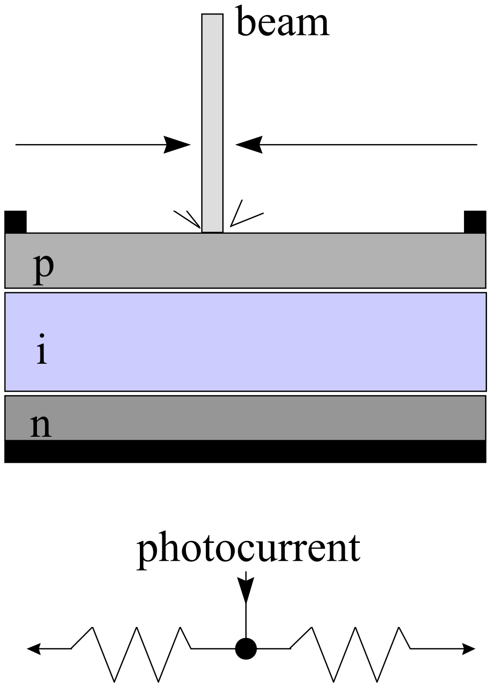
•Reasonably linear A/W output
•Also depends on wavelength
•Has some temperature coeff.



current = *f*(light)

Lateral effect photodiode

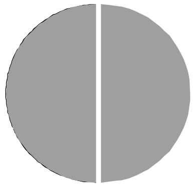
Lateral Effect Photodiodes



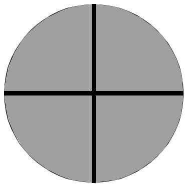
- "Photopots"
- Inject photocurrent onto the wiper of a potentiometer to measure position.
- In the case of the photopot, the resistor is doped silicon.

Split cells

- Two or more diodes on one common substrate.
- Very low noise, can detect to subnanometer
- Requires separate amplifiers.



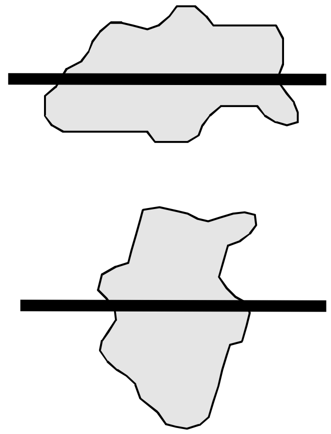
BI cell



QUAD cell

Linearity & split cell systems

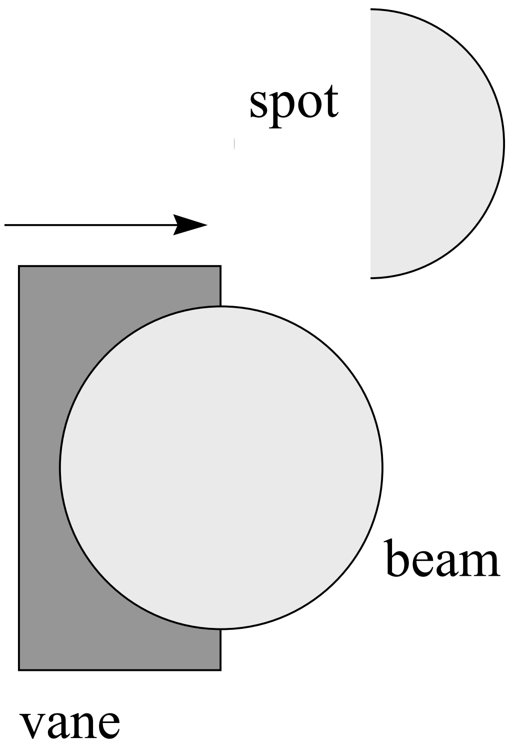
laser spot on bi-cell



- Shape and structure of spot can effect sensitivity.
- Detectors are flat, not spherical.
- Reflections from detector window can cause problems.

•This is the type of sensor used in AFM’s
•Best use is in a “nulling” system (like mass comparators, where you measure how much current (force) it takes to bring the UUT back to zero)

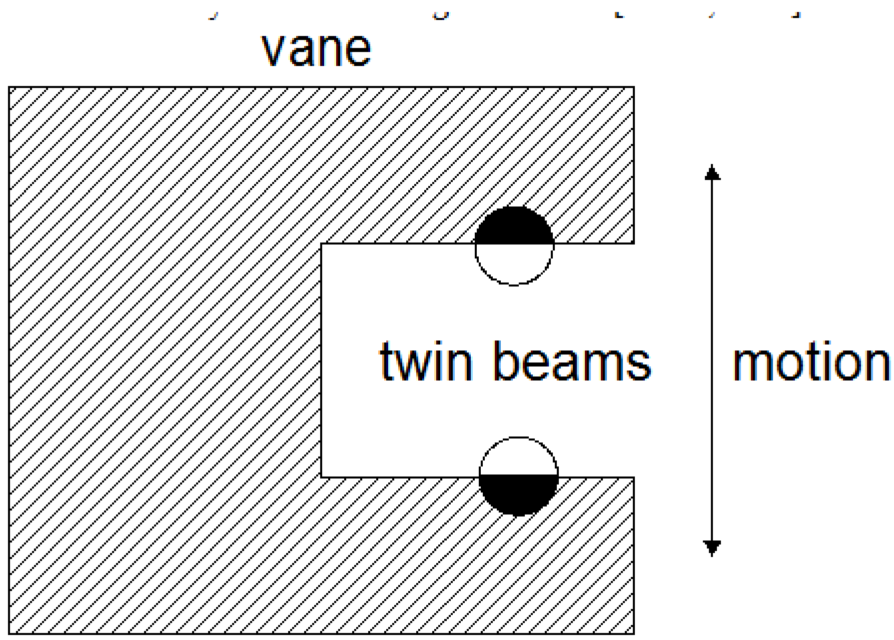
Shadow Sensors: I



- Moving vane type: power varies with position of vane.
- Only requires one detector.
- Intensity noise can not be removed.

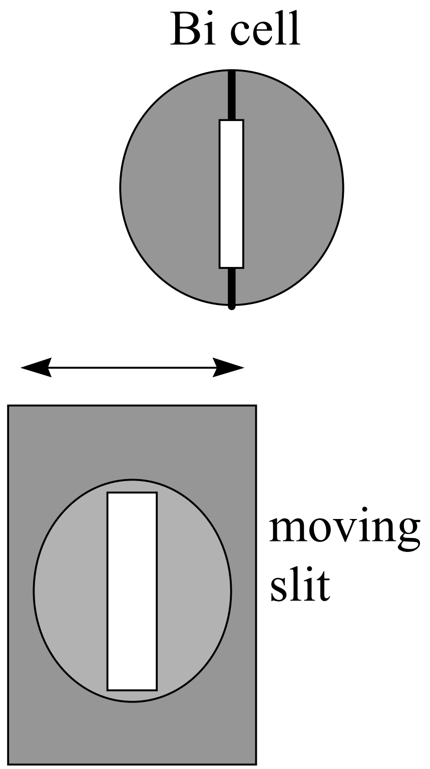
This is likely what is inside the mass comparators

Differential shadow sensor

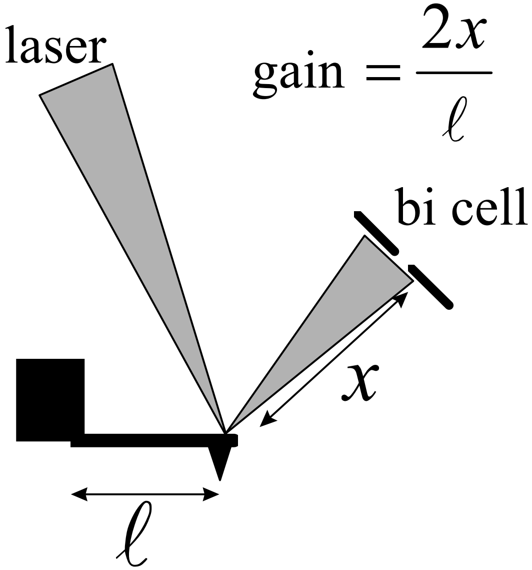


Shadow Sensors: II

- Moving slit type: position of spot moves.
- Requires position sensitive detector.
- Intensity noise can be reduced by normalization.



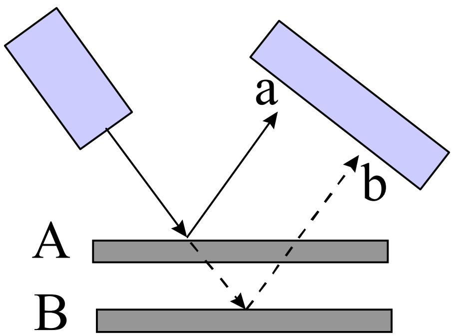
Lever Arms & Cantilevers



$$\text{gain} = \frac{2x}{\ell}$$

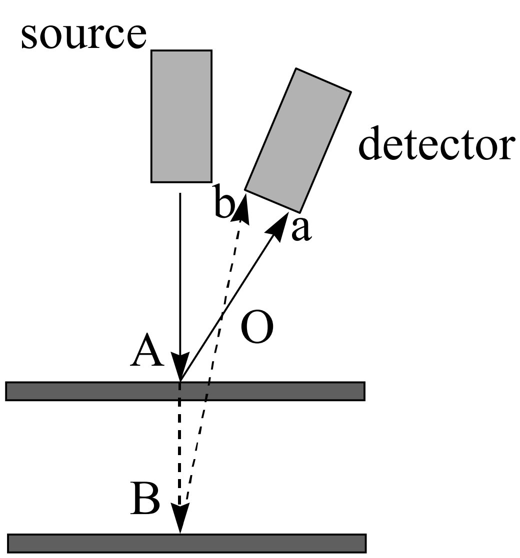
- Beam waist can be placed at infinity, the cantilever or the detector.
- Cantilever lever is a geometrical amplifier.
- A close cousin to specular triangulation.

Specular Triangulation



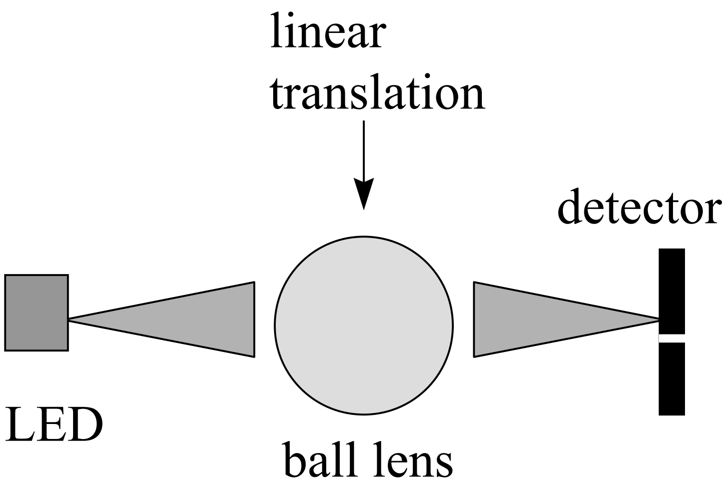
- Note: this is also a **very** sensitive angle detector. Surface should be flat or displacement should be small.
- Angle of incidence between 22 to 45 degrees.

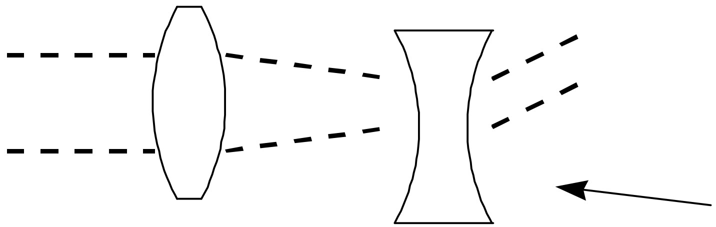
Diffuse Triangulation



- Assumes some scattered light from surface.
- If surface is not randomly rough, the situation becomes more complicated.
- A variety of detectors and sources can be used.

Spherical Lens: Translation





- Ball lens converts translations to angle changes.
- Insensitive to lens rotations.
- Used in JILA *SuperSpring* suspension for absolute gravimeter.
- Galilean Afocal pair.

Sources

- Plain old light bulb (also good as a heater!)
  - Note that this is an extended source, and not a point source!
  - Can use apertures or fiber optics to clean the source
- LED's
  - Also an extended source
  - Short(ish) coherence length
  - Reasonably monochromatic
- Lasers—both gas and diode lasers
  - Gas lasers (HeNe) have long coherence length, good wavelength stability, good beam shape, relatively poor pointing stability
  - diode lasers have good pointing stability, pretty poor beam shape
  - Don't use lasers if optical interference phenomena are undesireable!

Conclusions

- Think about averaging before you average!
- Lots of non-contact displacement sensors used in various dimensional/mechanical MMS's

Noise simulation, revisited

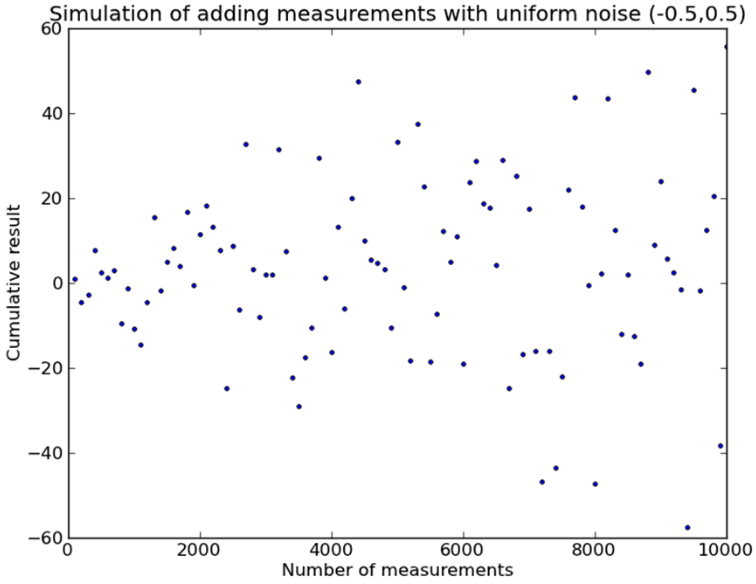
simulated repeats added together

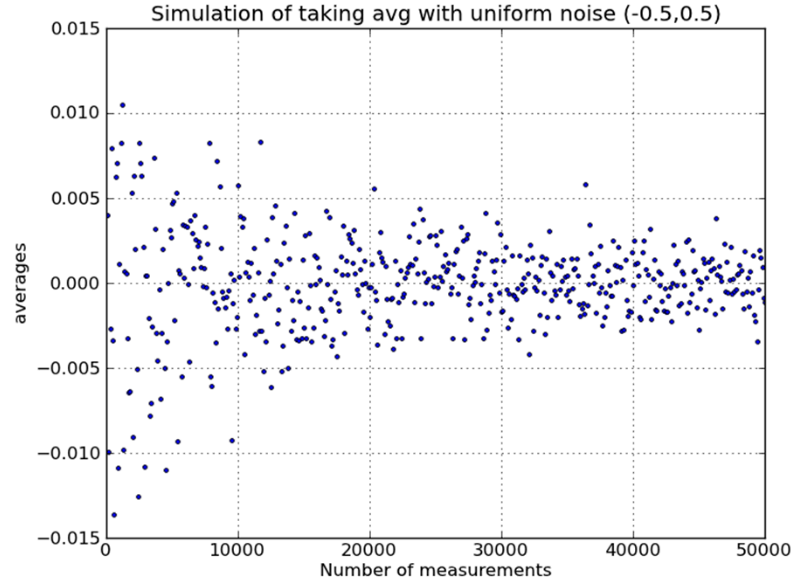
this diverges—but I made an intentional blunder

simulated repeats averaged

I'm not getting to zero while averaging 50,000 measurements?

Convergence goes as √n





Controlling your processes

- Typical equipment/procedures?
  - 1D universal comparator
    - Masters at external lab, comparator also are checked annually
  - Height gage
    - Gage blocks internally from masters
    - Check standards
  - CMM
    - Calibrated by vendor
    - We own check stds
  - What else? (SNL/PSL/LMF also has force, mass, etc.)

Controlling your measurement process: f-test for variance & t-test for means

- I have a well-characterized measurement process.
- I am measuring a new unknown.
- When measuring the new unknown, is my measurement process “in control”?
  - Are my “random” disturbances acceptably low?
  - Have my stds or equipment drifted without me knowing?

- See NIST e-statistics handbook (available on the web)

f-test for equal variances

- (from section 2.2/2.2.3 of NIST e-handbook)
- First test: Are my "random" disturbances (construction, earthquakes, manager walk-thrus) sufficiently low?
- Your process has a well-characterized historical standard deviation σ_p ; variance σ_p^2
- You are taking *n* new measurements. The standard deviation of the new measurements is σ_n ; variance σ_n^2
- Given a current variance σ_n^2 , is that consistent with the historical process variance σ_p^2 ?

f-test, continued (using f-table)

- Hypothesis: $\sigma_n^2 \leq \sigma_p^2$
- Use f-table: 5% level ($F_{0.05}$), ν_1=new; ν_2=historical
- Consider 6 new points*, 15 historical: $\sigma_n^2 \leq 2.901 \times \sigma_p^2$?

*6 points=5 DOFs

Upper critical values of the F distribution

for ν_1 numerator degrees of freedom and ν_2 denominator degrees of freedom

5% significance level

$F_{.05}(\nu_1, \nu_2)$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	
1	161.448	199.500	215.707	224.583	230.162	233.986	236.768	238.882	240.543	241.882	
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385	19.396	
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786	
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388	3.347	
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	

f-test using MS-Excel

- (version 2010; could also work with 2007; not sure about earlier versions)
- F.INV(0.95,5,20) ← multiplier for:
  - 95% confidence
  - 5 “new” DOF’s
  - 20 “historical” DOF’s
- If process is very well characterized, use large number for historical DOF’s (~100 or more)
- Remember—the f multiplier is for the ***variance***

Multiply your “recent” variance by f multiplier. If result ≤ “historical” variance, your process is likely to be “in control.”

Example of f-test

- My historical process has a std dev. = 1.0 (variance is also 1.0)
- I took 5 measurements, and get:

-0.18721
1.323309
1.115101
-2.08763
0.338451

- Calculate std dev: →1.365
- Var=1.863
- Is the likelihood that I am ***outside*** of my historical process > 5%?
- "current samples" variance=1.86
- "historical samples" variance=1.00
- Ratio (current/history)=1.86
  - Look up f-table, 5%, 4 DOF's to "many", or F.INV(0.95,4,100)→2.46
  - 1.86<2.46, so I am in control

t-test for means

- I have a well-characterized measurement process.
- I am measuring a new unknown.
- I have a check standard (a known item)
- My variance is “in control” (random disturbances in control)

- Do I have an unknown bias?

t-test, use excel & tables

- Calculate the test "t" value:

$$t = \frac{\overline{Y} - \mu_0}{s / \sqrt{N}}$$

Where $\overline{Y}$ is the measured mean; μ_0 is the known (historical, check) value; *s* is the sample std deviation, and *N* is the number of measurements

- Check the calculated *t* value against the critical value in the *t* table

t-test example

- Consider the following data:
  - Historical value = 50
  - 10 (new) measurements
  - Calculated mean, sample std dev:

50
48
44
56
61
52
53
55
67
51

Mean of measurements
53.7
Std dev of measurements (sample std dev)
6.566751

- Calculate t and critical t_{crit}:
- t_{crit} from T.INV.2T(0.05,9)→2.26

$$t = \frac{\overline{Y} - \mu_0}{s / \sqrt{N}} = 1.782$$

$t < t_{crit}$, so unlikely (< 5% probability) that process has shifted.

Concluding remarks; t & f

- There are other ways of doing this (for example, chi-squared instead of f); I've just shown you one way.
- You can use your own "assurance" level—you're not stuck with 5%/95%; you can choose 10%/90%, etc. Published tables commonly have 5%, 10%, but not necessarily 3%. Use the F.INV and T.INV functions (or equivalent). Verify the functions against published values before you strike out on your own!
- You can pose (and answer) different questions; e.g. "have I reduced my variance with a new measurement process", rather than "is my variance acceptable"
- If you are monitoring your measurement process (why aren't you?)—be sure to update your historical process std dev as you go. Likewise your check standards if you are using them.
- Consult statistician as needed