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UNIVERSITY OF WISCONSIN–MADISON
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# Determining Model Form Uncertainty of Reduced Order Models

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**Matthew Bonney**
**The University of Wisconsin-Madison**

**Daniel Kammer**
**The University of Wisconsin-Madison**

**Matthew Brake**
**Sandia National Laboratories***

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# Outline

- Background
  - ROM archetypes
  - Maximum Entropy
- Substructure Reduction Results
- ROM Synthesized Results
- Conclusions/ Remarks

# Motivation

- Majority of systems are constructed with ROMs
  - Large systems take too much time to analyze
- Uncertainty of ROM is difficult to describe
  - Even more difficult to propagate
- Typically, error associated in modal frequency difference
  - No direct relation to physical parameters
- Attempt to encompass nonlinear response in a linear system

# ROM Types

- Fixed-Interface Modes
  - Craig-Bampton
    - Truncated fixed-interface modes and interface displacement DOF
- Free-Interface Modes
  - To be used in Craig-Chang
    - Truncated free-interface modes and interface force DOF
  - Inertia Relief Residual Attachment Modes
- Modes truncated based on frequency

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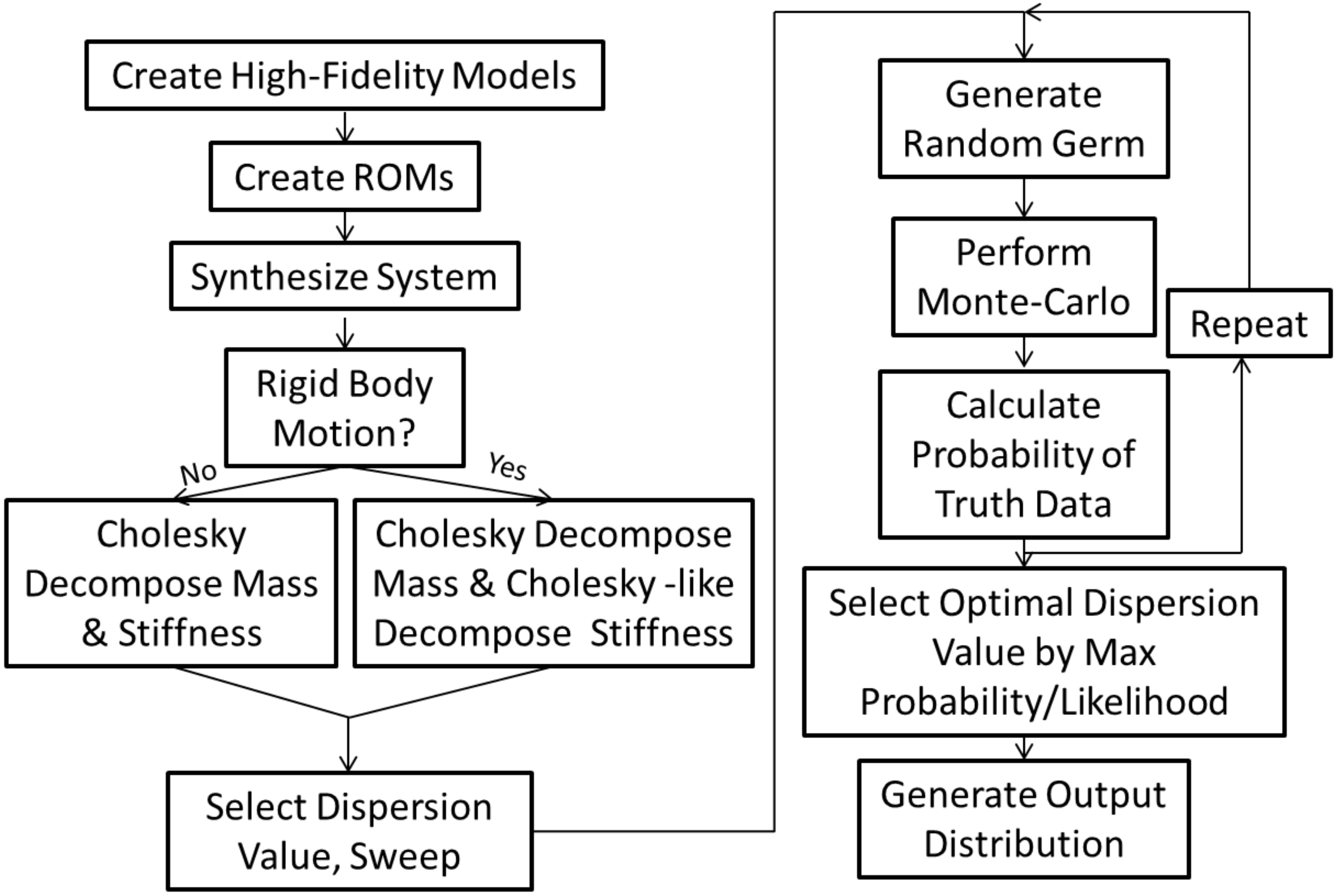
# Maximum Entropy

- Technique developed by Soize
- Incorporates random matrices with EOM
  - Assumes distribution that matches physical characteristics
  - Stiffness is positive semi-definite, etc.
  - Uses single parameter to characterize distribution
    - Dispersion parameter, δ
  - Utilizes Cholesky decomposition
    - $[K] = [L_K(X)]^T[G(\delta)][L_K(X)]$

# Maximum Entropy Cont.

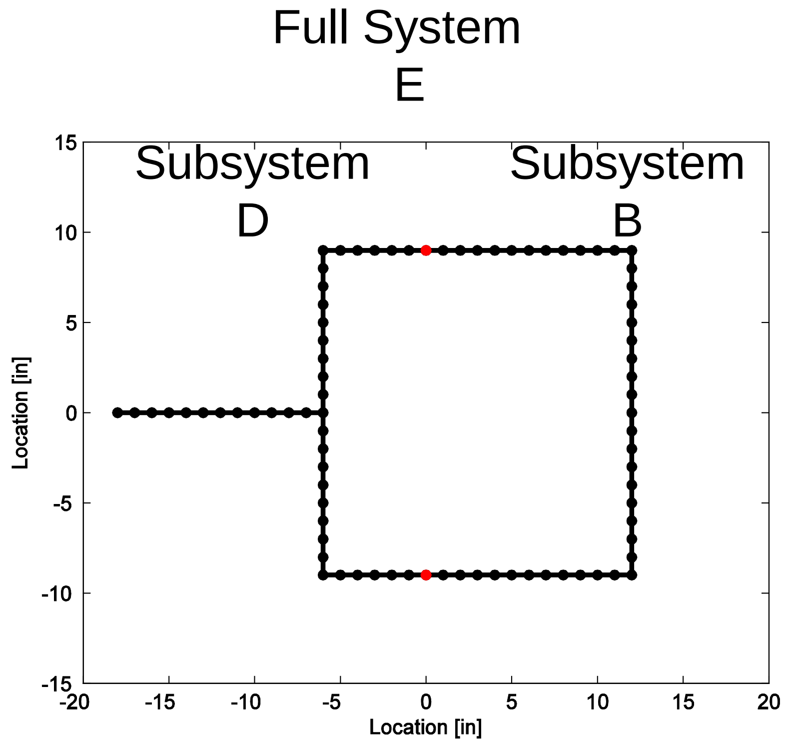
- Optimize dispersion parameter to best characterize the truth data
  - Truth data can be experimental or high-fidelity data
- Uses maximum likelihood estimate
  - $L(\delta) = \Pi_i P_r(\omega_i|\delta)$
  - $\delta$ is bounded $\in [0,1)$
- To reduce numerical error, empirical likelihood is used
  - $EL(\delta) = \sup_i L_i(\delta)$

# Analysis Flow

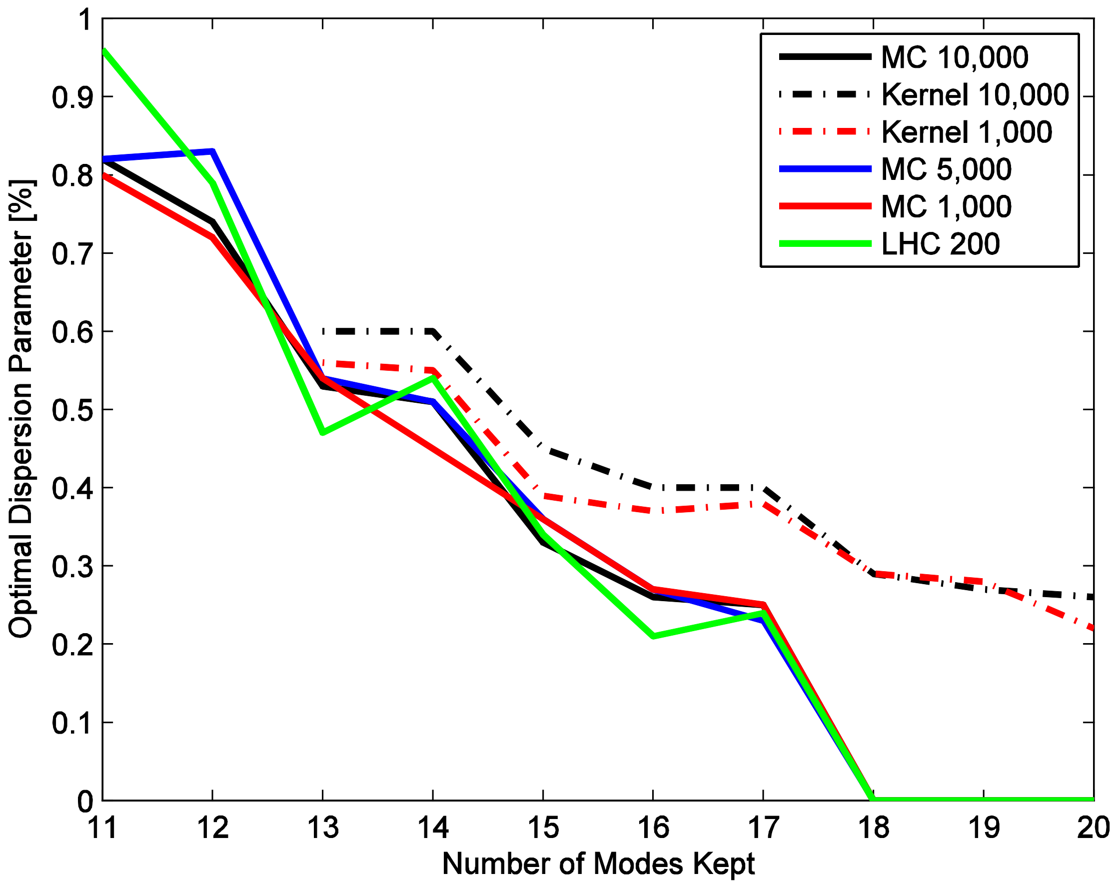


# Example System

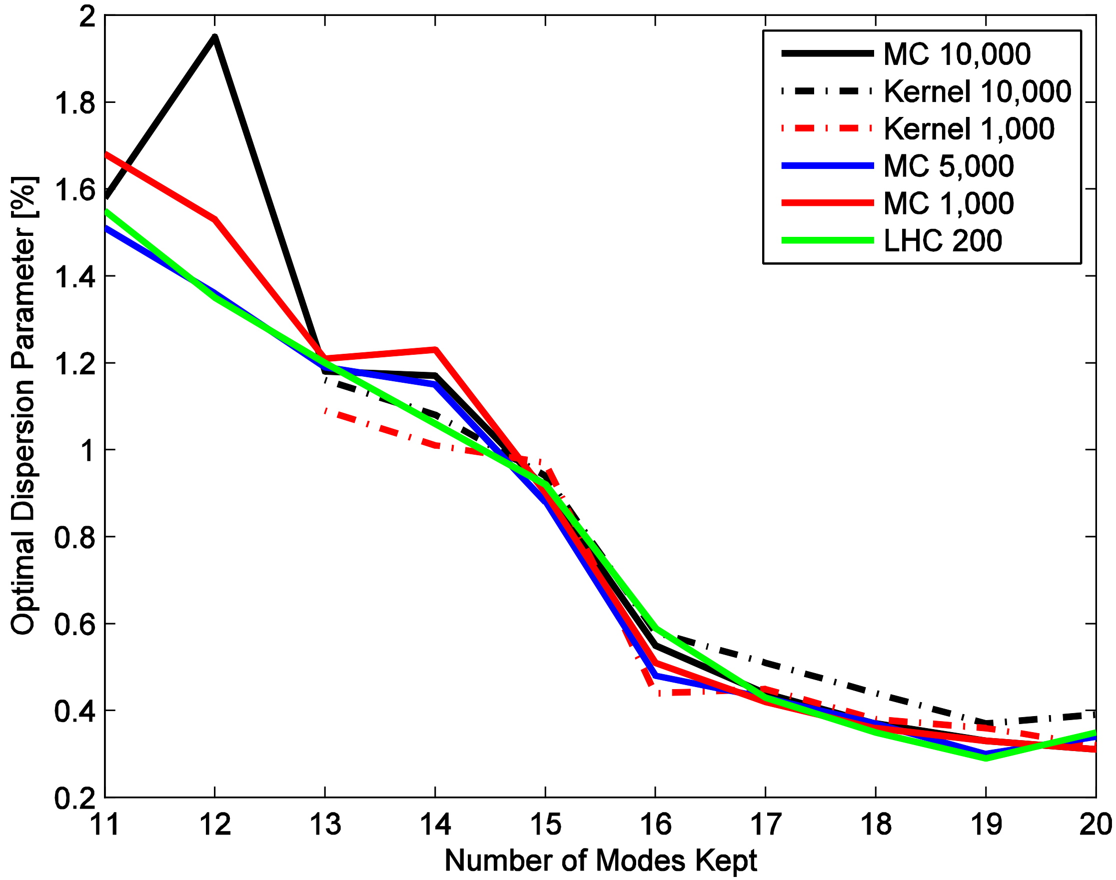
- 2 Substructures
  - 2 repeated nodes, in red
  - Defined as interface
- Planar Frame
- Euler beam elements
- Use first 11 elastic frequencies as truth data
  - For fixed-interface reduction, used free-free modes
  - For free-interface reduction, used fixed-interface modes



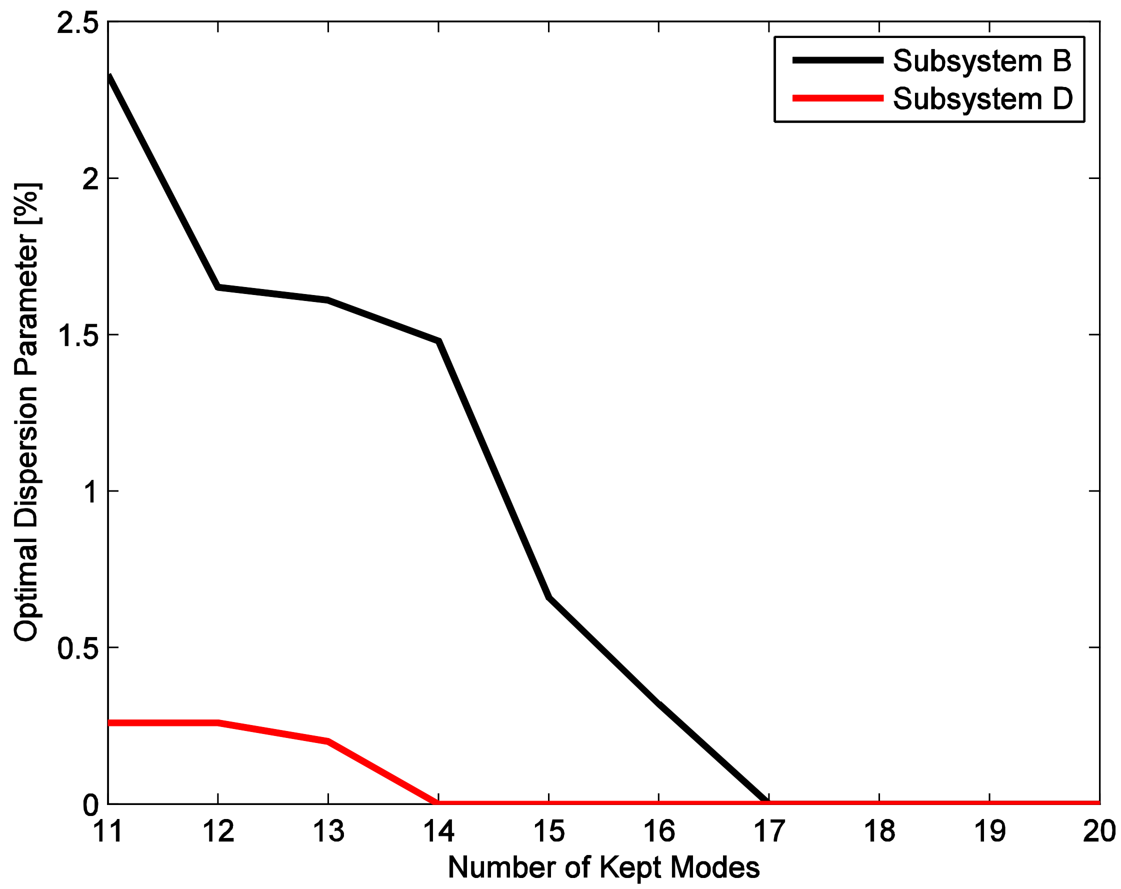
# Craig-Bampton Results Subsystem B



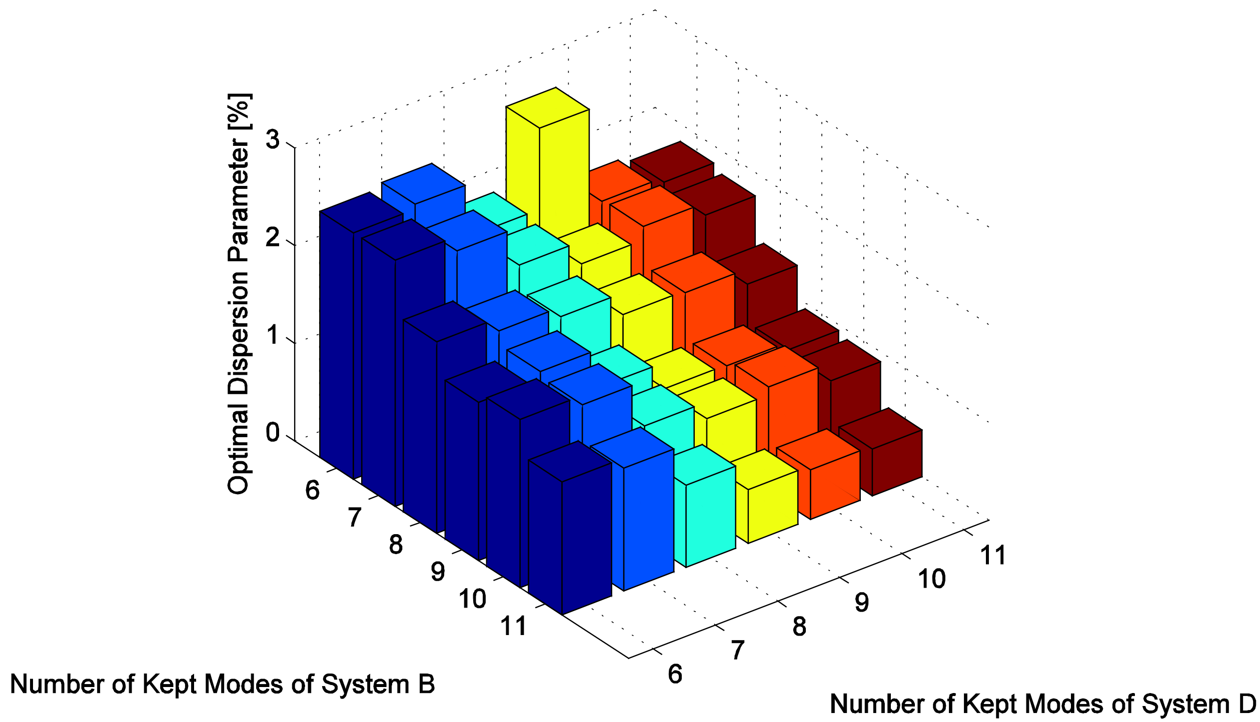
# Craig-Bampton Results Subsystem D



# Free-Interface Substructure Results

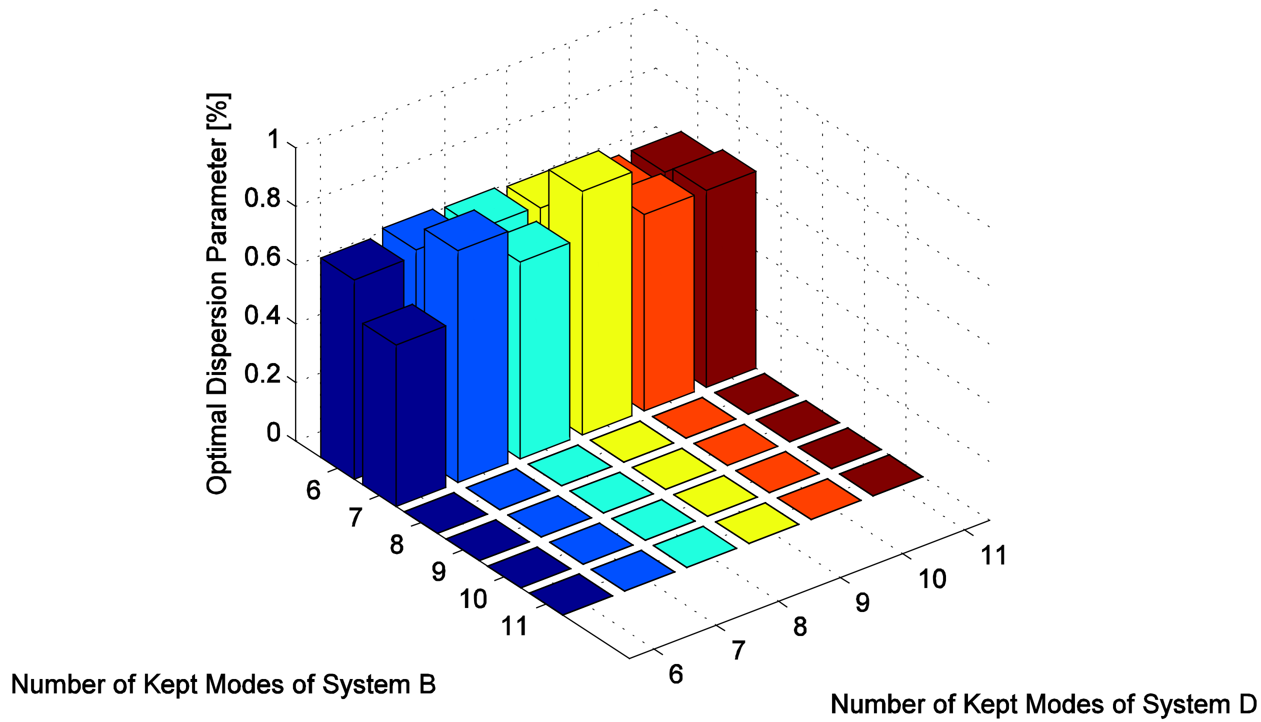


# Craig-Bampton Synthesized Results




| Subsystem Frequency Range | Optimal Dispersion Parameter |
|---|---|
| 1.5x | 1.69 % |
| 2.0x | 1.17 % |

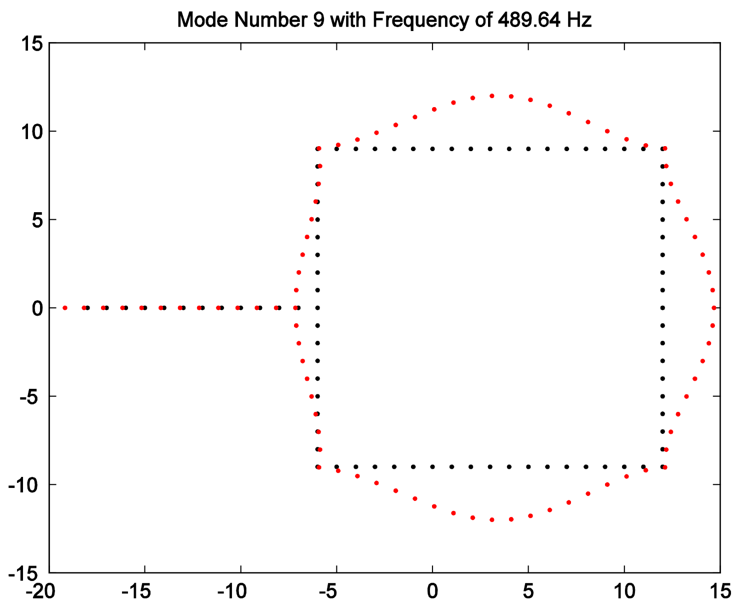
# Craig-Chang Synthesized Results

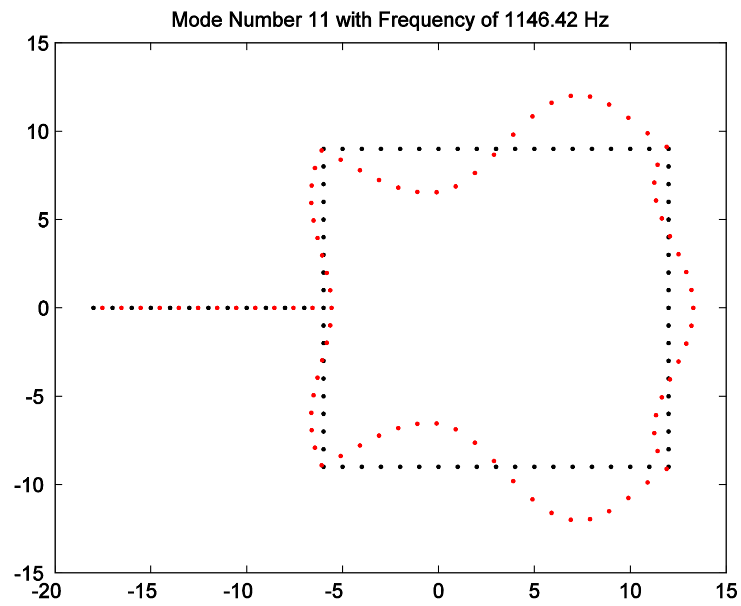



| Subsystem Frequency Range | Optimal Dispersion Parameter |
|---|---|
| 1.5x | 0.00 % |
| 2.0x | 0.00 % |

# Remarks

- In assembly, Craig-Chang produced less error than Craig-Bampton
  - Assembly modes better represented by free-interface modes



# Remarks Cont.

- On a subsystem level, fixed-interface modes can better represent free-interface modes than vise-versa
- Difficult to determine relationship between subsystem and assembled system dispersion parameters
- Technique able to describe uncertainty in the physical model

# Conclusions

- This method is able to characterize error not associated with parameter uncertainty
- Substructures using fixed-interface and free-interface are used and compared
- Different methods of generating likelihood are investigated
    - Sampling methods
    - Probability measures

# Special Thanks

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Questions?