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Estimate of electrical potential difference between plasmas with different degrees of ionization

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The electrical potential difference has been estimated across the mixing region of two plasmas with different degrees of ionization. The estimation has been carried out in two different contexts of a charge neutral mixing region and a charge non-neutral sheath. Ion energy gained due to the potential difference has also been estimated. In both analyses, ion energy gain is proportional to the degree of ionization, and a fairly large ionization appears to be needed for overcoming the potential energy barrier of strongly coupled plasmas.

I. INTRODUCTION

When two plasmas with different degrees of ionization are in contact, an electric field is developed, even in the absence of a net electrical current. This electric field limits the electron motion in such a way that electron and ion diffusion is coupled, maintaining the charge neutrality and zero current in plasma. We refer to this diffusion as "ambipolar" diffusion and the associated electric field as the ambipolar electric field [1]. Needless to say, the electrical potential is also different across the mixing region. (The potential is higher at the material with higher degree of ionization.)

A similar effect occurs when a plasma contacts a solid wall: an electric field is developed so that the electron and ion flux is balanced at the wall. The plasma boundary layer near the wall is referred to as the sheath in which charge neutrality is not satisfied. When a plasma faces another plasma with a different degree of ionization, instead of a solid wall, a sheath can also be developed. The size of the sheath is of order of a few Debye length λ_D, which usually is much smaller than the mixing region. Therefore, sheath development between two plasmas would not be relevant to most mixing (diffusion) of plasmas. However, a sheath would develop during the initial stage of mixing, and its impact of the ion motion should still be examined.

We present simple estimations of electric potential differences across mixing regions in a plasma state for both situations associated with ambipolar electric field (charge neutral) and sheath (charge non-neutral). We first estimate the potential difference caused by an ambipolar electric field in Sec. II. The ion energy loss of the plasma with lower degree of ionization across the mixing region is discussed in Sec. III. We then extend a simple collisionless sheath theory to the case of two plasmas in contact, in Sec. IV. A criterion similar to the well-known Bohm sheath criterion is developed. Added are comments regarding the potential energy barrier in strongly coupled plasmas, as diffusion is known to become smaller when plasma is strongly coupled [2].

Dense plasma effects such as Coulomb interactions between ions are not included in the present analyses. In fact, simplifying assumptions of zero collisions have been used in Sec IV. Nevertheless, the present analyses are expected to produce information useful in estimating the onset of diffusion in dense plasmas. Improved analyses are, of course, desired.

II. AMBIPOLAR ELECTRIC FIELD

Since $m_e \ll m_i$, where m_e is the electron mass and m_i is the ion mass, m_e/m_i can be ignored. The ambipolar electric field $\mathbf{E}$ can then be approximated as [1]

$$\mathbf{E} = -\frac{\nabla p_e}{n_e e} \tag{1}$$

where p is the pressure, e is the elementary charge (electrical charge of a proton), n is the number density, and subscripts i and e respectively denote the ions and free electrons.

Imagine a situation in which mixing progresses between two plasmas with different degrees of ionization, Z^H and Z^L, where superscripts H and L respectively denote higher and lower ionization, i.e., $Z^H > Z^L$. Assuming temperature and pressure are identical for electrons and ions, number densities are related as

$$n_e^H = Z^H n_i^H \tag{2}$$

$$n_i^L = \frac{Z^H+1}{Z^L+1} n_i^H \tag{3}$$

$$n_e^L = \frac{Z^H+1}{Z^L+1} Z^L n_i^H \tag{4}$$

Let us consider the case that the Z^H side is on the left (smaller x). The electric potential difference across the mixing region can be approximated as

$$\Delta\phi = \int_H^L \frac{1}{n_e e}\frac{dp_e}{dx}dx \approx -\frac{p_e^H - p_e^L}{e(n_e^H + n_e^L)/2} = -2\frac{Z^H - Z^L}{Z^H + Z^L + 2Z^H Z^L}\frac{k_B T}{e} \tag{5}$$

where k_B is the Boltzmann's constant, and T is the temperature.

The ion energy gained by the ambipolar electric field $\mathbf{E}$ is then given by

$$\Delta E_i^H = -Z^H e\Delta\phi = 2\frac{Z^H - Z^L}{Z^H + Z^L + 2Z^H Z^L} Z^H k_B T \tag{6}$$

The potential energy barrier of the plasma with Z^H is given by $\Gamma^H k_B T$, where Γ is the coupling coefficient. So when $\Delta E_i^H \geq \Gamma^H k_B T$, ions would have sufficient energy to overcome the potential energy barrier, i.e.,

$$2\frac{Z^H - Z^L}{Z^H + Z^L + 2Z^H Z^L} Z^H \geq \Gamma^H \tag{7}$$

When $Z^L = 1$ and $Z^H \gg Z^L$, we have

$$Z^H \geq \frac{3}{2}\Gamma^H \tag{8}$$

Equation (8) implies that fairly large value of Z^H (degree of ionization) is necessary to overcome the potential energy barrier of strongly coupled plasmas.

III. ENERGY LOSS OF IONS WITH LOWER DEGREE OF IONIZATION

The electrical potential difference given in Eq. (5) accelerates ions with higher degree of ionization Z^H. This implies that ions with lower degree of ionization Z^L need to overcome this electrical potential difference in order to diffuse into the plasma with Z^H. The energy required to climb up the potential difference is

$$\Delta E_i^L = -Z^L e\Delta\phi = 2\frac{Z^H - Z^L}{Z^H + Z^L + 2Z^H Z^L} Z^L k_B T \tag{9}$$

When $Z^L = 1$ and $Z^H \gg Z^L$, we have

$$\Delta E_i^L = \frac{2}{3} k_B T \tag{10}$$

Equation (10) implies that ions with less energy than $(2/3)k_B T$ would not be able to diffuse into the plasma with Z^H. That is, only ions with energy higher than $(2/3)k_B T$ in the Maxwell-Boltzmann distribution will overcome the electrical potential difference developed between plasmas with different degrees of ionization.

If the Z^L plasma is strongly coupled, additional energy is required to overcome the potential energy barrier $\Gamma^L k_B T$. That is, Eq. (10) is rewritten as

$$\Delta E_i^L = \left(\frac{2}{3} + \Gamma^L\right) k_B T \tag{11}$$

Equation (11) implies that substantial fraction of ions with lower degree of ionization would not diffuse into the other plasma, when the Z^L side plasma is moderately coupled.

IV. COLLISIONLESS SHEATH

Another way to estimate $\Delta\phi$ is using the approach based on the physics of the sheath. The sheath develops between plasma and a confining wall (usually solid). Electrons accumulate on the wall surface so that fluxes of ions and electrons will balance. In the sheath region, space is positively charged (i.e., plasma is not charge neutral), and an electrical field is developed. This electric field adds energy to the ion leaving the sheath edge. The velocity of ions flowing into the sheath from the bulk plasma (leaving the sheath edge) can be determined by the Bohm sheath criterion [3].

Theories regarding the sheath such as the Bohm sheath criterion, collisionless sheath, matrix and collisional sheath, and Child-Langmuir law are well summarized in textbooks, hence not repeated here. Here, we extend the analysis of collisionless sheath at the boundary between two plasmas. (The Z^H side is on the left.) That is, instead of solid wall, we have a plasma with lower degree of ionization Z^L, and a considerable number of electrons, $n_i^H Z^L(Z^H+1)/(Z^L+1)$, are already present there.

Ignoring collisions in the sheath, the energy conservation of ions in the sheath is given by

$$\frac{1}{2}m_i^H(u_i^{SH})^2 = \frac{1}{2}m_i^H(u_i^H)^2 - Z^H e\phi \tag{12}$$

where superscript S denotes inside the sheath region, and u is the speed of ions. Note that $\phi = 0$ at the Z^H side. The ion flux conservation is given by

$$n_i^H u_i^H = n_i^{SH} u_i^{SH} \tag{13}$$

We then have

$$n_i^{SH} = n_i^H \left(1 - \frac{Z^H e\phi}{E_i^H}\right)^{-1/2} \tag{14}$$

$$E_i^H = \frac{1}{2}m_i^H(u_i^H)^2 \tag{15}$$

where E_i^H is the initial ion energy (energy of an ion entering the sheath). Electron density distributions are given by the Boltzmann relation as

$$n_e^{SH} = n_e^H \exp\left(\frac{e\phi}{k_B T}\right) \tag{16}$$

$$n_e^{SL} = n_e^L \exp\left(\frac{e\phi - e\phi^L}{k_B T}\right) \tag{17}$$

$$\tag{18}$$

The electric potential would be lower on the Z^L side in this case, similar to the solid wall, and the ion flux from the Z^L side would be substantially reduced as discussed in Sec. III. For simplicity, this ion flux from the Z^L side is ignored in the present analysis. Using Eqs. (2)–(4), the Poisson equation for the electric potential in the sheath region is given by

$$\begin{aligned}\frac{d^2\phi}{dx^2} &= \frac{e}{\epsilon_0}\left(n_e^{SH} + n_e^{SL} - Z^H n_i^{SH}\right) \\ &= \frac{eZ^H n_i^H}{\epsilon_0}\left[\exp\left(\frac{e\phi}{k_B T}\right) + \frac{Z^H+1}{Z^L+1}\frac{Z^L}{Z^H}\exp\left(\frac{e\phi - e\phi^L}{k_B T}\right) - \left(1 - \frac{Z^H e\phi}{E_i^H}\right)^{-1/2}\right]\end{aligned} \tag{19}$$

Equation (19) can be integrated analytically with the result

$$\frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 = \frac{Z^H n_i^H}{\epsilon_0} f(\phi) \tag{20}$$

$$\begin{aligned}f(\phi) &= k_B T \exp\left(\frac{e\phi}{k_B T}\right) - k_B T + \frac{Z^H+1}{Z^L+1}\frac{Z^L}{Z^H}k_B T \exp\left(\frac{e\phi - e\phi^L}{k_B T}\right) \\ &- \frac{Z^H+1}{Z^L+1}\frac{Z^L}{Z^H}k_B T \exp\left(-\frac{e\phi^L}{k_B T}\right) + \frac{2E_i^H}{Z^H}\left(1 - \frac{Z^H e\phi}{E_i^H}\right)^{1/2} - \frac{2E_i^H}{Z^H}\end{aligned} \tag{21}$$

Note that f must be positive. Since $f(0)=0$ and $f'(0)=0$, $f''(0)$ must be positive. That is, we require

$$f''(0) = \frac{e^2}{k_BT} + \frac{Z^H+1}{Z^L+1}\frac{Z^L}{Z^H}\frac{e^2}{k_BT}\exp\left(-\frac{e\phi^L}{k_BT}\right) - \frac{Z^He^2}{2E_i^H} \geq 0 \tag{22}$$

or

$$E_i^H \geq \frac{Z^Hk_BT}{2}\left[1+\frac{Z^H+1}{Z^L+1}\frac{Z^L}{Z^H}\exp\left(-\frac{e\phi^L}{k_BT}\right)\right]^{-1} \tag{23}$$

We then have the sheath criterion, similar to the Bohm sheath criterion, as

$$u_i^H \geq u_i^B = \sqrt{\frac{Z^Hk_BT}{m_i^H}}\left[1+\frac{Z^H+1}{Z^L+1}\frac{Z^L}{Z^H}\exp\left(-\frac{e\phi^L}{k_BT}\right)\right]^{-1/2} \tag{24}$$

Electron and ion flux balance at $\phi=\phi^L$ is given by

$$n_i^Hu_i^B = \frac{n_e^Hv_e}{4}\exp\left(\frac{e\phi^L}{k_BT}\right) - \frac{n_e^Lv_e}{4} \tag{25}$$

where $v_e=\sqrt{8k_BT/(\pi m_e)}$ is the thermal speed of electrons. Note that electron flux from the Z^L side is in the opposite direction of the electron flux from the Z^H side. Using Eqs. (2)–(4) and (24), Eq. (25) is rewritten as

$$\sqrt{\frac{Z^Hk_BT}{m_i^H}}\left[1+\frac{Z^H+1}{Z^L+1}\frac{Z^L}{Z^H}\exp\left(-\frac{e\phi^L}{k_BT}\right)\right]^{-1/2} = \frac{Z^H}{4}\sqrt{\frac{8k_BT}{\pi m_e}}\left[\exp\left(\frac{e\phi^L}{k_BT}\right) - \frac{Z^H+1}{Z^L+1}\frac{Z^L}{Z^H}\right] \tag{26}$$

Since $m_e \ll m_i^H$, we can ignore the left hand side of Eq. (26). We then obtain ϕ^L and $\Delta\phi$ as

$$\Delta\phi = \phi^L = -\frac{k_BT}{e}\ln\left(\frac{Z^L+1}{Z^H+1}\frac{Z^H}{Z^L}\right) \tag{27}$$

Observe that $\phi^L<0$, since $Z^H>Z^L$. Substituting Eq. (27) in Eq. (24) yields

$$u_i^B = \sqrt{\frac{Z^Hk_BT}{2m_i^H}} \tag{28}$$

we can compare this u_i^B with the well-known Bohm sheath criterion given by $u_i^B=(Z^Hk_BT/m_i^H)^{1/2}$. The difference is caused by the fact that the plasma is facing a different material plasma with lower degree of ionization, not solid wall, and that electrons are fluxing into the sheath from this plasma.

When $Z^L=1$ and $Z^H \gg Z^L$, we have

$$\phi^L = -\frac{k_BT}{e}\ln 2 \tag{29}$$

The ion energy gained the potential difference across the sheath is

$$\Delta E_i^H = Z^Hk_BT\ln 2 \tag{30}$$

The degree of ionization needed to overcome the potential energy barrier of strongly coupled plasma is then given by

$$Z^H \geq \frac{1}{\ln 2}\Gamma^H \tag{31}$$

Observe that $\ln 2 = 0.69315$ is fairly close to 2/3 appearing in Eq. (8).

V. REMARKS

The electric potential difference and ion energy gain across a plasma mixing region have been examined using simple analyses. The ambipolar diffusion constraint presented in Sec. II implies the charge neutrality through the interface. On the contrary, charge non-neutrality is allowed in the sheath analysis. Although the sheath analysis should only be applicable to the early stage of mixing, both analyses produce similar results, which imply that fairly large degree of ionization is needed to overcome the potential energy barrier in strongly coupled plasmas. Although these results are not expected to be highly accurate due to simplifying assumptions, they nonetheless provide estimations of Z^H necessary for the onset of diffusion.

When plasma is facing a different plasma, not a solid wall, the Bohm sheath criteria needs to be redeveloped. This has been presented in Sec. IV of this report. The results are summarized in Eqs. (24), (27), and (28).

The present analysis shows that the diffusion of ions with lower degree of ionization (Z^L) appears to be reduced substantially. When the plasma with Z^L is very strongly coupled, diffusion of the Z^L plasma may stop completely. Since diffusion flux must be balanced, i.e., $\sum_i \mathbf{J}_i = \mathbf{0}$, where $\mathbf{J}_i$ is the diffusion flux of species i, diffusion of the Z^H plasma would also stop. Or diffusion may become one-way: the Z^H plasma diffuses into the Z^L plasma, but not the other way around. Improved analysis is needed.

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