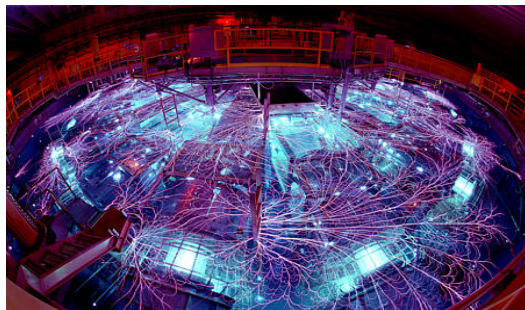
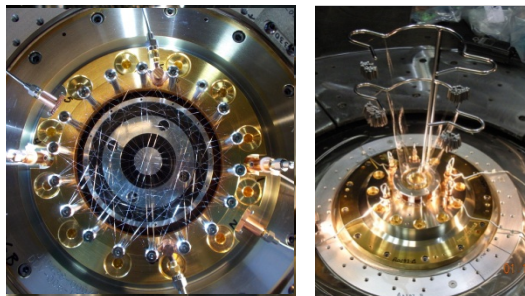


High Energy Density Physics with Pulsed Power



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*Exceptional
service
in the
national
interest*



Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

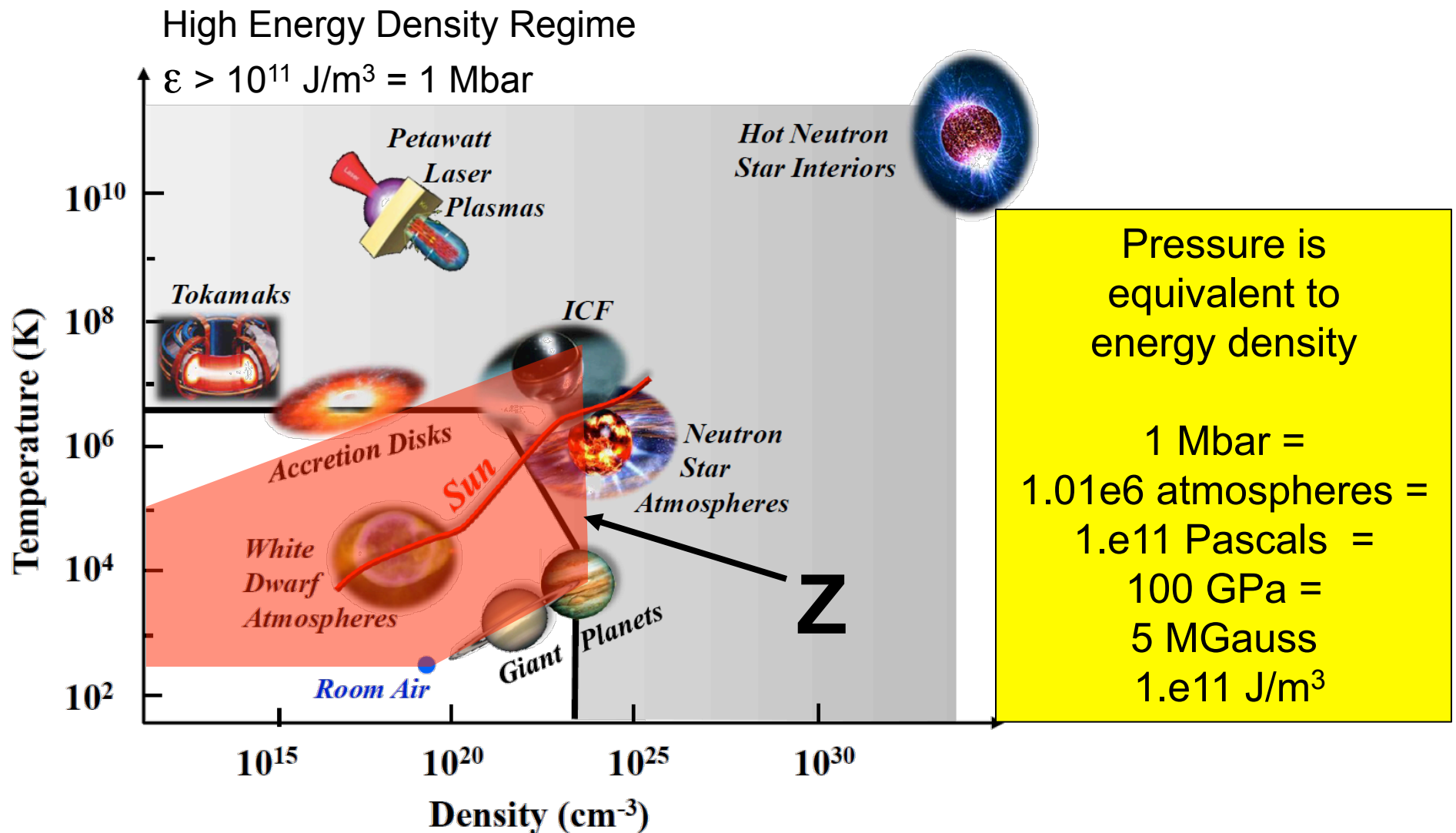
Summary

- What is high energy density science?
- How do we generate high energy density conditions?
- What do we use high energy density for?
 - Discovery science at the forefront of what's possible in the laboratory
 - Developing and validating predictive models
 - Nuclear weapon relevant scientific work
 - Fundamental science
- Some example scientific results

Key takeaway: when we make measurements, ***almost without exception*** the measurements disagree with scientific predictions or expectations before the experiments



High energy density is defined, somewhat arbitrarily, as a pressure of 1 Mbar



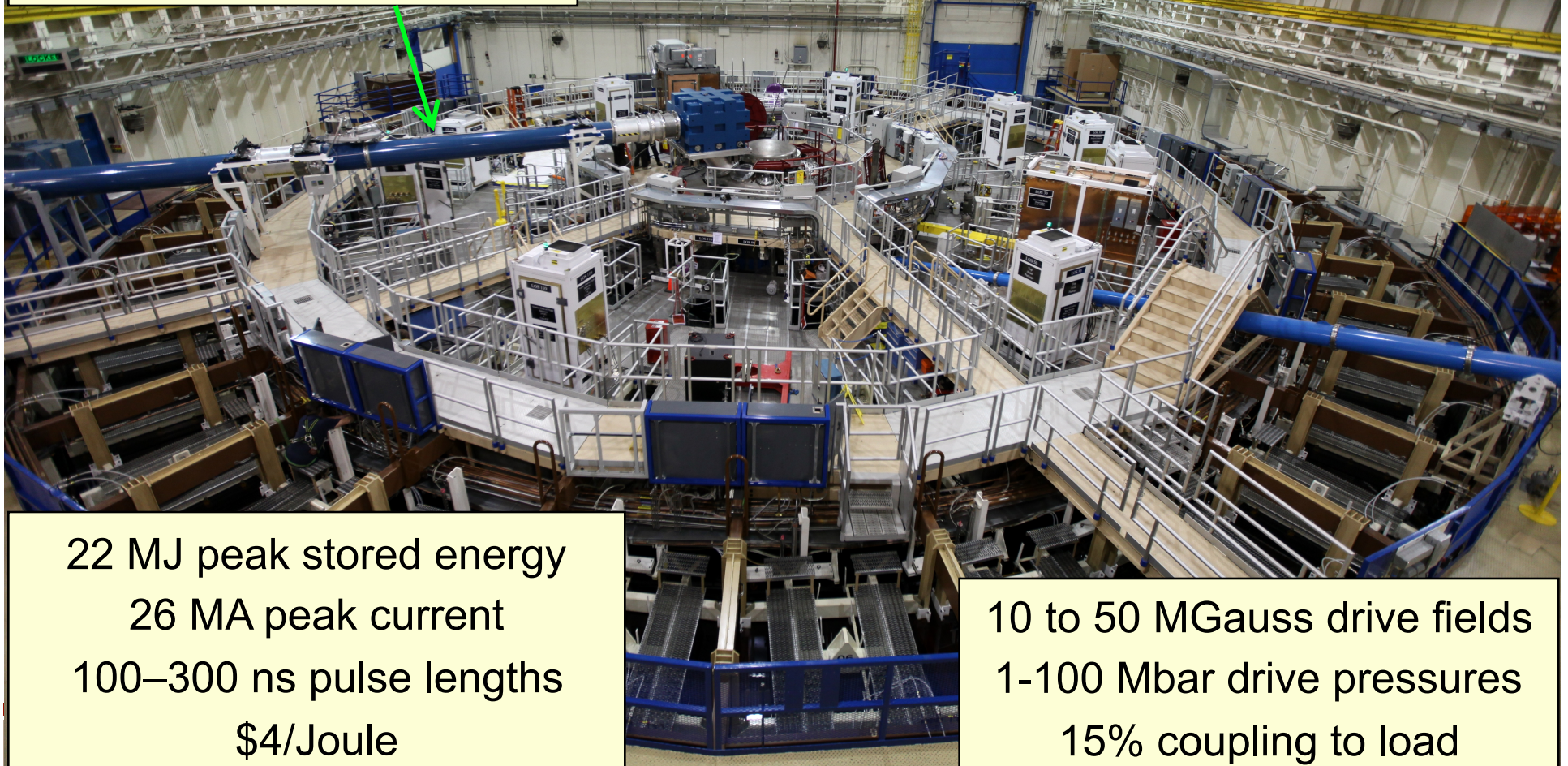
Extreme conditions are present at pressures of 1 Mbar (100 GPa)

- Material strength
 - internal energy density of a hydrogen molecule
 - bulk moduli of solids (glass ~ 45 GPa, steel ~ 160 GPa, diamond ~ 440 GPa)
- Energy density of electromagnetic field
 - photon intensity $\sim 3 \times 10^{15}$ W/cm²
 - thermal (blackbody) radiation field ~ 400 eV $\sim 4.6 \times 10^6$ °K
 - electric field $E \sim 1.5 \times 10^{11}$ V/m $\sim 1.5 \times 10^5$ MV/m
 - magnetic field $B \sim 500$ T
- Plasma pressure
 - plasma density $\sim 6 \times 10^{20}$ cm⁻³ for 1 keV temperature
 - plasma density $\sim 6 \times 10^{14}$ cm⁻³ for 1 GeV particle energy
- Electron degeneracy pressure $\sim 1.5 \times 10^{23}$ cm⁻³ for cold plasmas
- Ablation (rocket) pressure
 - 1 μ m laser $\sim 4 \times 10^{12}$ W/cm²
 - thermal radiation on plastic ~ 70 eV $\sim 8.1 \times 10^5$ °K
- Magnetic pressure
 - 26 MA at 1 cm
 - 2.6 MA at 1 mm

The Z pulsed-power facility combines a compact MJ-class target physics platform (the Z accelerator) with a TW-class laser (ZBL)

2-kJ Z Beamlet Laser (ZBL)
for radiography and
MagLIF fuel preheating

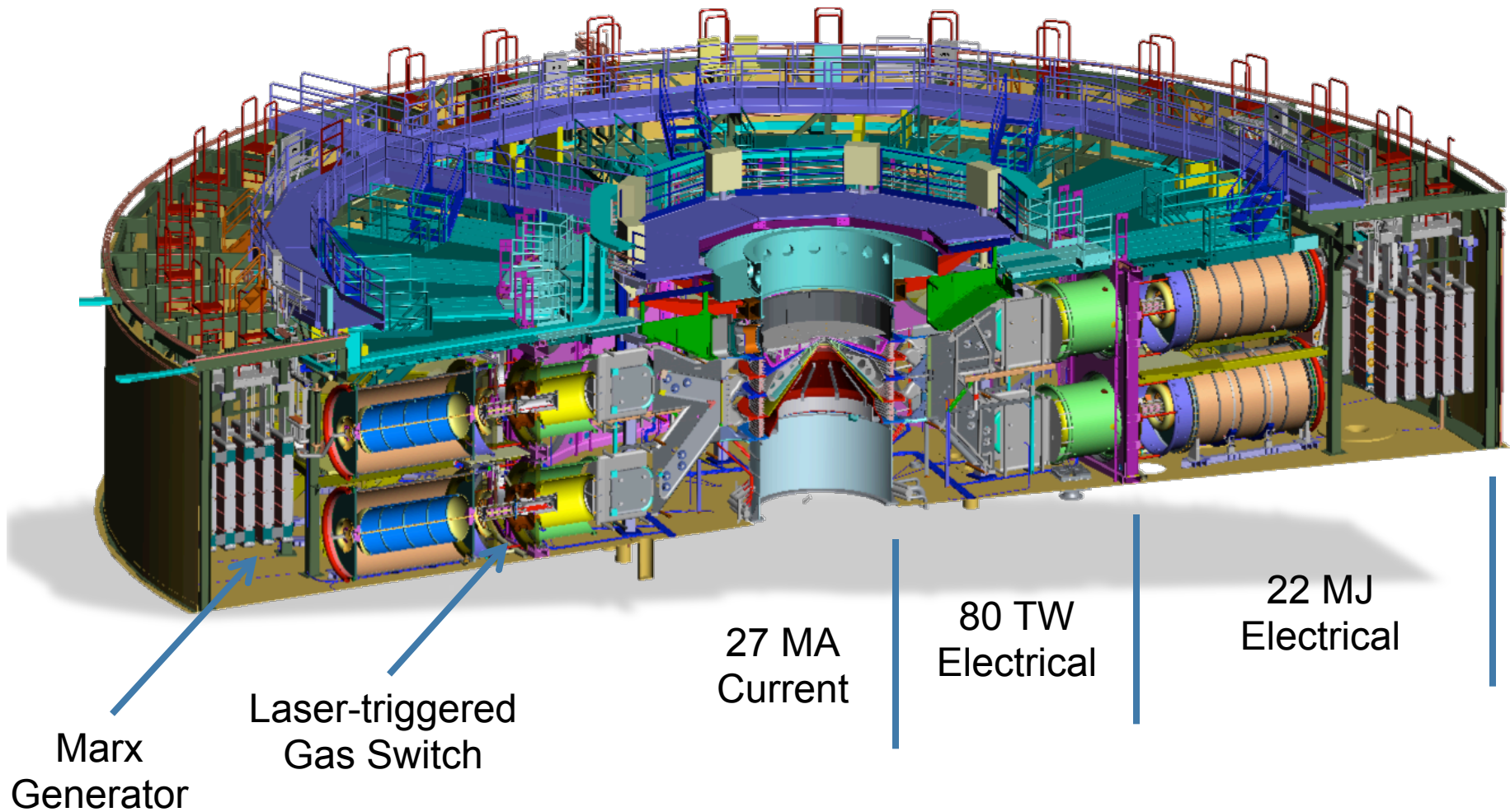
10,000 ft²



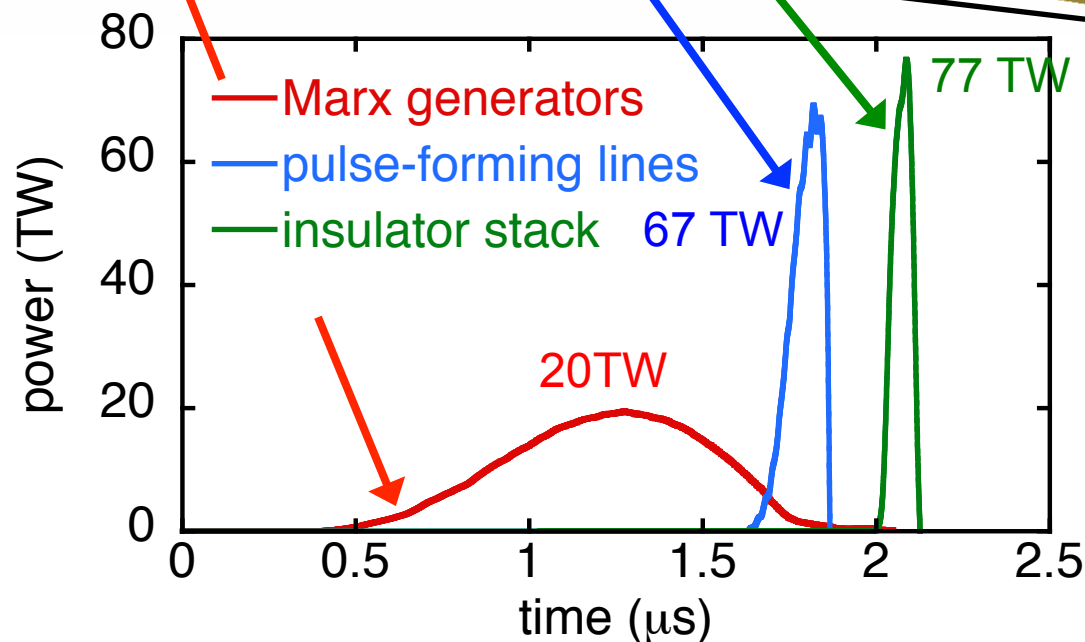
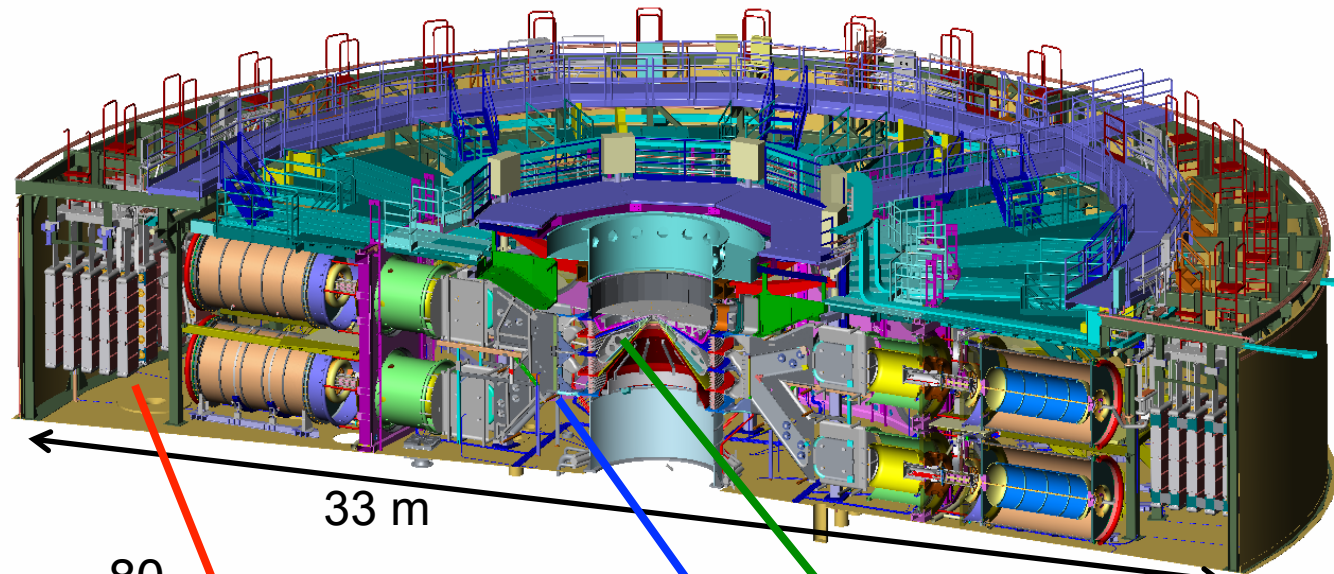
22 MJ peak stored energy
26 MA peak current
100–300 ns pulse lengths
\$4/Joule

10 to 50 MGauss drive fields
1-100 Mbar drive pressures
15% coupling to load

Cross section of the Z facility at Sandia National Laboratories



Z works by compressing energy in space and time to reach high energy densities (pressures)



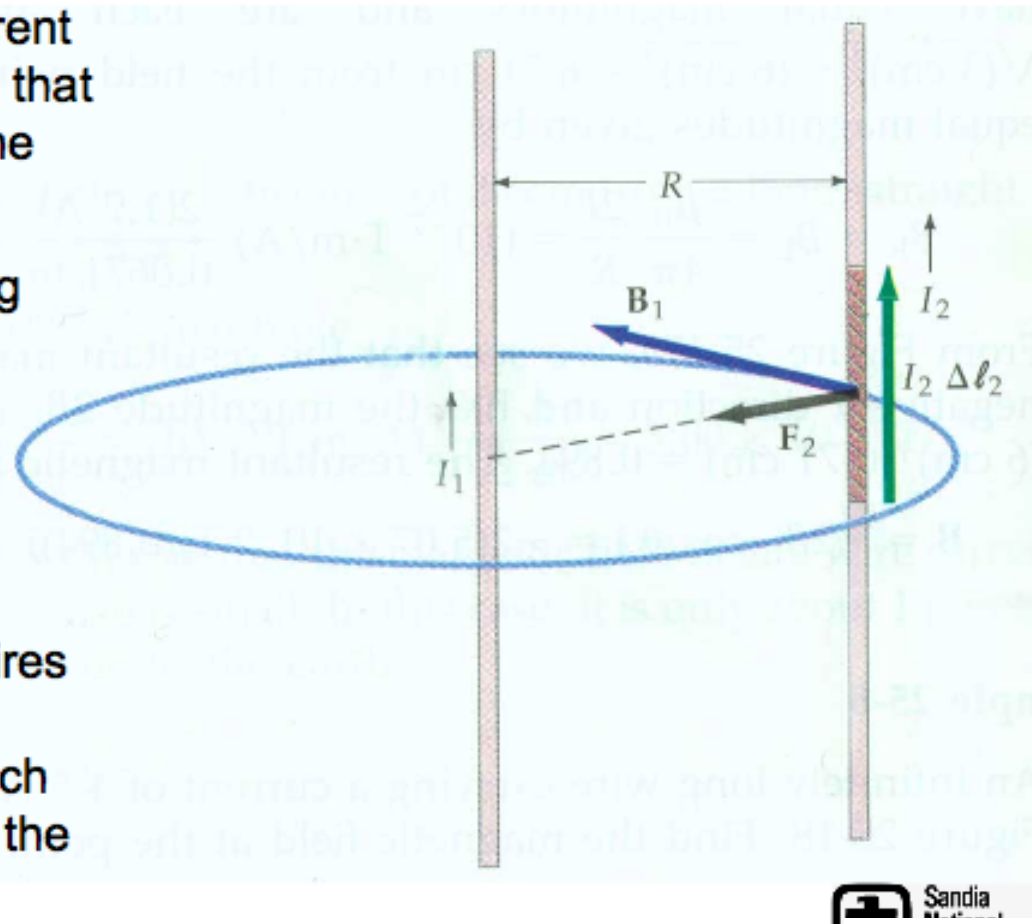
■ High electrical power density achieved through multiple stages of spatial and temporal pulse compression

1st year physics refresher: the “pinch effect”

A single wire carrying current produces a magnetic field that encircles it according to the right-hand rule

Two parallel wires carrying current along the same direction will attract each other (Biot-Savart Law, “ $\mathbf{J} \times \mathbf{B}$ force”)

Definition of an Ampere:
If two very long parallel wires 1 m apart carry equal currents, the current in each is defined to be 1 A when the force/length is 2×10^{-7} N/m



How strong is this pressure?

$N = kg\ m\ s^{-2}$ (mks) versus $dyne = g\ cm\ s^{-2}$ (cgs)

So $1\ N = 10^5\ dyne$, and, in pressure units:

$$1\ N\ m^{-2}\ (Pa) = 10\ dyne\ cm^{-2} = 10^{-5}\ bar$$

$$P_m(dyne / cm^2) = \frac{B(G)^2}{8\pi}$$

A typical refrigerator magnet is 100 gauss \sim 400 dyne/cm²

A 5000 G (0.5 T) magnetic field \sim 10⁶ dyne/cm² \sim 1 atmosphere \sim 1 Bar

A 5x10⁶ G (500 T) magnetic field \sim 1 Million atmospheres = 1 Megabar (MB)=
High energy density physics (“HEDP”)

A 5x10⁹ G (500 kT) magnetic field \sim 1 Trillion atmospheres = 1 Terabar (TB) >
pressure in the center of the sun

Note that high explosives have pressure \sim 100,000-300,000 atmospheres
 \sim 0.1-0.3 Mbar (not “HEDP”) \sim equivalent \sim 50-150 T or 5x10⁵-1.5x10⁶ G



Large currents can create large B fields!

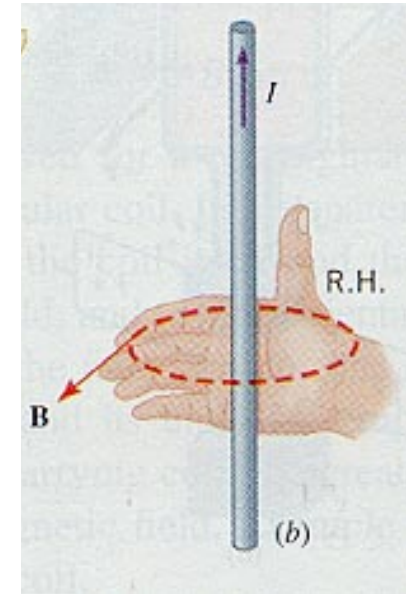
$$\nabla \times \mathbf{B} = \frac{4\pi \mathbf{J}}{c}$$

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \frac{4\pi}{c} \iint_S \mathbf{J} \cdot d\mathbf{S} \quad \text{Ampere's law}$$

For an axial current I :

$$2\pi r B_\theta = \frac{4\pi}{c} I \quad B_\theta = \frac{2}{c} \frac{I}{r} \quad (\text{cgs})$$

$$B_\theta (\text{G}) = \frac{I(\text{A})}{5 r(\text{cm})} \longrightarrow \mathbf{P_{mag} \sim B^2 \sim I^2 r^{-2}}$$

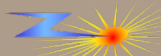


100 A at 2 mm radius is 100 G

1.0×10^7 A (**10 MA**) at **4 mm** radius is 5×10^6 G = **1 MBar** of pressure!

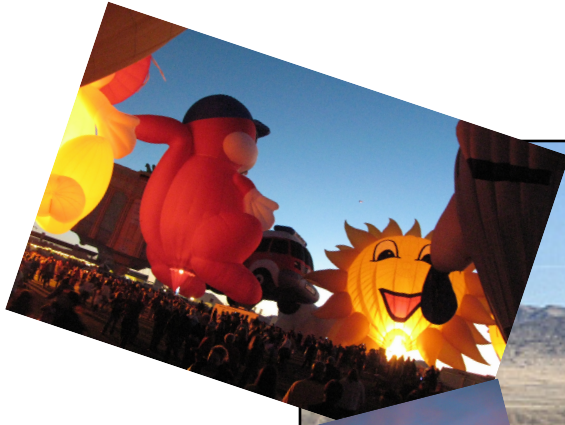
2.5×10^7 A (**25 MA**) at **1 mm** radius is 5×10^7 G = **100 MBar** of pressure!! **← Z Machine**
(~1000x more than high explosives)

LARGE CURRENTS → LARGE MAGNETIC FIELDS → LARGE PRESSURES!

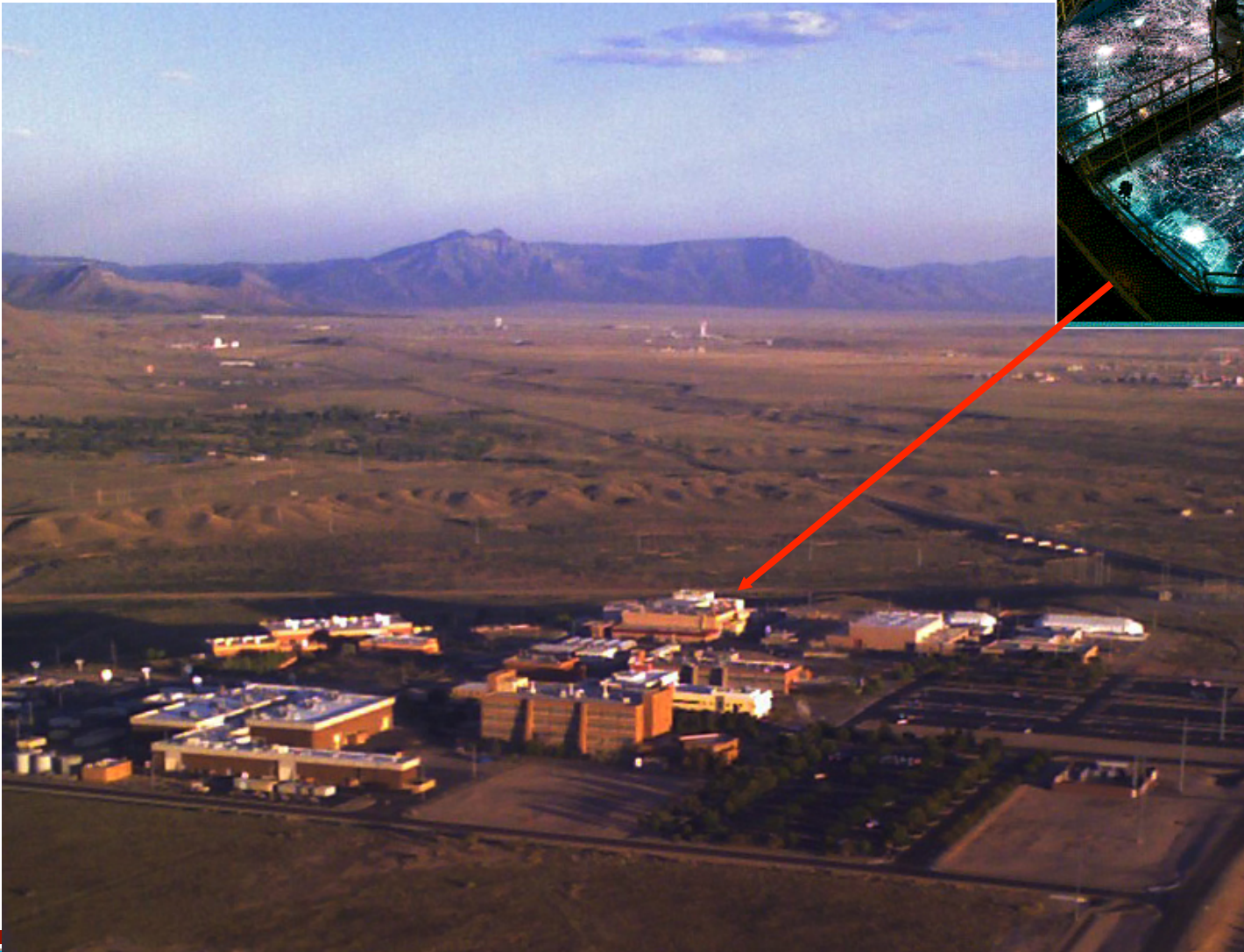


The “Z” pulsed-power facility is located at Sandia National Laboratories in Albuquerque, New Mexico

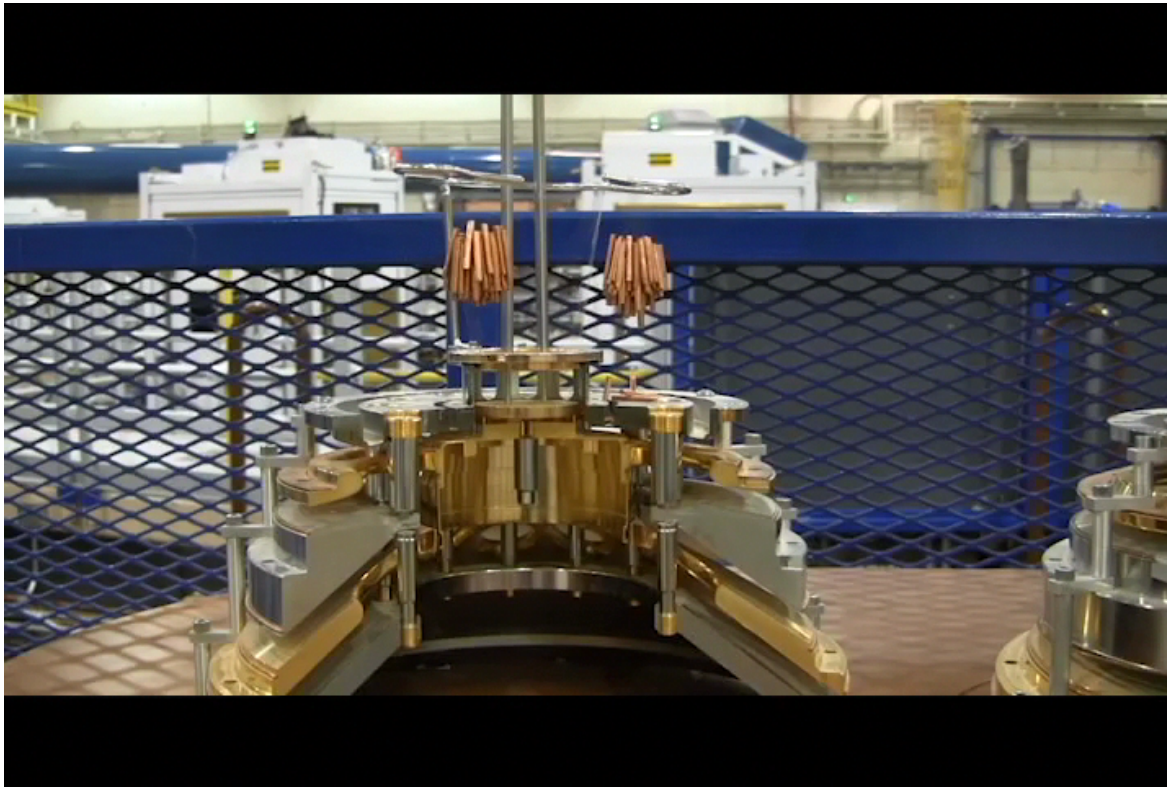
Youtube.com: search for the BBC TV show:
“Horizon: Can we make a star on earth?”



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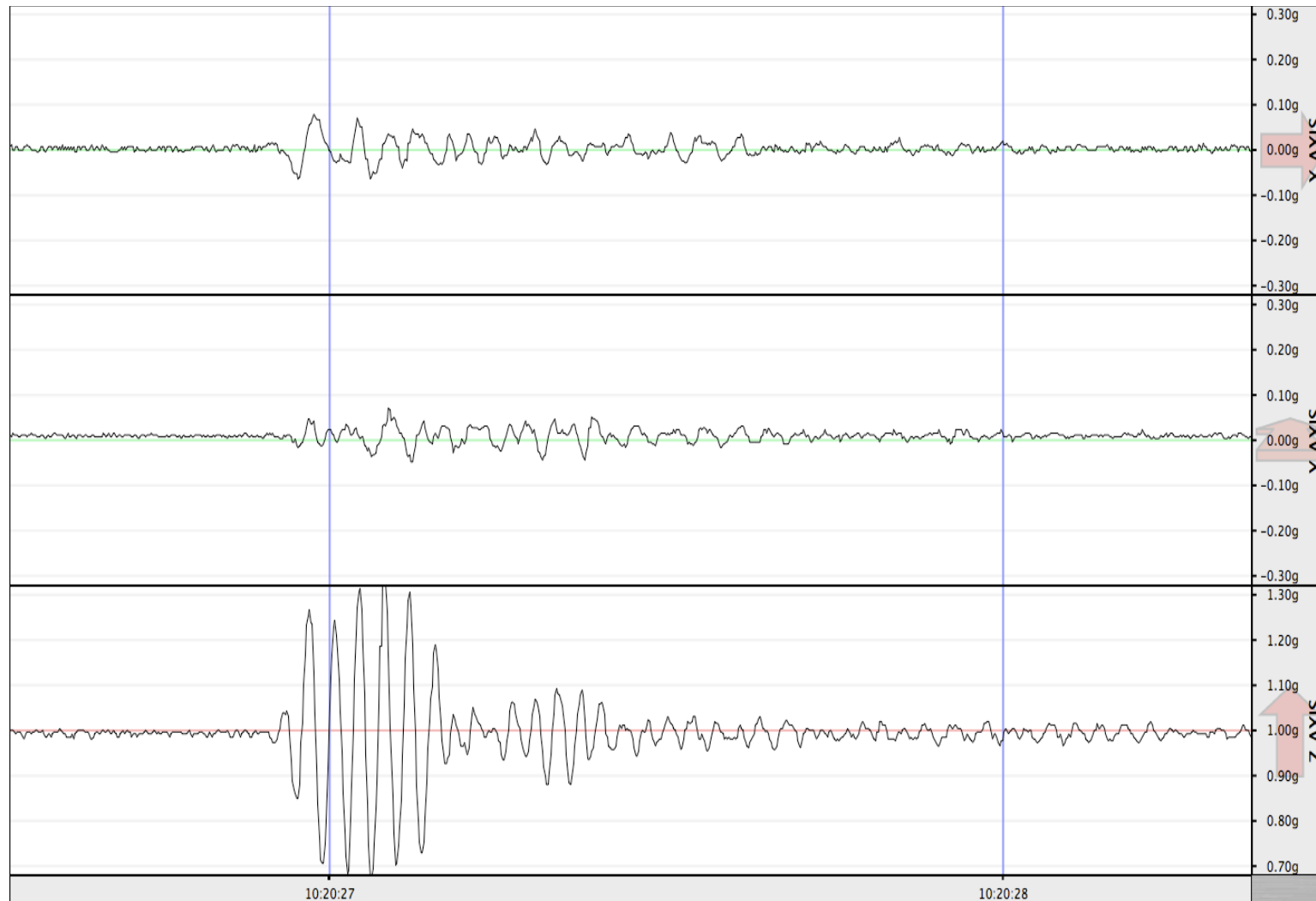


Z poses a challenging environment for experiments



- 30 – 100 g instantaneous shock at center

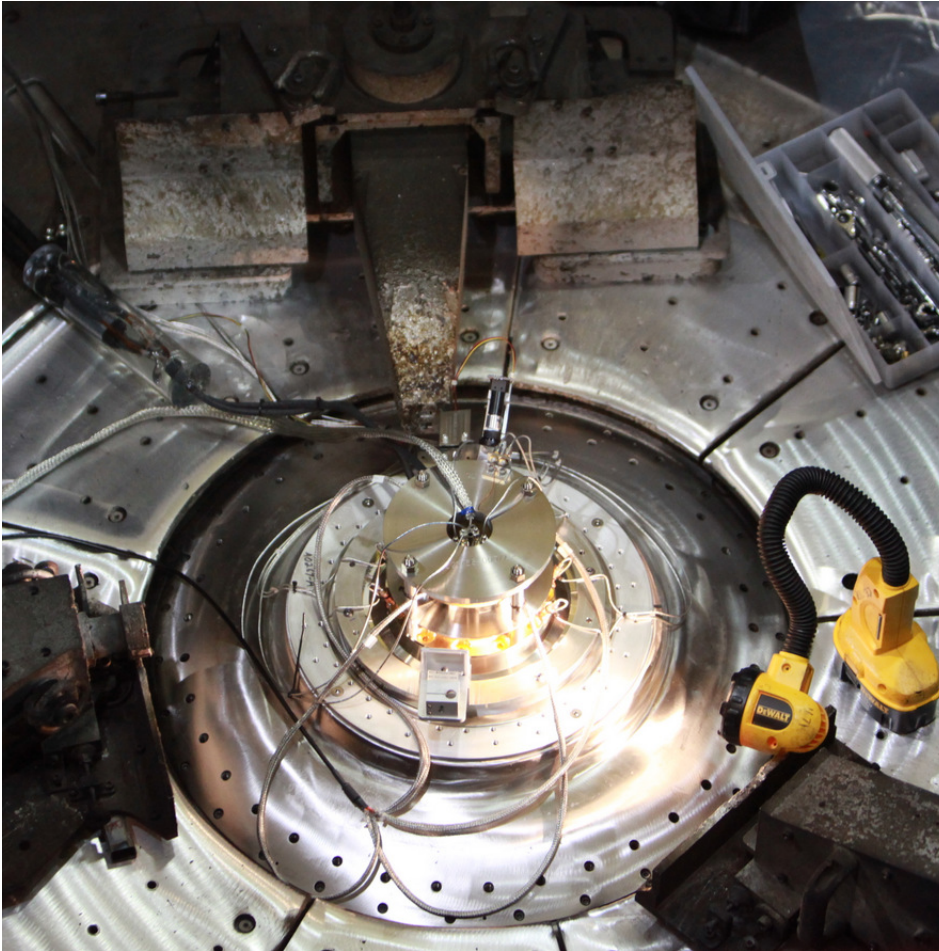
The ground shock is $\pm 1g$ of acceleration over 200 ms



Seismac program



Debris from all experiments must be carefully managed (several MJ energy release equivalent to few sticks of dynamite)



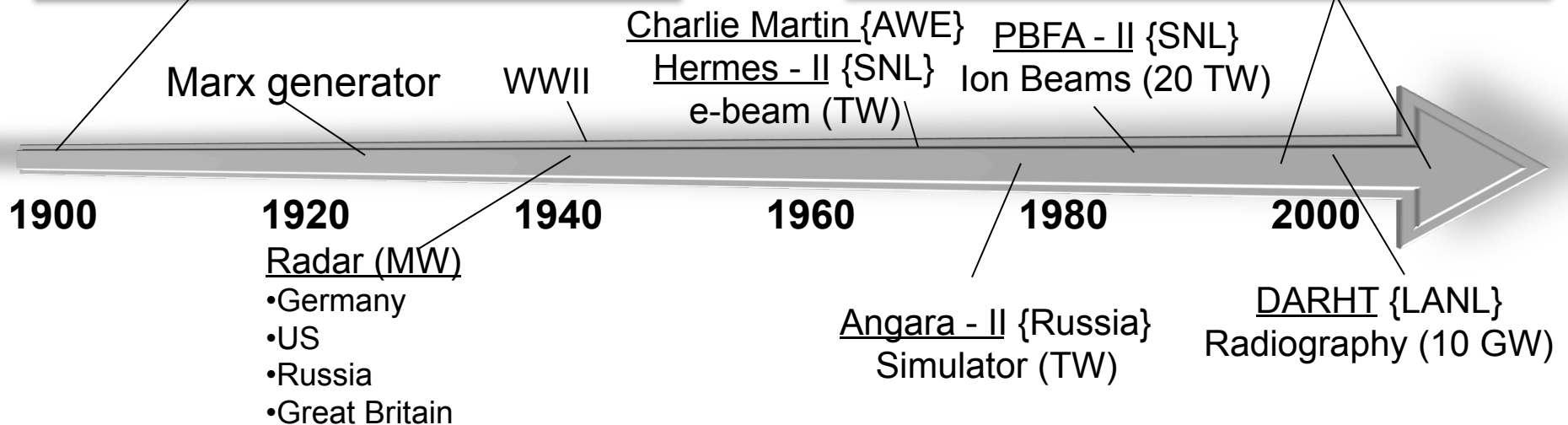
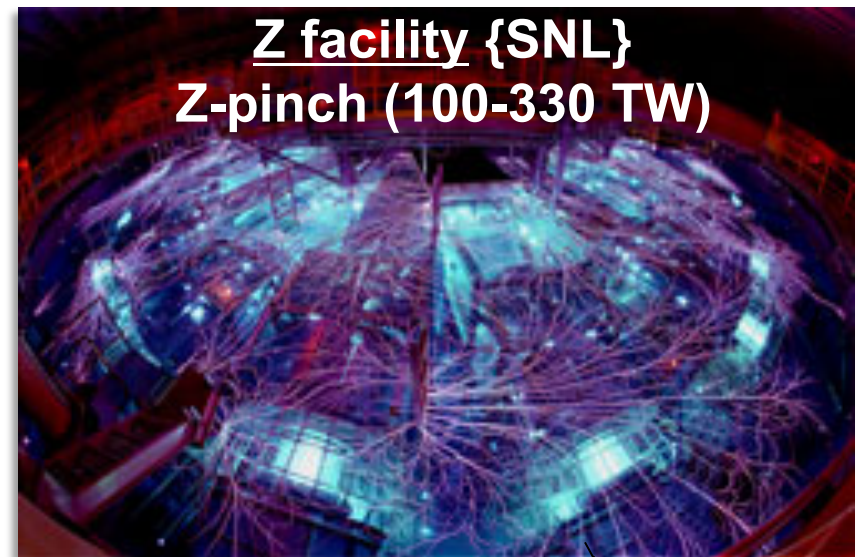
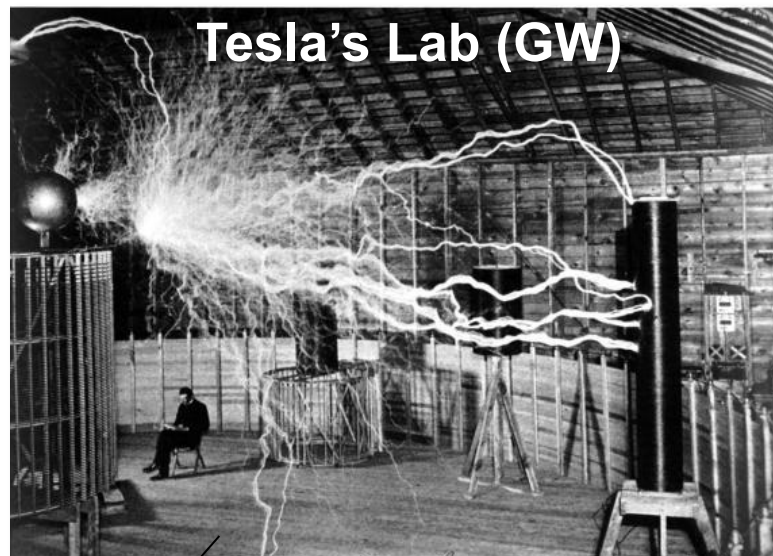
Pre-shot photo of coils & target hardware



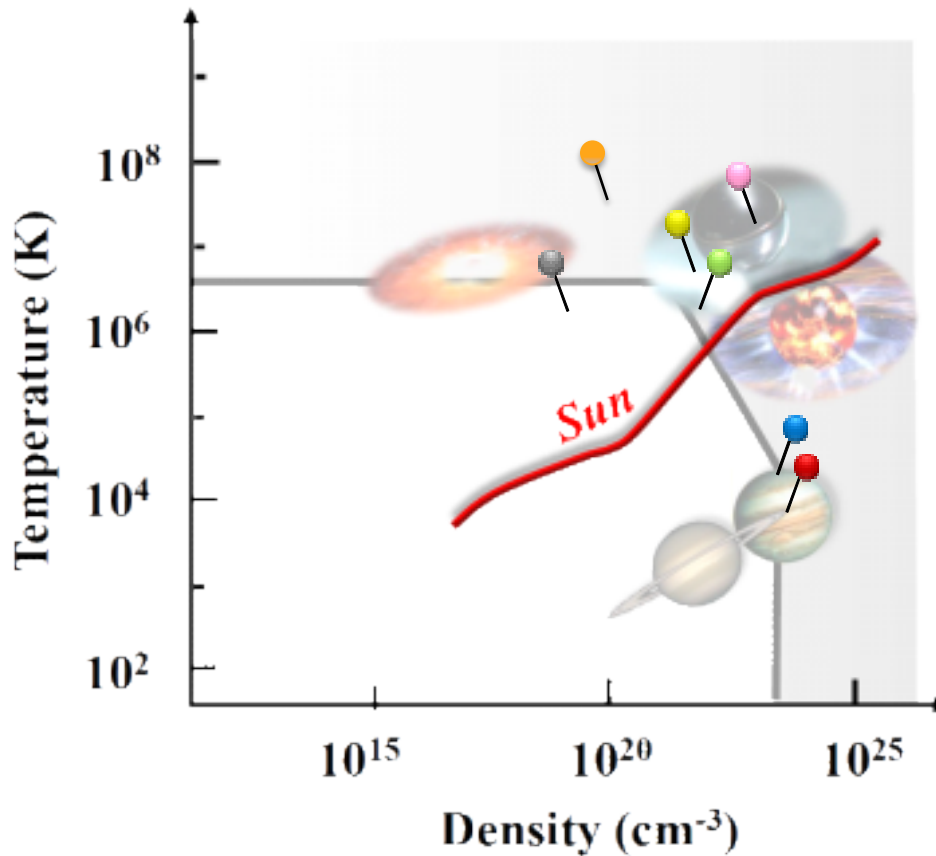
Post-shot photo

MagLIF Experiment

The accumulation and transmission of electromagnetic energy, called “pulsed power”, has been investigated for more than a century



HED experiments on Z address issues of fundamental importance



Diamond at 10 Mbar

Knudson et al., Science 322 (2008)

D₂ EOS at 1 Mbar

Knudson et al., PRL 87 (2001)

Photoionized Plasmas

Foord et al., PRL 94 (2004)

Radiating Shocks

Rochau et al., PRL 100 (2008)

Opacity at $T_e > 150$ eV

Bailey et al., PRL 99 (2007)

Fusing Plasmas

Bailey et al., PRL 93 (2004)

High-Z Ion Radiating Plasmas at 40 Mbar

Ampleford et al., submitted PoP (2013)

Pulsed power generates amazing conditions in the laboratory

Peak currents of 10 – 25 Mega Amperes

Electromagnetic energy of 1 – 2 Mega Joule

Magnetic fields of 0.1 – 5 Mega Gauss

Magnetic pressures of 1 – 100 Mega bar

Accelerations and decelerations of 1 Tera g's

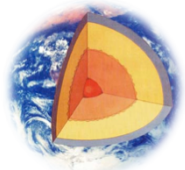
Implosion velocities of 0.1 – 1 Mega meters/sec

Plasma stagnation temperatures of 10 – 50 Mega kelvin

Plasma stagnation pressures of 1 Mega Bar to 1 Giga Bar

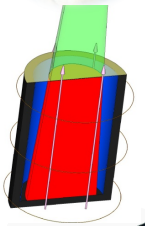


The stockpile stewardship is divided into a broad portfolio of “science campaigns” with diverse requirements



Dynamic Materials Properties

Materials of interest at pressure and temperatures of interest



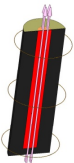
Inertial Confinement Fusion

Researching into creating “self-heating or burning plasmas”



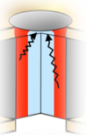
Radiation Effects Sciences

Study the impact of radiation on circuits and components?



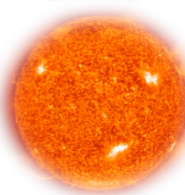
Primary Assessment

Simulate and study process of importance in NW primaries



Secondary Assessment

Simulate and study process of importance in NW secondaries

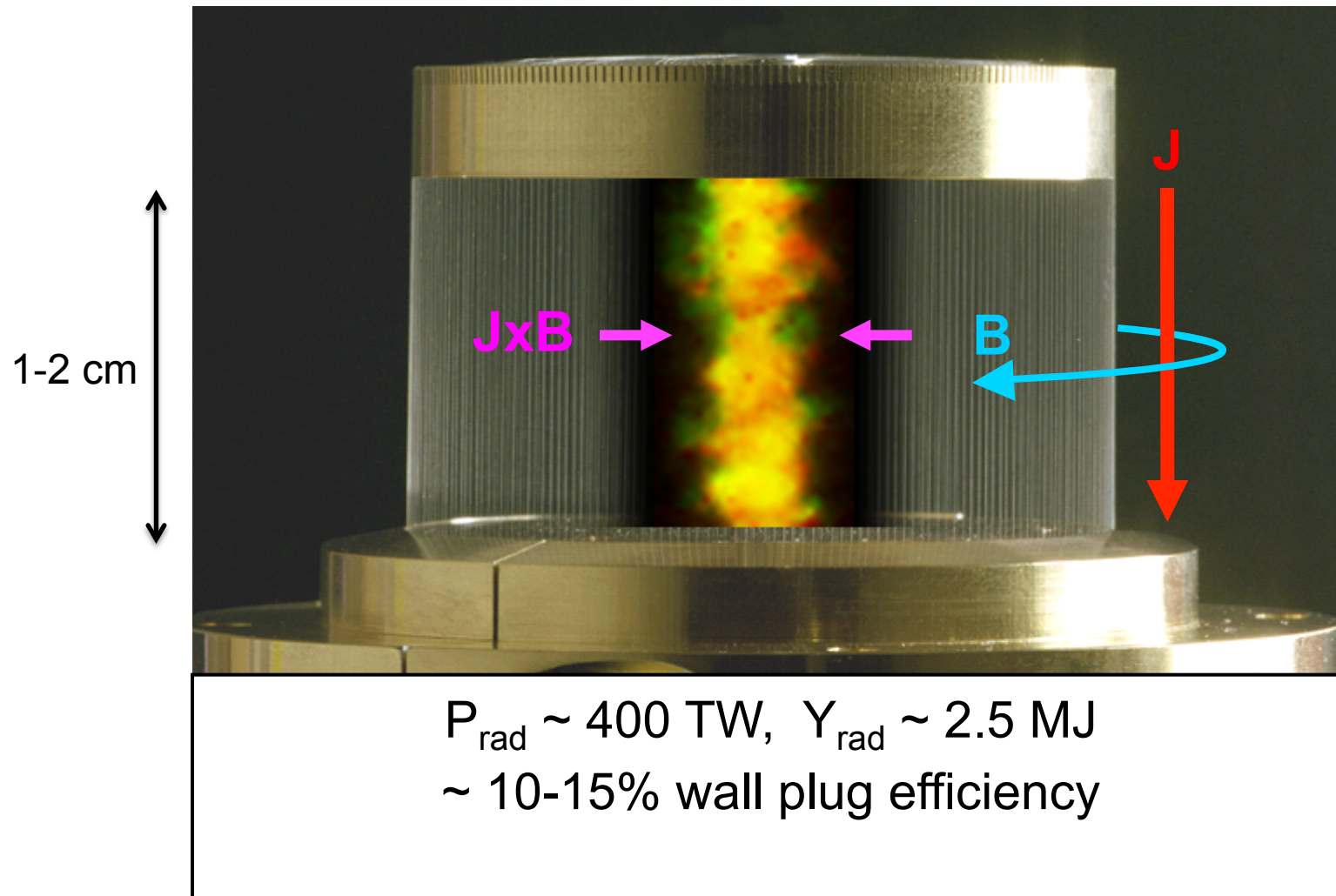


Fundamental Science

Use the conditions to study basic science questions in astrophysics, geophysics, plasma, atomic physics



Magnetically driven implosions are efficient, powerful, x-ray sources from 0.1 to 10 keV



We employ energies of ~ 1 MJ in every day objects



An energy of 1 MJ:

- Kinetic energy of F150 at ~ 60 mph
- 0.48 x energy in a stick of dynamite
- 100 W incandescent light bulb uses 1 MJ in 4.3 hours

A velocity of 500 km/sec:

- $\sim 1,100,000$ miles per hour
- New York to LA in ~ 8 seconds
- 1/600 speed of light

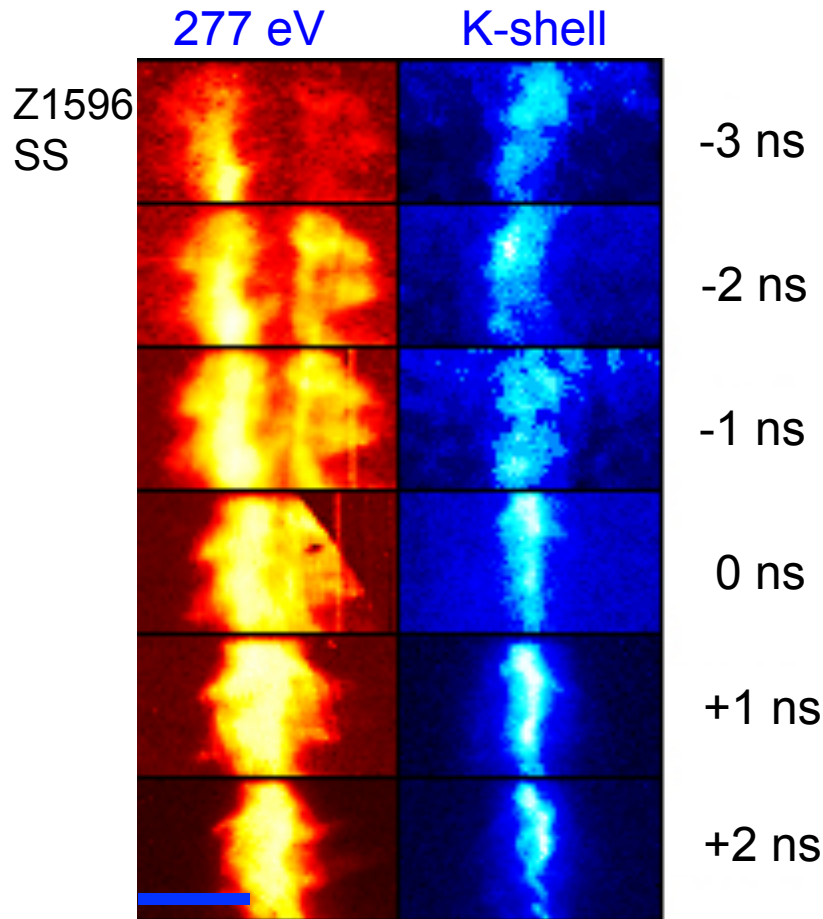
- $m_{\text{F150}} = 2950 \text{ kg}$
- $v_{\text{F150}} = 94 \text{ km/hour (58 mph)}$
- $E = 1 \text{ MJ}$
- In a typical z-pinch, this 1 MJ is released in 5 ns



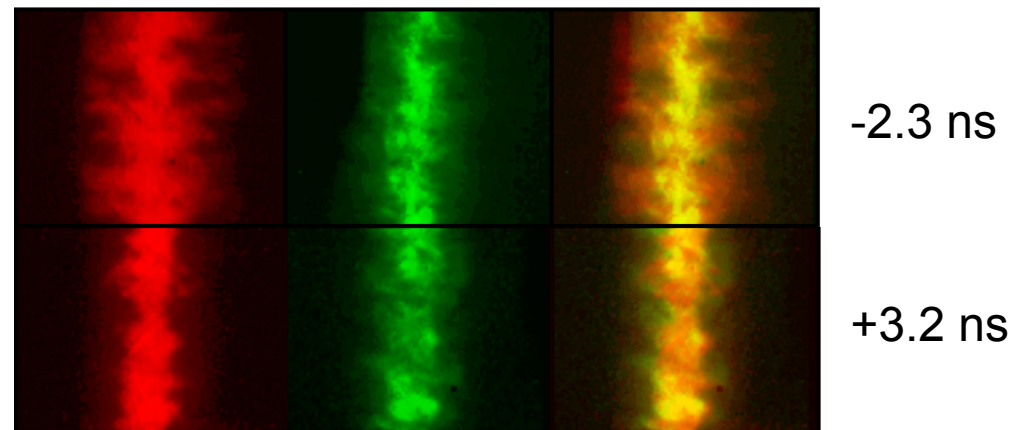
Self-emission x-ray imaging shows that magnetically driven implosion generates large volumes (10 to 100 mm³) of hot plasma

Z1519, Al

277 eV 1.7 keV Al Overlay
K-shell K-shell



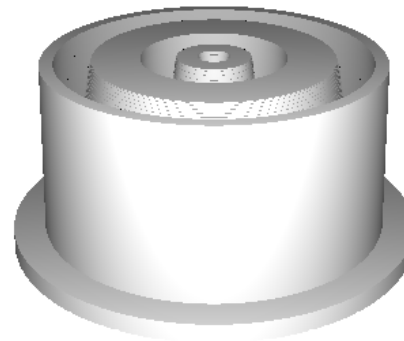
5 mm



- As noted previously,
 - Evidence of instability at stagnation
 - Observed structure varies with atomic number
- Hot, dense column emitting K-shell on axis with colder material still imploding

Numerical modeling tools are mature and complement Z gas puff experiments

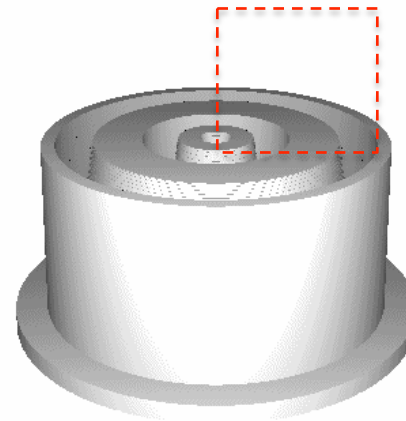
- Cold gas flow models may be validated using experimental interferometer data
- Benchmarked simulated profiles can be used to initiate MHD simulations
- Tabulated atomic data are used to estimate K-shell x-ray outputs
- Pre-shot NRL modeling [Thornhill *et al.*, HEDP **8**, 197 (2012)] was consistent with SNL Gorgon simulations (Jennings)



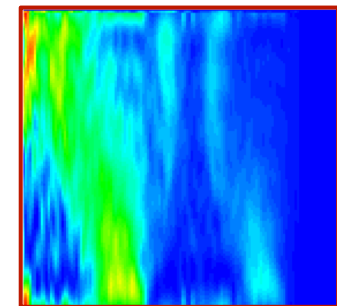
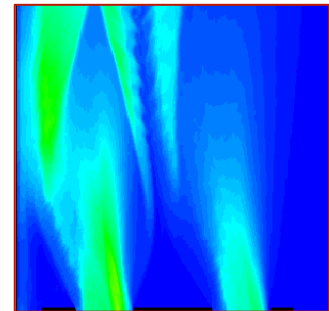
C.A. Jennings, SNL, GORGON

Numerical models are being used to design experiments and benchmarked post-shot to gain physics insight

- Cold gas flow models may be validated using experimental interferometer data
- Benchmarked simulated profiles can be used to initiate MHD simulations
- Tabulated atomic data are used to estimate K-shell x-ray outputs
- Pre-shot NRL modeling [Thornhill *et al.*, HEDP **8**, 197 (2012)] was consistent with SNL Gorgon simulations (Jennings)



Simulation

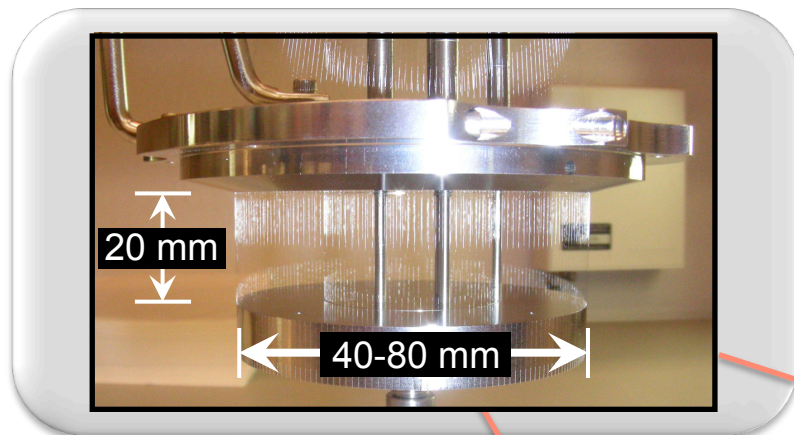


Interferometer

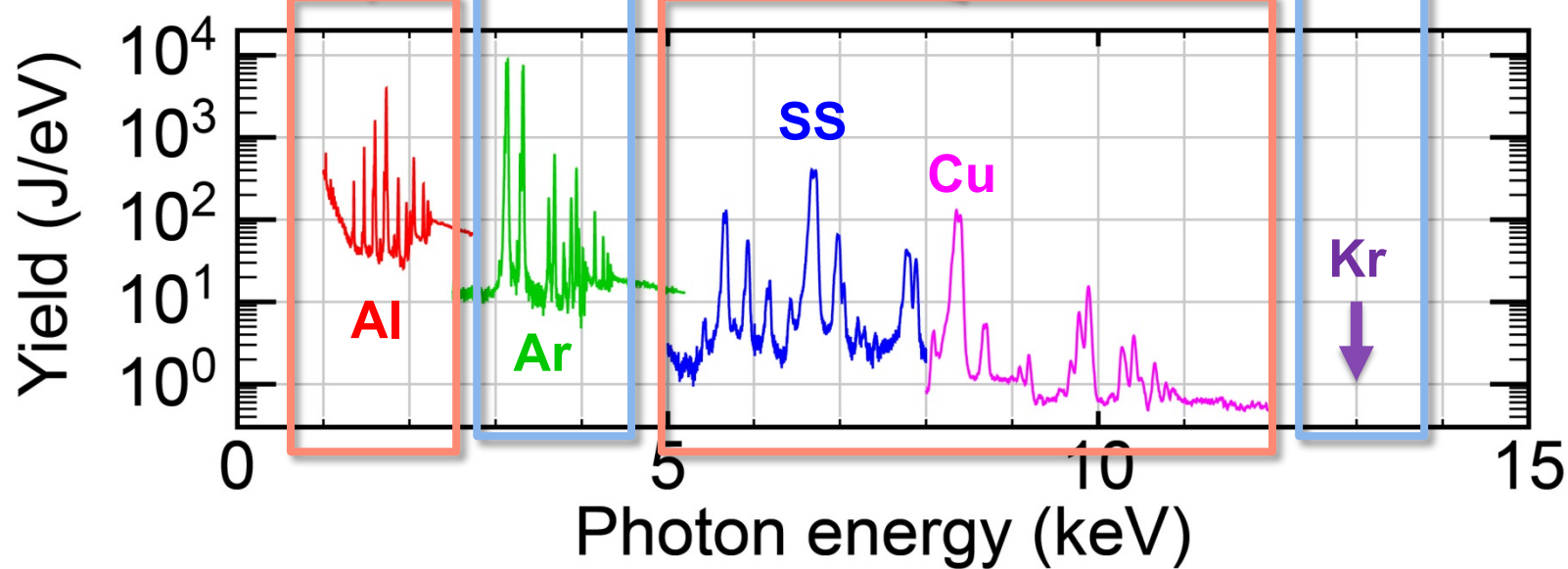
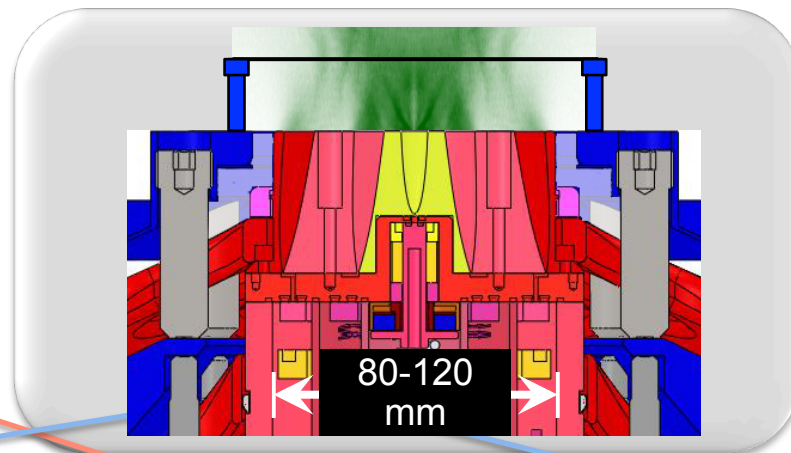
C.A. Jennings, SNL, GORGON

Magnetically-driven implosions to $\sim 500\text{--}1000\text{ km/s}$ are used to access K-shell emissions from different materials

Nested Wire Arrays

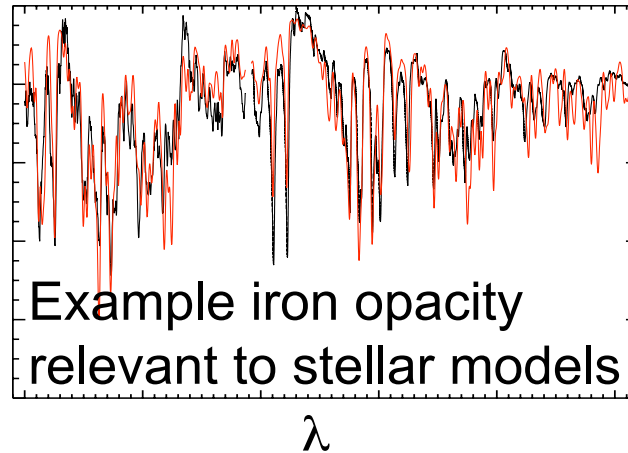


Structured Gas Puffs

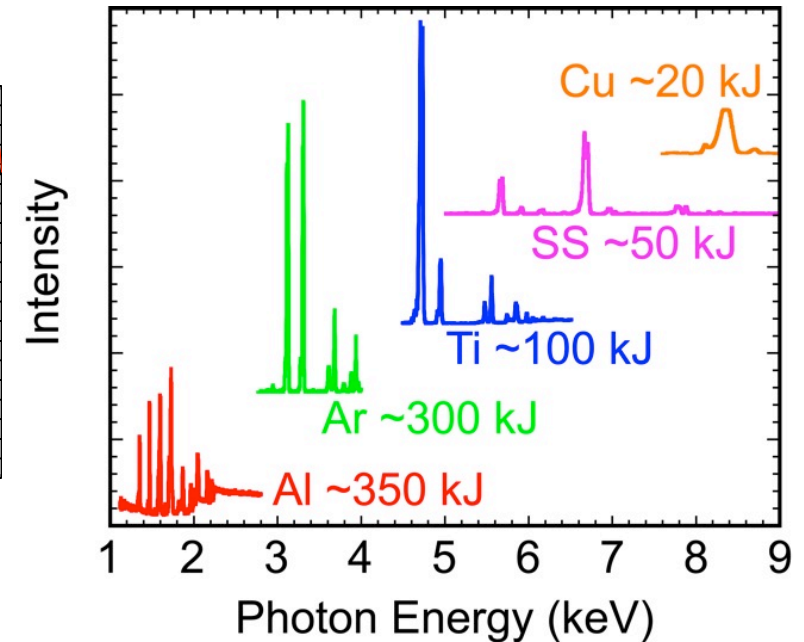


Magnetically-driven x-ray sources are being used for a variety of fundamental and applied science applications

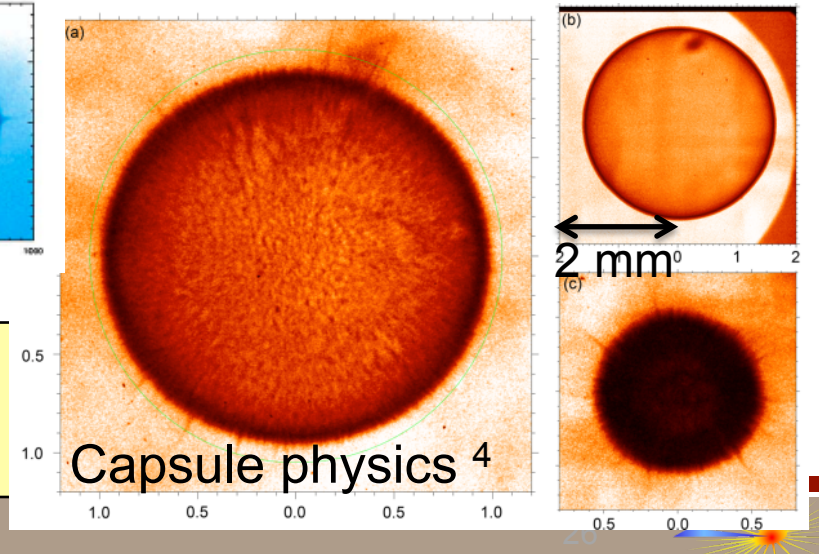
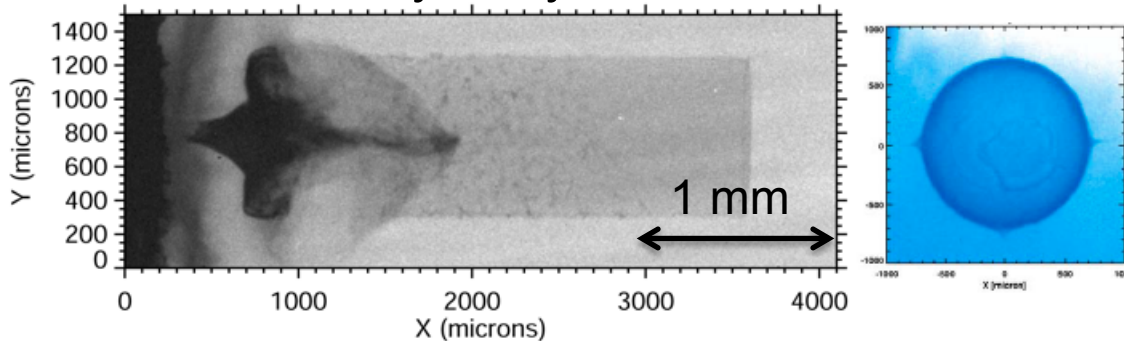
Opacity Measurements and Photo-ionized plasmas ¹



K-Shell Sources ³



Radiation Hydrodynamics ²

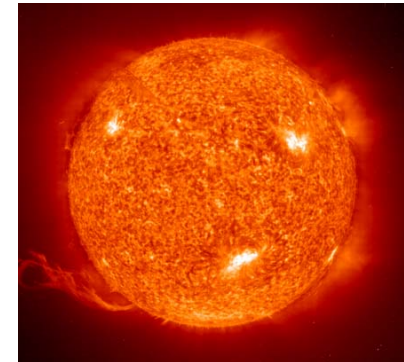
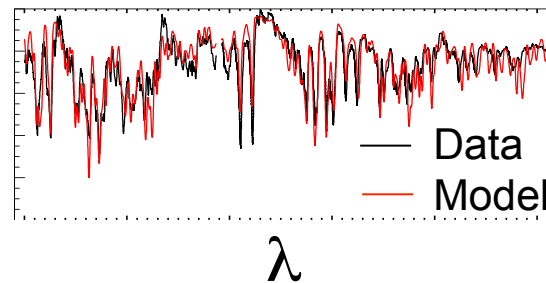


- ¹ J.E. Bailey, et al, Phys. Rev. Lett. 99, 265002 (2007).
- ² G.R. Bennett, et al., Phys. Rev. Lett. 205003 (2007).
- ³ C. A. Coverdale et al., IEEE T. Plas. Sci. 35, 582 (2007).
- ⁴ M.E. Cuneo et al., IEEE T. Plas. Sci. 40, 3222 (2012).

Magnetically-driven x-ray sources are being used for a variety of fundamental and applied science applications

Solar opacity at $T_e > 150\text{-}170$

Bailey et al., PRL 99 (2007)



White dwarf physics

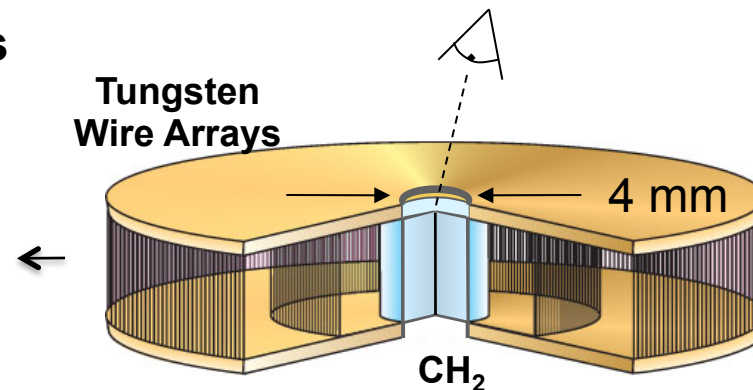
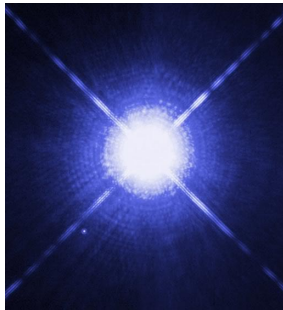
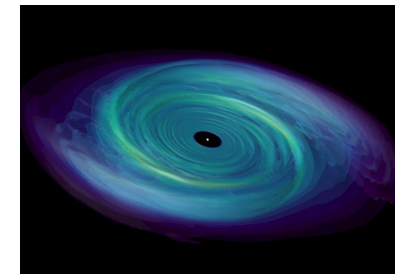


Photo-ionized plasmas

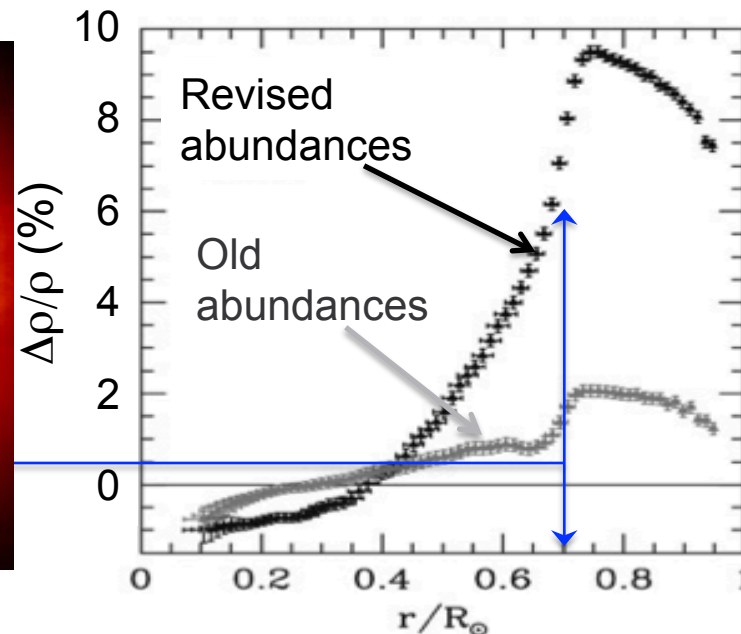
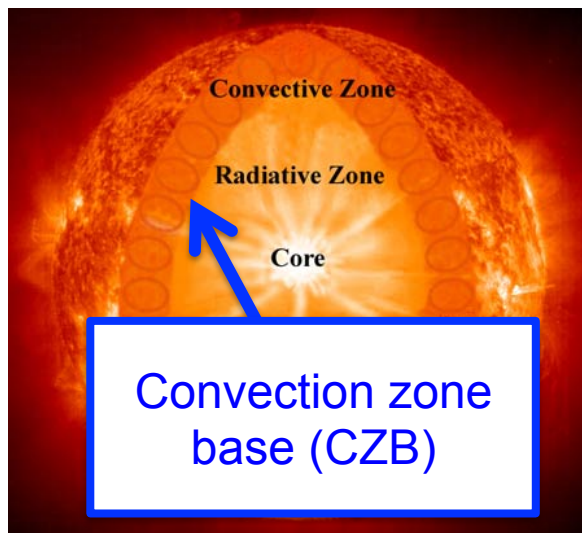


- $1 \mu\text{g}$ of stellar interior at $R \sim 0.7R_{\text{sol}}$
- 10^{-3} liters of accretion disk at $R \sim 100 - 1000 \text{ km}$ from black hole
- ~ 0.1 liters of white dwarf photosphere
- weapons science



Z iron opacity experiments refine our understanding of the sun, stars, and laboratory HED systems

- Opacity is the fundamental property that controls photon absorption in matter
- Opacity model uncertainty has been proposed as an explanation for the decade old problem that solar models do not match helioseismic observations



Solar problem arose with abundance revision in ~ 2000

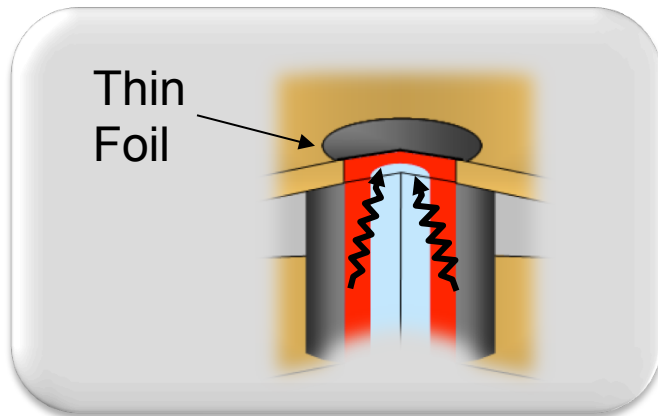
Stellar composition and opacity are intertwined

Problem resolved if mean opacity ~15% higher than predicted

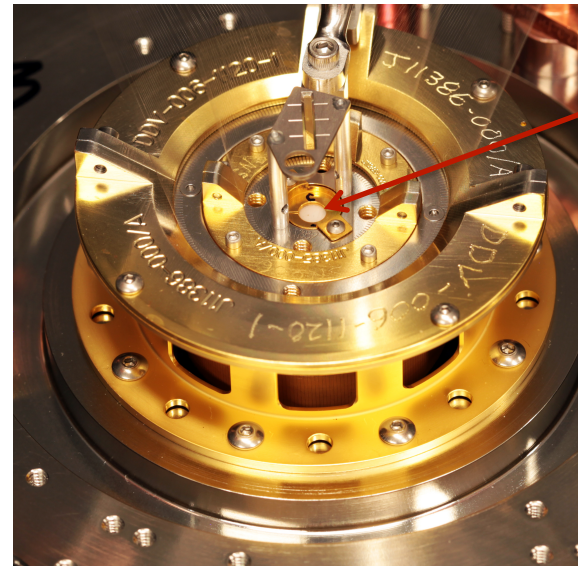
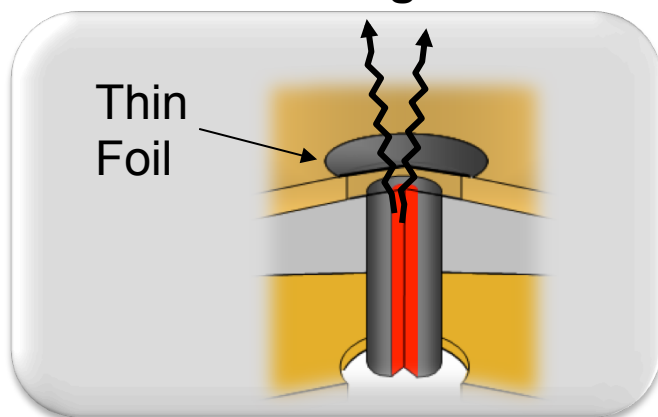
- **Conditions at the solar CZB : 182 eV, $9e22$ electrons/cc**
- **Similar conditions exist in many astrophysical and laboratory plasmas**
- **Thus, solar opacity model refinements have far-reaching implications**

The ZPDH radiating shock is used to both heat and backlight samples to stellar interior conditions.

Foil is heated during the ZPDH implosion

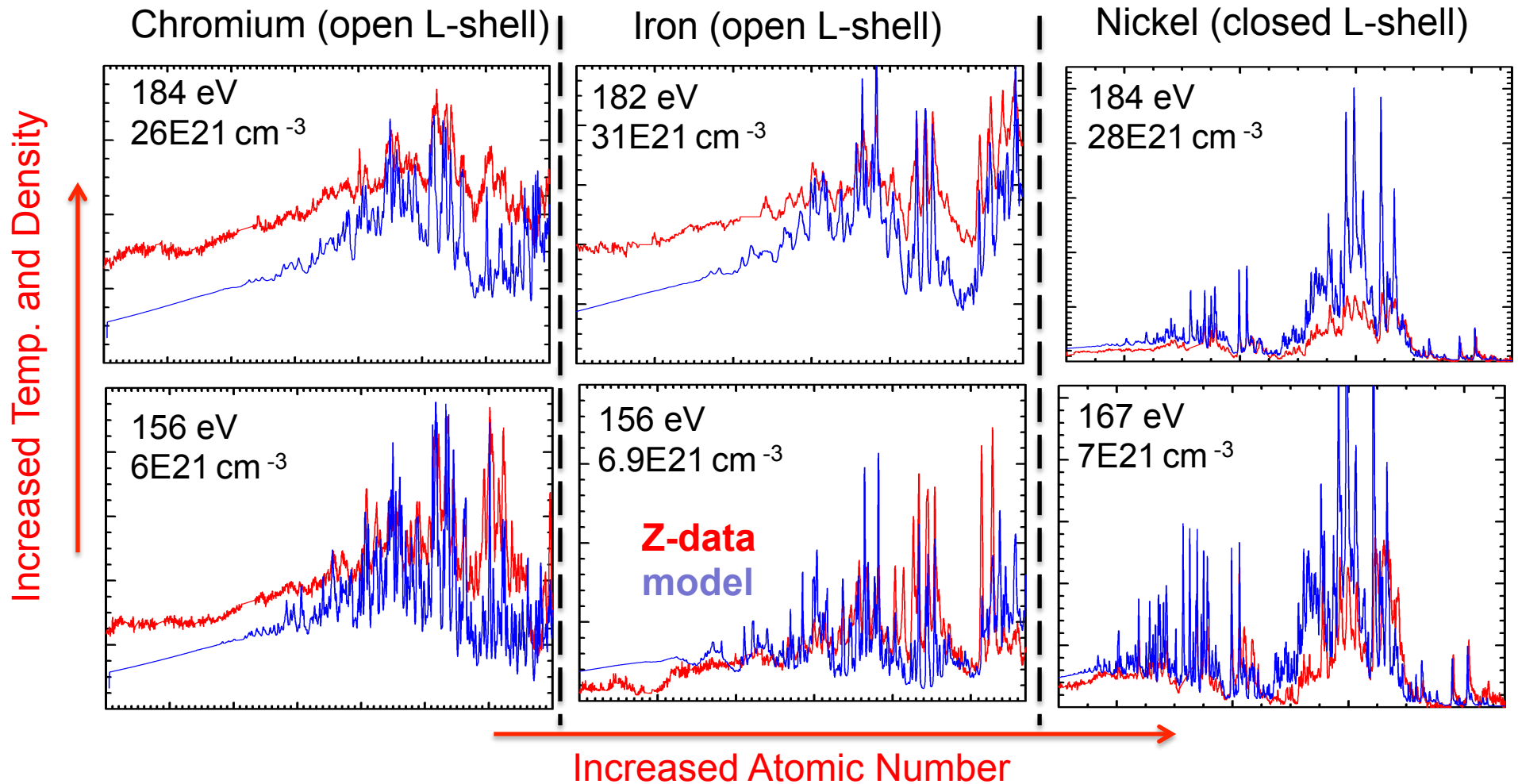


Foil is backlit at shock stagnation



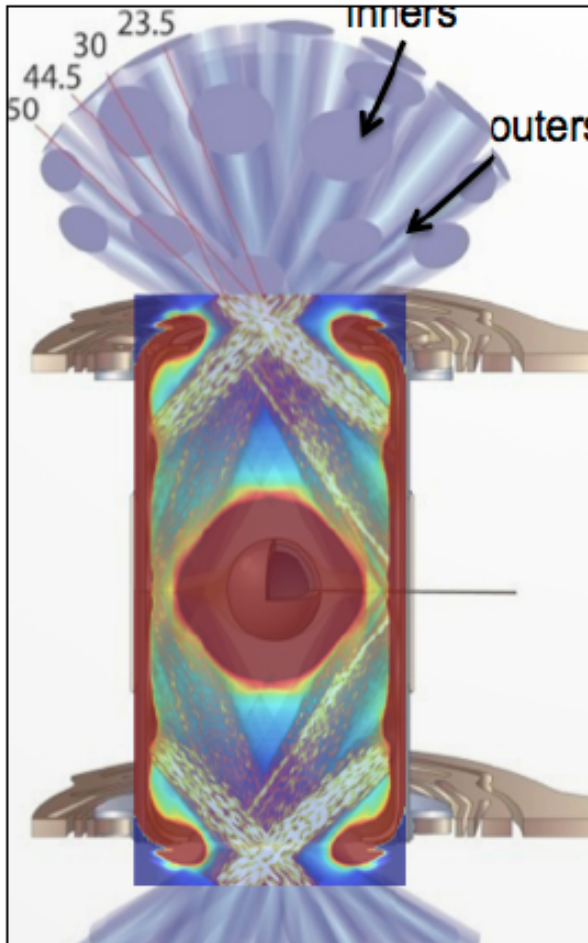
Photon absorption in HED matter is different than expected: atomic physics models must be improved

fewer L-shell vacancies, lower excited state populations

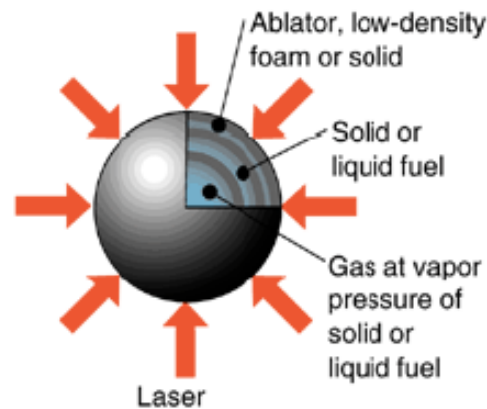


The United States ICF program has focused on three main approaches

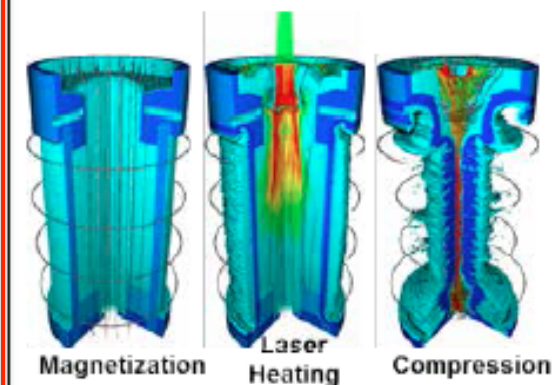
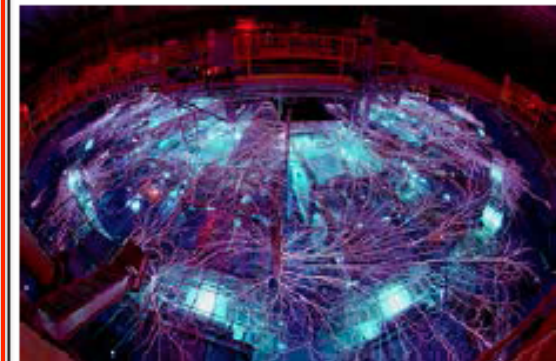
Radiation-driven implosions



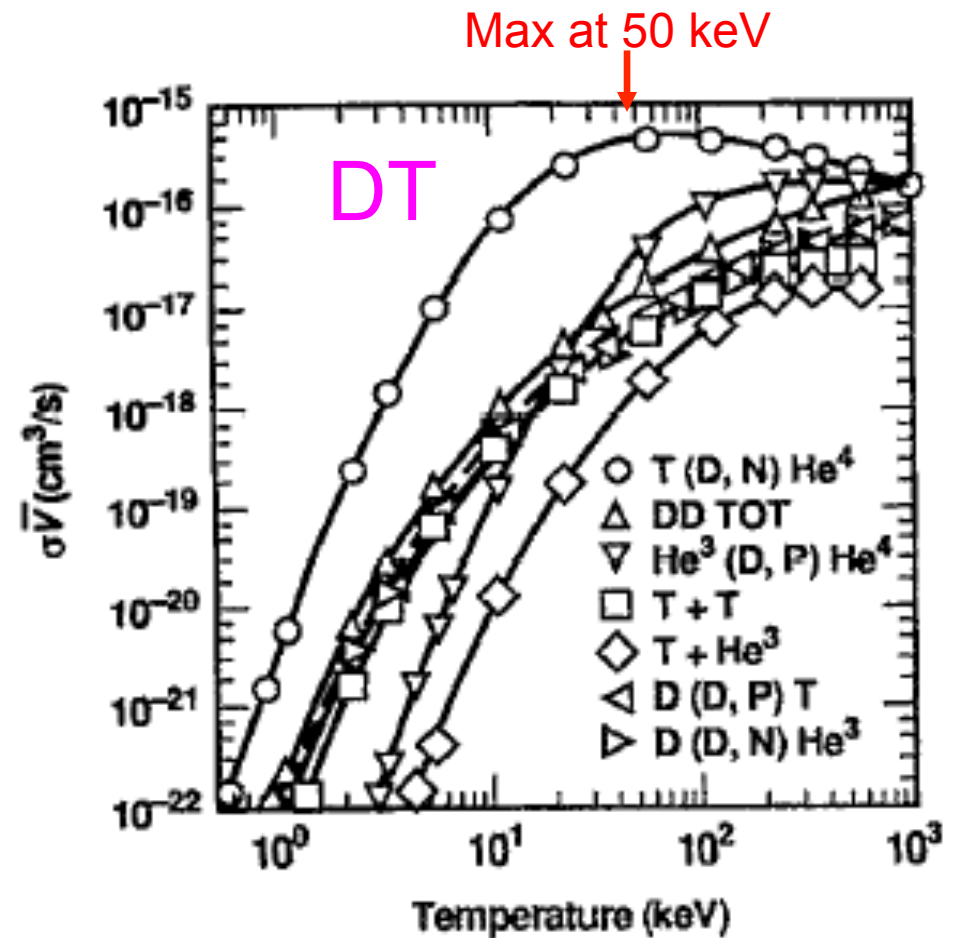
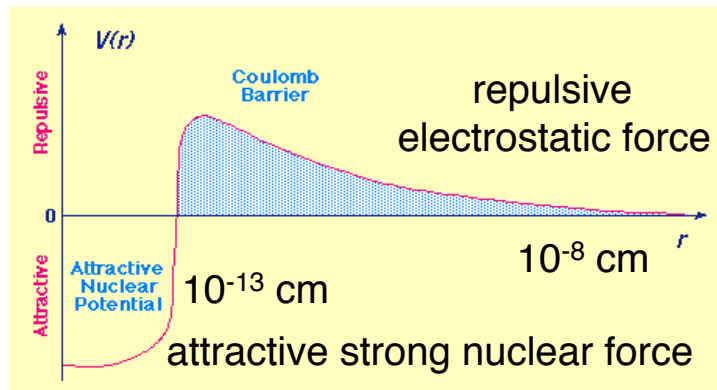
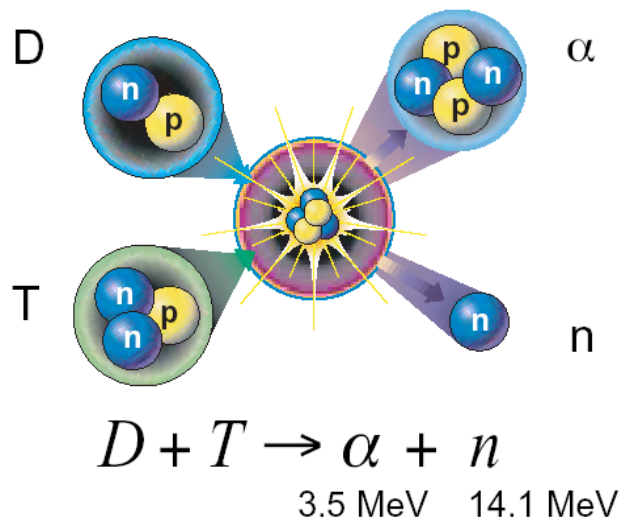
Laser-driven implosions



Magnetically-driven implosions



Thermonuclear fusion requires high temperatures to overcome the Coulomb barrier

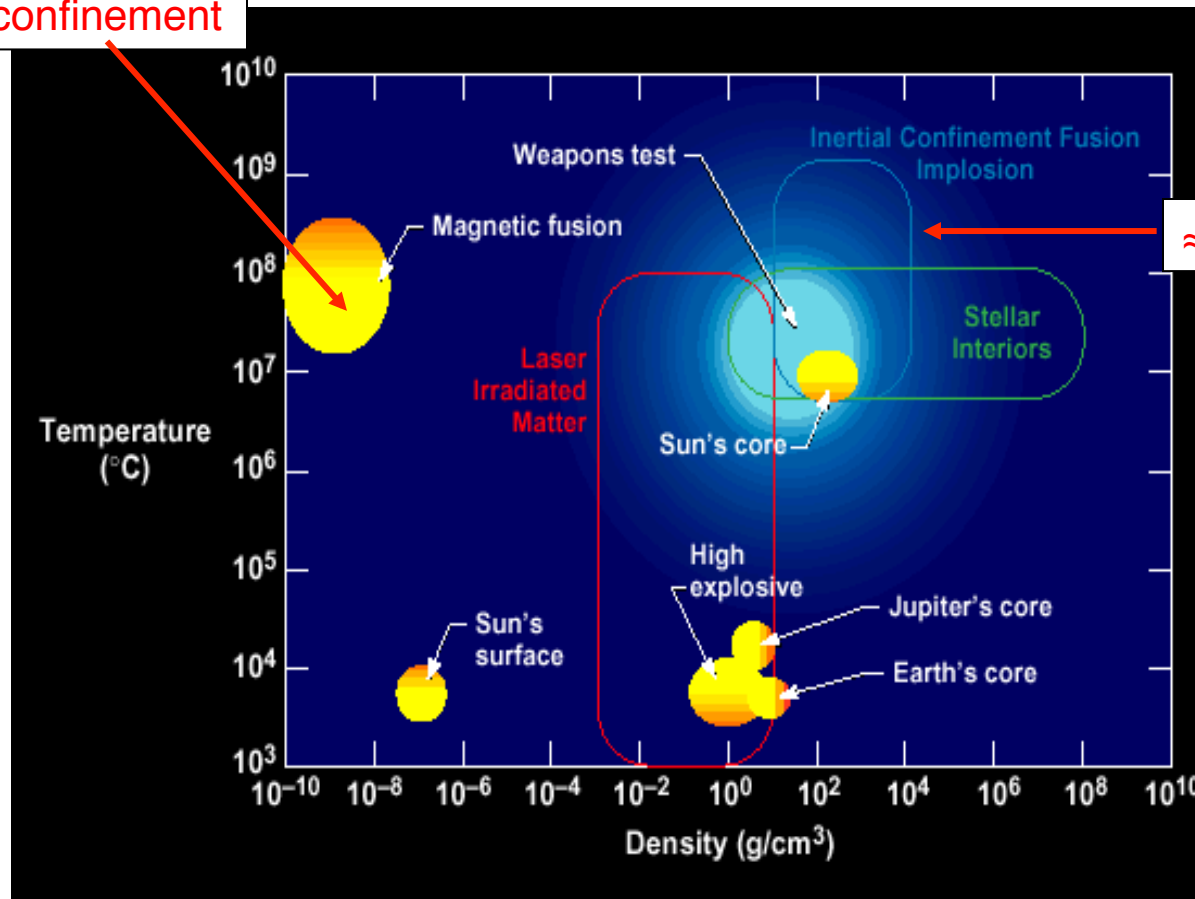


■ $Q_{DT \text{ fusion}} = 17.6 \text{ MeV}/5 \text{ AMU} = 3.3 \times 10^{11} \text{ J/g}$



Fusion can produce net energy with sufficient plasma density and confinement time at temperatures > 10 keV

\approx second confinement

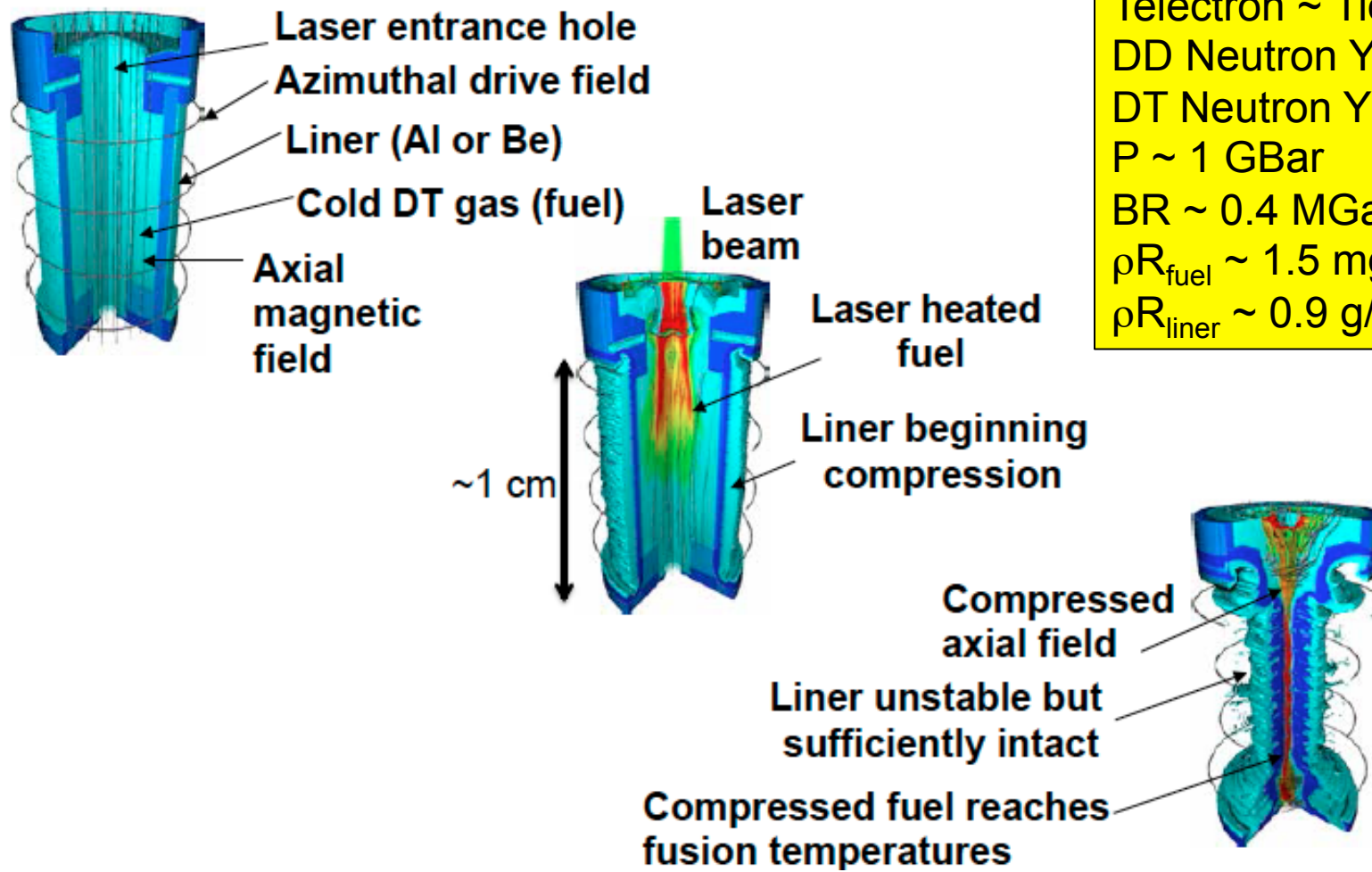


≈ 40 -100 ps confinement

- $n\tau > 2.5 \times 10^{14} \text{ cm}^{-3}\text{s}^{-1}$ at $T \sim 20$ keV for magnetic confinement (Lawson criterion)
- $\rho r > 0.3 \text{ gcm}^{-2}$ for ignition of ICF hot spot, equivalent to $n\tau > 2.5 \times 10^{14} \text{ cm}^{-3}\text{s}^{-1}$
- $\rho r > 3.0 \text{ gcm}^{-2}$ for $\sim 30\%$ fuel burn-up at 30 keV, equivalent to $n\tau > 2.5 \times 10^{15} \text{ cm}^{-3}\text{s}^{-1}$



We are evaluating Magnetized Liner Inertial Fusion (MagLIF) – a concept well suited to pulsed power and that may lower requirements for plasma self-heating

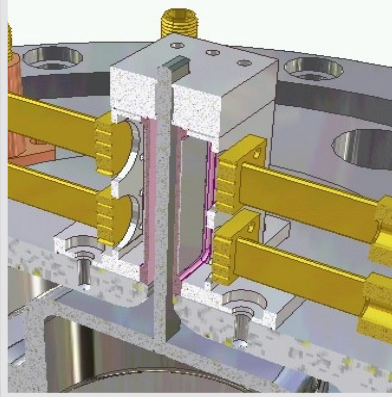


Telectron \sim Tion \sim 3 keV
DD Neutron Yield $\sim 2.e12$
DT Neutron Yield $\sim 5.e10$
 $P \sim 1$ GBar
 $BR \sim 0.4$ MGauss-cm
 $\rho R_{\text{fuel}} \sim 1.5$ mg/cm²
 $\rho R_{\text{liner}} \sim 0.9$ g/cm²

Magnetically-driven planar and cylindrical geometries are used for the dynamic materials program

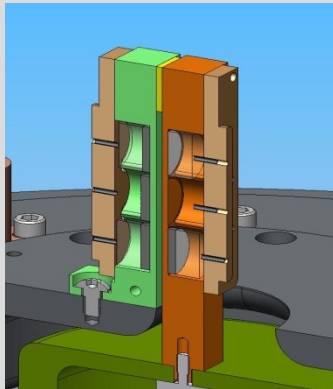
Shock Compression

Velocities > 40 km/s



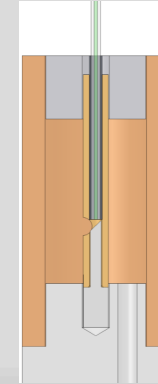
Planar Ramp-Compression

continuous compression up to ~4 Mbar



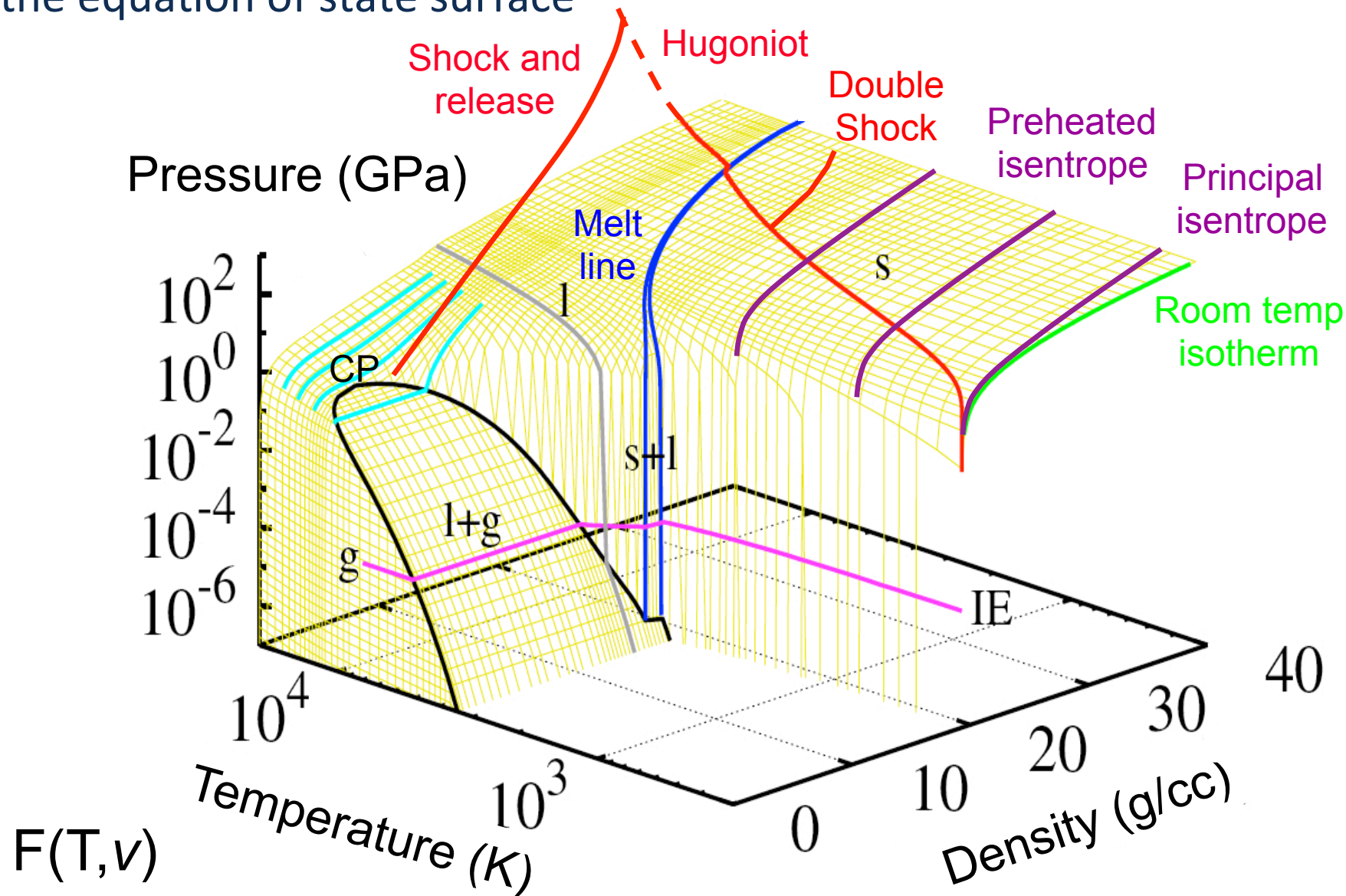
Cylindrical Ramp-Compression

Possibly continuous compression up to ~10-20 Mbar



- Shock or ramp compression of materials increases densities by 2 to **10:1**

Magnetic pressure on Z allows access to a large region of the equation of state surface



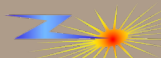
The dynamic materials program measures equations of state and other properties (strength) for elements and compounds of interest

<div>hydrogen 1 H 1.0079</div>		<div>D₂ LiD CO₂ H₂O</div>										<div>MgO CaF LiF SiO₄ (e.g. quartz)</div>										<div>boron 5 B 10.811</div>					<div>carbon 6 C 12.011</div>	<div>nitrogen 7 N 14.007</div>	<div>oxygen 8 O 15.999</div>	<div>fluorine 9 F 18.998</div>	<div>helium 2 He 4.0026</div>
<div>lithium 3 Li 6.941</div>		<div>beryllium 4 Be 9.0122</div>											<div>aluminium 13 Al 26.982</div>	<div>silicon 14 Si 28.086</div>	<div>phosphorus 15 P 30.974</div>	<div>sulfur 16 S 32.065</div>	<div>chlorine 17 Cl 35.453</div>	<div>argon 18 Ar 39.948</div>													
<div>sodium 11 Na 22.990</div>		<div>magnesium 12 Mg 24.305</div>	<div>potassium 19 K 39.098</div>	<div>calcium 20 Ca 40.078</div>	<div>scandium 21 Sc 44.956</div>	<div>titanium 22 Ti 47.867</div>	<div>vanadium 23 V 50.942</div>	<div>chromium 24 Cr 51.996</div>	<div>manganese 25 Mn 54.938</div>	<div>iron 26 Fe 55.845</div>	<div>cobalt 27 Co 58.933</div>	<div>nickel 28 Ni 58.693</div>	<div>copper 29 Cu 63.546</div>	<div>zinc 30 Zn 65.39</div>	<div>gallium 31 Ga 69.723</div>	<div>germanium 32 Ge 72.61</div>	<div>arsenic 33 As 74.922</div>	<div>selenium 34 Se 78.96</div>	<div>bromine 35 Br 79.904</div>	<div>krypton 36 Kr 83.80</div>											
<div>rubidium 37 Rb 85.468</div>		<div>strontium 38 Sr 87.62</div>	<div>caesium 55 Cs 132.91</div>	<div>barium 56 Ba 137.33</div>	<div>yttrium 39 Y 88.906</div>	<div>zirconium 40 Zr 91.224</div>	<div>niobium 41 Nb 92.906</div>	<div>molybdenum 42 Mo 95.94</div>	<div>technetium 43 Tc [98]</div>	<div>ruthenium 44 Ru 101.07</div>	<div>rhodium 45 Rh 102.91</div>	<div>palladium 46 Pd 106.42</div>	<div>silver 47 Ag 107.87</div>	<div>cadmium 48 Cd 112.41</div>	<div>indium 49 In 114.82</div>	<div>tin 50 Sn 118.71</div>	<div>antimony 51 Sb 121.76</div>	<div>tellurium 52 Te 127.60</div>	<div>iodine 53 I 126.90</div>	<div>xenon 54 Xe 131.29</div>											
<div>francium 87 Fr [223]</div>		<div>radium 88 Ra [226]</div>	<div>57-70 ★</div>	<div>57-70 ★</div>	<div>lutetium 71 Lu 174.97</div>	<div>hafnium 72 Hf 178.49</div>	<div>tantalum 73 Ta 180.95</div>	<div>tungsten 74 W 183.84</div>	<div>rhenium 75 Re 186.21</div>	<div>osmium 76 Os 190.23</div>	<div>iridium 77 Ir 192.22</div>	<div>platinum 78 Pt 195.08</div>	<div>gold 79 Au 196.97</div>	<div>mercury 80 Hg 200.59</div>	<div>thallium 81 Tl 204.38</div>	<div>lead 82 Pb 207.2</div>	<div>bismuth 83 Bi 208.98</div>	<div>polonium 84 Po [209]</div>	<div>astatine 85 At [210]</div>	<div>radon 86 Rn [222]</div>											
<div>89-102 ★ ★</div>		<div>89-102 ★ ★</div>	<div>lawrencium 103 Lr [262]</div>	<div>rutherfordium 104 Rf [261]</div>	<div>dubnium 105 Db [262]</div>	<div>seaborgium 106 Sg [266]</div>	<div>bohrium 107 Bh [264]</div>	<div>hassium 108 Hs [269]</div>	<div>meitnerium 109 Mt [268]</div>	<div>unnilium 110 Uun [271]</div>	<div>ununium 111 Uuu [272]</div>	<div>unubium 112 Uub [277]</div>	<div>ununquadium 114 Uuq [289]</div>																		

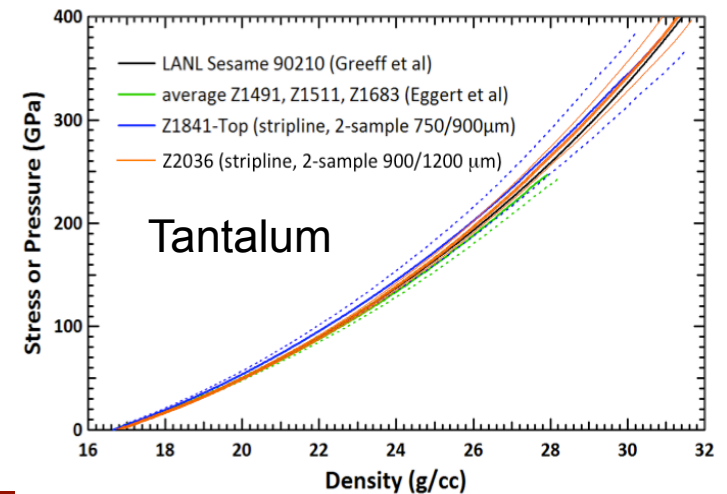
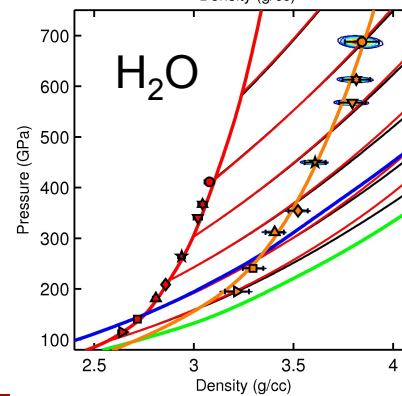
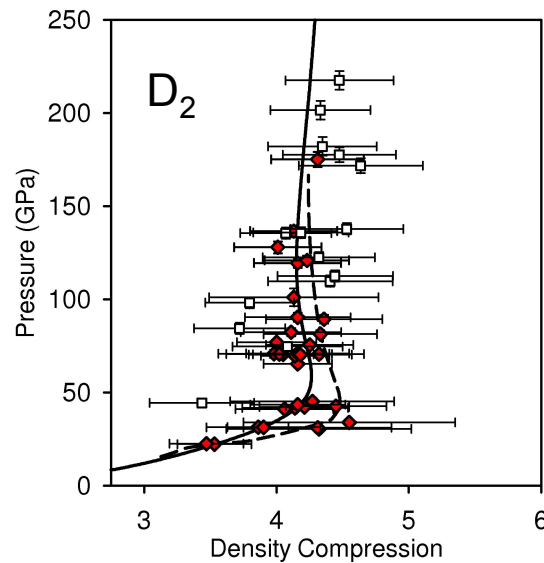
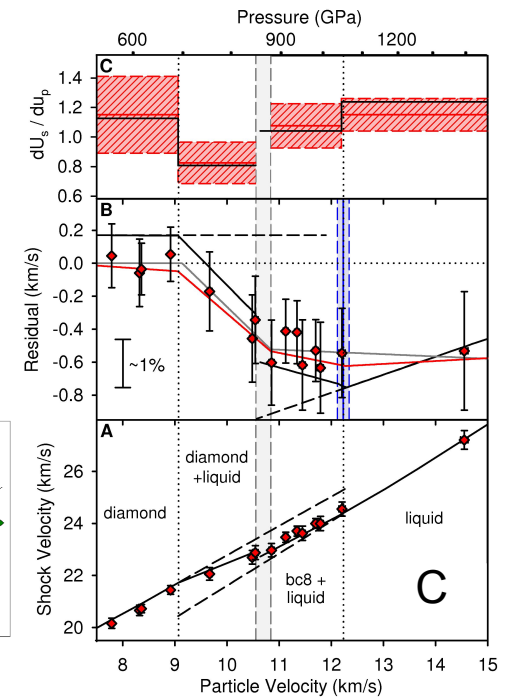
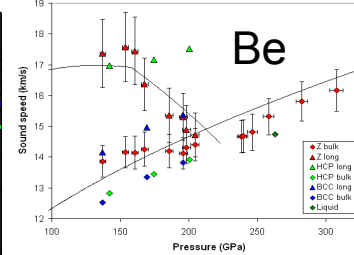
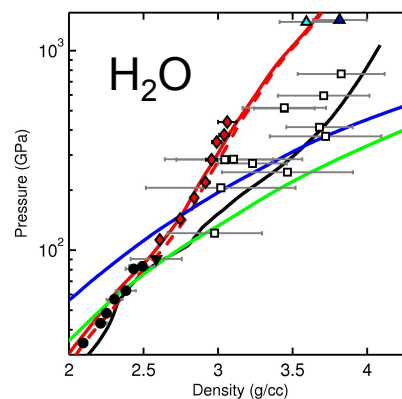
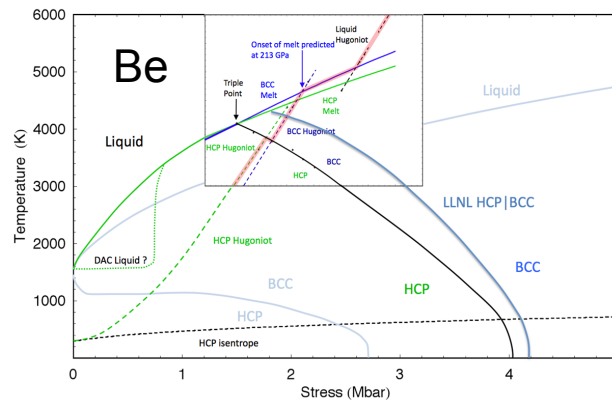
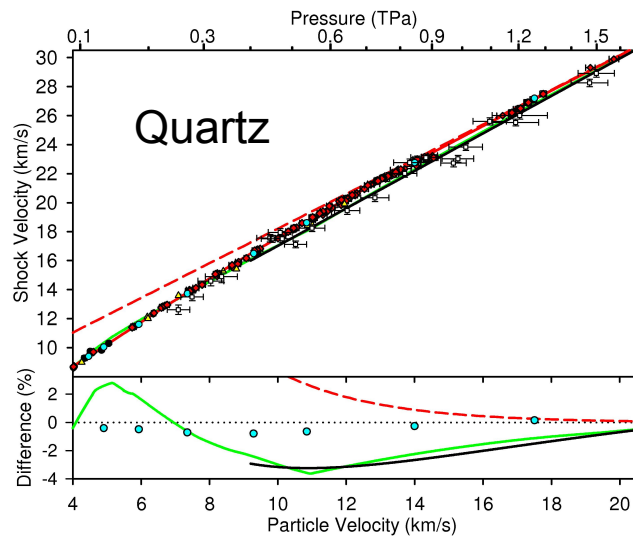
*Lanthanide series

**** Actinide series**

lanthanum 57	cerium 58	praseodymium 59	neodymium 60	promethium 61	samarium 62	europium 63	gadolinium 64	terbium 65	dysprosium 66	holmium 67	erbium 68	thulium 69	ytterbium 70
La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb
138.91	140.12	140.91	144.24	[145]	150.36	151.96	157.25	158.93	162.50	164.93	167.26	168.93	173.04
actinium 89	thorium 90	protactinium 91	uranium 92	neptunium 93	plutonium 94	americium 95	curium 96	berkelium 97	californium 98	einsteinium 99	fermium 100	mendelevium 101	nobelium 102
Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No
[227]	232.04	231.04	238.03	[237]	[244]	[243]	[247]	[247]	[251]	[252]	[257]	[258]	[259]



Z has been used to address several interesting problems in the multi-Mbar regime

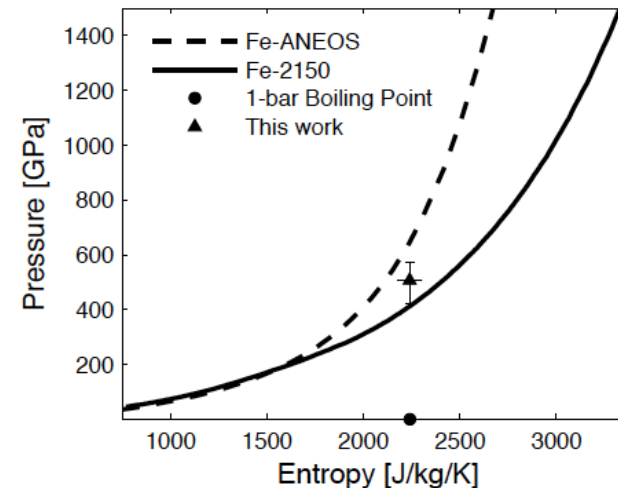
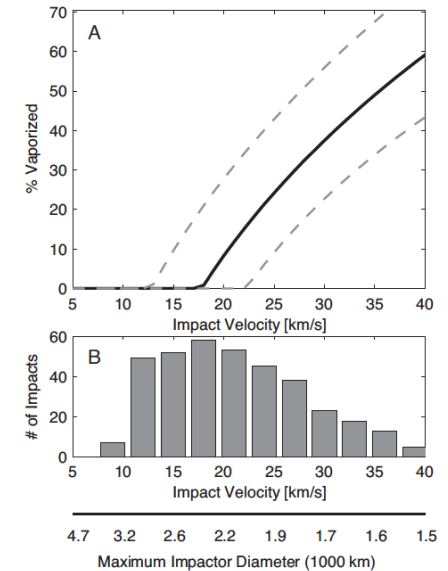
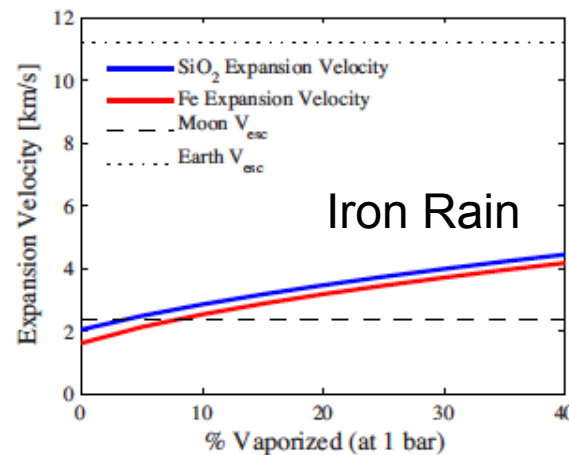


Planetesimal impacts played an important role in shaping the solar system

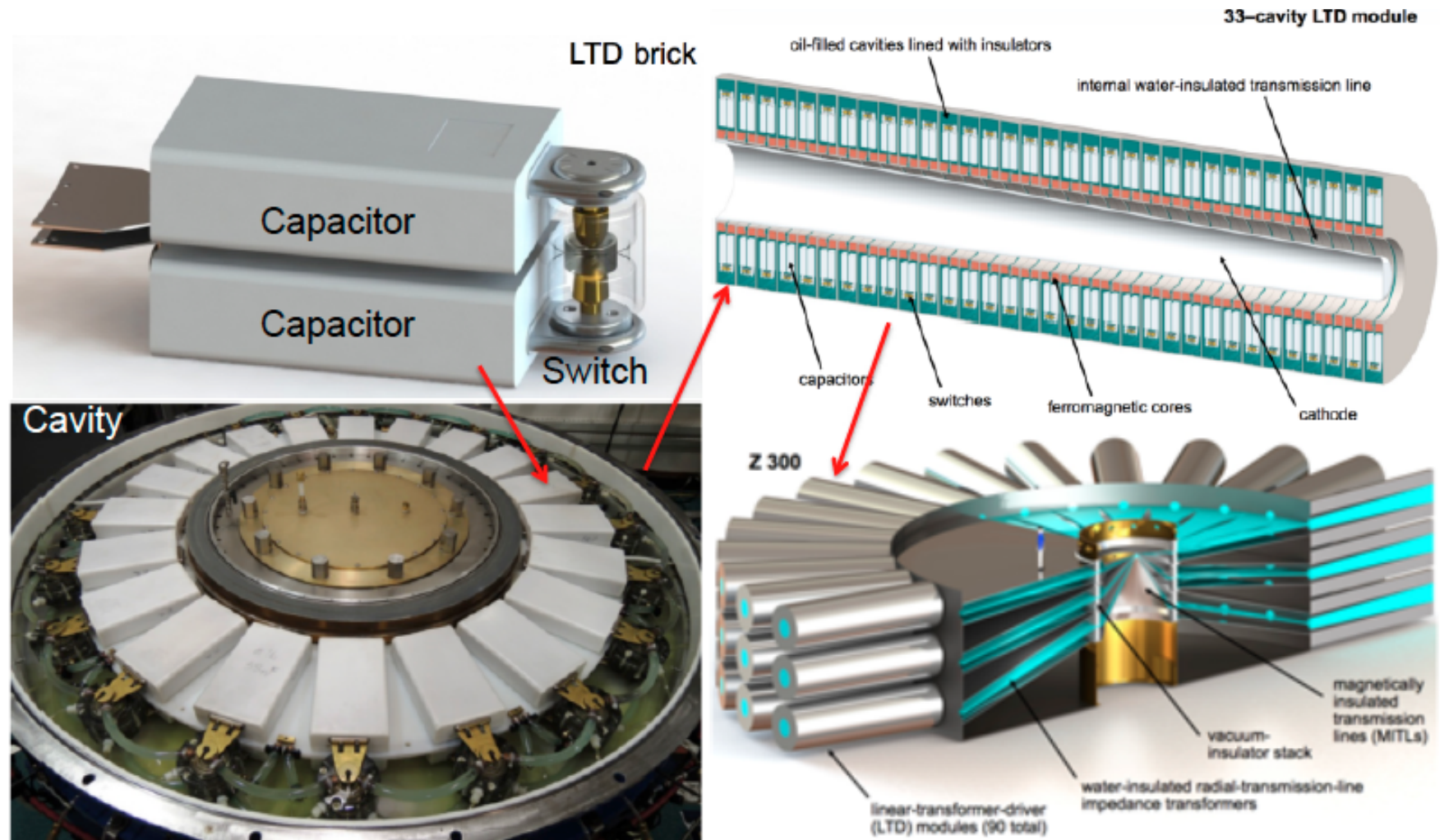


R. G. Kraus et al., Nature Geoscience
04/2015; 8(April):269-272

A significantly lower shock pressure (500 GPa) is required to vaporize iron upon release than previously used in planetary modeling (890 GPa)

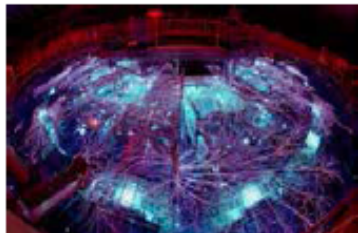


We have developed a linear transformer driver architecture that can scale to 800-1000 TW and that is twice as efficient and twice as compact as Z



We are currently exploring target designs and pulsed power architectures that may be on the path to 0.5-1 GJ yields that also meet the needs of the science campaigns

Yield = $E_{\text{fuel}}?$
(~100kJ_{DT eq})
Physics Basis for Z300



Z

- 80 TW
- 33 Meter diameter
- 26 MA
- 22 MJ Stored Energy

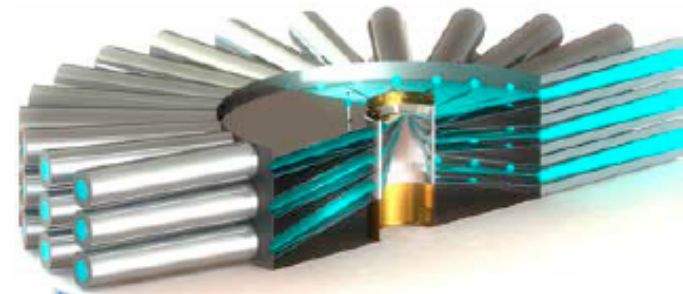
Yield = $E_{\text{target}}?$
(About 3-4 MJ)
 α -dominated plasmas



“Z300”

- 300 TW
- 35 Meter diameter
- 47 MA
- 47 MJ Stored Energy

Fusion Yield 0.5-1 GJ?
Burning plasmas

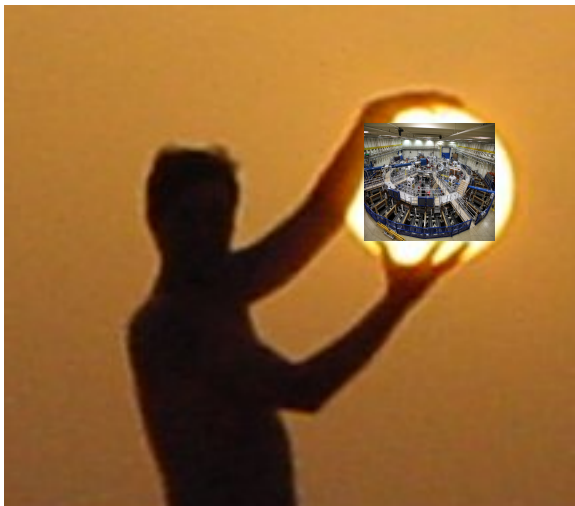


“Z800”

- 800 TW
- 52 Meter diameter
- 61 MA
- 130 MJ Stored Energy

The “bold outcome” of the
First to High Yield Fusion
Research Challenge

Large HED facilities are as close as we can get to “holding the Sun” on Earth



- We make sufficient enough macroscopic quantities of high energy density, hot-dense matter to allow accurate study
- Pulsed power, in particular, allows large energy delivery to the largest samples or targets, generating large scale size, hot, dense plasmas with large volume-to-surface area ratios, and at low cost per shot
- We may someday be able to “create a star” in the laboratory



Backup

Scales of energy density found in nature

- 0.001 MJ/kG water at 100 m dam height
- 0.7 MJ/kG Li ion battery
- 1.968 MJ/kG water
- 7.5 MJ/kG stick of dynamite
- 33 MJ/kG Low Earth Orbit
- 46 MJ/kG gasoline
- ~100 MJ/kG internal energy of the hydrogen molecule
- ~100 MJ/kG 1 MBar on unshocked plastic
- ~170,000 MJ/kG stagnation energy density typical z-pinch implosion at 20 MA
- 3.5 million MJ/kG fission of 3.5% enriched U-235
- 337 million MJ/kG DT fusion
- 645 million MJ/kG hydrogen fusion (Sun)
- 89.9 billion MJ/kG ($E=mc^2$, antimatter-matter annihilation)
- (see http://en.wikipedia.org/wiki/Energy_density)

