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A Comparison of Reduced Order Modeling Techniques Used in Dynamic Substructuring

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Outline

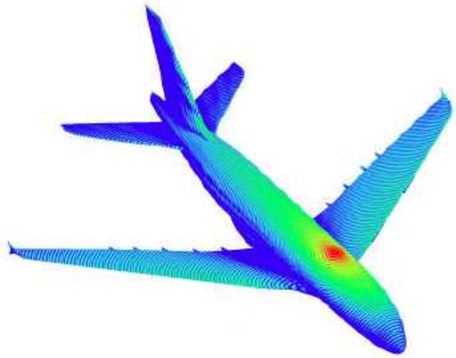
- NOMAD Institute
- Motivation
- Substructuring Theory
- Cylinder-Plate-Beam Case Study: Transmission Simulator
- Cylinder-Plate-Beam Case Study: Craig-Mayes
- Conclusions

NOMAD Institute

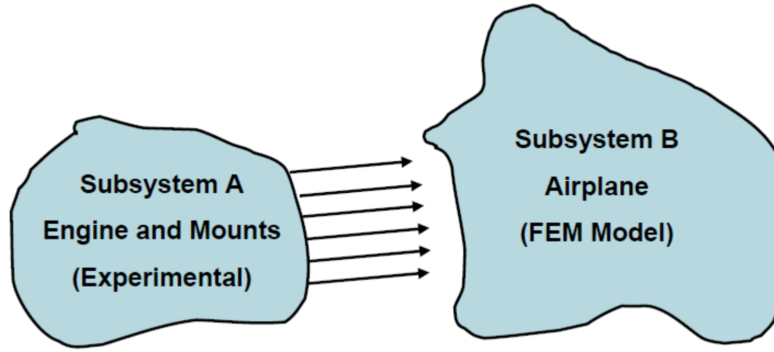
- This project is a part of the **No**nlinear **M**echanics **a**nd **D**ynamics (NOMAD) summer institute hosted at Sandia National Laboratories in the Summer of 2015
- Graduate students from 5 schools and mentors contributed over the summer
- For information on the 2016 NOMAD institute see Matt Brake

Motivation

Subsystem A
Engine and Mounts
(Experimental)

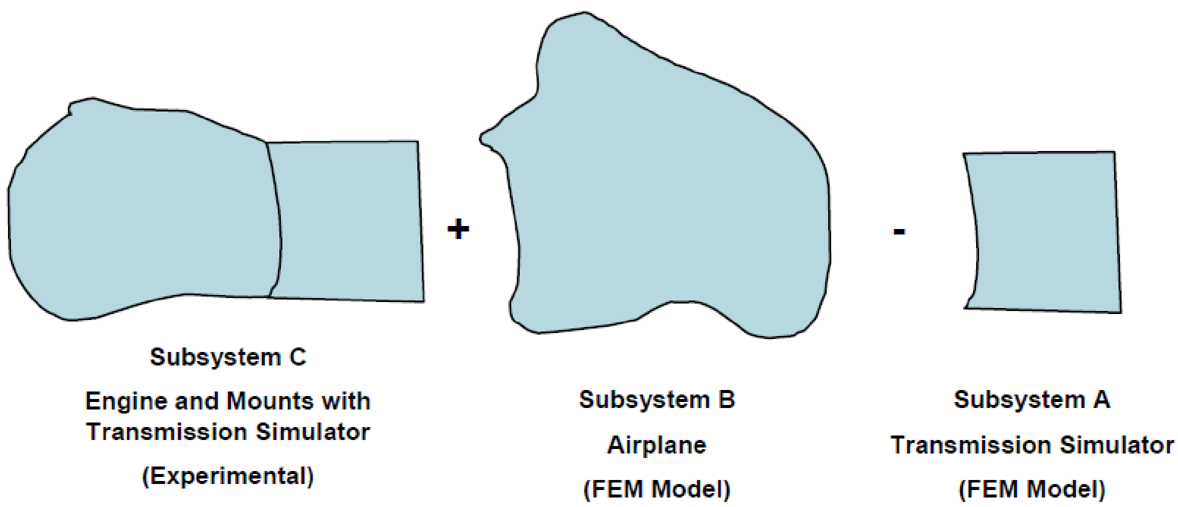


Subsystem B
Airplane
(FEM Model)



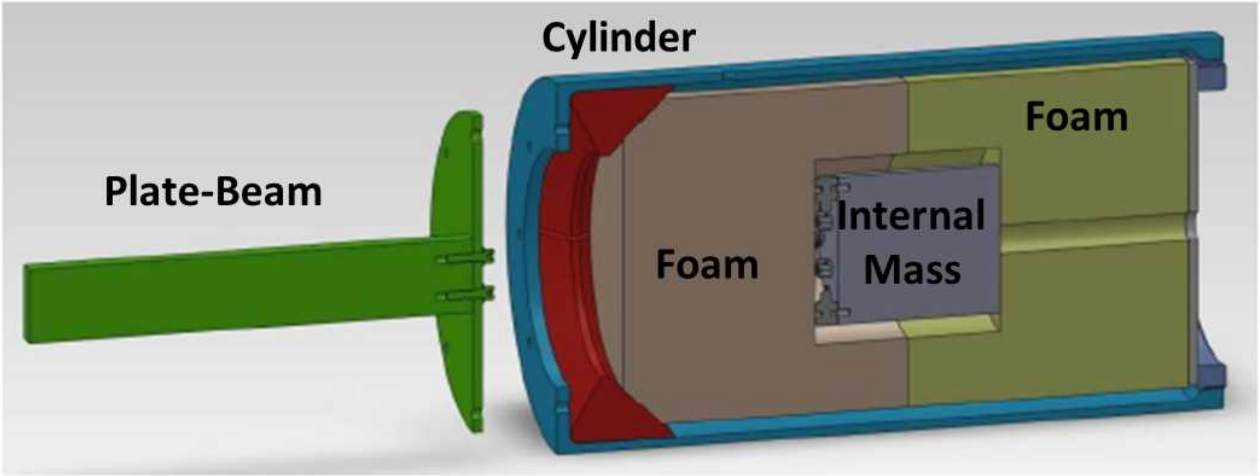
- In modern system design details about neighboring hardware is not always known. It may be designed by an outside vendor or have unknown material properties and internal geometry.
- Experimental-analytical substructuring allows one to connect the measurements from a test on this hardware to a model of the known substructure

Transmission Simulator Method



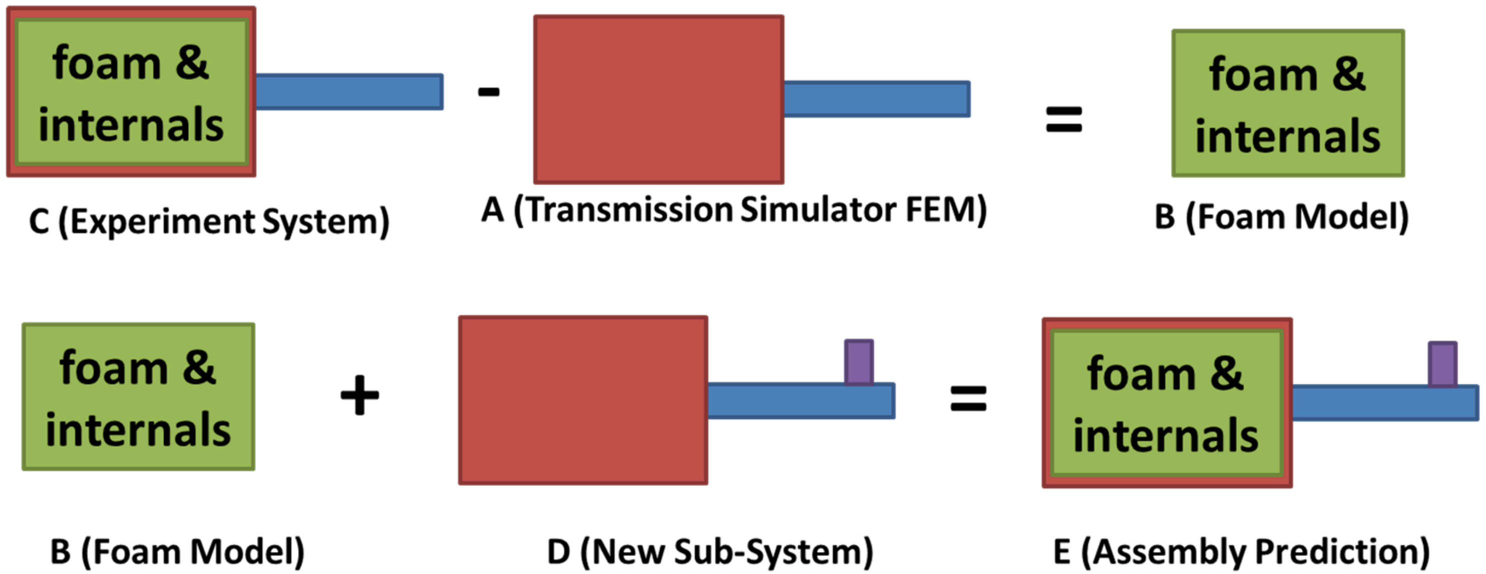
- The free-free modes of a structure often provide an inadequate modal basis for substructuring.
- The Transmission Simulator method helps by mass loading the interface DOF during the experiment. This provides loading at the interface that is similar to the assembled structure.

Cylinder-Plate-Beam (CPB) Case Study



- System consists of a cylinder with an internal mass packed in foam. A plate and beam are attached to one end of the cylinder
- Nonlinearity sources include the joint, foam compression, and internal contact loads between the internal mass and foam
- An additional case study was completed on a more academic beam system but is not covered today

Substructuring Overview



- To complete the Transmission Simulator method measurements were taken of the CPB system with internals.
- A finite element model of the Cylinder, Plate and Beam was subtracted acting as the Transmission Simulator, creating an experimental model of the foam and internals.
- To validate the model a new CPB system with an additional mass was added to the system which was compared to truth test data.

Substructuring Methodology

Equations of Motion:

$$\begin{bmatrix} I_C & 0 & 0 \\ 0 & I_D & 0 \\ 0 & 0 & -I_A \end{bmatrix} \begin{Bmatrix} \ddot{q}_C \\ \ddot{q}_D \\ \ddot{q}_A \end{Bmatrix} + \begin{bmatrix} 2\zeta_c\omega_c & 0 & 0 \\ 0 & 2\zeta_D\omega_D & 0 \\ 0 & 0 & -2\zeta_A\omega_A \end{bmatrix} \begin{Bmatrix} \dot{q}_C \\ \dot{q}_D \\ \dot{q}_A \end{Bmatrix} + \begin{bmatrix} I_C & 0 & 0 \\ 0 & I_D & 0 \\ 0 & 0 & -I_A \end{bmatrix} \begin{Bmatrix} q_C \\ q_D \\ q_A \end{Bmatrix} = \begin{Bmatrix} \phi_C^T F_C \\ \phi_D^T F_D \\ \phi_A^T F_A \end{Bmatrix}$$

Physical Constraints → Modal Coordinate Constraints

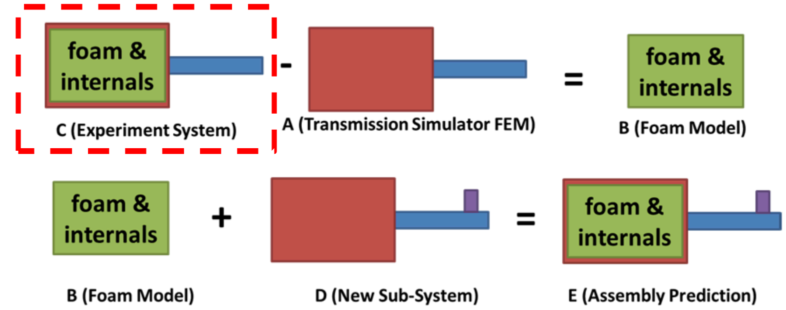
$$\begin{bmatrix} I & 0 & -I \\ 0 & I & -I \end{bmatrix} \begin{Bmatrix} q_C \\ q_D \\ q_A \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \rightarrow \quad \begin{bmatrix} \phi_C & 0 & -\phi_A \\ 0 & \phi_D & -\phi_A \end{bmatrix} \begin{Bmatrix} q_C \\ q_D \\ q_A \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

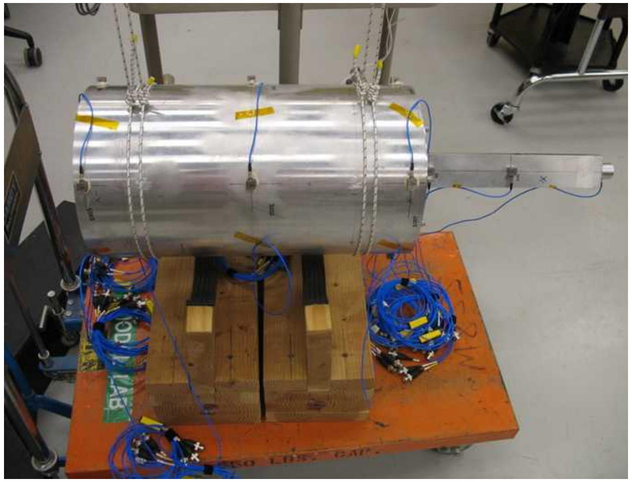
Premultiply by Psuedo-Inverse to Soften Constraints

$$\begin{bmatrix} \phi_A^+ & 0 \\ 0 & \phi_A^+ \end{bmatrix} \begin{bmatrix} \phi_C & 0 & -\phi_A \\ 0 & \phi_D & -\phi_A \end{bmatrix} \begin{Bmatrix} q_C \\ q_D \\ q_A \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Then follow standard component-mode-synthesis procedure using a transformation matrix to synthesize the system.

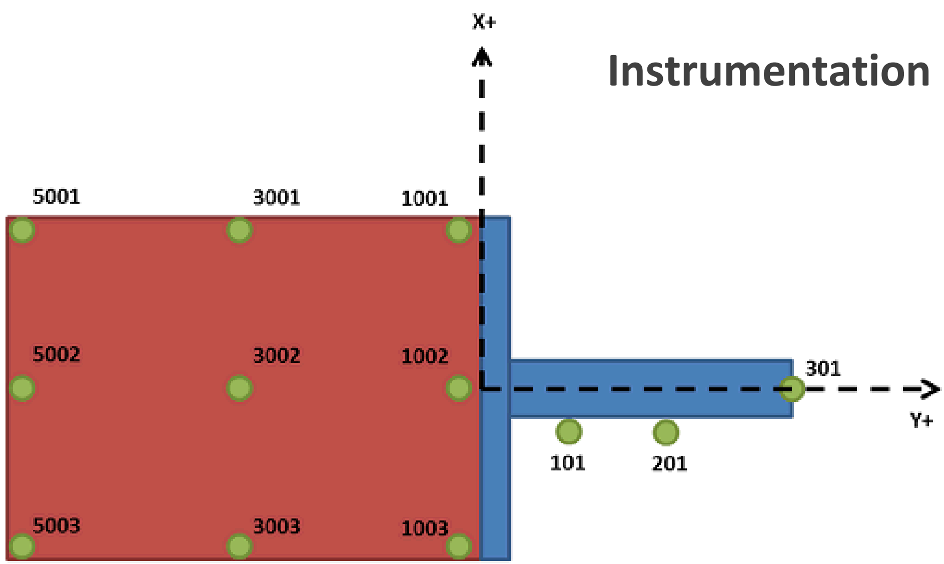
Experimental Set-up

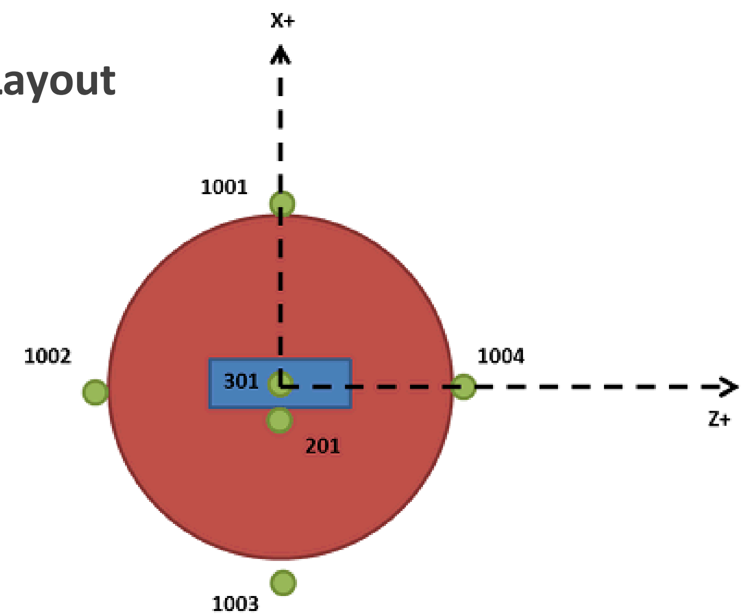




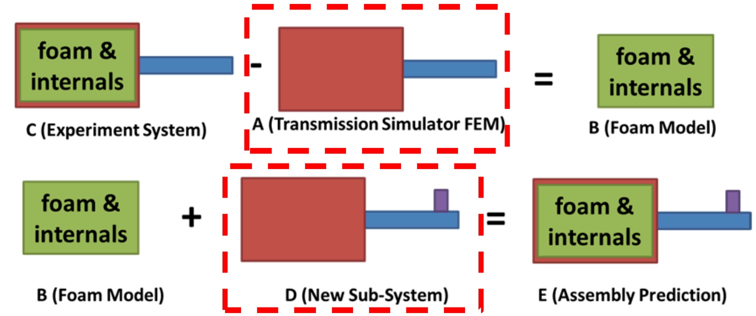
- Modal test completed on subsystem (C)
- Modes considered up to 1000 Hertz including 14 elastic modes
- 6 rigid body modes were analytically computed

Instrumentation Layout

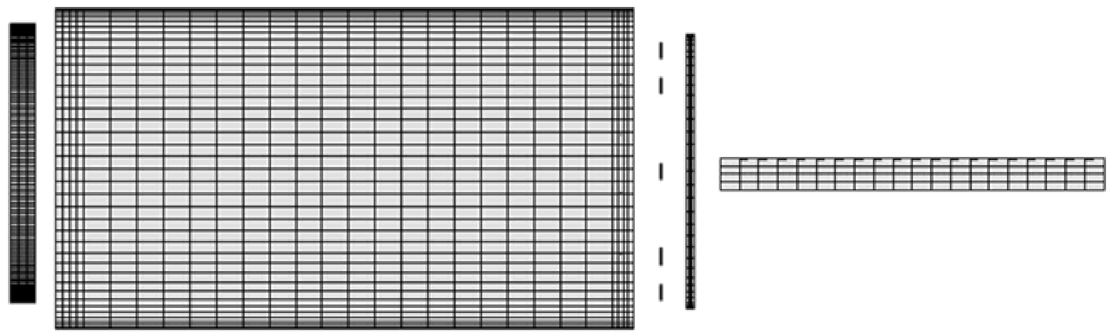


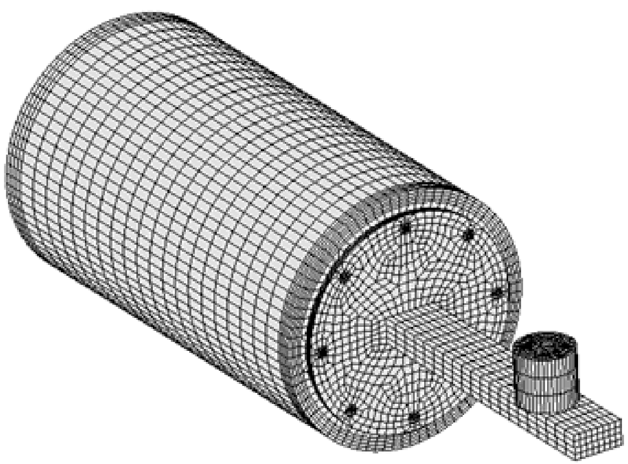


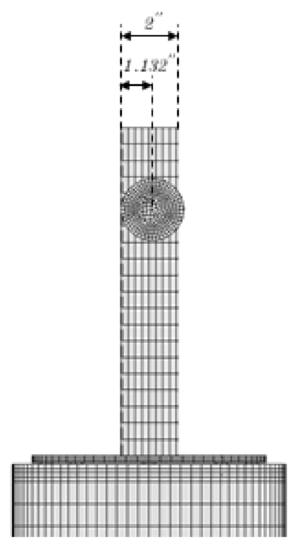
Finite Element Model



- A finite element model was constructed for the transmission simulator
- This model was updated to a previous test of the Cylinder-Plate-Beam without any internals

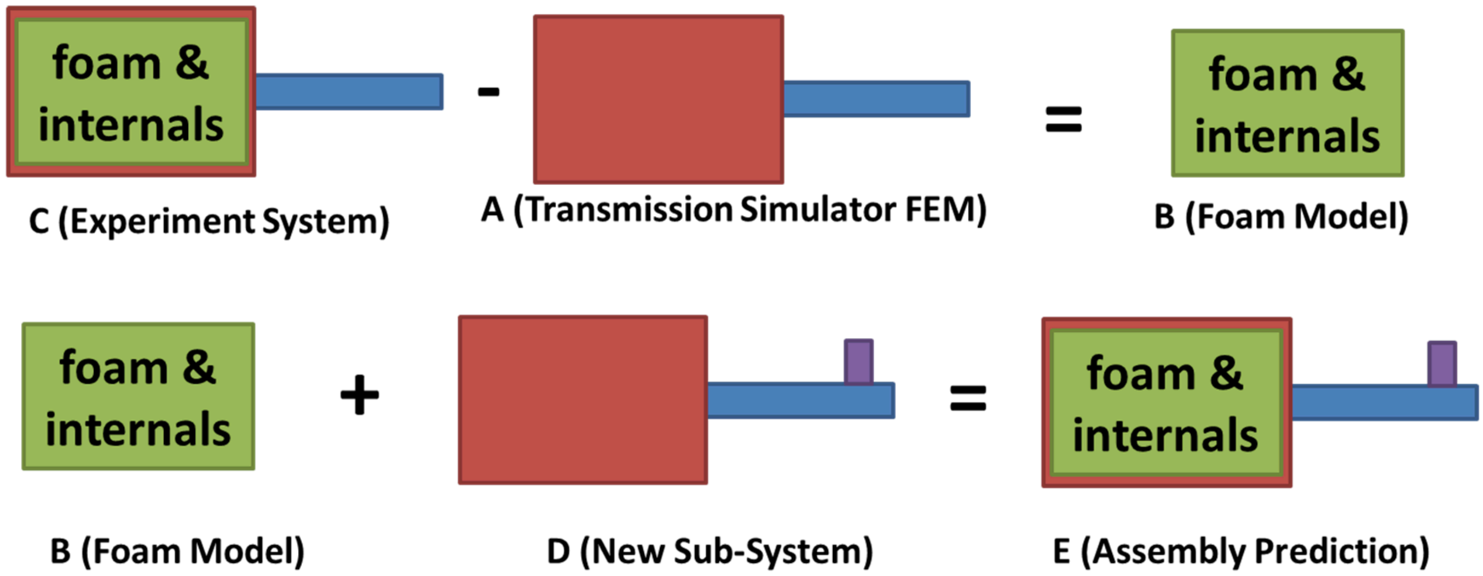






- This modeled was modified to create the new Cylinder-Plate-Beam system with the additional beam mass loading
- The placement and mass of this loading was matched to the experimental truth test set-up

Substructuring Overview



- 20 modes of Subsystem C were used (6 rigid and 14 elastic).
- The Transmission Simulator was modelled with 10 modes (6 rigid and 4 elastic)
- In Subsystem D 13 modes were retained (6 rigid and 7 elastic)

Substructuring Predictions

Mode	Truth Frequency [Hz]	Substr. Frequency [Hz]	Frequency Error [%]	Truth Damping Ratio	Substr. Damping Ratio	Damping Ratio Error [%]	MAC
7	88.33	86.59	-1.96	0.00196	0.00215	9.38	0.9803
8	115.8	115.06	-0.64	0.00163	0.00207	26.83	0.9929
9	275.97	276.11	0.05	0.02468	0.02466	-0.1	0.9006
10	283.32	283.24	-0.03	0.02151	0.02168	0.8	0.9995
11	301.4	301.77	0.12	0.02327	0.0229	-1.61	0.9957
12	346.25	349.76	1.01	0.00291	0.00359	23.47	0.9867
13	584.71	583.2	-0.26	0.02119	0.02135	-0.77	0.9963
14	635.16	634.89	-0.04	0.02037	0.01897	-6.87	0.9948
-	NA	670.72	NA	NA	0.00504	NA	NA
15	688.92	690.36	0.21	0.01515	0.01363	-10.03	0.932
-	NA	717.45	NA	NA	0.00537	NA	NA
16	758.36	NA	NA	0.01131	NA	NA	NA
17	769.71	770.99	0.17	0.01191	0.01201	0.84	0.8827

Modal Assurance Criteria

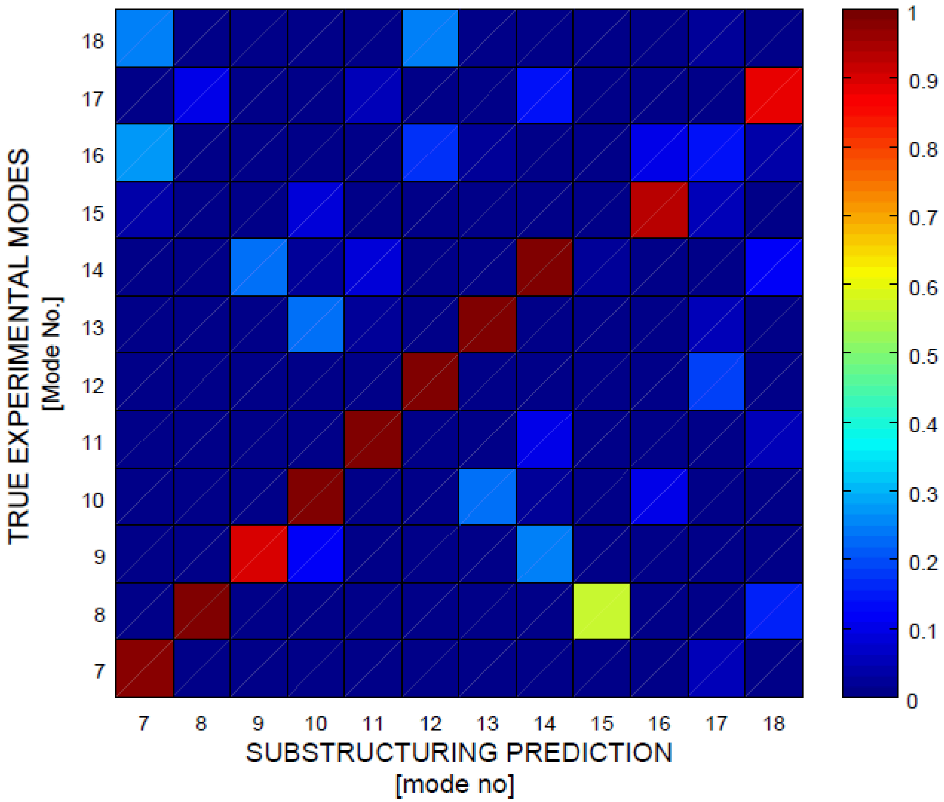


Figure 11: MAC traditional TS method vs. Truth

Mode	Truth Frequency [Hz]	Substr. Frequency [Hz]	MAC
7	88.33	86.59	0.9803
8	115.8	115.06	0.9929
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16	758.36	NA	NA
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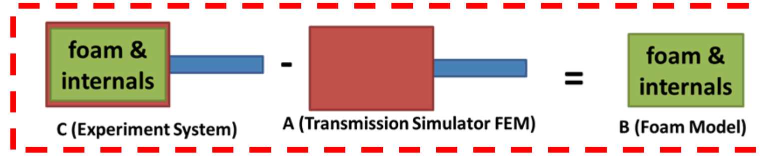
Craig-Mayes Formulation



- Another way to come up with the experimental model for the internals and foam (Subsystem B) is to use the Craig-Mayes formulation [1]
- This method brings the modal coordinates extracted from a measurement into a Craig-Bampton like form:

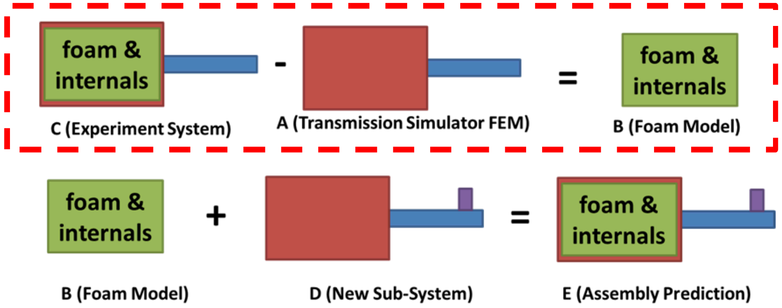
$$q = \boldsymbol{T} \begin{bmatrix} p \\ s \end{bmatrix}$$

where p are the fixed-interface modal degrees of freedom, which are found by fixing the TS degrees of freedom, and s are coordinates that describe the TS motion, which are found by setting the TS motion equal to the motion of the same degrees of freedom on the experimental system.

$$\left[\begin{bmatrix} K_{pp} & K_{ps} \\ K_{sp} & K_{ss} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{pp} & M_{ps} \\ M_{sp} & M_{ss} \end{bmatrix} \right] \begin{Bmatrix} p \\ s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

[1] R. Mayes, "A craig-bampton experimental dynamics substructure using the transmission simulator method," in *Proceedings of the 33rd International Modal Analysis Conference*, January 2015.

Craig-Mayes Formulation



- Once a transformation to these coordinates is completed the mass and stiffness of the transmission simulator can be subtracted from the lower right partition of the matrices.

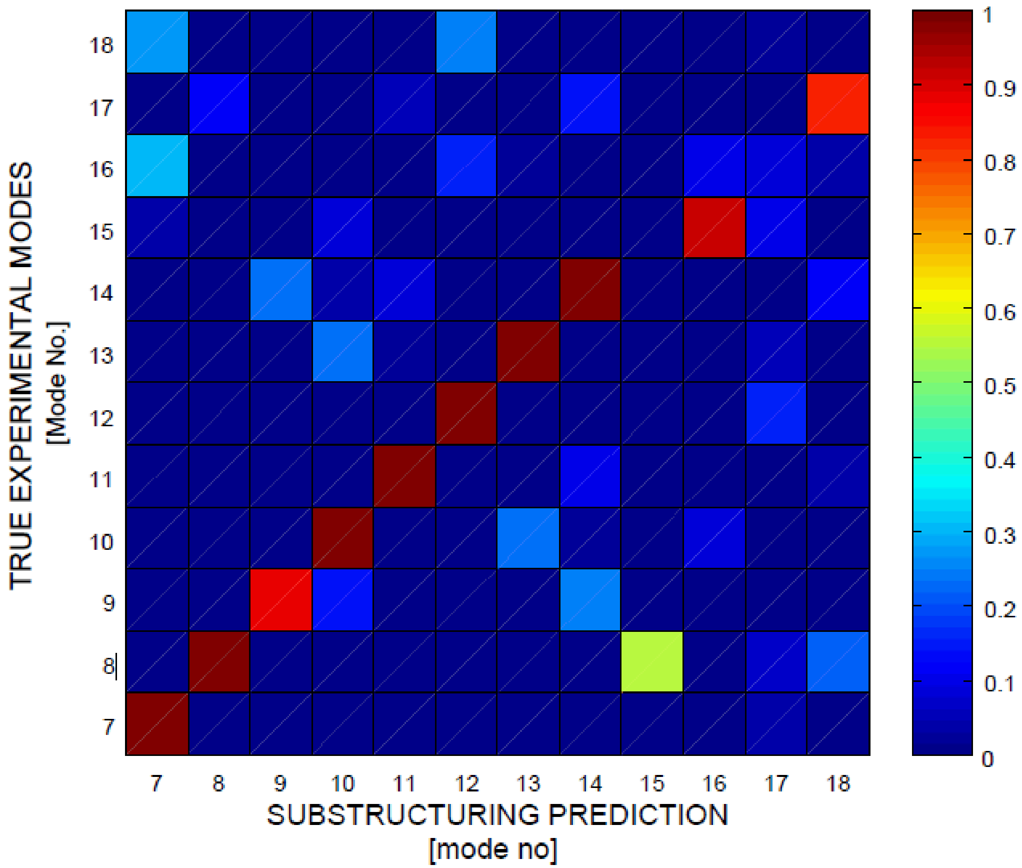
$$\left[\begin{bmatrix} K_{pp} & K_{ps} \\ K_{sp} & K_{ss} - \omega_{TS}^2 \end{bmatrix} - \omega^2 \begin{bmatrix} M_{pp} & M_{ps} \\ M_{sp} & M_{ss} - I \end{bmatrix}\right] \begin{Bmatrix} p \\ s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

- This form can be easily coupled the a Craig-Bampton formulation from a finite element program. This new formulation for the internals an foam model (Subsystem B) was coupled to a Craig-Bampton formulation for the mass-loaded CPB system.

Substructuring Predictions (Craig-Mayes)

Mode	Truth Frequency [Hz]	Substr. Frequency [Hz]	Frequency Error [%]	Truth Damping Ratio	Substr. Damping Ratio	Damping Ratio Error [%]	MAC
7	88.33	89.58	1.42	0.00196	0.00208	5.84	0.9861
8	115.8	115.25	-0.47	0.00163	0.00204	25.14	0.9975
9	275.97	275.83	-0.05	0.02468	0.02459	-0.36	0.8886
10	283.32	282.61	-0.25	0.02151	0.02156	0.23	0.9986
11	301.4	301.48	0.03	0.02327	0.02284	-1.84	0.9966
12	346.25	350.61	1.26	0.00291	0.00343	17.96	0.9897
13	584.71	583.64	-0.18	0.02119	0.02141	1.05	0.9971
14	635.16	634.85	-0.05	0.02037	0.019	-6.69	0.9961
-	NA	679.99	NA	NA	0.00491	NA	NA
15	688.92	691.85	0.43	0.01515	0.01351	10.83	0.9075
-	NA	707.57	NA	NA	0.00574	NA	NA
16	758.36	NA	NA	0.01131	NA	NA	NA
17	769.71	760.69	-1.17	0.01191	0.01125	-5.54	0.8308

Modal Assurance Criteria



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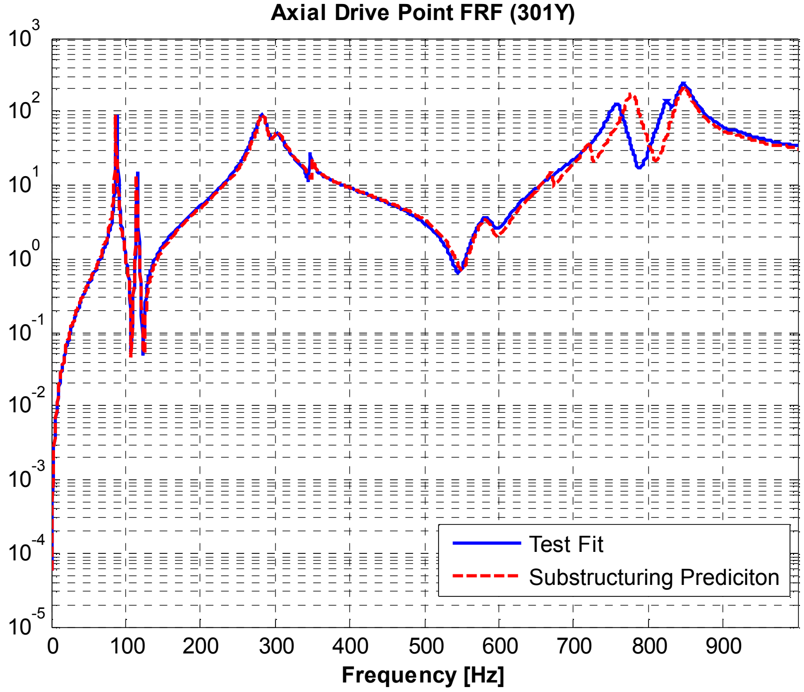
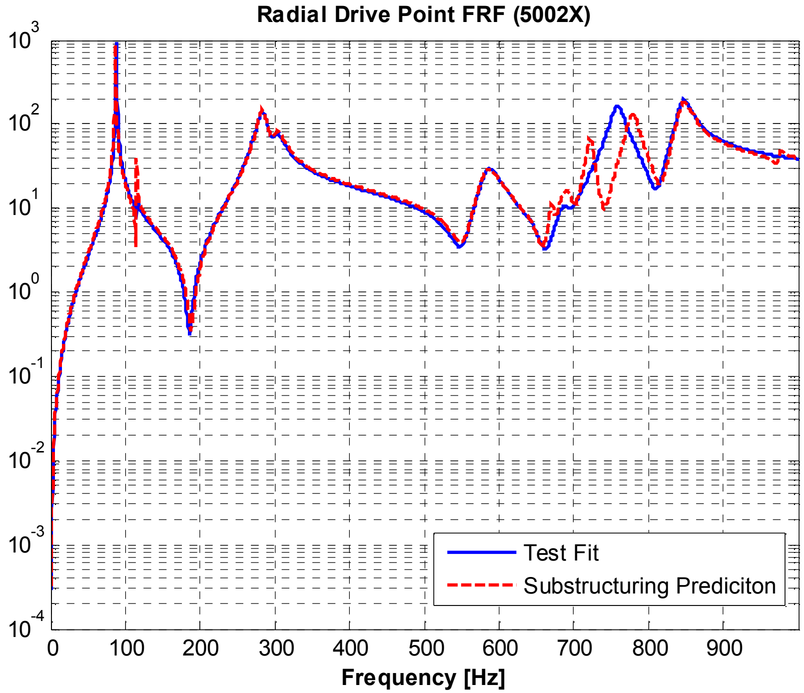
8	115.8	115.25	0.9975
9	275.97	275.83	0.8886
10	283.32	282.61	0.9986
11	301.4	301.48	0.9966
12	346.25	350.61	0.9897
13	584.71	583.64	0.9971
14	635.16	634.85	0.9961
-	NA	679.99	NA
15	688.92	691.85	0.9075
-	NA	707.57	NA
16	758.36	NA	NA
17	769.71	760.69	0.8308

Remarks

- The traditional Transmission Simulator method had frequency errors of up to 1.96% and damping ratio errors of up to 26.83%
- Similarly, the Craig-Mayes method had frequency errors of up to 1.42% and damping ratio errors of up to 25.14%
- Both methods also had very similar MAC values and had strong correlation with each other





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