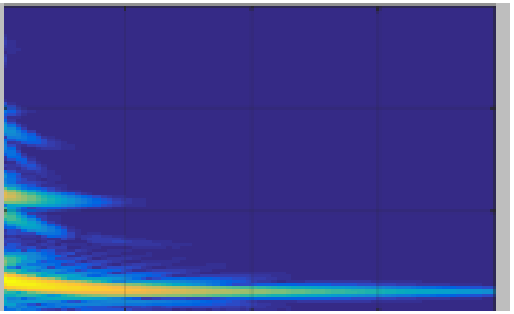
SAND2016-0140C

Instantaneous Frequency and Damping from Transient Ring-Down Data



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Sandia National Laboratories

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Exceptional service in the national interest

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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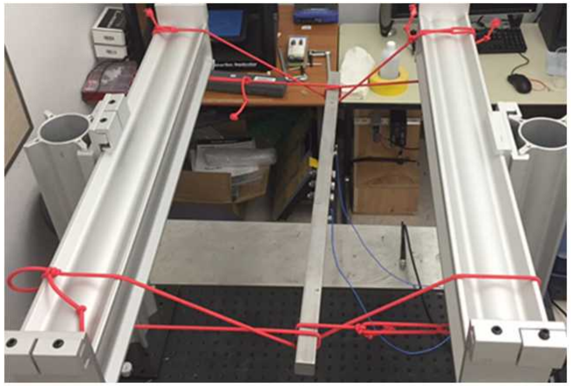
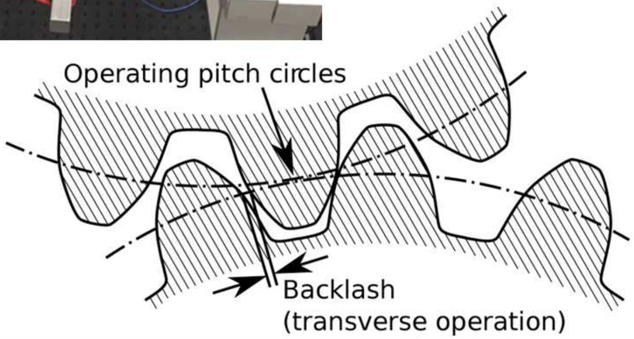
What is our goal?

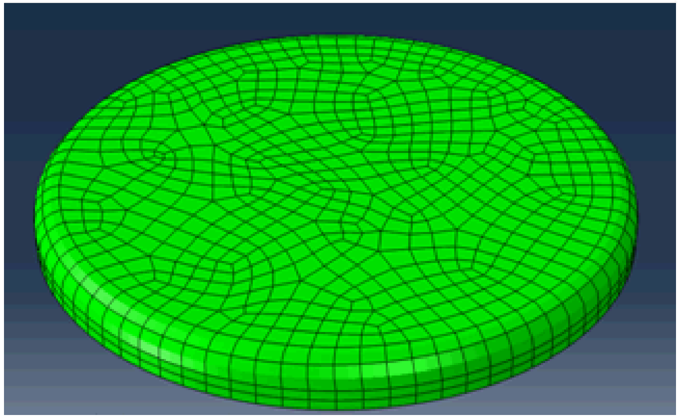
- Characterize *instantaneous* frequency and damping ratios of nonlinear mechanical structures.
- Use broadband excitation to obtain the measured response data.
- Seek nonlinear system metrics that are invariant under load type, amplitude, input location, etc…

Time-Frequency Analysis of Nonlinear Vibrations of Mechanical Structures

- Traditional frequency domain analysis based on discrete Fourier transforms only applicable to linear systems.
- Nonlinear vibration signals require new tools to characterize frequency content of the non-stationary signals (i.e. frequencies changing over time).
- Various signal processing tools exist to perform time-frequency analysis [1].







[1] Neild et al. "A review of time-frequency methods for structural vibration analysis", Engineering Structures 25 (2003) 713-728.

Review of Time-Frequency Analysis Tools

- **Hilbert Transform**:
  - Compute the phase & decay envelope from analytic signal of "monocomponent", transient ring-down data.
  - Requires band-pass filter or empirical mode decomposition.

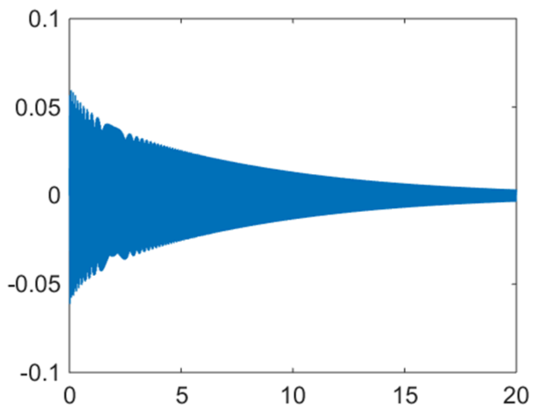
- **Zero-Crossing Detection**:
  - Find time at zero-crossing of resonant decay response following appropriated mode at steady state.

- **Wavelet Transform**:
  - Applied to MDOF responses and extracts frequency/damping from ridges of transformed signals

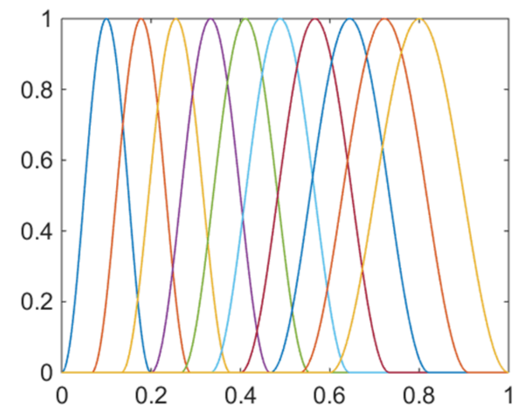
- **Others include**: Wigner-Ville, Evolutionary Spectrum, Auto-regressive Model, etc..

Extracting Instantaneous Frequency and Damping Using (modified) Short-Time Fourier Transform

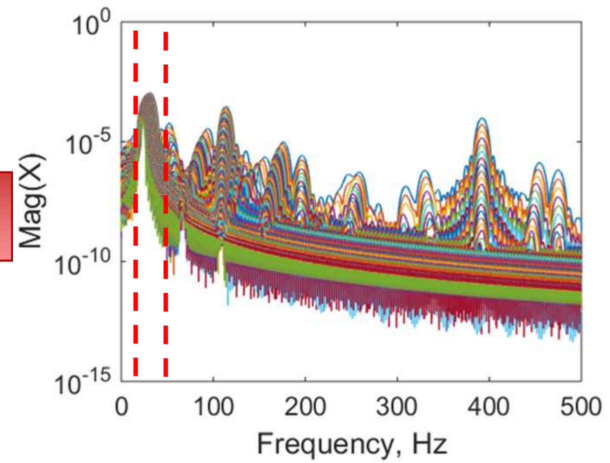
Transient Ring-Down



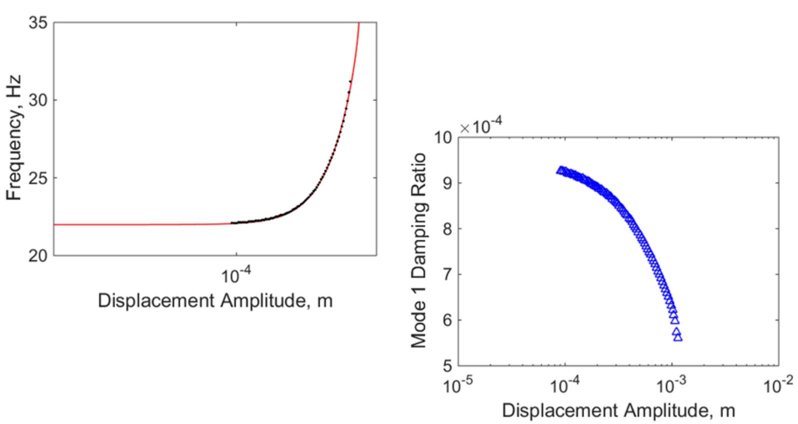
Expanding/Contracting Short-Time Windows



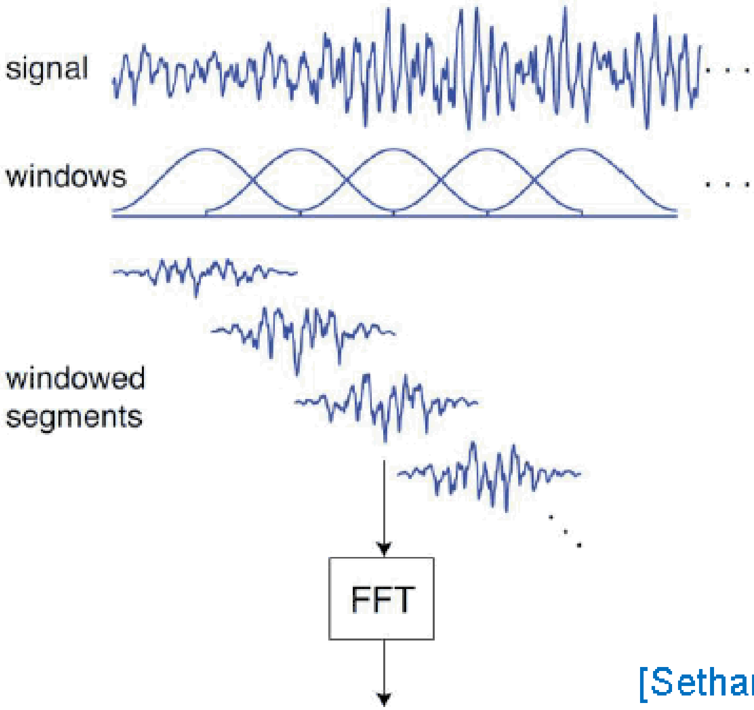
Spectra of Windowed Fourier Transforms



Amplitude Dependent Estimates of Frequency and Damping



Short-time Fourier Transform (STFT)



[Sethares, 2007]

- Take Fourier transform of windowed section of signal as

$$X(\omega,\tau) = \int_{-\infty}^{\infty} x(t) w(t-\tau) e^{-i\omega t} dt$$

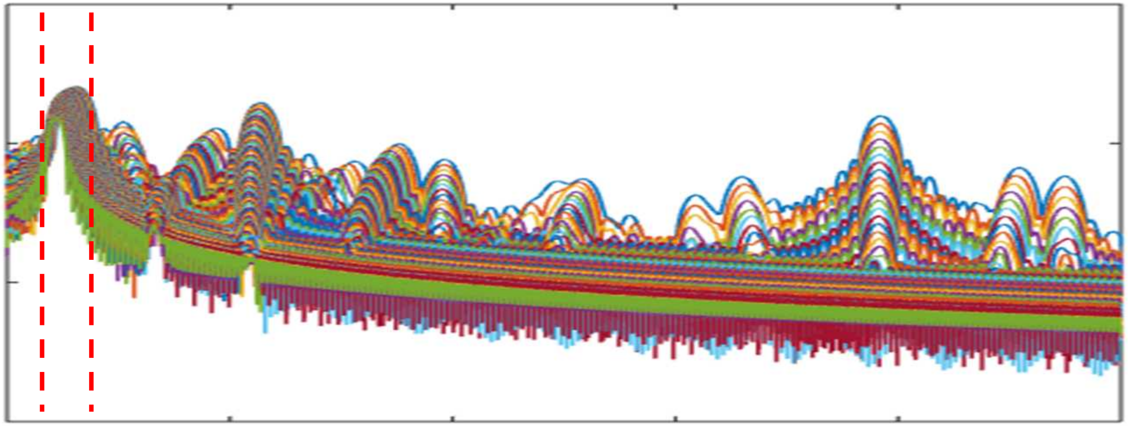
- STFT modified to allow window to expand or contract such that

$$w(t) = w\left(\frac{t-\tau}{\delta(t)}\right)$$

τ - shifting parameter

$\delta(t)$ - dilation parameter

Estimating Decaying Harmonic Functions



- Damped, free response represented as a summation of decaying harmonic functions:

$$x(t) = \sum_{r=1}^{p} \mathrm{Re}\left(A_{r,0} e^{-\zeta_r(t)\omega_r(t)t} e^{i\omega_{r,D}(t)t} \right)$$

Magnitude of Fourier coefficient

$$\omega_{r,D}(t) = \omega_r(t)\sqrt{1-\zeta_r(t)}$$

Frequency at peak magnitude of Fourier coefficient

Comparison between Undamped Nonlinear Normal Modes and Instantaneous Frequencies

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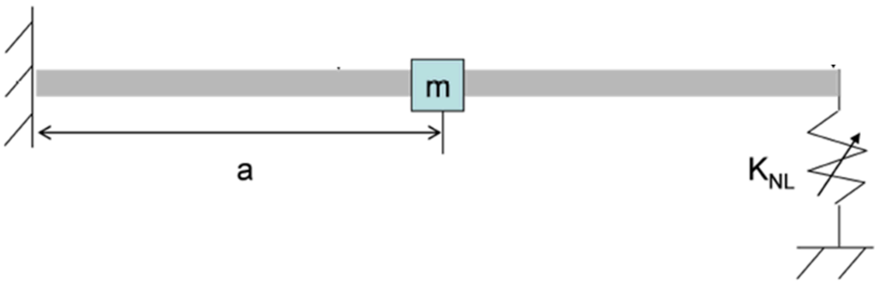
- Nonlinear Normal Modes (NNMs) are invariant properties of the undamped, unforced nonlinear mechanical system.
  - "Not-necessarily synchronous periodic response of the undamped equations of motion".

$$x_{NNM}(t, E) = \sum_{k=1}^{m} \mathrm{Re}\left(X_{k,NNM}(E) e^{ik\omega_{NNM}(E)t}\right)$$

- In the presence of light damping, the undamped NNM closely approximates the damped invariant manifold.

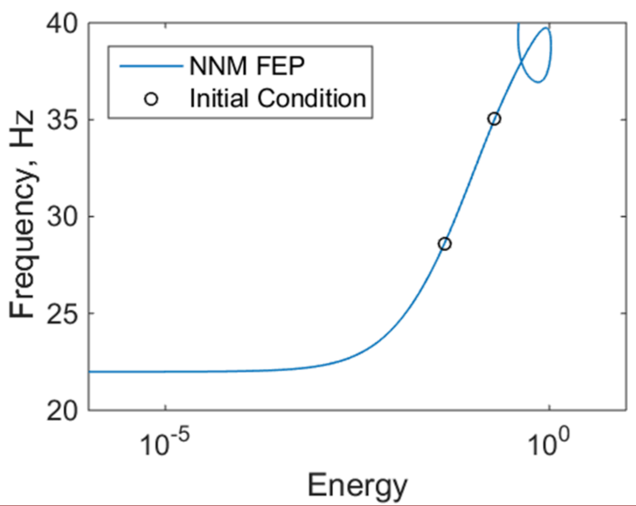
- How well does the freely decaying response follow the undamped nonlinear normal modes?

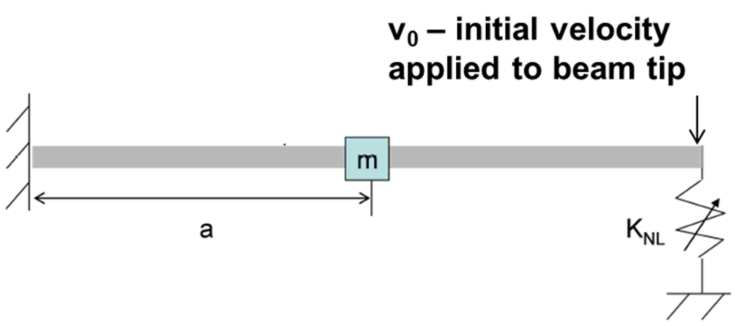
Application to Nonlinear Beam Model



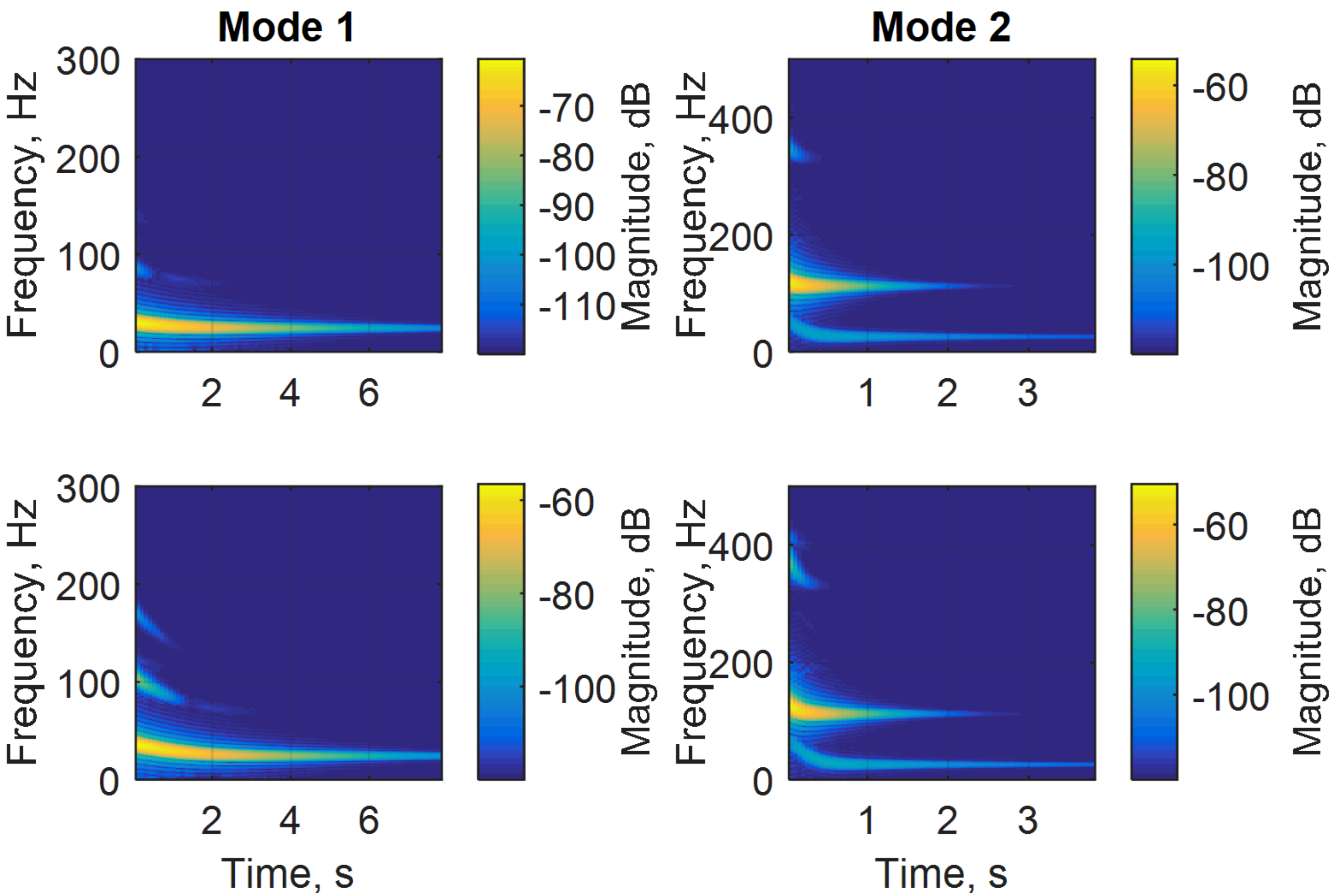
- Cubic spring attachment
- 20 Euler-Bernoulli beam elements
- 0.5 % modal damping

Case 1: Free Decay from Initial Conditions of NNMs 1 and 2	Case 2: Broadband Excitation via Initial Velocity at Beam Tip

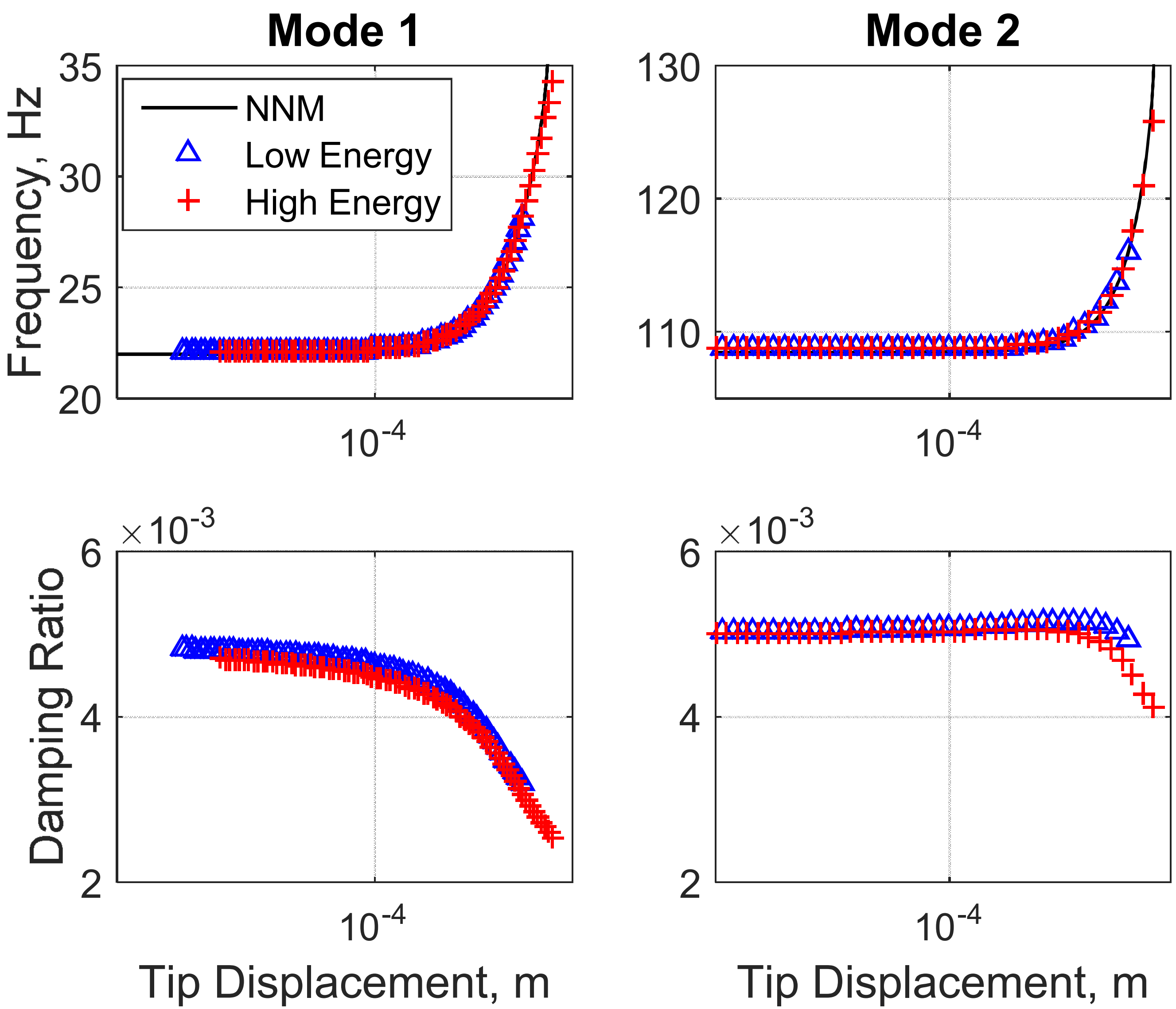




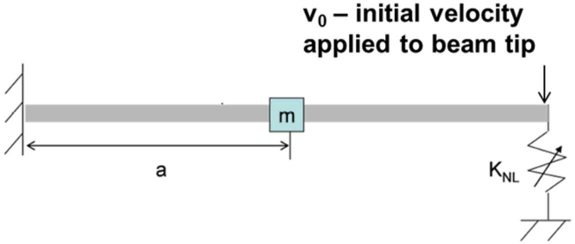
Case 1: Free Decay along NNM Backbone

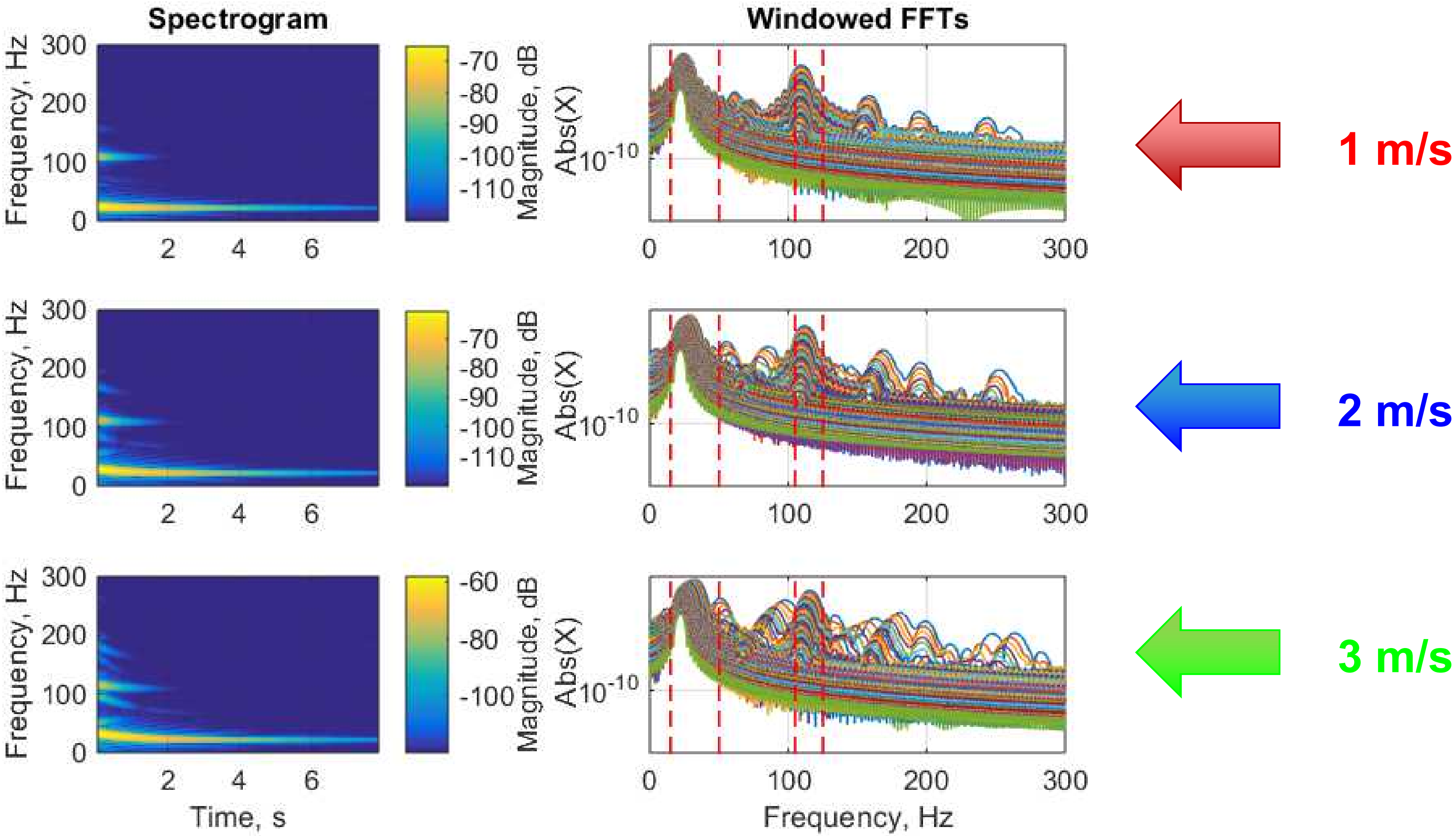


Case 1: Free Decay along NNM Backbone



Case 2: Broadband Excitation





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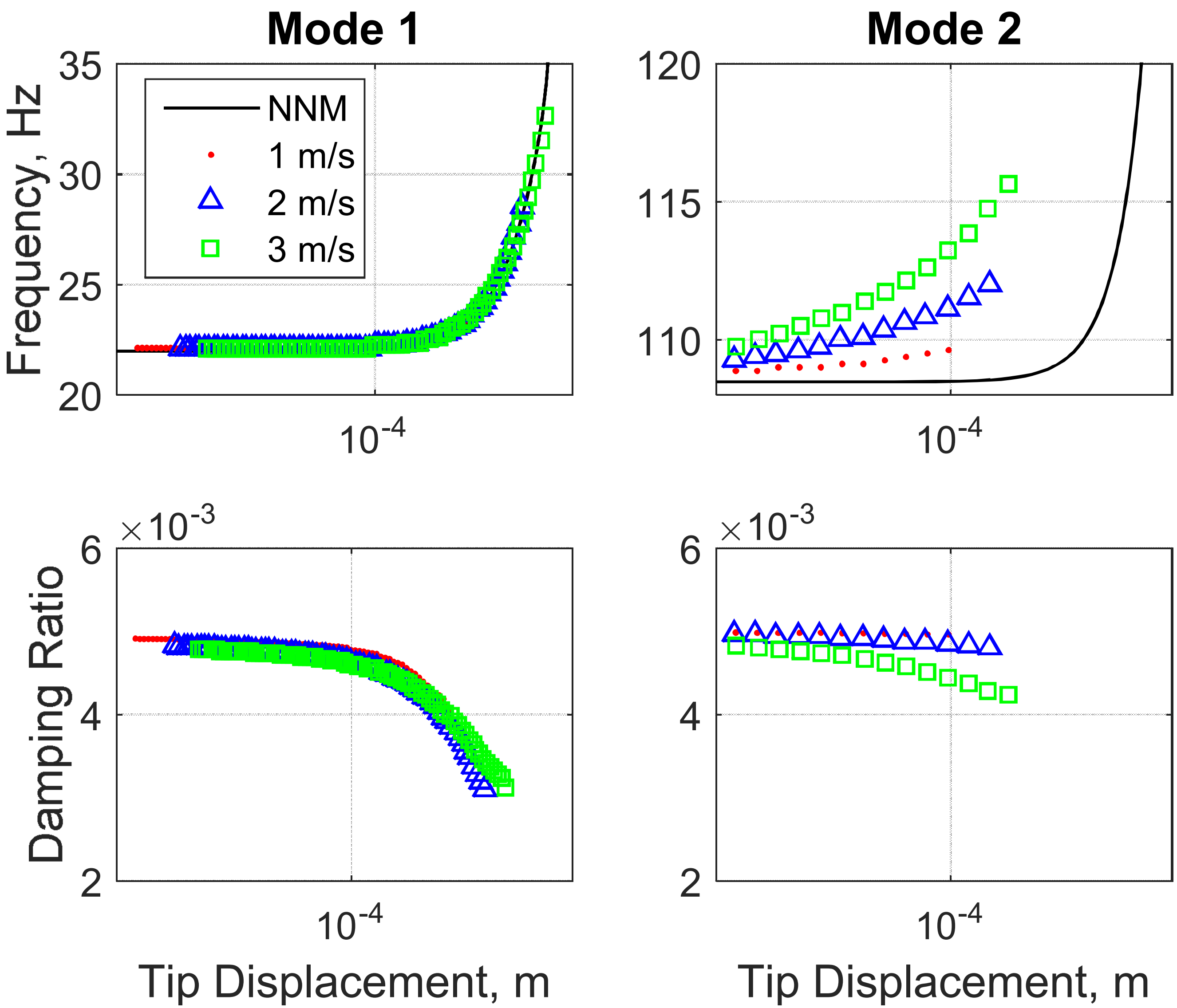

Discussion of Results/Conclusions

- As expected, **free response from initial conditions on NNM closely follow the underlying NNM** due to their invariance property.

- Instantaneous frequency versus amplitude for **broadband excitation** did not agree well for NNM 2.
  - At each windowed section, the decaying harmonic functions are assumed to satisfy **linear superposition**.
  - Recent work by Ardeh [1] showed that NNMs ($\mathbf{x}_i$) can be used to reconstruct (undamped) free response to any initial condition using a **nonlinear connecting function**.

$$\Phi = \mathbf{d} + \sum_{i=1}^{m} \mathbf{A}_i \mathbf{x}_i + \sum_{i=1}^{m} \mathbf{B}_i \dot{\mathbf{x}}_i + \sum_{k=1}^{n} \sum_{i=1}^{m} \sum_{j=1}^{m} \left[\mathbf{x}_i^{\mathrm{T}} \mathbf{C}_{ijk} \dot{\mathbf{x}}_i \right] \mathbf{e}_k$$

[1] Ardeh, H.A., "Geometrical Theory of Nonlinear Modal Analysis", Ph.D Dissertation, 2014.

Any Questions?

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- The authors would like to acknowledge Scott Smith and Caroline Nielsen for their assistance with this research.

- Contact Information

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Extra Slides

Application to Experimental Data

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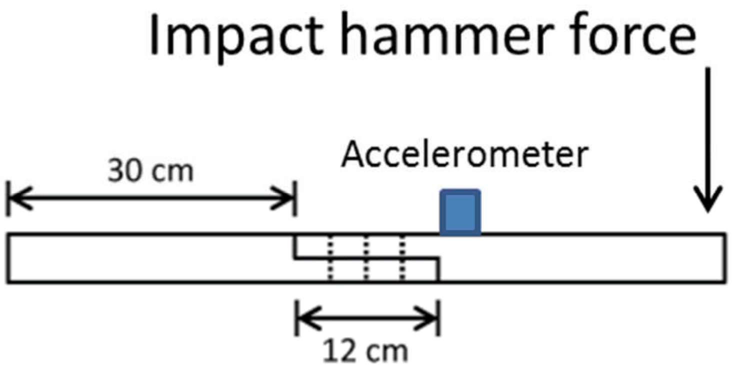

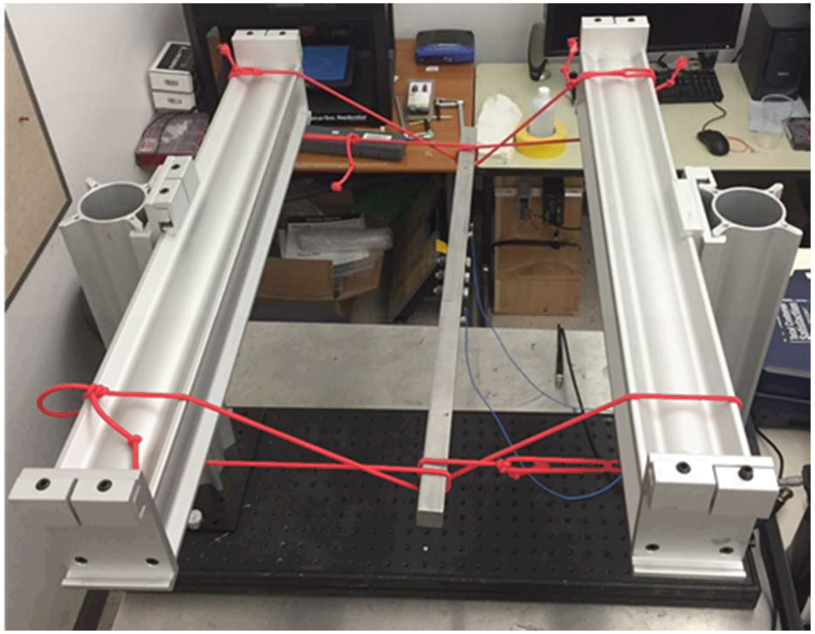

○ 100 N

- - - - 1000 N

– · – · – 2000 N

Two beams assembled with three bolts, creating a lap joint interface between the two.

Application to Experimental Data