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Antenna phase center locations in tapered aperture subarrays

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ABSTRACT

Antenna apertures are often parsed into subapertures for Direction of Arrival (DOA) measurements. However, when the overall aperture is tapered for sidelobe control, the locations of phase centers for the individual subapertures are shifted due to the local taper of individual subapertures. Furthermore, individual subaperture gains are also affected. These non-uniform perturbations complicate DOA calculations. Techniques are presented to calculate subaperture phase center locations, and algorithms are given for equalizing subapertures' gains.

Keywords: radar, antenna, subapertures, array, sidelobes

1 INTRODUCTION

The ability of a radar system to make Direction-of-Arrival (DOA) measurements, especially of moving targets, requires the ability to generate multiple antenna beams and compare responses from the collection. Techniques for this are well established in the literature. When more than two receive antenna beams are required along any one axis, the typical antenna architecture is to use distinct antenna subapertures with multiple separated phase centers in a general phase-monopulse configuration.[1] We also generally desire low sidelobe responses from the antenna, including its main reference beam, and to some extent even any other beams the antenna may generate in order to make its DOA calculations. Low sidelobe response generally requires beam shaping via antenna aperture tapering.

We will assume herein that we are dealing with a monostatic antenna configuration, wherein the same overall aperture is used for transmit and receive signals. We assume that the transmit signal will use the entire aperture, but the receive signal is collected by subapertures, wherein the subapertures are parsed from the larger overall sum aperture. That is, the sum of the individual receive subapertures equals the transmit aperture. This architecture complicates the nature of the receive subapertures in that they inherently have different characteristics, perhaps in beamwidth, gain, or both. Nevertheless, of ultimate interest to the subsequent signal processing are the locations of the phase centers of the various subapertures, and the gains imparted to the subapertures' received signals. Knowledge of these is required for optimal DOA calculations.

Herein we discuss locating the phase centers of uneven aperture illuminations, and of subapertures in overall tapered apertures. We confine our discussion to one-dimensional analysis and examples. We furthermore ignore many aspects of practical antenna design such as mutual coupling in array antennas, etc. This paper presents an abridged analysis detailed in a previous report.[2]

2 PHASE CENTER OF ARBITRARY APERTURE ILLUMINATION

We begin by considering a linear aperture with illumination

$$w(z) = \text{normalized (domain) aperture illumination function,} \quad (1)$$

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where the illumination function is limited to a finite extent such that

$$\int_{-\infty}^{x} w(z)\,dz = \int_{-\infty}^{x} w(z)\,\mathrm{rect}(z)\,dz\,, \tag{2}$$

Where the bounding rect( ) function is defined as

$$\mathrm{rect}(z) = \begin{cases} 1 & |z| < 1/2 \\ 1/2 & |z| = 1/2 \\ 0 & else \end{cases}. \tag{3}$$

We define the rect( ) function this way to make later mathematics in this report more precise. We now define a point on the linear axis, specified as the *x* axis, about which we rotate the aperture as

$$x_0 = \text{rotation point of the aperture.} \tag{4}$$

We furthermore define x_1 as the physical center the aperture, and *L* as the actual length of the aperture. This geometry is identified in Figure 1. The far-field pattern can then be calculated as

$$G(\theta) = \int_{x_1 - L/2}^{x_1 + L/2} w\left(\frac{x - x_1}{L}\right) e^{-j\frac{2\pi \sin\theta}{\lambda}(x - x_0)}\,dx\,, \tag{5}$$

where θ is the Direction of Arrival (DOA) incidence angle, and λ is the wavelength of the sinusoidal signal. Note that this is just the well-known axiom that the far-field antenna pattern is essentially a Fourier transform of the aperture illumination function. In general, $G(\theta)$ is complex-valued. Our concern here is with a phase change in $G(\theta)$ as we vary θ. We desire the value of x_0 for which there is no phase change in $G(\theta)$ as θ is varied. The value of x_0 that accomplishes this is the "phase center" of the aperture. We now examine two example cases.

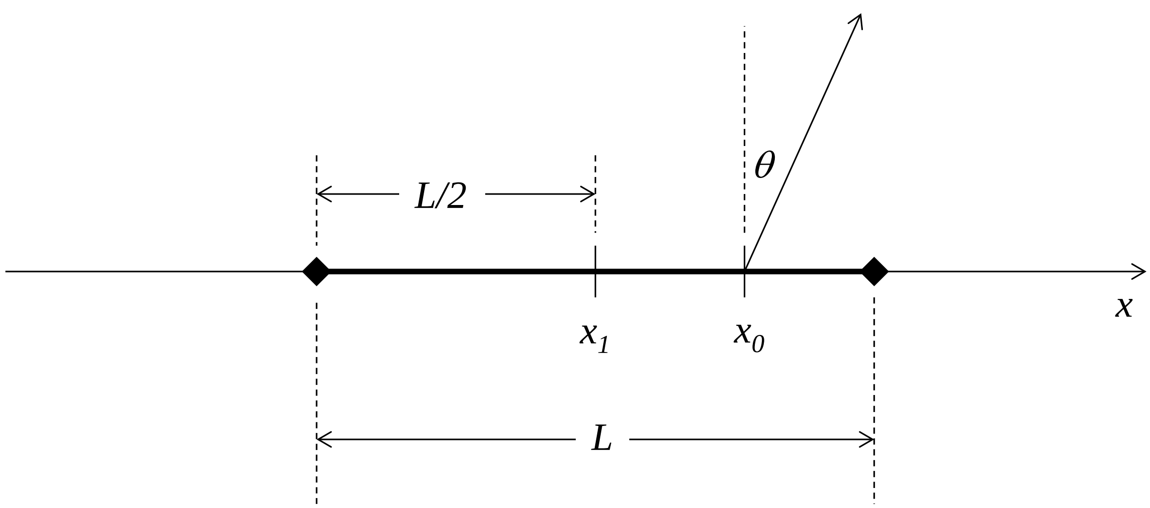

Figure 1. One-dimensional aperture geometry definitions. The aperture illumination is essentially the current distribution within the finite aperture.

2.1 Uniform Aperture Illumination

Consider the case where we have a uniformly illuminated aperture described by

$$w(z) = \text{rect}(z). \tag{6}$$

In this case, then the far-field pattern is given by

$$G(\theta) = \int_{x_1 - L/2}^{x_1 + L/2} e^{-j\frac{2\pi \sin\theta}{\lambda}(x - x_0)} dx, \tag{7}$$

where θ = Direction of Arrival (DOA) incidence angle, which can be solved and simplified to

$$G(\theta) = e^{-j\frac{2\pi \sin\theta}{\lambda}(x_1 - x_0)} L \,\text{sinc}\left(\frac{L}{\lambda}\sin\theta\right), \tag{8}$$

where the sinc( ) function is given by

$$\text{sinc}(z) = \frac{\sin(\pi z)}{\pi z}. \tag{9}$$

We identify the phase term as the exponent of the exponential factor, which shows a dependence on angle θ whenever $x_0 \neq x_1$. Consequently, the far-field pattern phase is independent of rotation angle θ when $x_0 = x_1$. Therefore, we identify the phase center for this aperture as

$$x_{pc} = x_1. \tag{10}$$

The aperture illumination and the phase center location for this example are illustrated in Figure 2, where we have arbitrarily presumed $x_1 = 0$. We may furthermore stipulate that Fourier properties suggest that any even-symmetric and real $w(z)$ will result in $x_{pc} = x_1$.

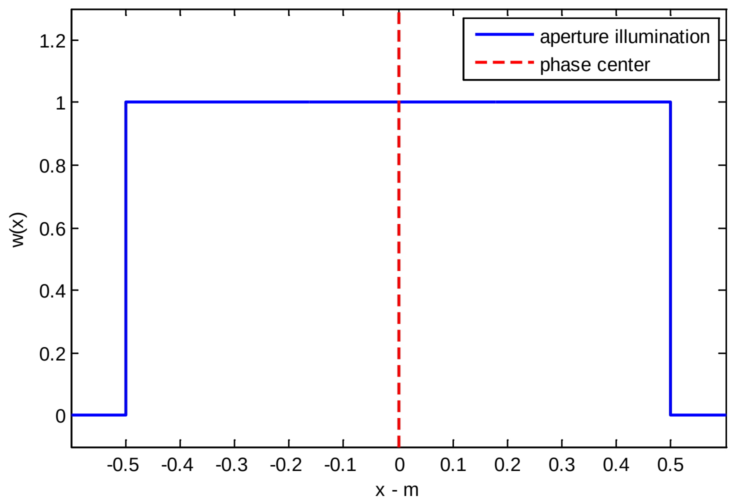

Figure 2. Uniform aperture illumination.

2.2 Linear-Gradient Aperture Illumination

Now consider an aperture illumination of the form

$$w(z) = (1+\alpha z)\,\mathrm{rect}(z) \tag{11}$$

where α = constant such that $0 < \alpha < 1$. Note that this is not an even function, and definitely not uniform. In this case, the far-field pattern is then given by

$$G(\theta) = \int_{x_1 - L/2}^{x_1 + L/2} \left(1 + \alpha\left(\frac{x - x_1}{L}\right)\right) e^{-j\frac{2\pi \sin\theta}{\lambda}(x - x_0)}\, dx \,. \tag{12}$$

With no loss of generality, we will assume that $x_1 = 0$. We will then rewrite the far-field pattern as

$$G(\theta) = e^{j\frac{2\pi\sin\theta}{\lambda}x_0} \int_{-L/2}^{L/2} \left(1 + \frac{\alpha}{L}x\right) e^{-j\frac{2\pi\sin\theta}{\lambda}x}\, dx \,. \tag{13}$$

This can be solved to yield

$$G(\theta) = e^{j\frac{2\pi\sin\theta}{\lambda}x_0} \left\{ \begin{array}{l} L\,\mathrm{sinc}\left(\dfrac{L}{\lambda}\sin\theta\right) \\ -j\left(\dfrac{\alpha L}{2}\right)\left(\dfrac{\sin\left(\pi\frac{L}{\lambda}\sin\theta\right)}{\left(\pi\frac{L}{\lambda}\sin\theta\right)^2} - \dfrac{\cos\left(\pi\frac{L}{\lambda}\sin\theta\right)}{\left(\pi\frac{L}{\lambda}\sin\theta\right)}\right) \end{array} \right\}. \tag{14}$$

Recall that we desire to identify a phase center that is a single constant value for x_0 that results in a phase that is independent of angle θ. We may solve for the phase as

$$\mathrm{Phase}(G(\theta)) = \mathrm{atan}\left\{\left(\frac{\alpha}{2}\right)\left(\cot\left(\pi\frac{L}{\lambda}\sin\theta\right) - \frac{1}{\left(\pi\frac{L}{\lambda}\sin\theta\right)}\right)\right\} + \frac{2\pi\sin\theta}{\lambda}x_0 \,. \tag{15}$$

We can solve for a position x_0 that causes the phase to go to zero, and designate this as the phase center of the aperture. Doing so yields the phase center location as

$$x_{pc} = \frac{-\lambda}{2\pi\sin\theta}\mathrm{atan}\left\{\left(\frac{\alpha}{2}\right)\left(\cot\left(\pi\frac{L}{\lambda}\sin\theta\right) - \frac{1}{\left(\pi\frac{L}{\lambda}\sin\theta\right)}\right)\right\}. \tag{16}$$

We note that phase center x_{pc} is not a constant, and in fact actually depends on DOA angle θ. No single value of x_{pc} causes the phase to go to zero for all angles θ. That is, the phase center moves as a function of DOA angle θ. In the neighborhood of the center of the beam where $\theta \approx 0$ we may nevertheless choose a constant x_{pc} that minimizes dependence of $G(\theta)$ on angle θ by setting the derivative of Eq. (15) to zero and solving for the x_0 that causes a nearly constant phase in the neighborhood of $\theta = 0$. We identify the derivative of the phase in the direction of $\theta = 0$ as

$$\lim_{\theta \to 0}\left(\frac{d}{d\theta}\text{Phase}\left(G(\theta)\right)\right) = -\frac{\pi}{6\lambda}\left(\alpha L - 12x_0\right). \tag{17}$$

Consequently, we designate the phase center as the value of x_0 that causes this to go to zero, namely

$$x_{pc} = \frac{L\alpha}{12}. \tag{18}$$

Essentially, the phase slope of $G(\theta)$ as a function of θ is zero with this value for x_0 at the angle $\theta = 0$. We observe that the non-uniform aperture illumination in Figure 3 has the phase center indicated, as calculated above. Note that it is off-center with respect to the aperture center at $x_1 = 0$. Furthermore, it is shifted in the direction of heavier weighting of the aperture, as we might expect. The far-field antenna pattern is illustrated in Figure 4, and the received signal phase as a function of rotation position and incidence DOA angle is shown in Figure 5. Note that rotating about the nominal phase center exhibits a fairly constant phase value within the mainlobe of the far-field pattern.

2.3 Comments

We offer the following additional comments.

- More complicated aperture illumination functions might cause us to use numerical techniques to estimate the phase center location.
- More complicated aperture illumination functions will generally have phase centers that are DOA angle dependent. However, some locations are more well-behaved than others, in minimizing phase variations over DOA angles of interest.
- Phase centers will be pulled in the direction of more weighting in the aperture.

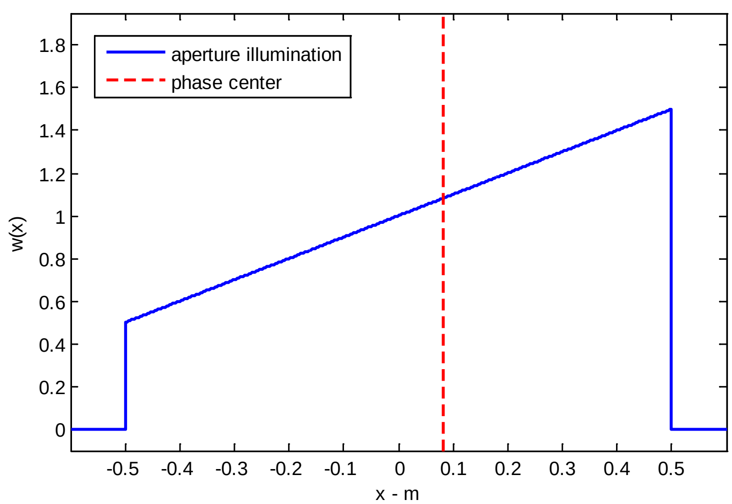

Figure 3. Example non-uniform aperture illumination.

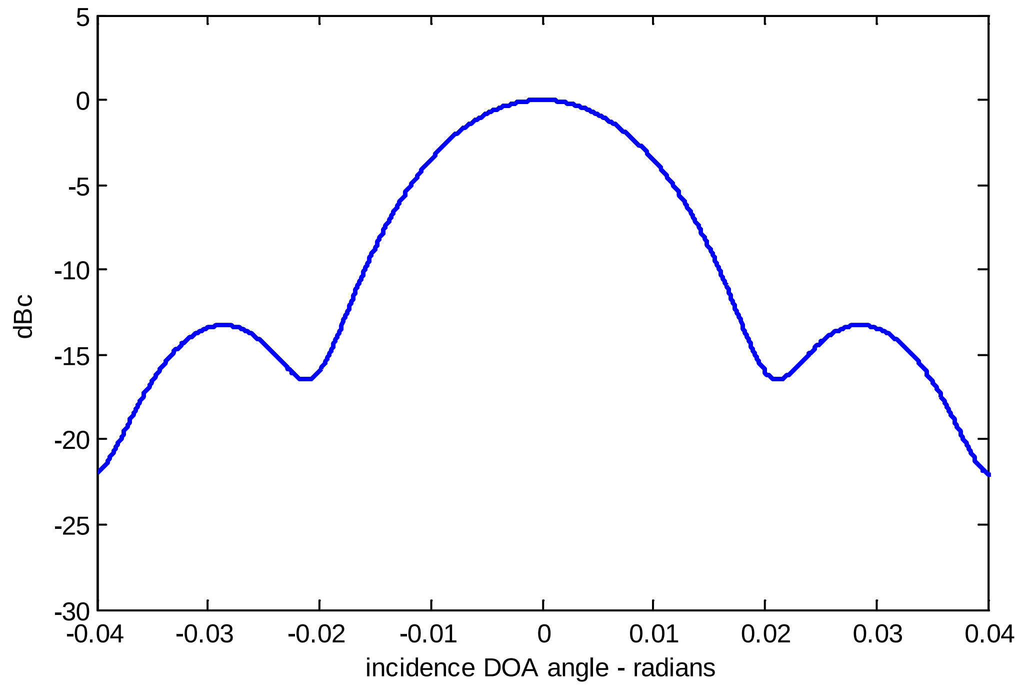

Figure 4. Far-field antenna pattern for illumination of Figure 3.

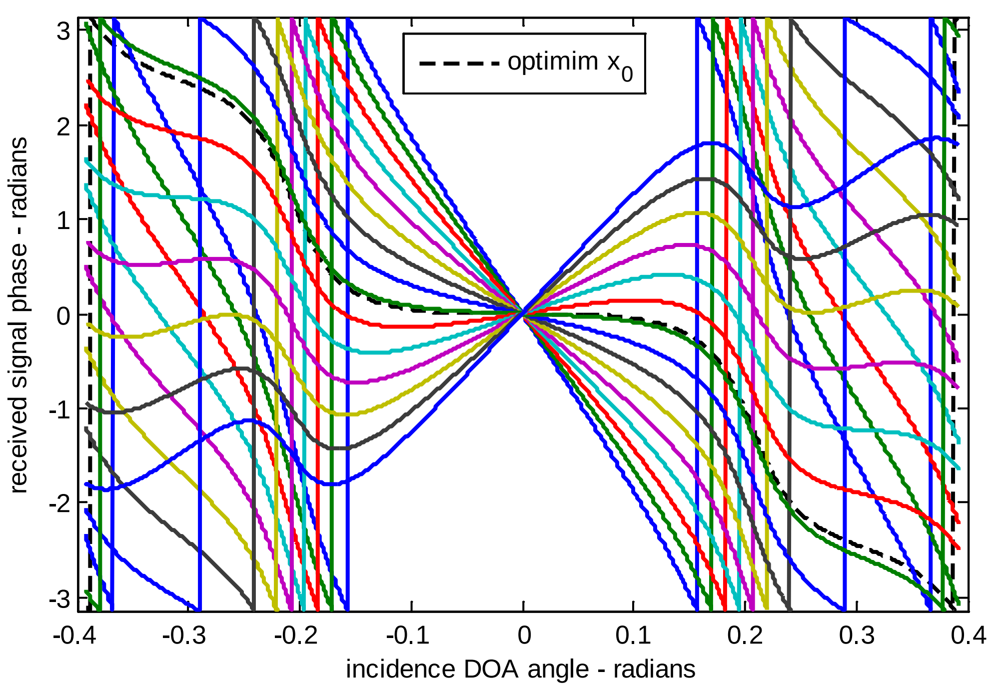

Figure 5. Received signal phase versus incidence DOA angle for various rotation positions x_0. The optimum x_0, that which is most flat at a zero DOA angle, indicates the phase center.

3 SUBAPERTURES IN A TAPERED SUM PATTERN

We now build upon the previous section and examine a larger aperture composed of contiguous but non-overlapping subapertures, wherein the larger aperture has a specified taper function for sidelobe control of the overall combined, or sum, antenna beam. We now modify and add the following definitions.

$$
\begin{aligned}
&L_{ap} = \text{physical length of the overall aperture,}\\
&w_{ap}\left(x\right) = \text{illumination (taper) function of the overall aperture,}\\
&i = \text{index of phase centers, } i \in \left\{0,1,2,...,\left(I-1\right)\right\},\\
&I = \text{number of subapertures,}\\
&x_{1,i} = \text{physical center of the } i^{\text{th}} \text{ subaperture,}\\
&x_{pc,i} = \text{phase center of the } i^{\text{th}} \text{ subaperture,}\\
&L_i = \text{physical length of the } i^{\text{th}} \text{ subaperture.}
\end{aligned}
\tag{19}
$$

We will assume that $w_{ap}\left(x\right)$ is real and even, and nonzero over the interval $\left[-0.5,0.5\right]$. Without loss of generality, we will also assume that the overall aperture is physically centered at zero. The contiguous but non-overlapped nature of the subapertures is then such that

$$\text{rect}\left(\frac{x}{L_{ap}}\right) = \sum_{i=0}^{I-1} \text{rect}\left(\frac{x - x_{1,i}}{L_i}\right). \tag{20}$$

This basically means that they don't overlap, and there are no gaps. The far-field pattern for the overall aperture, also referred to as the "sum" beam, is calculated as

$$G_{ap}\left(\theta\right) = \int_{-\infty}^{\infty} w_{ap}\left(\frac{x}{L_{ap}}\right) e^{-j\frac{2\pi\sin\theta}{\lambda}x} dx , \tag{21}$$

whereas the far-field patterns of the individual subapertures are calculated as

$$G_i\left(\theta\right) = \int_{-\infty}^{\infty} \left[\text{rect}\left(\frac{x - x_{1,i}}{L_i}\right) w_{ap}\left(\frac{x}{L_{ap}}\right)\right] e^{-j\frac{2\pi\sin\theta}{\lambda}x} dx . \tag{22}$$

Note that the quantity in the square brackets represents the taper function of the individual subapertures. These individual subaperture tapers are real but not by themselves even. Interestingly, a consequence of this is that the sum of the individual subaperture beams equals the overall sum pattern beam shape. Recall that we have not yet specified the lengths of the individual subapertures; merely that they add up to the overall aperture length.

Furthermore, we will assume that subapertures with indices i and $I-1-i$ are mirror images of each other, and consequently will each have the same shapes of their beam patterns; same in magnitude but not necessarily in phase. This follows from the Fourier property that the transform of a real function will have symmetry in the magnitude of its spectrum. This is why in some of the following plots some subapertures' patterns are not visible; they are basically covered up by other subapertures' responses.

Now we examine some specific cases.

3.1 Uniform Width Subapertures

We illustrate with an example that divides the aperture into 5 equal-length subapertures. We shall assume an overall aperture weighting defined by a Taylor window with –30 dB sidelobes and $\bar{n} = 5$. Furthermore, we shall assume L_{ap} = 1 m, and λ = 0.02 m. Subaperture phase centers are calculated using numerical integration.

Relevant measures are given in Table 1. Figure 6 plots the subapertures and their phase centers. Figure 7 plots the one-way patterns. Figure 8 illustrates the two-way pattern, with the transmitted signal using the sum pattern. We observe that for this example there is nearly a 9 dB gain difference between center and outer subapertures.

Table 1. Subaperture characteristics for 5 subapertures using -30 dB Taylor window, $\bar{n} = 5$.

Subaperture	*Physical Center (m)*	*Phase Center (m)*	*Width (m)*	*Gain (dBc)*
0	–0.4	–0.3856	0.2	–19.1214
1	–0.2	–0.1913	0.2	–12.5004
2	0	0	0.2	–10.3295
3	0.2	0.1913	0.2	–12.5004
4	0.4	0.3856	0.2	–19.1214

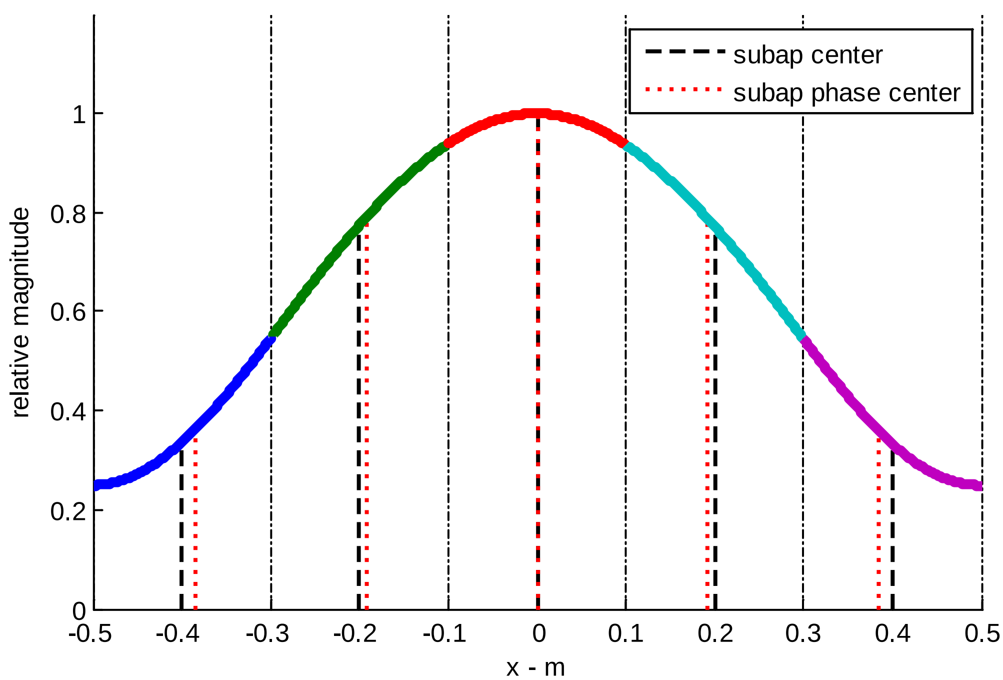

Figure 6. Subaperture definitions and parameters. Overall aperture weighting is –30 dB Taylor window with $\bar{n} = 5$.

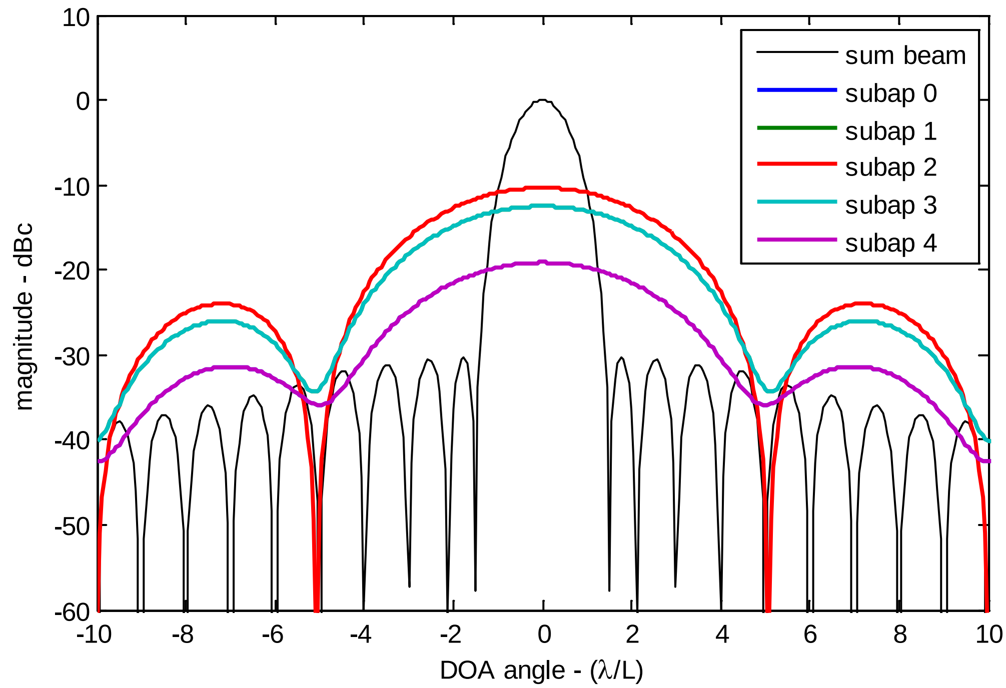

Figure 7. One-way beam patterns for overall aperture and subapertures.

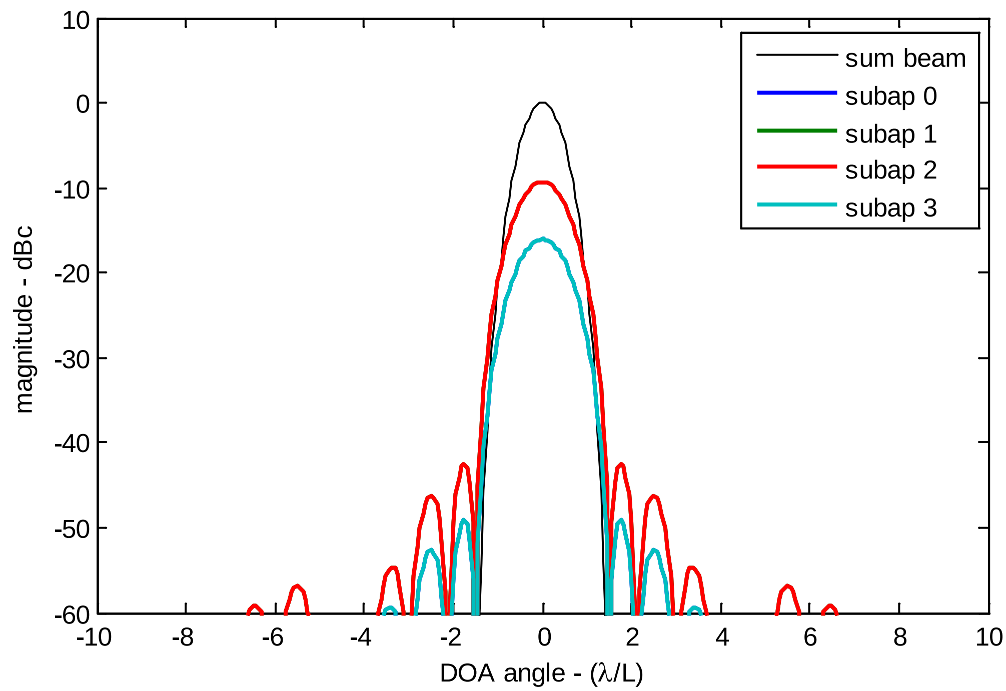

Figure 8. Two-way beam patterns for overall aperture and subapertures.

3.2 Uniform Gain Subapertures

We illustrate with an example that divides the aperture into 5 equal-gain subapertures. To accomplish this, we will give up the constraint of equal widths. We illustrate a specific example. We shall assume an overall aperture weighting defined by a Taylor window with –30 dB sidelobes and $\bar{n} = 5$. Furthermore, we shall assume L_{ap} = 1 m, and λ = 0.02 m. Subaperture phase centers are calculated using numerical integration.

Relevant measures are given in Table 2. Figure 9 plots the subapertures and their phase centers. Figure 10 plots the one-way patterns. Figure 11 illustrates the two-way pattern, with the transmitted signal using the sum pattern.

Table 2. Subaperture characteristics for 5 subapertures using -30 dB Taylor window, $\bar{n} = 5$.

Subaperture	*Physical Center (m)*	*Phase Center (m)*	*Width (m)*	*Gain (dBc)*
0	–0.3558	–0.3265	0.2885	–13.9794
1	–0.1382	–0.1351	0.1466	–13.9794
2	0	0	0.1298	–13.9794
3	0.1382	0.1351	0.1466	–13.9794
4	0.3558	0.3265	0.2885	–13.9794

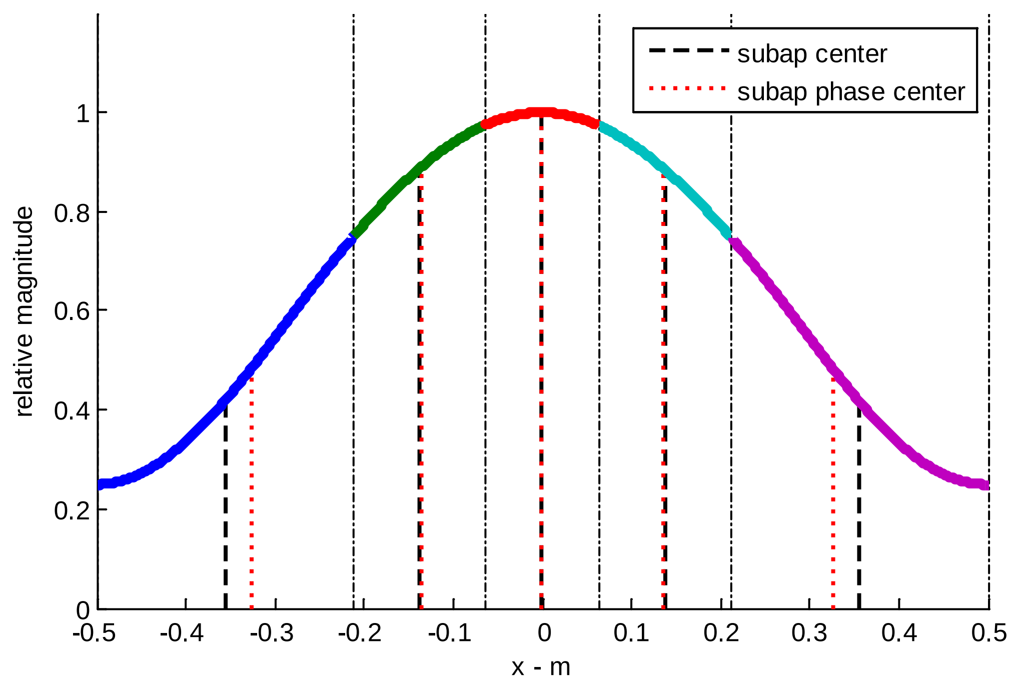

Figure 9. Subaperture definitions and parameters. Overall aperture weighting is –30 dB Taylor window with $\bar{n} = 5$.

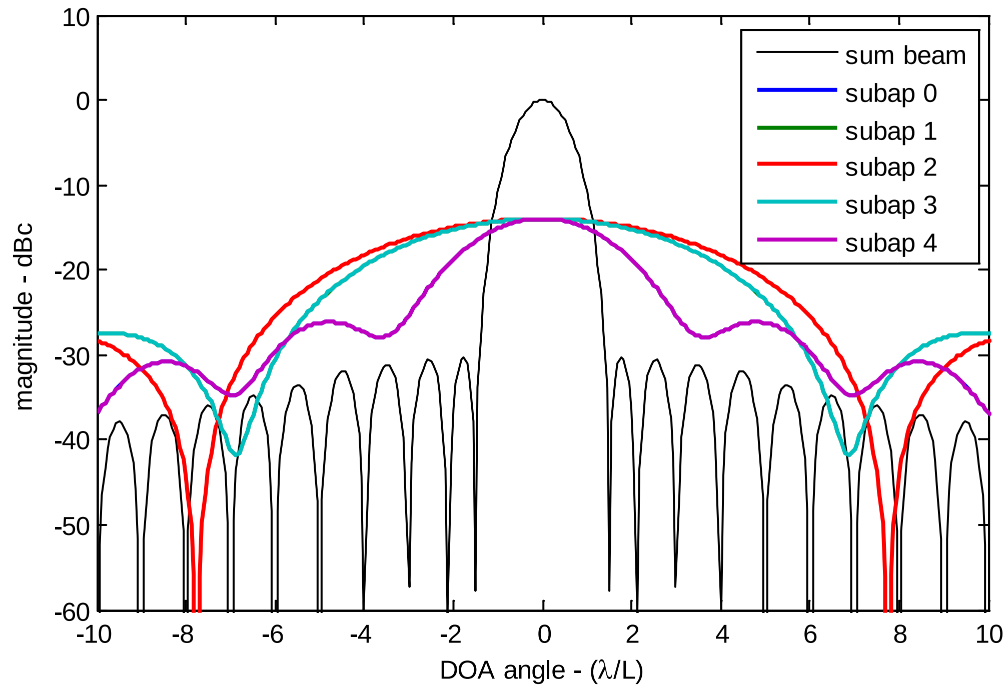

Figure 10. One-way beam patterns for overall aperture and subapertures.

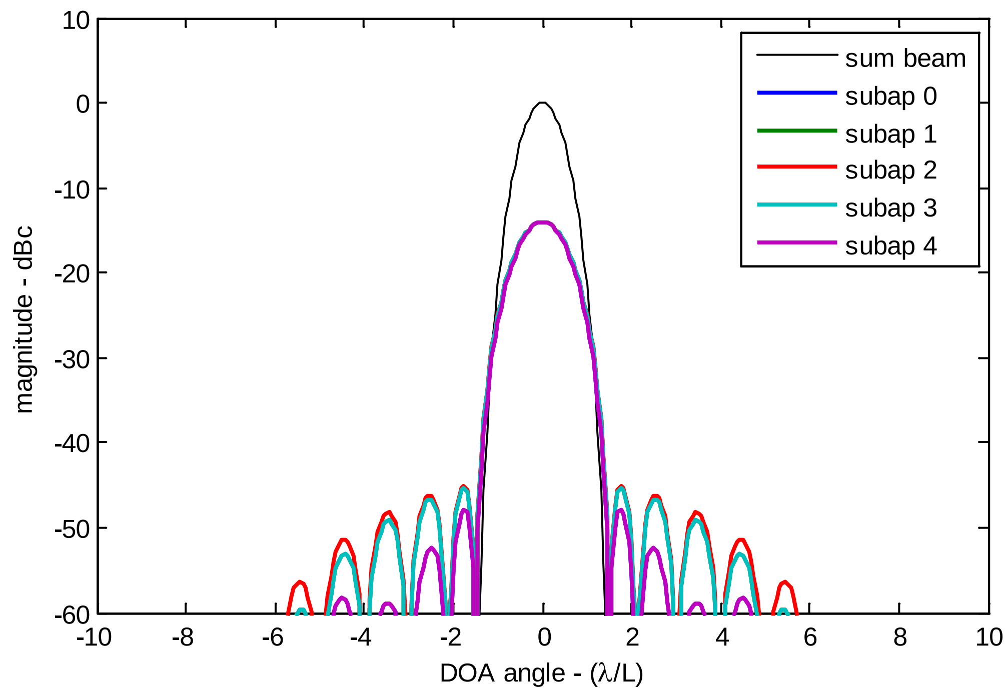

Figure 11. Two-way beam patterns for overall aperture and subapertures.

3.3 Comments

We offer the following comments.

- The width of the mainlobe responses from the individual subaperture beams will always be wider than the width of the mainlobe response of the sum beam.
- For a tapered sum beam with segmented subapertures, uniform-width subapertures will have different maximum gains, whereas uniform-maximum-gain subapertures will have different physical widths.
- Tapering the sum beam will draw the phase centers of the individual subapertures inwards, towards the center of the overall aperture.
- Uniform-maximum-gain subapertures will have the phase centers of their individual subapertures moved inwards somewhat from those of uniform-width subapertures, towards the center of the overall aperture.
- Although the previous analysis dealt with non-overlapping subapertures, we can just as easily analyze feathered or blended subapertures. We refer the reader to the earlier report for details and examples.[2]

4 BRIEF COMMENTS ON PERFORMANCE

Recapping, subapertures in antenna arrays are used to estimate DOA. The subaperture weighting functions affect both the subarray gain and the relative position of phase centers. In DOA estimation, the performance is a function of the Signal-to-Noise Ratio (SNR) and the (projected) separation distance between phase centers, B. In fact the angle estimation noise depends on these as

$$\sigma_\theta \propto \frac{1}{B\sqrt{snr}} \tag{23}$$

The SNR, is in turn a function of the product of the transmit and receive antenna gains (plus any additional processing gain between subarrays for the DOA estimation). As we have just seen in the previous analysis, the subaperture weightings affect both the gains and the phase center separations. Therefore, the radar designer needs to account for these in performing the trade-off between different subaperture weightings.

5 CONCLUSIONS

We summarize herein that uneven subaperture tapering will shift the phase center away from the center of the subaperture, and generally in the direction with greater weighting. Subaperture phase center spacing is not guaranteed to be constant (i.e. uniform), even with equal-width subapertures. Furthermore, subaperture gain will not necessarily be identical for all subapertures, although subapertures can be defined to specifically make them so, albeit with other consequences.

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