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# A Scalable Multi-chain Markov Chain Monte Carlo Method for Inverting Subsurface Hydraulic and Geological Properties

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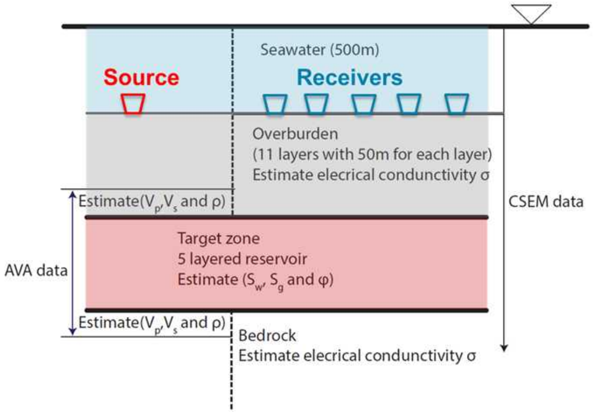
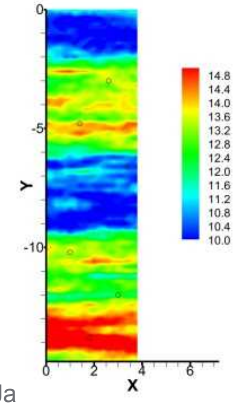
# Outlines

- Scalable Multi-chain Markov Chain Monte Carlo Method
- Case 1: Reservoir porosity and saturation through invert marine seismic amplitude versus angle (AVA) and controlled-source electro-magnetic (CSEM) data




- Case 2: Soil moisture variations through ground penetrating radar (GPR) travel time data

# Scalable Multi-chain Markov Chain Monte Carlo Method
## Bayesian Formulation

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- Generate posterior distributions on model parameters, given
  - Experimental data
  - A prior distribution on model parameters
  - A presumed probabilistic relationship between experimental data and model output that can be defined by a likelihood function

$$\pi(\theta \mid d) \propto \pi(\theta) L(d \mid \theta)$$

Posterior parameter distribution

Model parameters

Observed Data

Prior parameter distribution

Likelihood function which Incorporates the model

# Bayesian Formulation



- Experimental data = Model output + error

$$d_i = G(\boldsymbol{\theta}_i) + \varepsilon_i$$

- If we assume error terms are independent, zero mean Gaussian random variables with variance $\sigma^2$, the likelihood is:

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[ -\frac{(d_i - G(\boldsymbol{\theta}_i))^2}{2\sigma^2} \right]$$

- Markov Chain Monte Carlo (MCMC)

  Generating a sampling density that is approximately equal to the posterior.

# Markov Chain Monte Carlo



- MCMC generates samples that approximate the posterior distribution
- MCMC requires a "proposal density" which is used for generating $\theta_{i+1}$ in the sequence, conditional on $\theta_i$.
- Metropolis-Hastings is a commonly used algorithm
  - Sample a candidate $Y$ from the proposal density function $q(Y|\theta_i)$
  - Calculate the acceptance ratio $\alpha(\theta_i, Y) = \min\left[1, \frac{\pi(Y)q(Y|\theta_i)}{\pi(\theta_i)q(\theta_i|Y)}\right]$
  - If $\alpha(\theta_i, Y) > U$, set $\theta_{i+1} = Y$, else set $\theta_{i+1} = \theta_i$.
  - Increment $i$.

# Scalable Multi-chain Markov Chain Monte Carlo Method
## Markov Chain Monte Carlo

- MCMC requires more than 10,000 evaluations of forward simulation model
- We want to avoid surrogates

**COMPUTATIONALLY VERY EXPENSIVE**

- Parallel MCMC

MCMC is inherently sequential

**SaChES: Scalable Adaptive Chain-Ensemble Sampling**

# Scalable Multi-chain Markov Chain Monte Carlo Method
## SaChES: Scalable Adaptive Chain-Ensemble Sampling



- Hybrid method that incorporates:
  - DREAM (DiffeRential Evolution Adaptive Metropolis) to utilize multiple chains to obtain high-quality proposal densities
  - DRAM (Delayed Rejection Adaptive Metropolis ) to obtain posterior distributions efficiently
  - Parallel chains to accelerate computations

**More details about the method is available on the poster**

**Bayesian calibration of the Community Land Model using a multi-chain Markov chain Monte Carlo method**

Jaideep Ray, Laura Swiler, Maoyi Huang, Zhangshuan Hou

Thursday, 17 December, 13:40 – 18:00

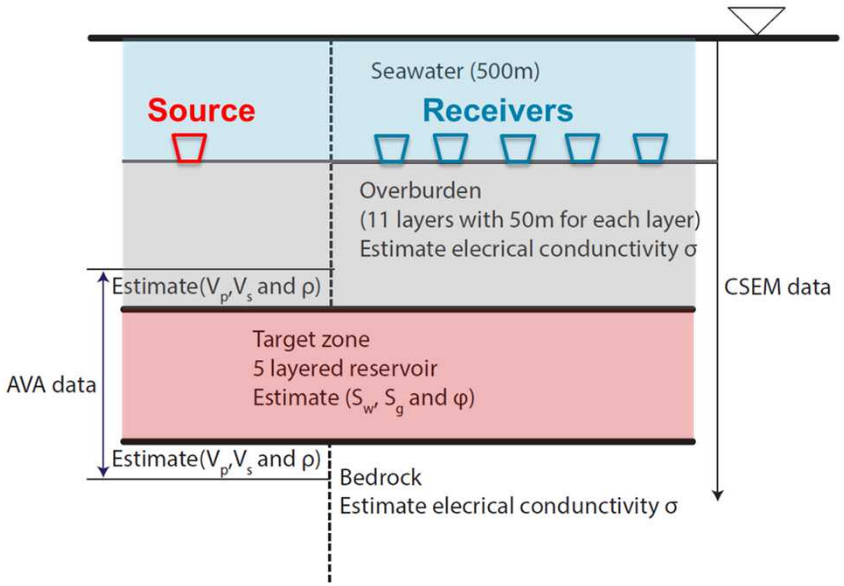
Moscone South – Poster Hall

# Case 1: Gas Saturation Estimation

- Inversion domain

Seismic amplitude versus angle(AVA)

Controlled-source electro-magnetic(CSEM)




5-layered reservoirs from the upper to bottom with water saturations:

0.95, 0.05, 0.6, 0.9 and 0.1

and the porosity:

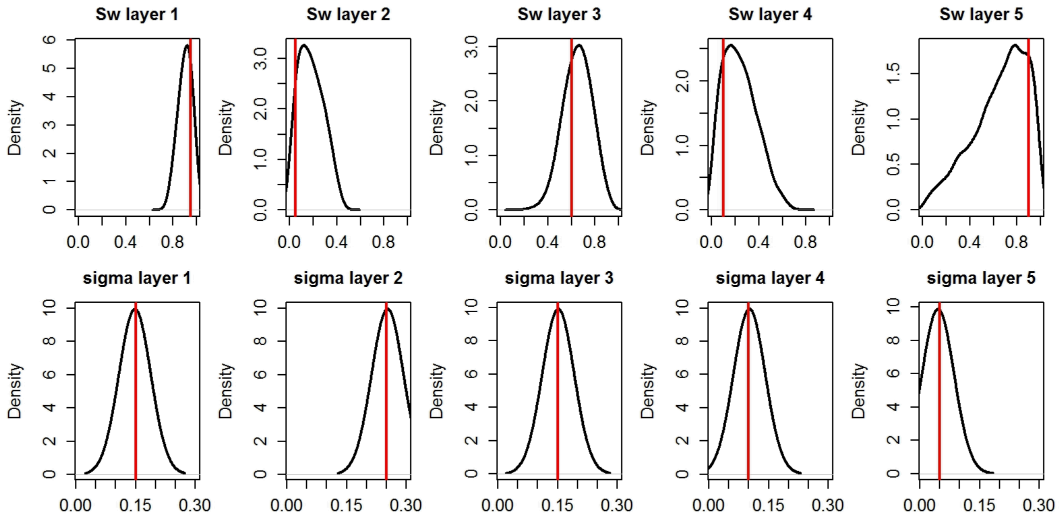
0.15, 0.25, 0.15, 0.1 and 0.05

The source and receivers were both located 50m above the seafloor. 21 receivers were away from electrodes from 500m to 5000m.

# Case 1: Gas Saturation Estimation

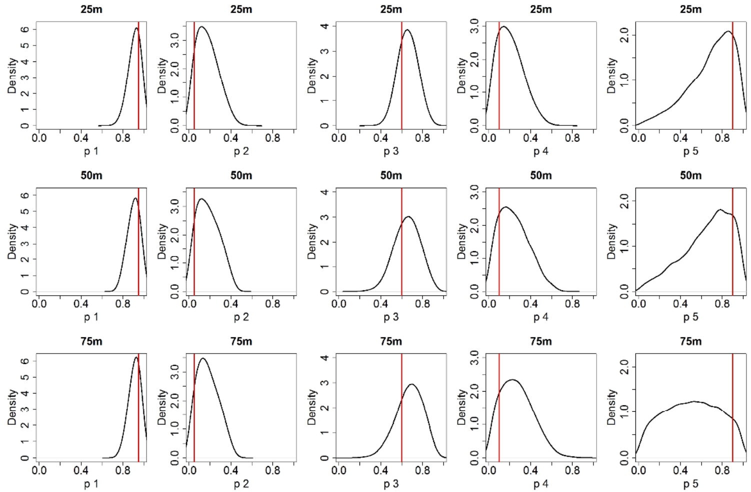
- Seismic AVA data (80 time steps) was used to estimate porosity and narrow bounds were obtained for each layer, then estimate water saturation.
- The reservoir thickness is 50m
- CSEM data were obtained from 2Hz channel

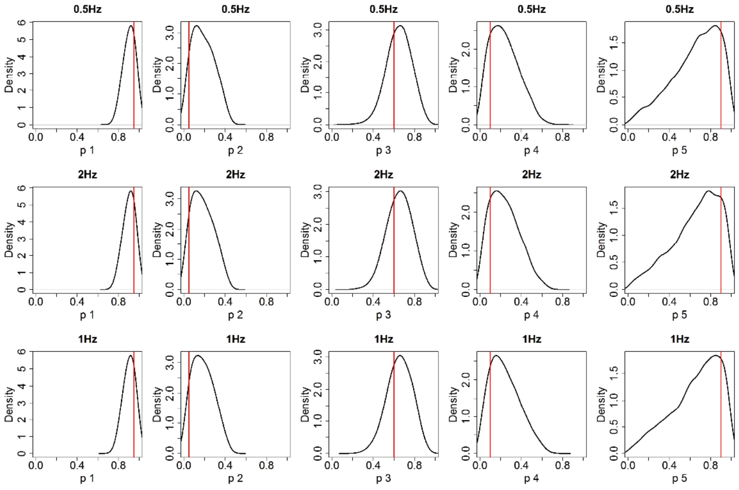
Posterior distribution of the parameters



# Case 1: Gas Saturation Estimation

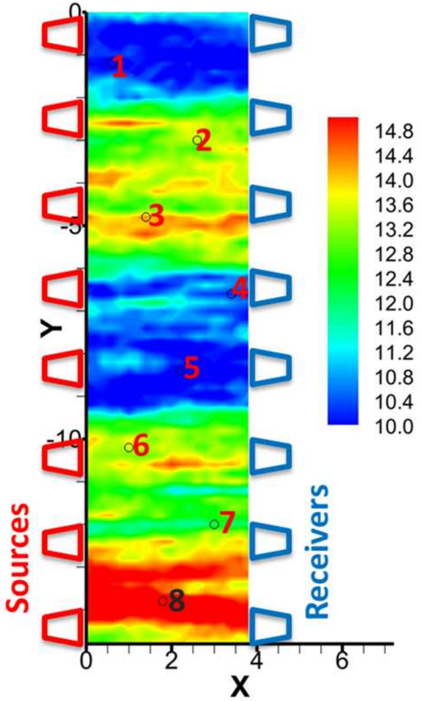
- The effect of thickness for each reservoir layer
  - Each layer thickness: 25m, 50m and 75m
- The effect of CSEM data frequency
  - CSEM data frequency: 0.5Hz, 1Hz and 2Hz



# Case 2: Soil Moisture Variations

- "True" dielectric permittivity field




- Synthetic test case

  3.8 X 15 m; 20 X 75 points

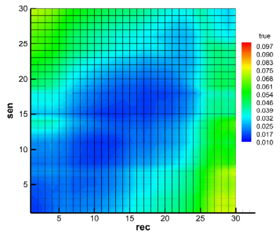
  8 pilot points, and range (correlation length)

  Generate a random dielectric field in SGSIM

- Ground penetrating radar (GPR) travel time simulation

  Velocity: $v = \frac{v_l}{\sqrt{\varepsilon}}$

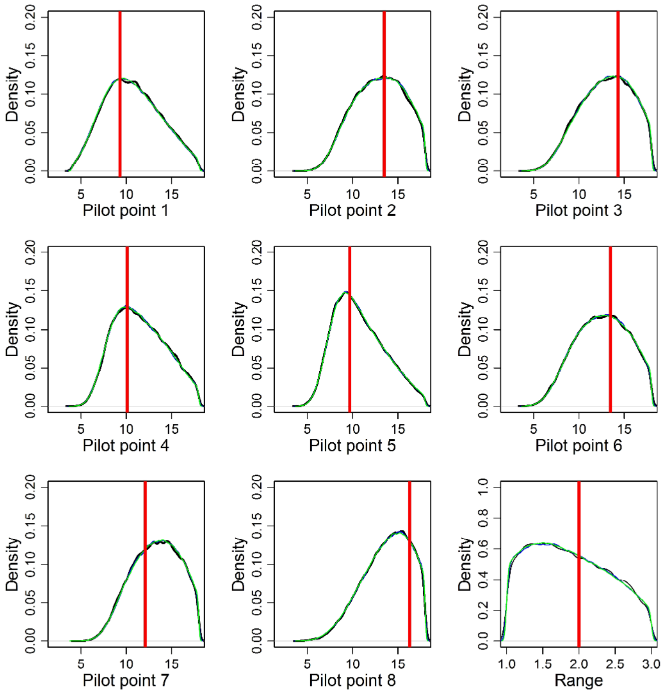
  Calculate the radar signal travel time between each source and receiver



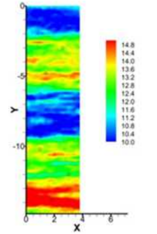

"Observations"
Travel time between 30 sources and 30 receivers

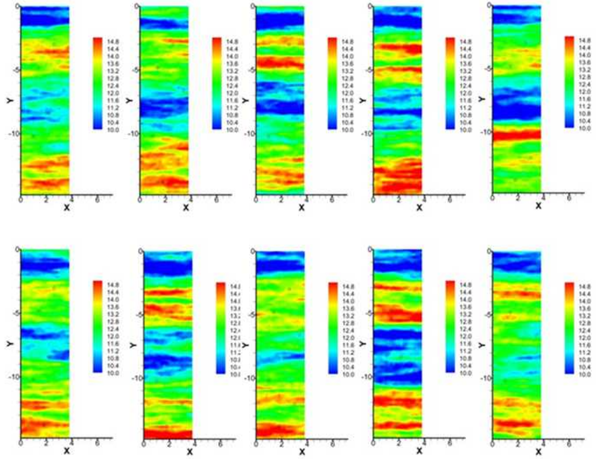
# Case 2: Soil Moisture Variations

- Posterior distribution of the parameters




- Inversed dielectric permittivity field



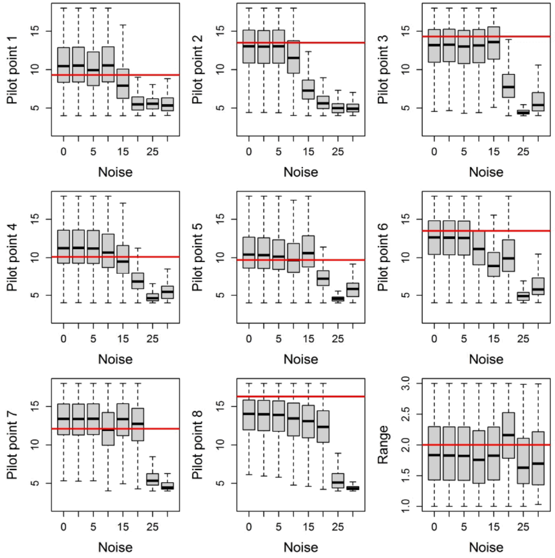
10 best inversed fields

# Case 2: Soil Moisture Variations



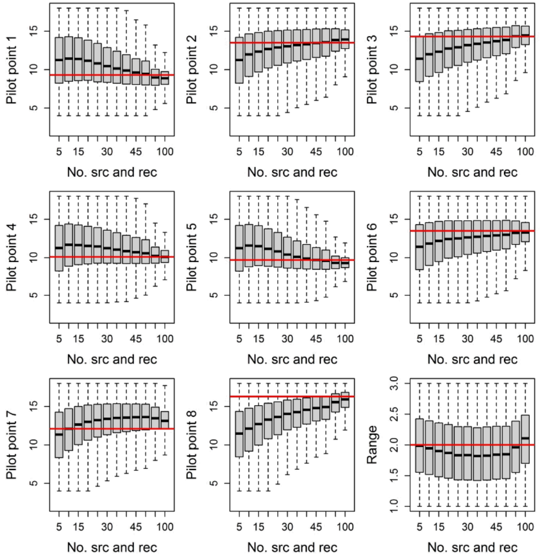
- **Noise on observation**

  Noise's standard deviation is defined as the percentage of the mean of the true observation




- **Number of sources and receivers**

  5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 75, and 100 sources and receivers



# Questions?