IMAC XXXIV
A Modal Model to Simulate Typical Structural Dynamic Nonlinearity

Randall Mayes, Benjamin Pacini, Dan Roettgen

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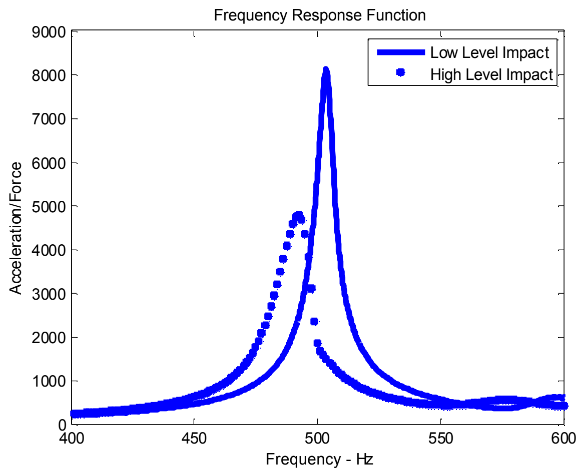
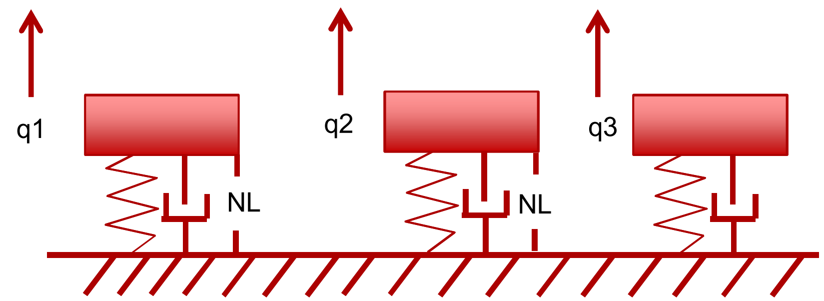
Motivation

- Many structural dynamic systems are mildly nonlinear in stiffness (few percent modal frequency change) and significantly nonlinear in damping (hundreds of percent damping ratio change) as a function of amplitude (e.g. in figure)
- It is difficult to validate local physical models for such nonlinearities because there are so many materials and interfaces with different degrees of nonlinearities
- One simulation approach is to reduce the number of nonlinearities down to the number of modes active in the system. In this way, one nonlinear element captures many nonlinear effects on a single modal response
- This project seeks to demonstrate this capability experimentally with 3 nonlinear pseudo-modal models
  - Iwan
  - FREEVIB (FV)
  - Cubic stiffness and damper



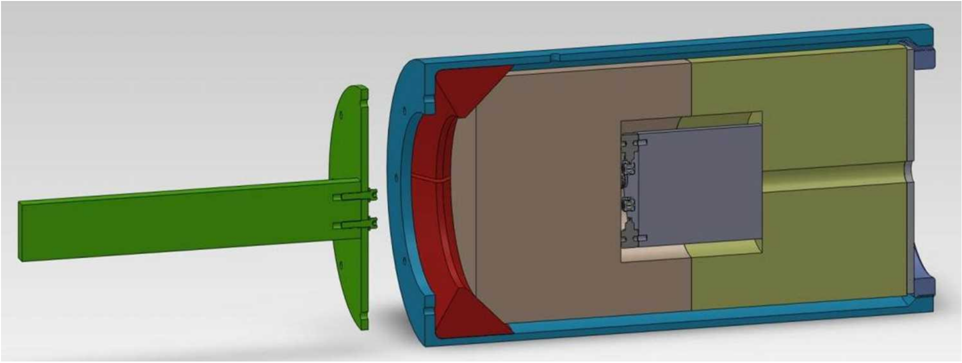
Assumptions for Pseudo-Modal Model

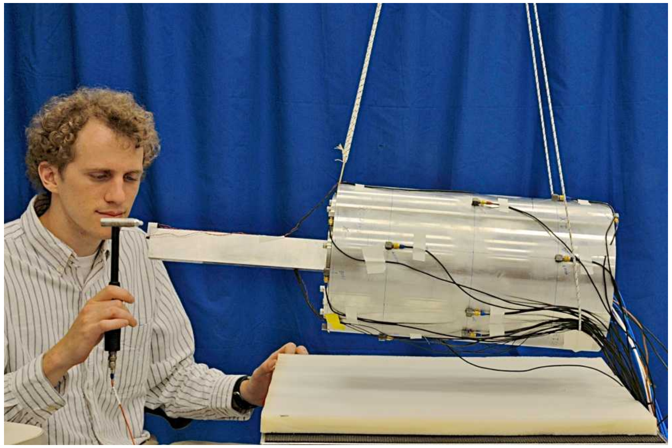
- The **mode shapes** do not change with amplitude of response so $\bar{\mathbf{x}} = \mathbf{\Phi}\bar{\mathbf{q}}$
- Nonlinear modes do not interact
- Significant nonlinearity is captured by adding nonlinear elements supporting each modal mass



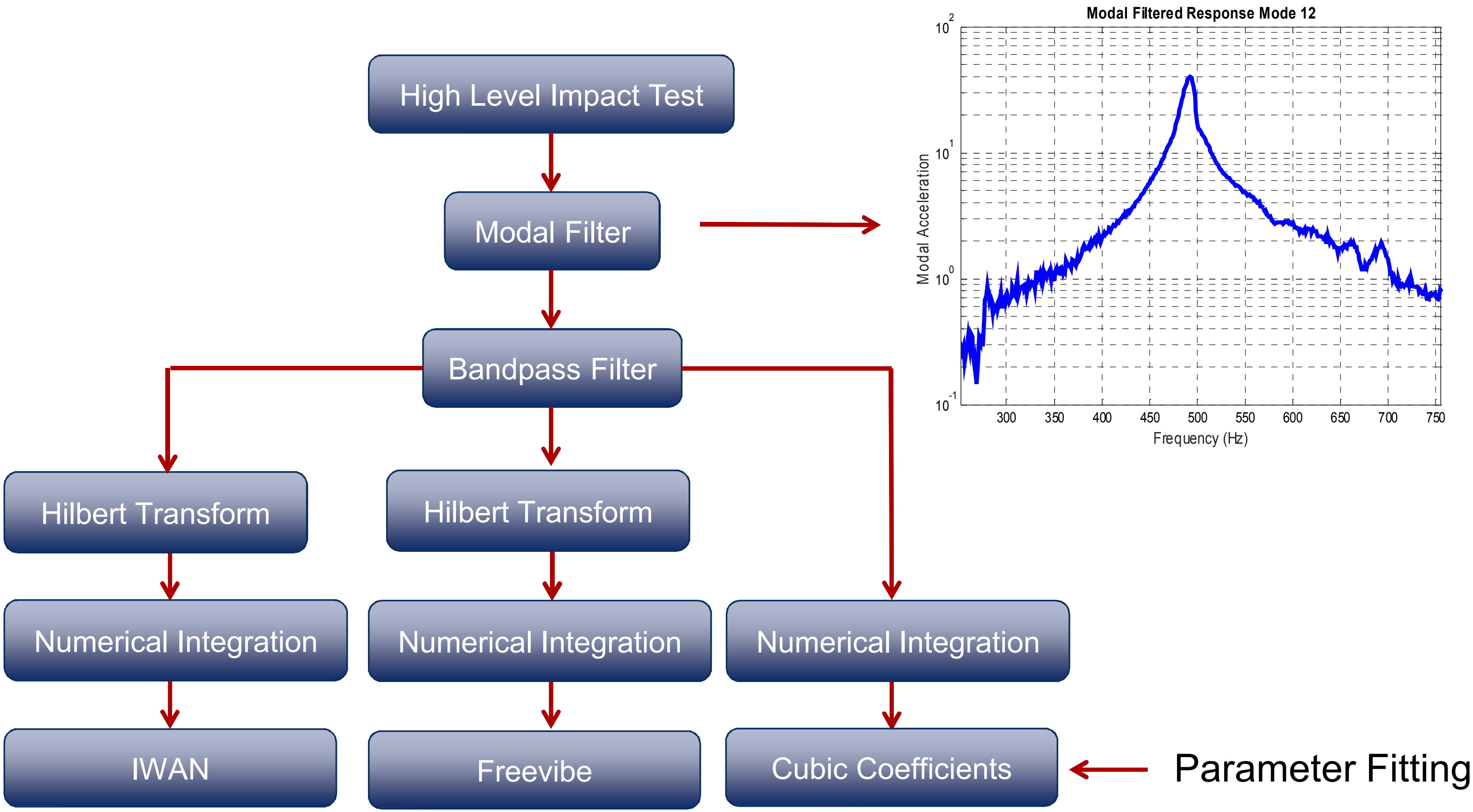
Hardware and Testing

- The pictured hardware has a
  - Nonlinear bolted joint
  - Nonlinear foam supporting an instrumented internal component
- Impact testing was performed at one axial and two lateral input locations
- A low level modal test was performed to generate a linear modal model
- High level impact data was used to identify nonlinear parameters for a nonlinear modal model





Nonlinear Identification of 3 Models For Each Mode



Modal Filter

- Necessary to isolate the nonlinear effects on each mode
- Data from all accelerometers is weighted and summed with a modal filter to obtain a single mode response

$$\overline{\mathbf{\Psi}}^T \overline{\mathbf{x}} = q_i$$

- Modal filter calculated using the Synthesize Modes And Correlate (SMAC) parameter estimation algorithm
  - SMAC obtains filter coefficients from high level FRFs, estimate of modal frequency and damping
  - Generally eliminated non-targeted modes better than the other methods
  - SMAC modal filter chosen for this work

Nonlinear Models

- All models were parameterized with 6 parameters per mode for fair comparison
- Iwan
  - $\ddot{q}(t) + C\dot{q}(t) + K_\infty q(t) = \mathbf{\Phi}^T F_{ext} + F_j$
  - F_j is a function of four parameters representing a distribution of Jenkins elements
  - Requires 6 parameters
    - C – linear damping
    - K_∞ – linear stiffness
    - F_j – function of four parameters F_s, K_T, χ, β
- FREEVIB (FV)
  - $\ddot{q}(t) + 2c(A_q)\dot{q}(t) + k(A_q)q(t) = 0$
  - Uses Hilbert Transform of free decay modal response to derive damping and stiffness as a function of amplitude
  - Parameterized to obtain cubic stiffness and damping forces
    - $k(A_q) = k_0 + k_1 A_q + k_2 A_q^2$
    - $c(A_{\dot{q}}) = c_0 + c_1 A_{\dot{q}} + c_2 A_{\dot{q}}^2$

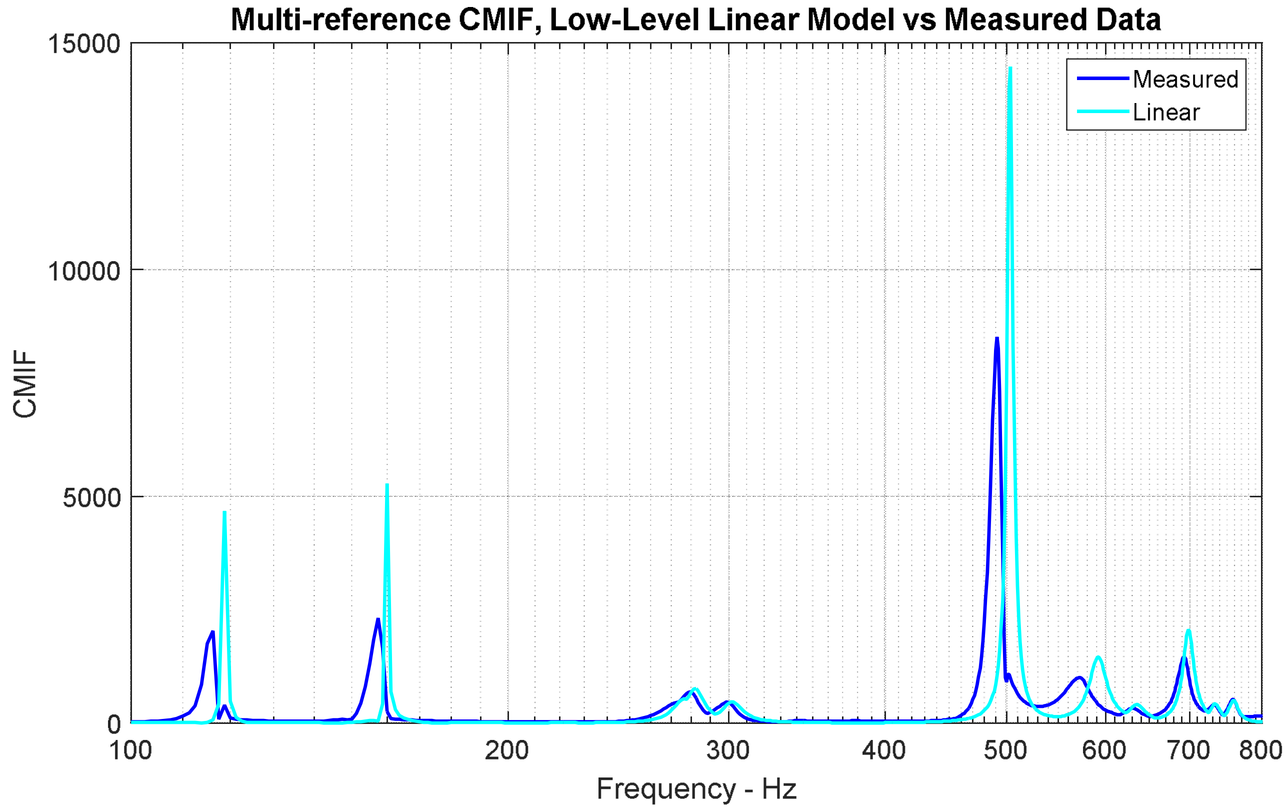
Nonlinear Models

- Restoring Force Surface (RFS)
  - $\ddot{q}(t) + F_r(q(t), \dot{q}(t)) = F(t)$
  - Since $\ddot{q}(t)$ and $F(t)$ are known, $F_r(q(t), \dot{q}(t))$ can be calculated
  - We assumed the $F_r(q(t), \dot{q}(t))$ as a cubic polynomial for damping and stiffness in terms of response amplitude.
    - $F_r(q(t), \dot{q}(t)) = c_0\dot{q}(t) + c_1|\dot{q}(t)|\dot{q}(t) + c_2\dot{q}^3(t) + k_0 q(t) + k_1|q(t)|q(t) + k_2 q^3(t)$
  - We know k_0 and c_0 from our low-level modal test frequency and damping ratio
  - Four parameters solved from linear system of equations in frequency domain

$$\begin{bmatrix} \overline{\dot{q}|\dot{q}|} & \overline{\dot{q}^3} & \overline{q|q|} & \overline{q^3} \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ k_1 \\ k_2 \end{Bmatrix} = \overline{F}_r - c_0\overline{\dot{q}} - k_0\overline{q}$$

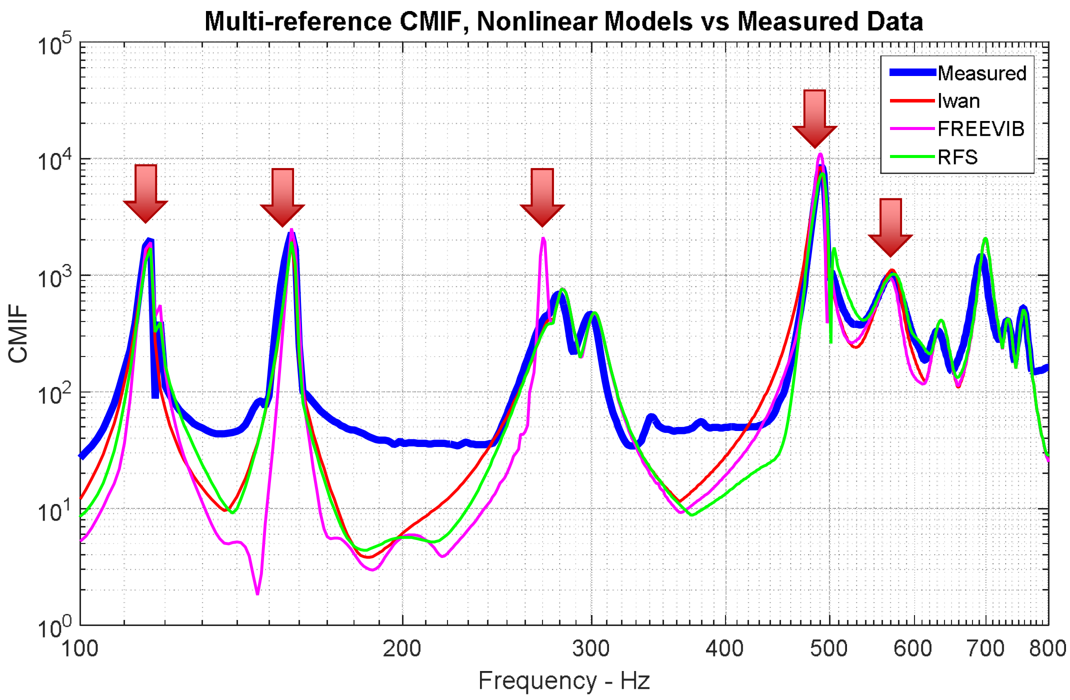
Why this matters – Linear vs Nonlinear Response

- Complex Mode Indicator Function (CMIF) for linear system compared with high level CMIF shows linear system over-predicts some modes by factor of 2



Simulation Results Compared with Measured

- Nonlinear simulations for all accelerometer locations compressed in principal CMIF – 17 modes simulated response to 800 Hz
- 5 modes modeled as nonlinear (red arrows); 12 modeled as linear (6 rigid bodies and 6 elastic)
- Either Iwan or cubic RFS model provide excellent simulation



Simulation Results Compared with Measured

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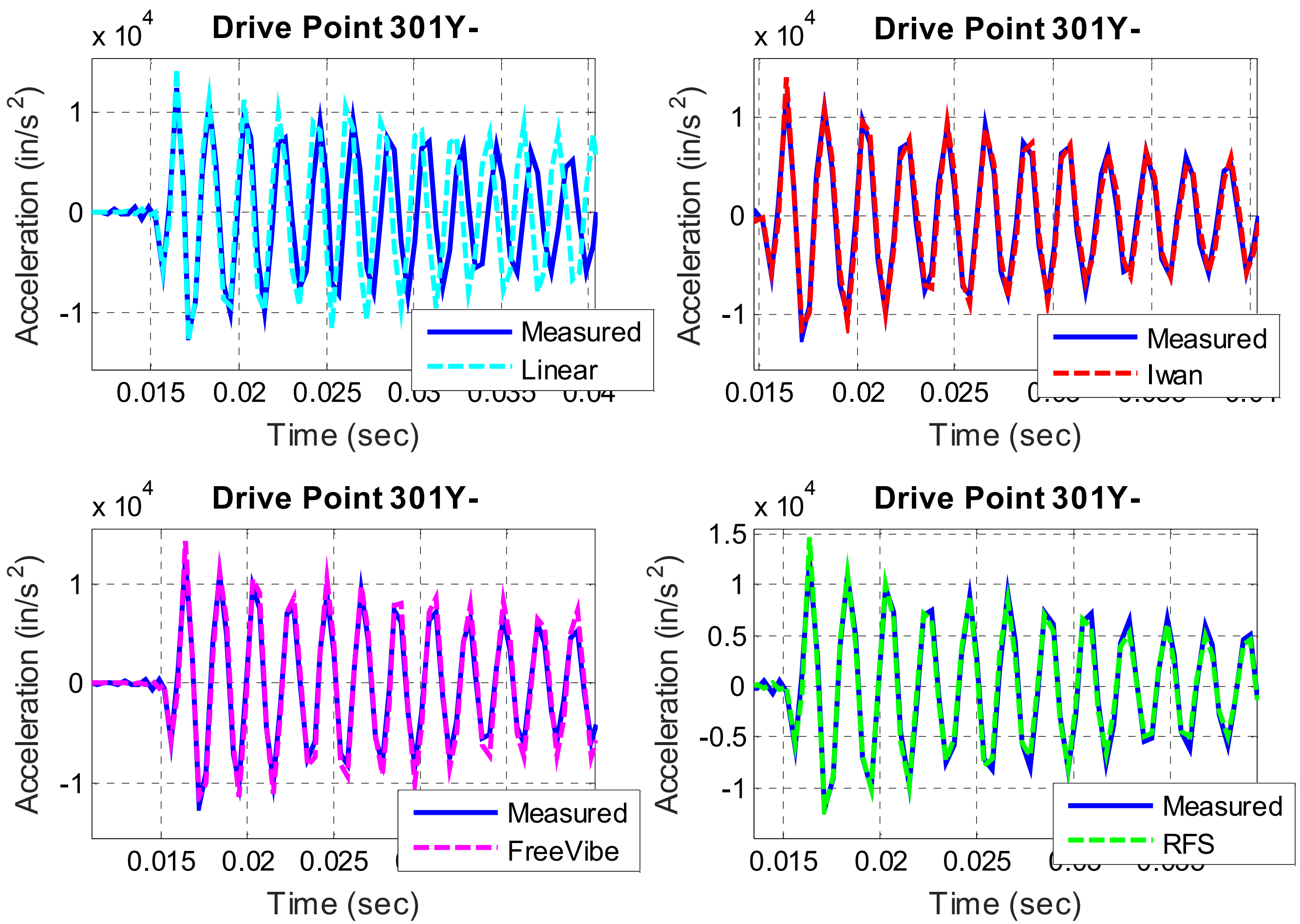
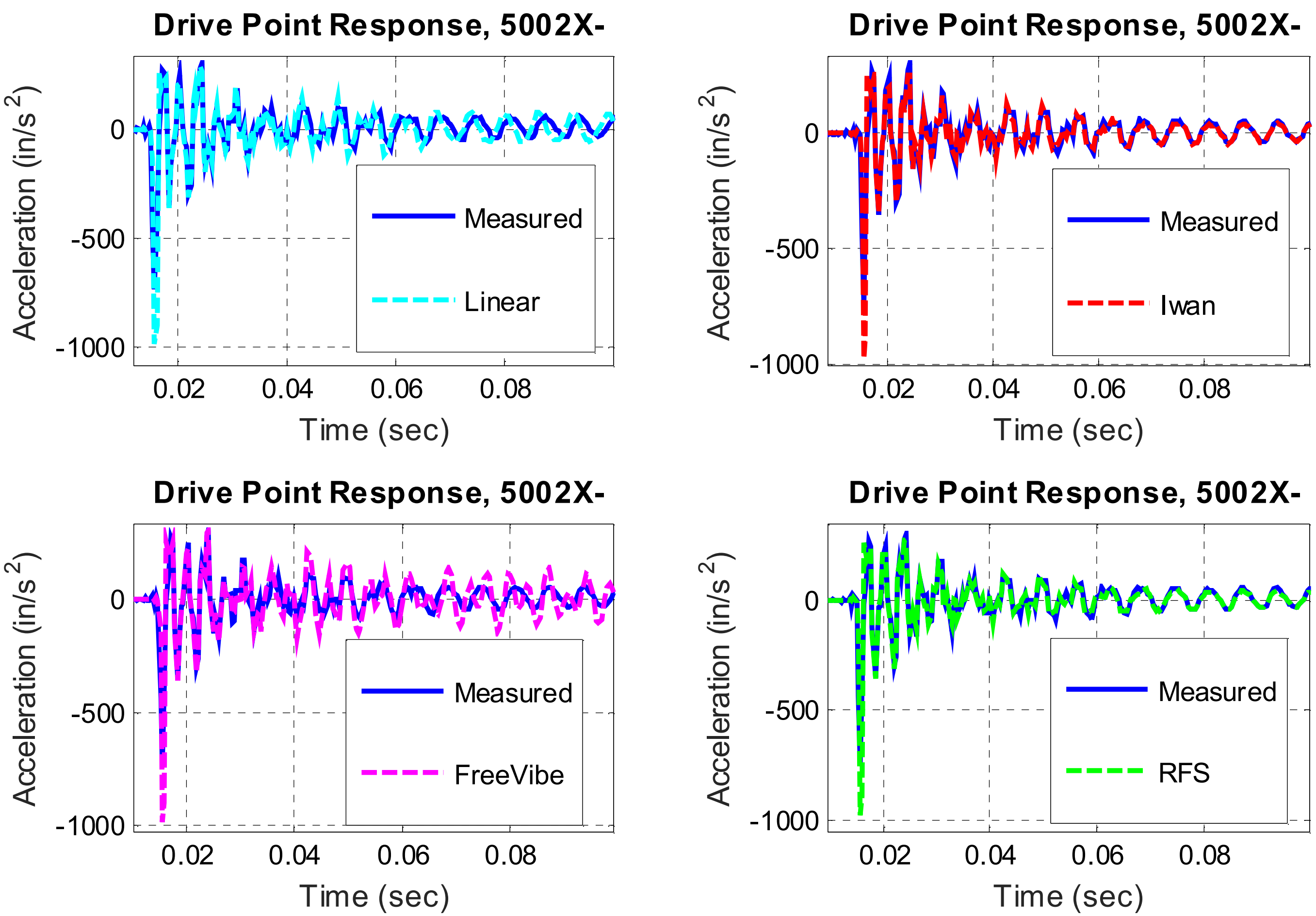
- Axial drive point accelerometer time histories look good for all nonlinear models



Simulation Results Compared with Measured

- Radial point accelerometer time histories look good for 2 of 3 nonlinear models



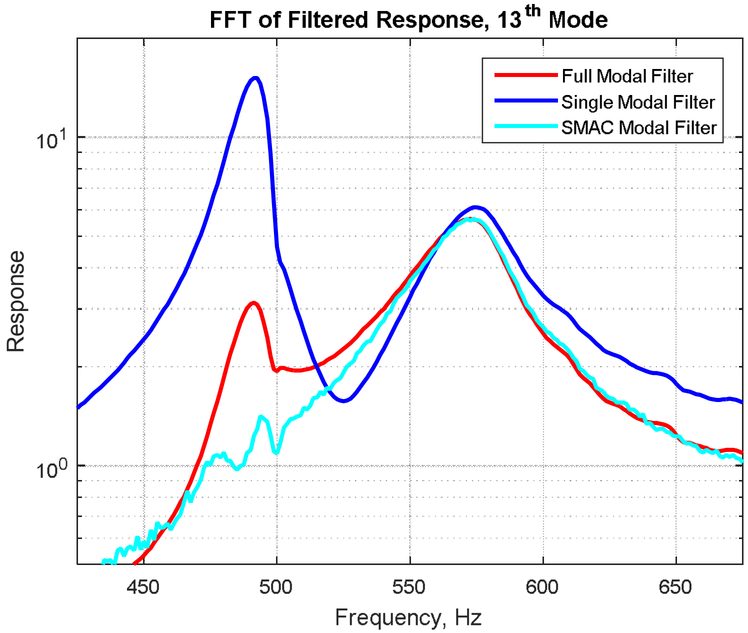
Conclusions

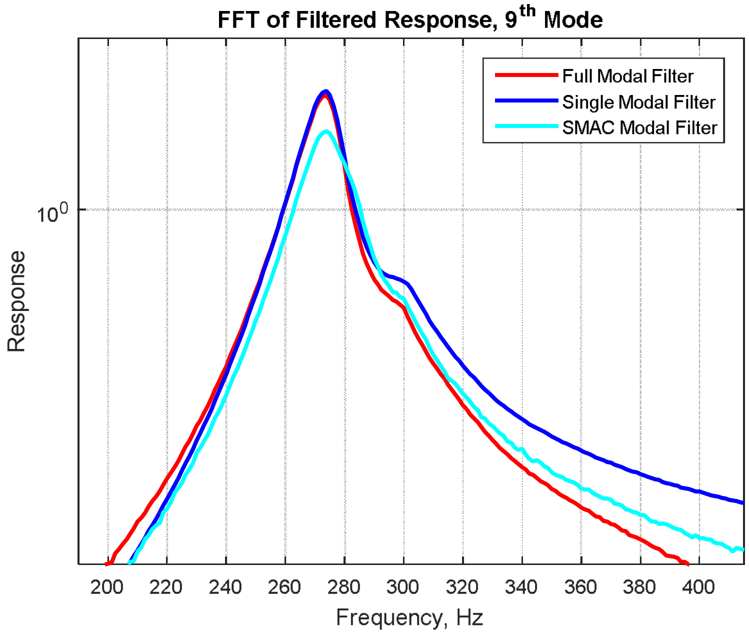
- Successfully identified and simulated nonlinearities with three pseudo-modal models that generally provided better results than the typical linear model
- Iwan and RFS provided better results than FREEVIB, but more time was invested in the former pair of models
  - With some user interaction, FREEVIB can provide comparable results
- Iwan
  - Simulation results very good
  - Required Hilbert Transform and a lot of user art to get good parameter fits
  - Understanding of parameters is complex vs other models
- FREEVIB
  - Required Hilbert Transform and some user art to get good parameter fits
  - Can only use data after force has been removed
- RFS with cubic stiffness and damping force
  - Simulation results very good
  - Does NOT require Hilbert Transform nor as much user interaction in fitting as others
  - Cubic stiffness and damping force is easy for engineers to understand

Backup Slides

Modal Filter

- Sample of results





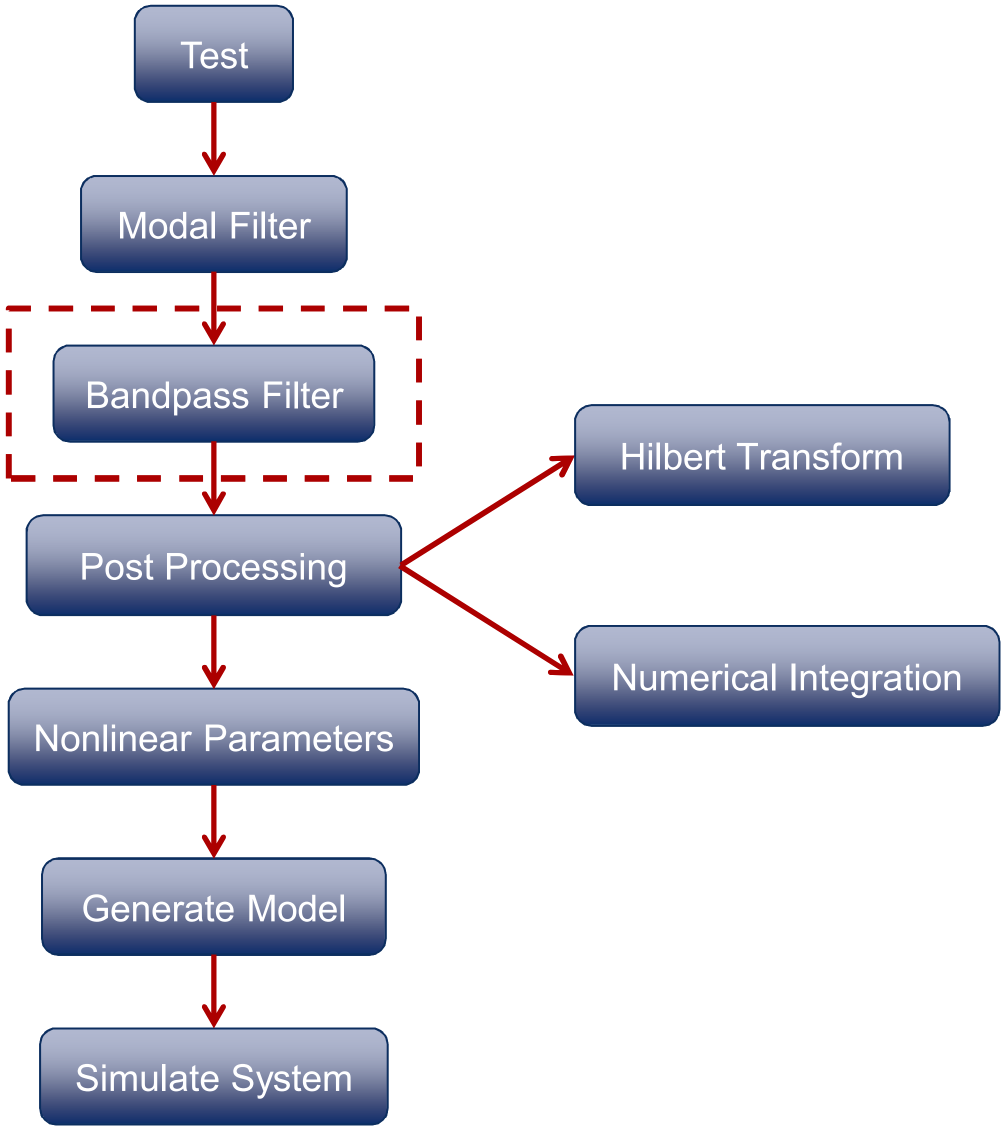
Notes

- Target Mode: 570 Hz
- FMF and SMF struggle to attenuate unwanted mode at ~490 Hz

Notes

- Target Mode: 276 Hz
- SMAC struggles to attenuate unwanted mode at 282 Hz
- All struggle with mode at 300 Hz

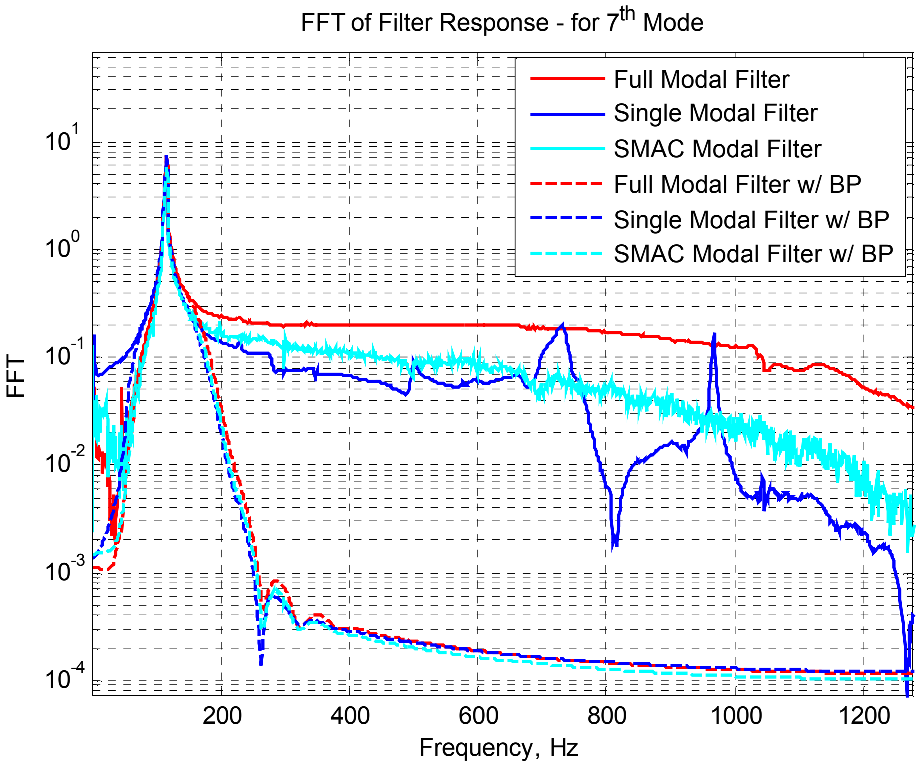
Process Flow

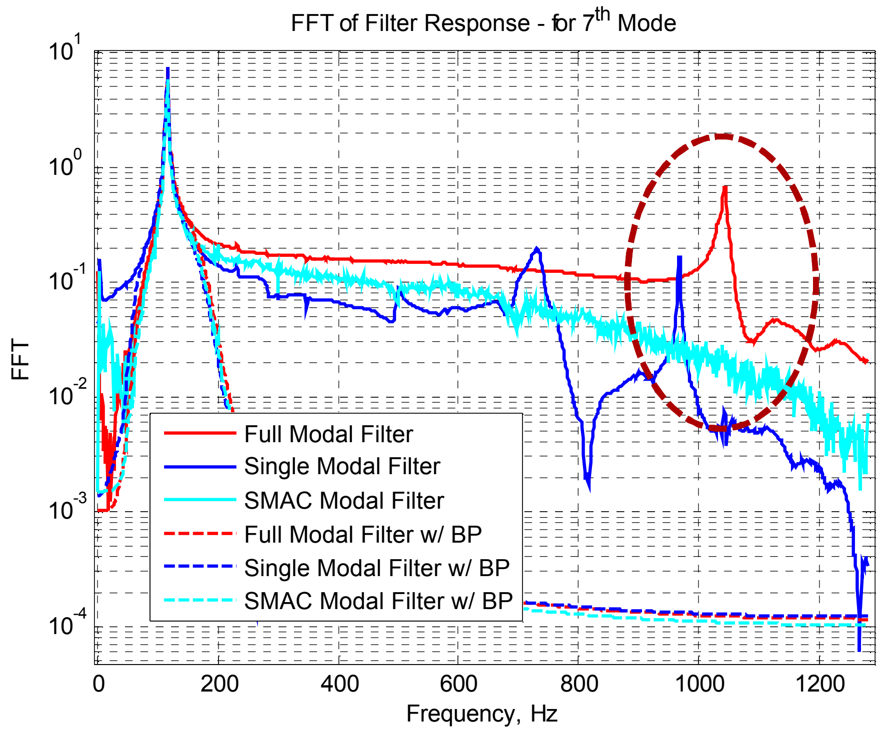


Bandpass Filtering

- Iwan and FREEVIB require the use of the Hilbert Transform in the calculation of nonlinear parameters
  - HT is very sensitive to unwanted frequency content
    - Distortions in envelope and instantaneous frequency calculations
- Bandpass filtering used to assist modal filter to further attenuate non-targeted frequency content
- Brief study conducted to determine effects on damping calculations
  - Desired outcome: passband narrow enough to eliminate unwanted frequency content without distorting damping
  - Passband varied from ±10% to ±50% of resonance
  - With SMAC modal filter, we were able to use ±50%

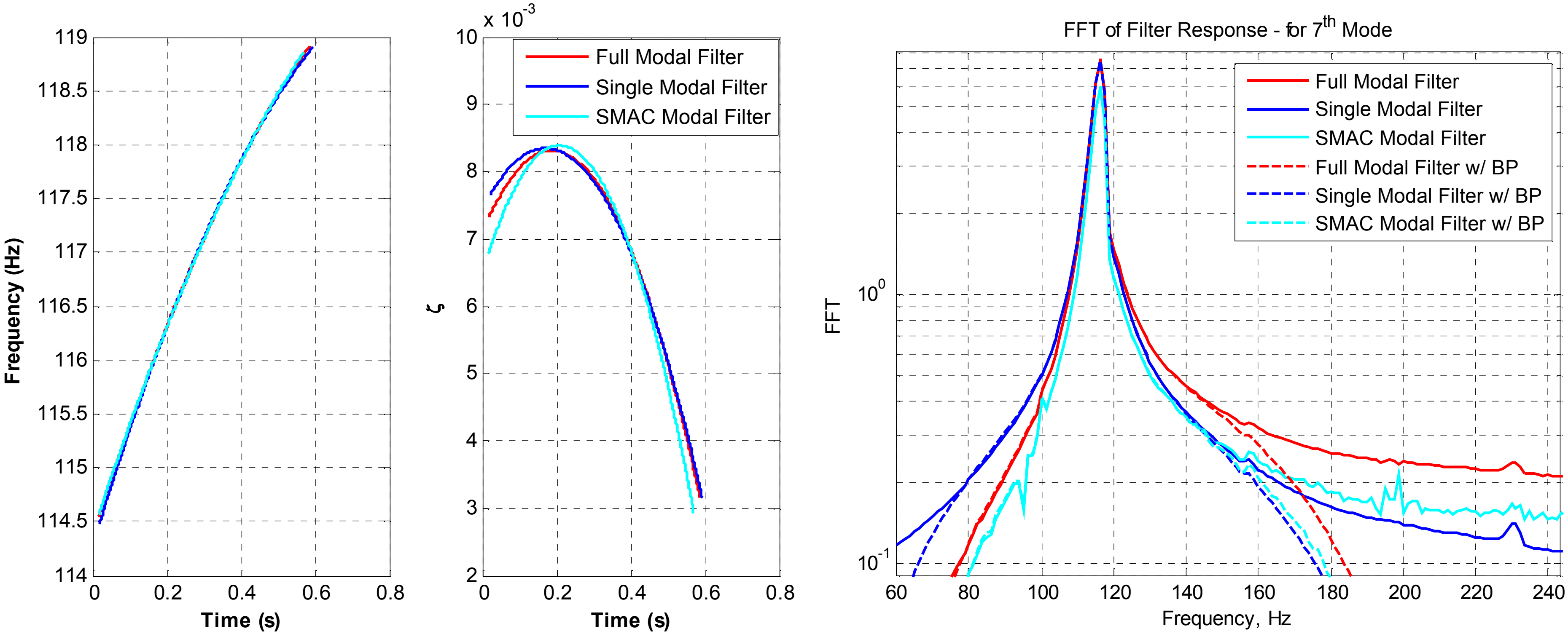
Comparing Filter Types Mode 7 – 119 Hz





- First system elastic mode (bending of the beam)
- All filters perform well especially after band-pass filtering
- High frequency content is important, sometimes modes fall between f_{max} and nyquist frequency that effect filter fits.

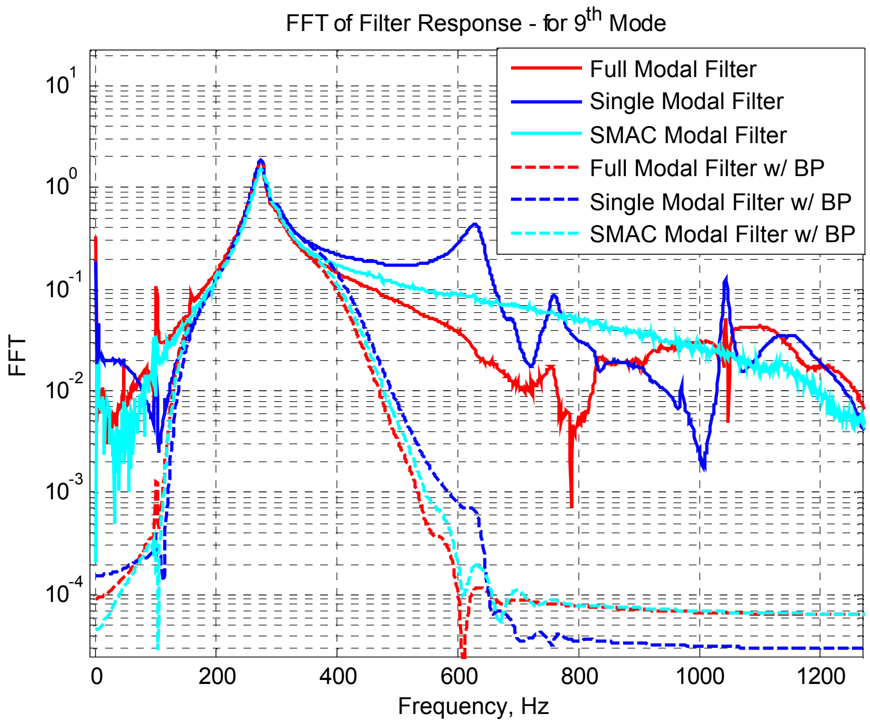
Instantaneous f_n and ζ Mode 7 – 119 Hz

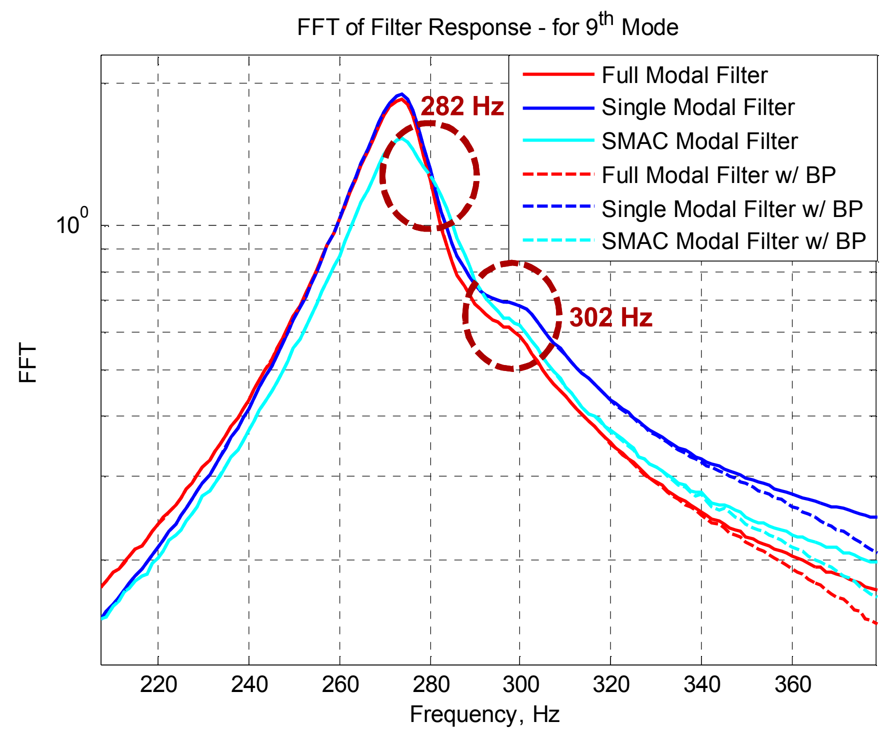


- Hilbert transform can be used to obtain instantaneous natural frequency and damping
- These results with well fit modes agree with experimental results

	f_n [Hz]	ζ
Linear Test	119.0	.0036
Non-Linear test	115.8	.0094

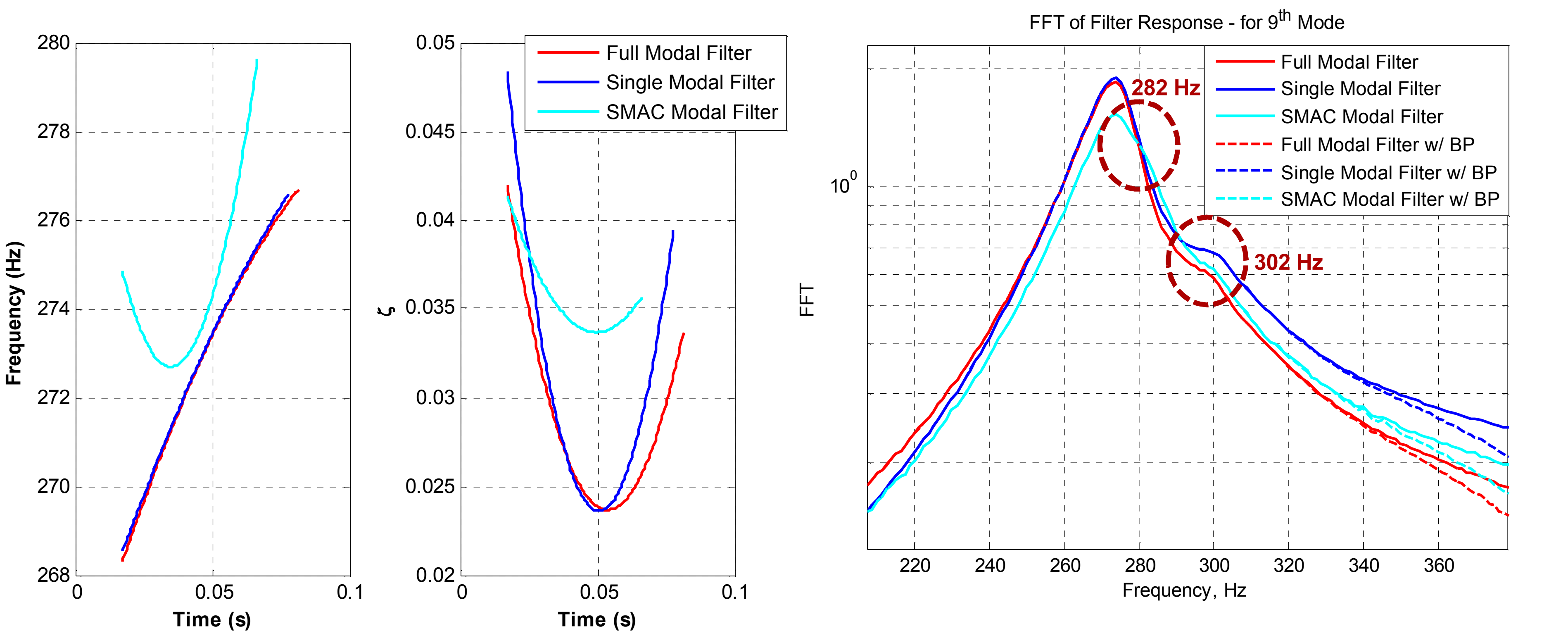
Comparing Filter Types Mode 9 – 276 Hz





- Internals torsion mode
- Full and Single (red and blue) Modal Filters ignore contamination from 282 Hz mode but SMAC Modal Filter (cyan) struggles to knock it out.
- All Filters still see slight contamination from 302 Hz mode.

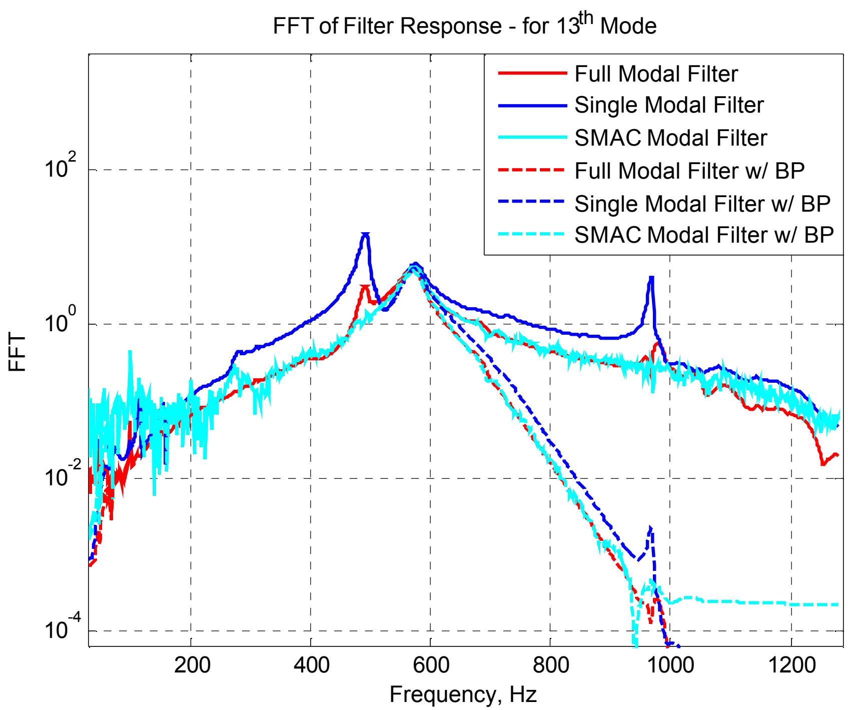
Instantaneous f_n and ζ Mode 9 – 276 Hz

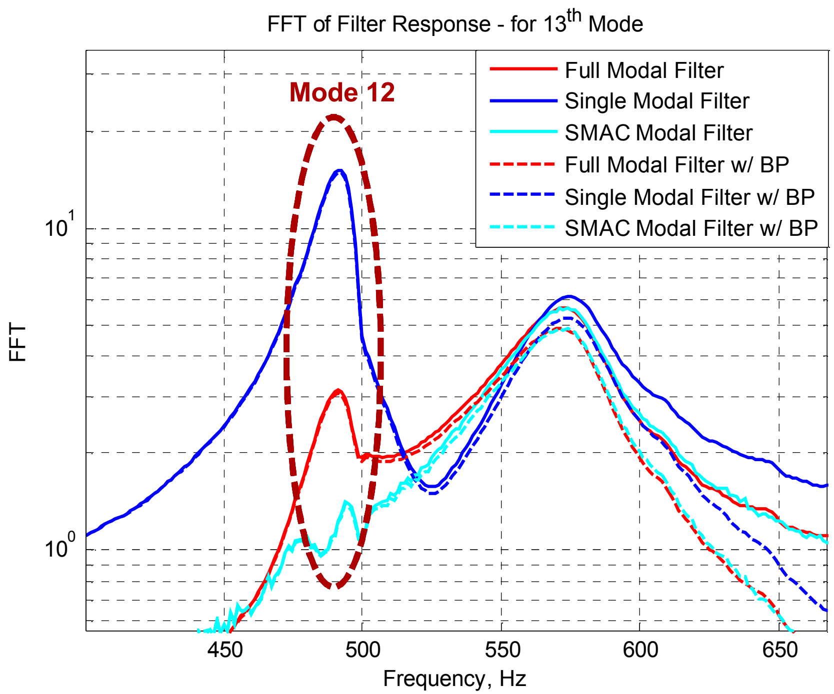


- Damping is actually linear but we find a parabola (due to cubic fit)
- Frequency on ψ Filter gets pulled up due to neighboring mode form FFT

	f_n [Hz]	ζ
Linear Test	276.0	.0246
Non-Linear test	272.8	.0239

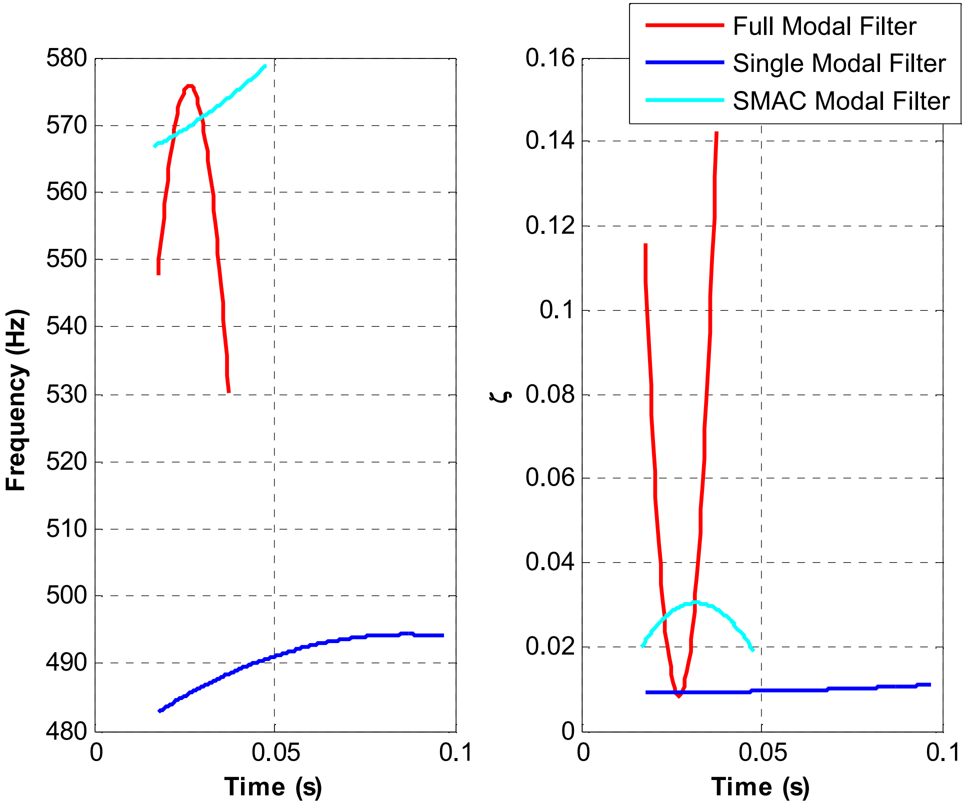
Comparing Filter Types Mode 13 – 592 Hz

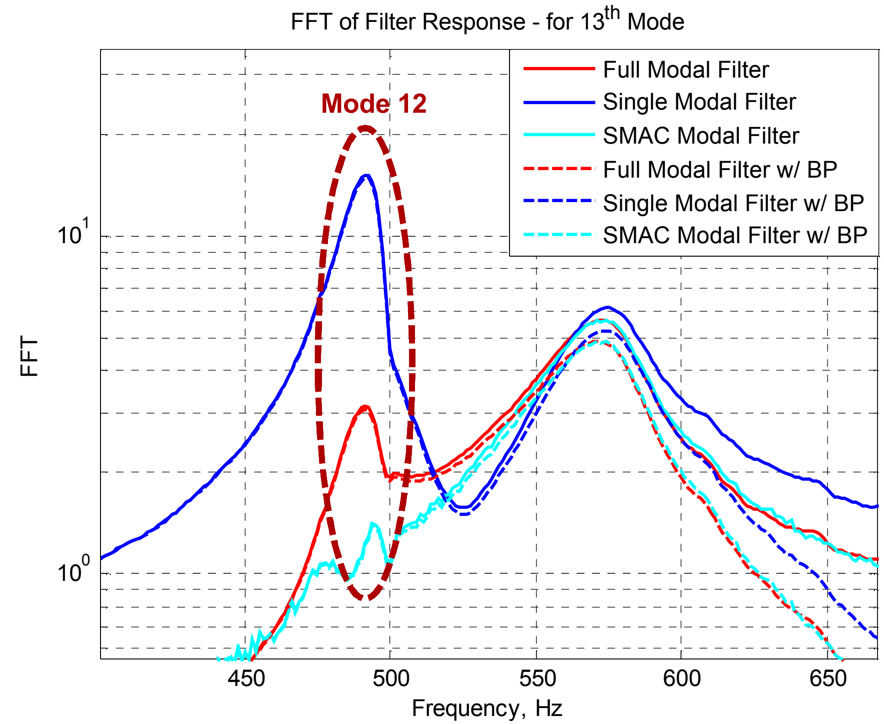




- Rotation of internals about Z
- Full and Single (red and blue) Modal Filters have contamination from Mode 12 but SMAC Modal Filter (cyan) knocks it out.

Instantaneous f_n and ζ Mode 13 – 592 Hz

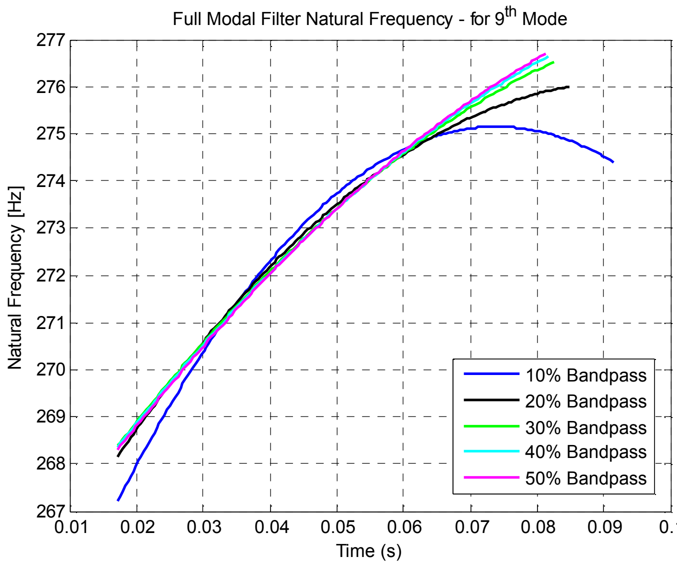


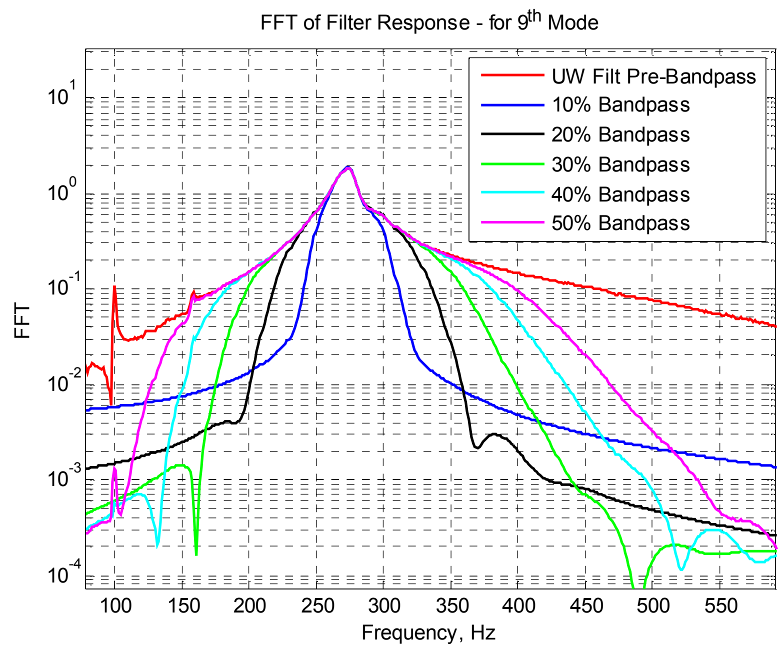


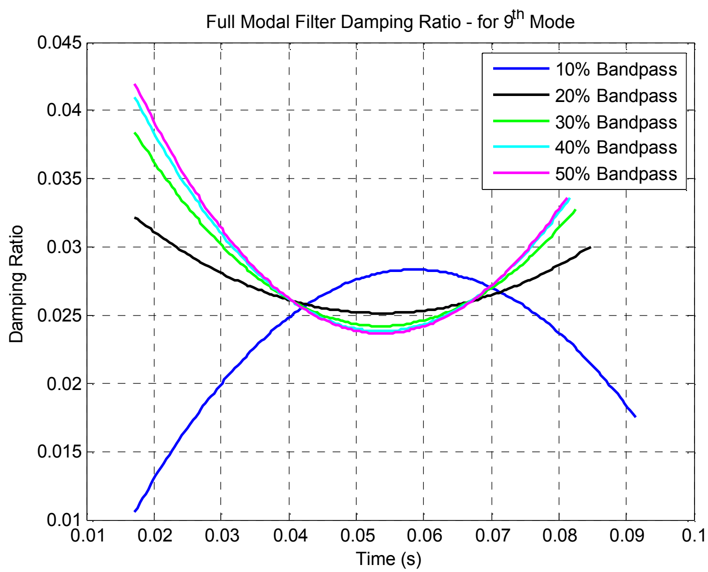
- Complete mode shape ϕ^+ (red) Filter competes between two peaks while (blue) Filter is dominated by contamination from 503 Hz mode but ψ Filter (cyan) adequately knocks it out and gives reasonable results

	f_n [Hz]	ζ
Linear Test	592.0	.0202
Non-Linear test	570.1	.0291

Band Pass Convergence

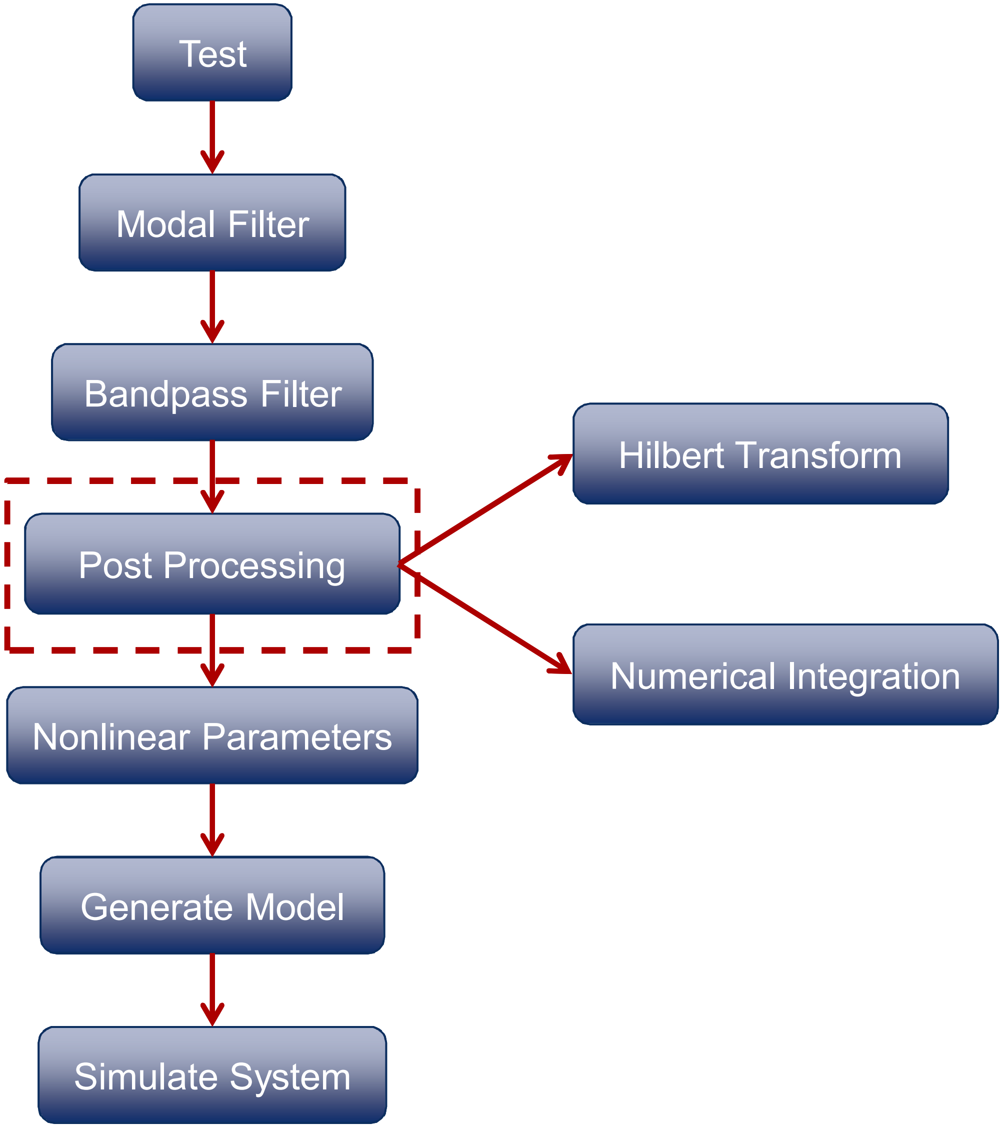






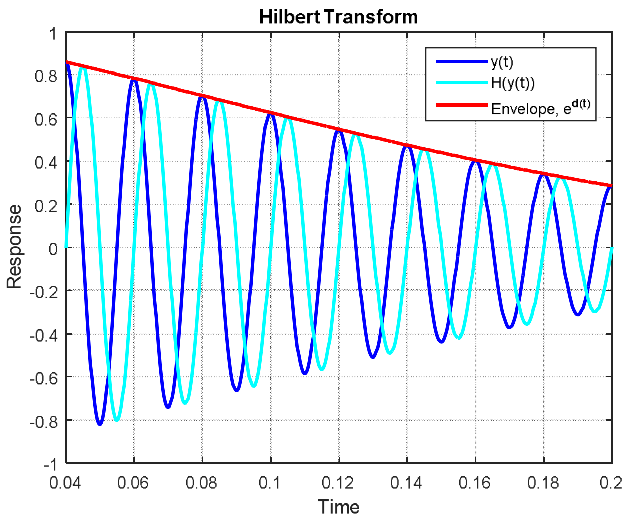
- Modal parameters sensitivity to band pass windows was investigated
- With too narrow of a passband (blue) the damping struggle to fit properly
- Too high of a passband is sometimes unacceptable depending on the quality of the modal filter

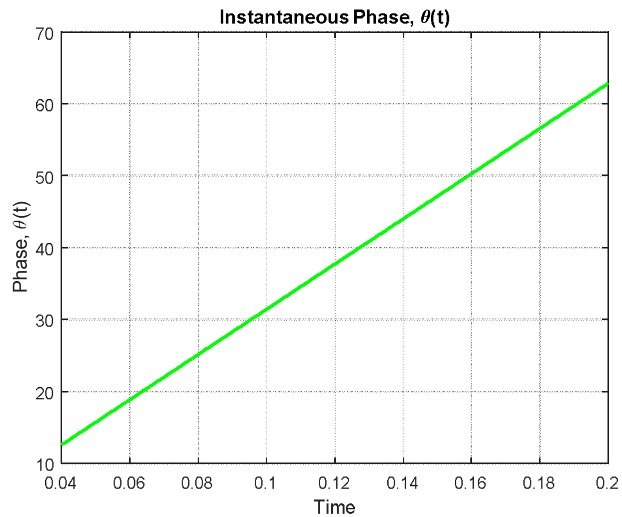
Process Flow



Post Processing: Hilbert Transform

- The Hilbert Transform is a 90 degree phase shift that facilitates calculation of instantaneous amplitude and frequency of a signal as functions of time
- Iwan and FREEVIB require the use of the HT to determine damping and natural frequency as a function of response amplitude
  - $\ddot{q}(t) = e^{d(t)}\cos[\theta(t)]$
    - $e^{d(t)}$ is the instantaneous amplitude, aka envelope (red)
    - $\theta(t)$ is the instantaneous phase (green) (Note: $\omega_d(t) \triangleq \dot{\theta}(t)$)





- For measured data, d(t) and θ(t) must be fit with polynomials
  - We fit these terms with cubic polynomials

Hilbert Transform

The Hilbert Transform fits a time signal into a time dependent decaying envelope [$e^{d(t)}$] and a time dependent phase [$\cos(\phi(t))$]

$$q(t) = e^{d(t)}\cos(\phi(t))$$

We fit these functions to cubic polynomials in order to simulate our system.

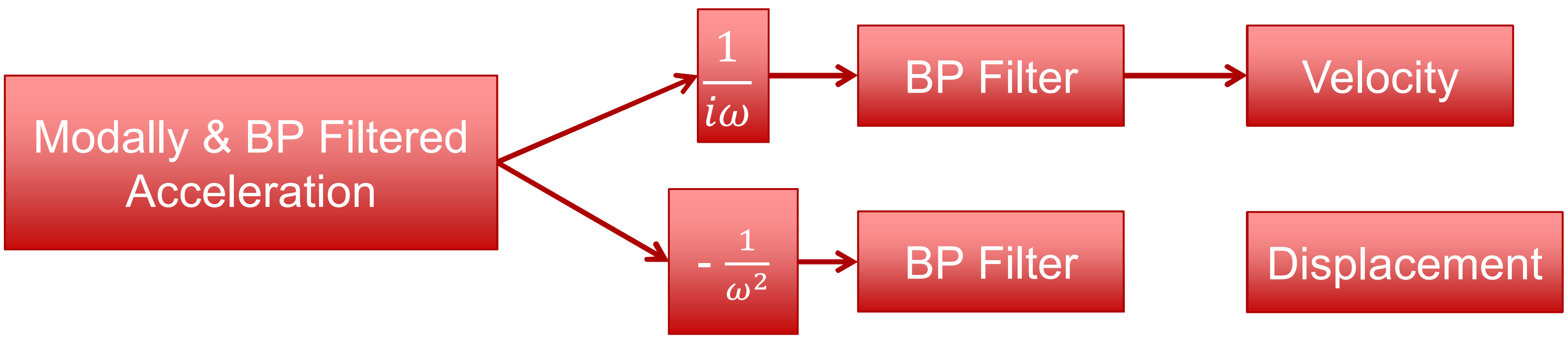
$$q(t) = e^{a_0 + a_1\left(\frac{t}{t_{max}}\right) + a_2\left(\frac{t}{t_{max}}\right)^2 + a_3\left(\frac{t}{t_{max}}\right)^3} \cos\left(b_0 + b_1\left(\frac{t}{t_{max}}\right) + b_2\left(\frac{t}{t_{max}}\right)^2 + b_3\left(\frac{t}{t_{max}}\right)^3\right)$$

Instantaneous damping and frequency can then be computed from these polynomials in order to characterize a non-linear model to best simulate the original data

f_n and ζ can be found by manipulating the $d(t)$ and $\phi(t)$ polynomials

Post Processing: Numerical Integration

- Restoring Force Surface method does not require the use of the HT
- RFS requires displacement, velocity, and acceleration at each time instant
- We estimated displacement and velocity from acceleration by integrating in the frequency domain



Process Flow