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Analytical Method to Determine a Tip Loss Factor for Highly-Loaded Wind Turbines

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Analytical Method to Determine a Tip Loss Factor for Highly-Loaded Wind Turbines

- "Background & Motivation"
- "Analytical Tip Loss Factor for Given Φ"
- "Numerical Methods"
  - BEMT Method (*XTurb-PSU*)
  - Higher-Order Free-Wake Method (*WindDVE*)
- "Results - NREL Phase VI Rotor"
  - Sequences T,U,J,S,X

“Background & Motivation”

– Today’s BEMT methods use classical tip loss factor due to Prandtl (or Glauert correction)

– Persistent issue of over-predicting blade tip loads

– **Why ?** Classical tip loss factor assumes ‘light loading’, i.e. no tip vortex rollup.

– Some work since 2000: Sørensen & Shen (2005), Lindenberg (2008), Branlard et al. (2012), …

– **This work has a new idea !**

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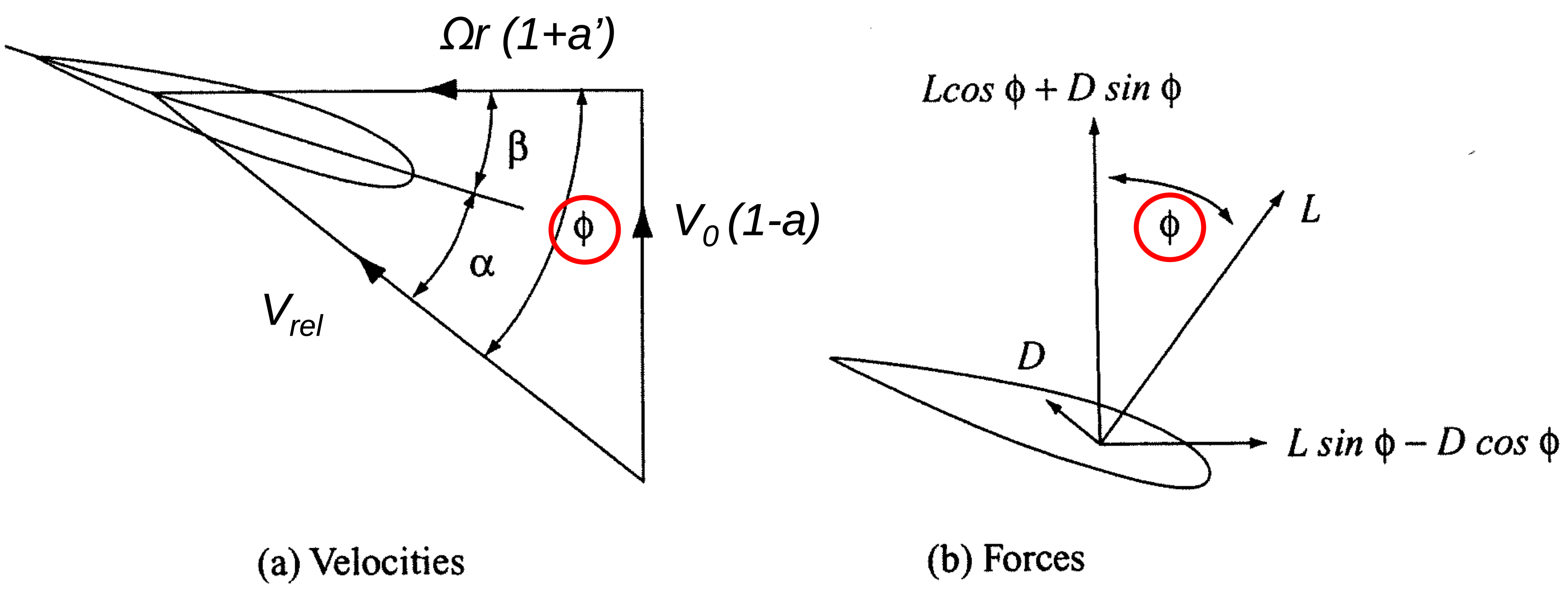
"Analytical Tip Loss Factor for Given Φ"

Section of a Wind Turbine Blade



Φ = Blade Flow Angle

α = Angle of Attack β = Blade Twist Angle

“Analytical Tip Loss Factor for Given Φ”

The common tip loss factor (Glauert correction)

$$F_T = \left(\frac{2}{\pi}\right)\cos^{-1}\left[\exp\left(-\left\{\frac{\left(\frac{BN}{2}\right)\left[1-\frac{r}{R}\right]}{\left(\frac{r}{R}\right)sin\Phi}\right\}\right)\right]$$

Can you solve directly a new F_T from a more accurate Φ ?

- **No**, because Φ is coupled to a, a’, and F_T through the BEMT equations.
- The above equation alone will **not** lead to a new F_T.

“Analytical Tip Loss Factor for Given Φ”

‘Ansatz’ for an extended tip loss factor

Concept of *g* function:
[Sørensen & Shen, 2005]

$$F_T = \left(\frac{2}{\pi}\right)\cos^{-1}\left[\, \boldsymbol{g}\, \exp\left(-\left\{\frac{\left(\frac{BN}{2}\right)\left[1-\frac{r}{R}\right]}{\left(\frac{r}{R}\right) sin\Phi}\right\}\right)\right]$$

Introduce the g function :

- Find an “analytical” solution for the correct F_T for a given Φ.
- Then solve for the *g* function & implement into BEMT code.

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"Analytical Tip Loss Factor for Given Φ"

How do you do that ?

$$dT = \rho {V_o}^2 4a(1-a)\pi r dr$$

$$dQ = \rho V_o 4a'(1-a)\pi r^3 \Omega dr$$

Momentum Theory

$$dT = B\frac{1}{2}\rho V_{rel}^2 (c_l \cos(\phi) + c_d \sin(\phi))c\, dr$$

$$dQ = B\frac{1}{2}\rho V_{rel}^2 (c_l \sin(\phi) + c_d \cos(\phi))cr\, dr$$

Blade Element Theory

"Analytical Tip Loss Factor for Given Φ"

– Combined these are 2 equations for 2 unknowns

– Typically, we solve iteratively for a and a'.

$$a = \left[\frac{1 + 4F(r)\sin^2(\phi)}{\sigma'(C_l cos\phi + C_d \sin\phi)}\right]^{-1}$$

$$a' = \left[\frac{-1 + 4F(r)sin(\phi)\cos(\phi)}{\sigma'(C_l sin\phi - C_d \cos\phi)}\right]^{-1}$$

– **But … can we solve for V_{rel}/V_0 and F_T instead ?**

… believe it or not. YES !

"Analytical Tip Loss Factor for Given Φ"

- For a given Φ $\gamma_1 = \sigma'(C_l \cos\phi + C_d\, sin\phi)$

  $\gamma_2 = \sigma'(C_l \sin\phi + C_d\, sin\phi)$

- Transform to obtain 2 equations for 2 other unknowns

$$4\sin(\Phi)\left(1-\boxed{\frac{V_{rel}}{V_0}}\sin(\Phi)\right)F(r) - \gamma_1 \boxed{\frac{V_{rel}}{V_0}} = 0$$

$$-4\sin(\Phi)\left(1-\frac{1}{\lambda_r}\boxed{\frac{V_{rel}}{V_0}}\cos(\Phi)\right)\lambda_r F(r) - \gamma_2 \boxed{\frac{V_{rel}}{V_0}} = 0$$

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"Analytical Tip Loss Factor for Given Φ"

- Solve to obtain …

$$V_{rel} = \frac{\gamma_1 \lambda_r + \gamma_2}{\gamma_1 \cos\phi + \gamma_2 \sin\phi}$$

- Total Loss Factor

$F = F_a = F_b$

$$F_a = \frac{\gamma_1 V_{rel}}{4 sin\phi(1 - V_{rel} sin\phi)}$$

$$F_b = \frac{-\gamma_2 V_{rel}}{4 sin\phi(\lambda_r - V_{rel} cos\phi)}$$

$F = F_R \times F_T^*$

= (Root Loss Factor) x (New Tip Loss Factor)

“Analytical Tip Loss Factor for Given Φ”

- Using … $F(r) = F_T^* \cdot F_R$

$$F_T^* = \frac{2}{\pi}\cos^{-1}\left[\exp\left(-\underline{\boldsymbol{g}(r)}\left\{\frac{B}{2}\left[1-\frac{r}{R}\right] / \left(\frac{r}{R}\sin(\Phi)\right)\right\}\right)\right]$$

$$F_R = \frac{2}{\pi}\cos^{-1}\left[\exp\left(-\left\{\frac{B}{2}\left[\frac{r}{R}-\frac{r_R}{R}\right] / \left(\frac{r_R}{R}\sin(\Phi)\right)\right\}\right)\right]$$

- Solve for *g* function

$$\boxed{\boldsymbol{g}(r) = -\frac{2\sin(\Phi)\, r/R}{B(1-r/R)}\, ln\left[\cos\left(\frac{\pi}{2}F(r)/F_R(r)\right)\right]}$$

Note: One obtains $g(r) = 1$ for classical BEMT solution.

Significance of Analytical Tip Loss Factor :

1. A BEMT tip loss factor can be computed directly from high-fidelity solutions that provide blade flow angle Φ
2. Methodology can be applied to any higher-fidelity analysis, e.g. free-wake and CFD methods
3. Simple *g* function can be used in BEMT analyses to account for tip vortex rollup w/o computational overhead
4. Potential to find ‘universal’ *g* function for highly-loaded wind turbines

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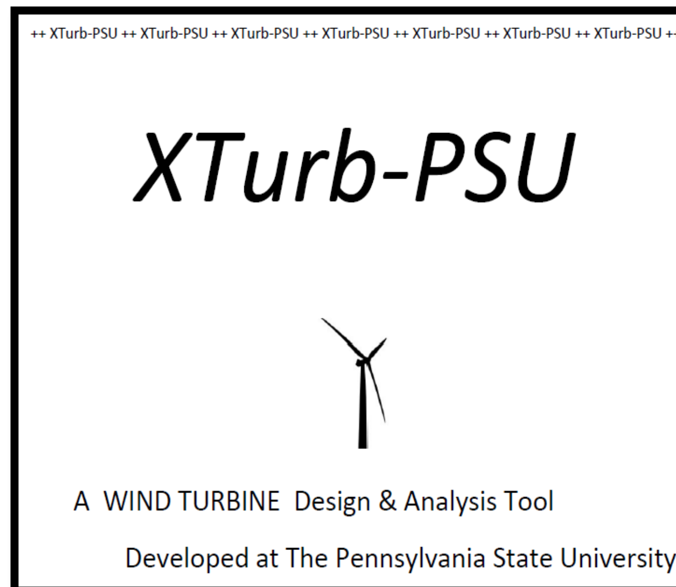
XTurb-PSU

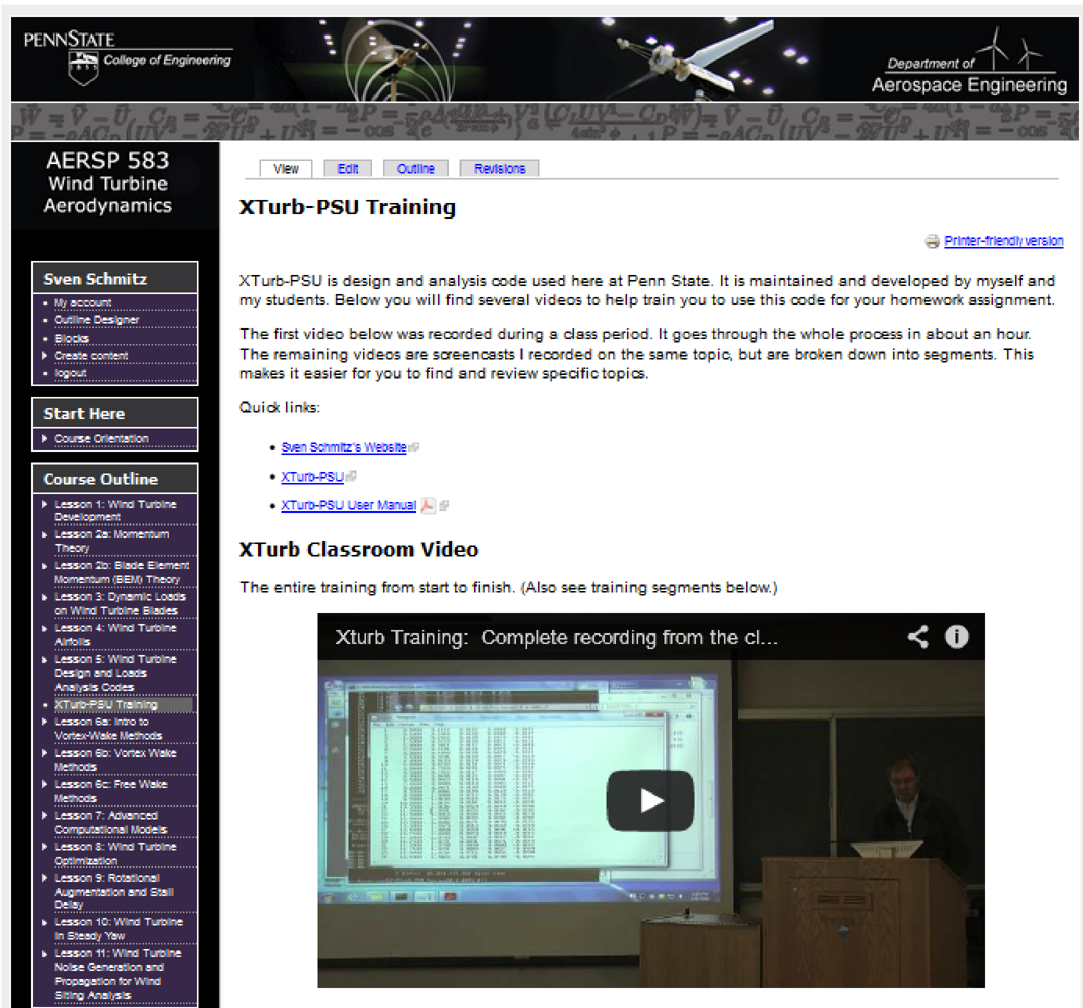
A Wind Turbine Design & Analysis Code

- **Computational Methods**
  - Blade-Element Momentum Theory (BEMT)
  - Helicoidal Vortex Method (HVM)
    [Chattot, *Computers & Fluids*, 2003]
- **Other Features**
  - VITERNA correction
  - Du & Selig stall delay model
  - Solution-Based Stall Delay (SBSD)
    [Dowler & Schmitz, *Wind Energy*, 2015]



XTurb-PSU

A Wind Turbine Design & Analysis Code



XTurb-PSU Training

Introduction
http://youtu.be/ILrvo7HN0HI

Blade Input List
http://youtu.be/mnIUiRf4rho

Blade Operation List
http://youtu.be/8WKRvnhnruk

Operation Modes
http://youtu.be/Y59VhVY77x0

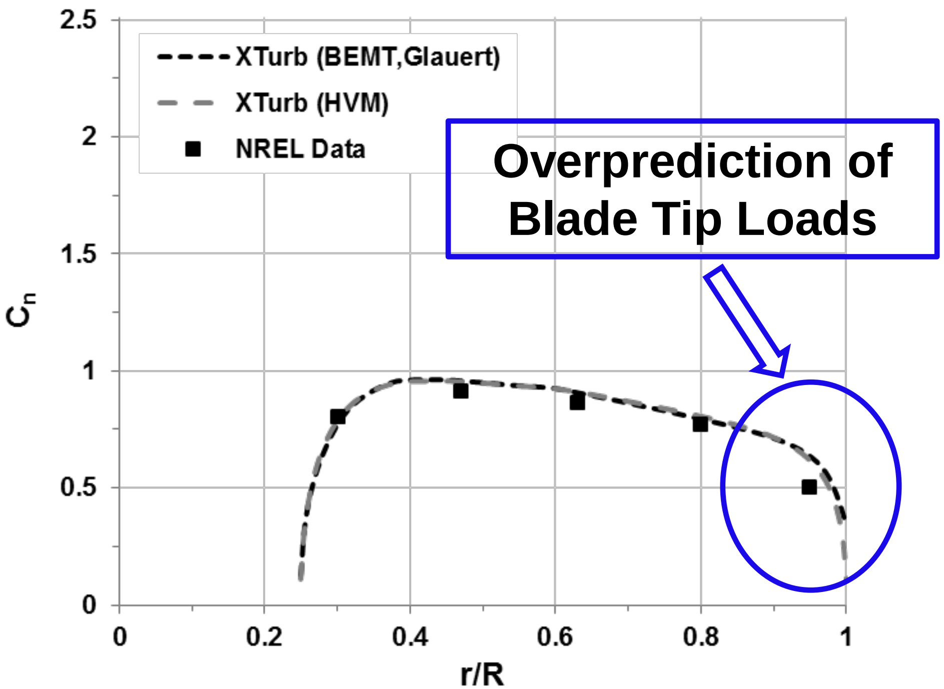
Solver Settings
http://youtu.be/AjOgLStyjy8

Running Cases
http://youtu.be/62uNH8ogjbo

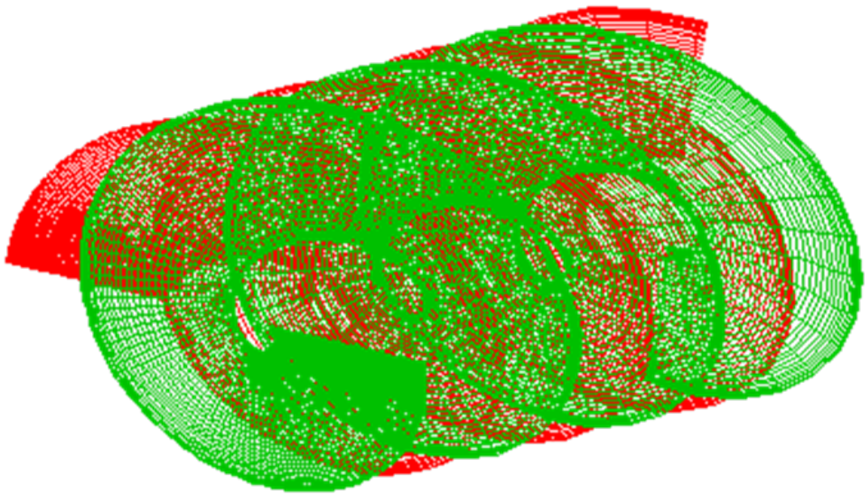
XTurb-PSU

A Wind Turbine Design & Analysis Code

- Why do we need an analytical *g* function ?



Normal force coefficient, Cn



Prescribed vortex structure, XTurb (HVM)

NREL Phase VI rotor (S-Sequence, V_0 = 7m/s, TSR = 5.42)

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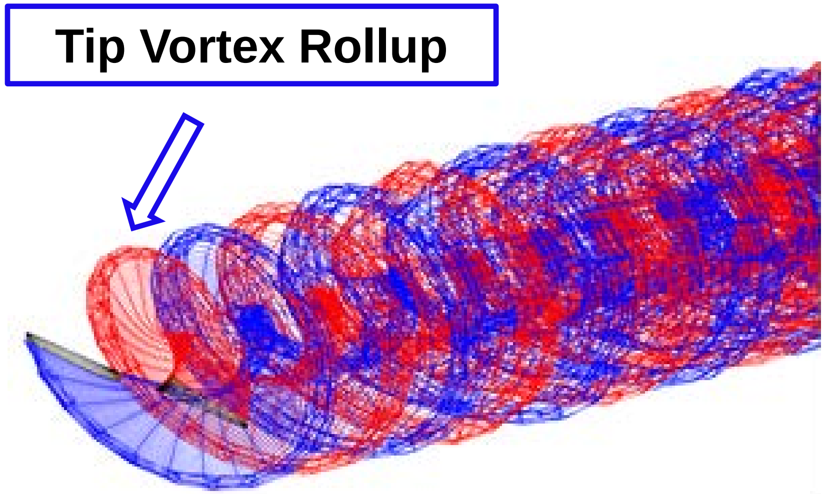
WindDVE

A Free-wake Vortex Method Wind Turbine Design & Analysis Code

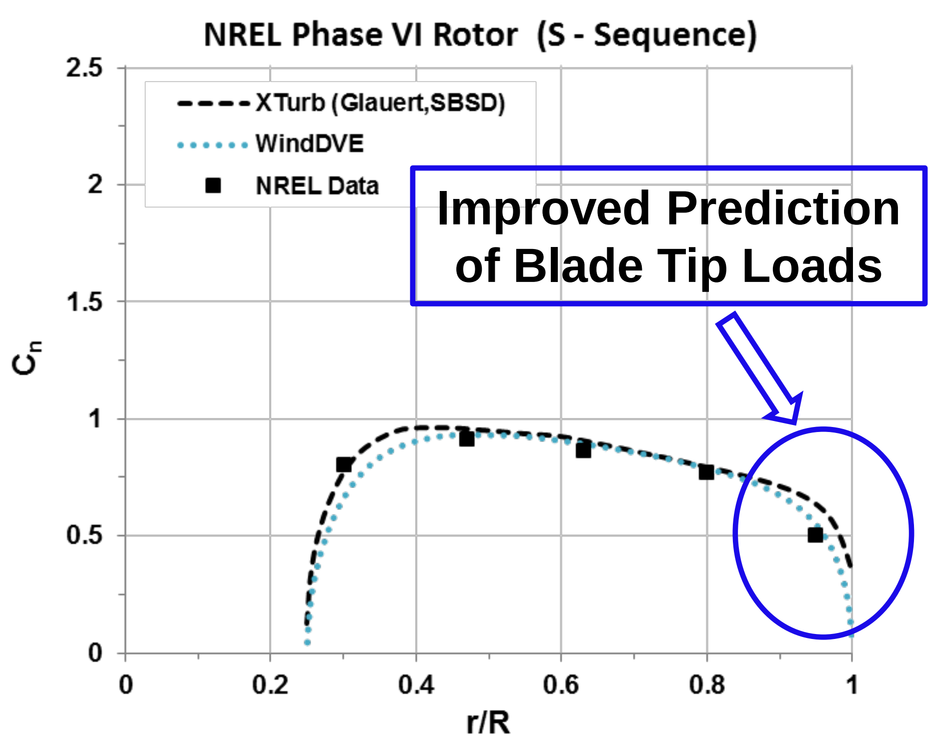
- **Computational Methods**
  - Multiple lifting-line vortex elements of Hortsmann with spanwise parabolic circulation
  - Blade-element corrections account for profile drag and stall
  - Distributed vorticity elements (DVEs) used to model wake vorticity distribution and advection

    [Bramesfeld and Maughmer, *Journal of Aircraft*, 2008]
  - **Computes Vortex Rollup !**

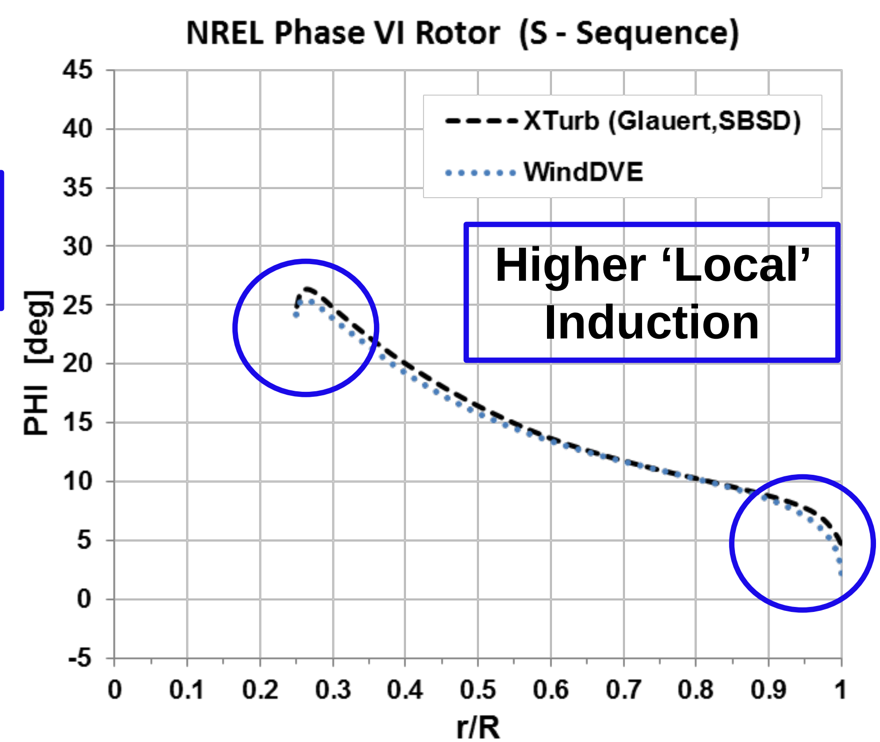


Free Wake vs. BEMT

- Effect of Computing 'Tip Vortex Rollup'



Normal force coefficient, Cn

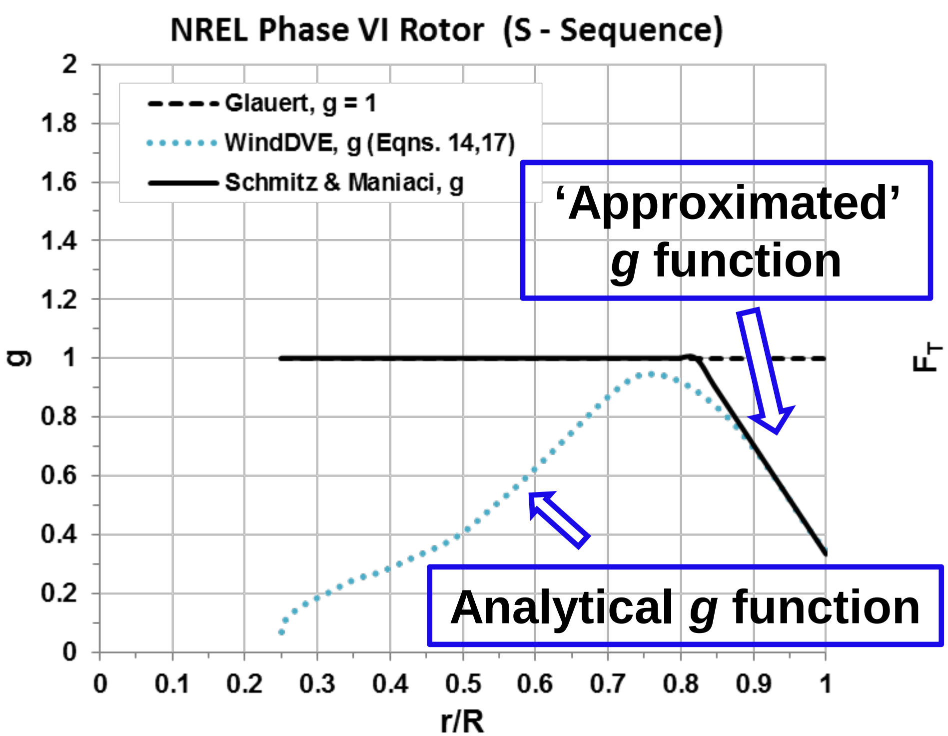


Blade flow angle, ϕ

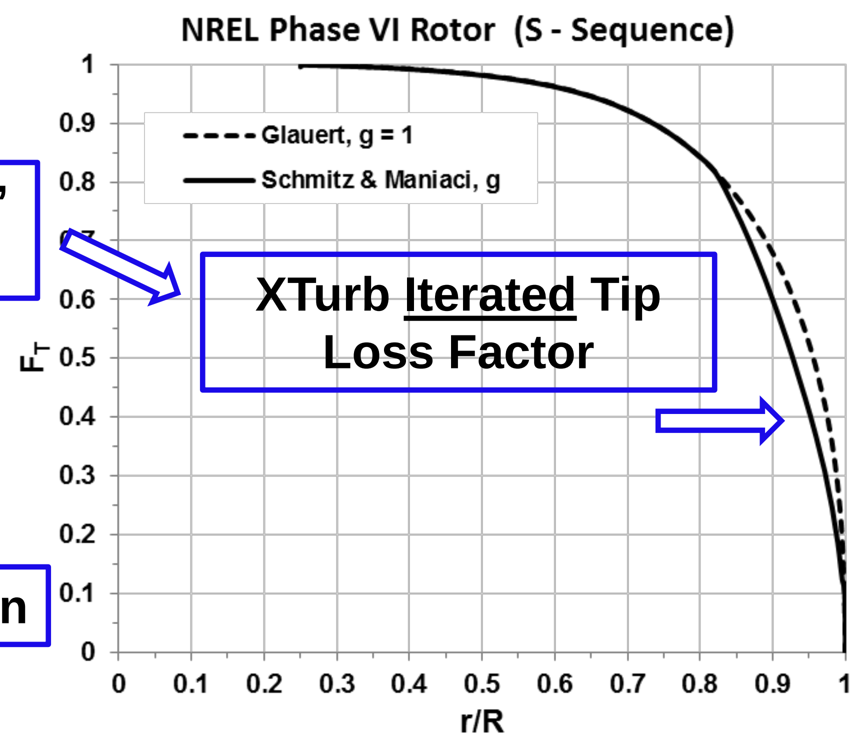
Baseline comparisons - TSR = 5.42, V_0 = 7m/s

Free Wake vs. BEMT

- Modify BEMT (XTurb) to include 'Tip Vortex Rollup'



g function, computed from WindDVE ϕ

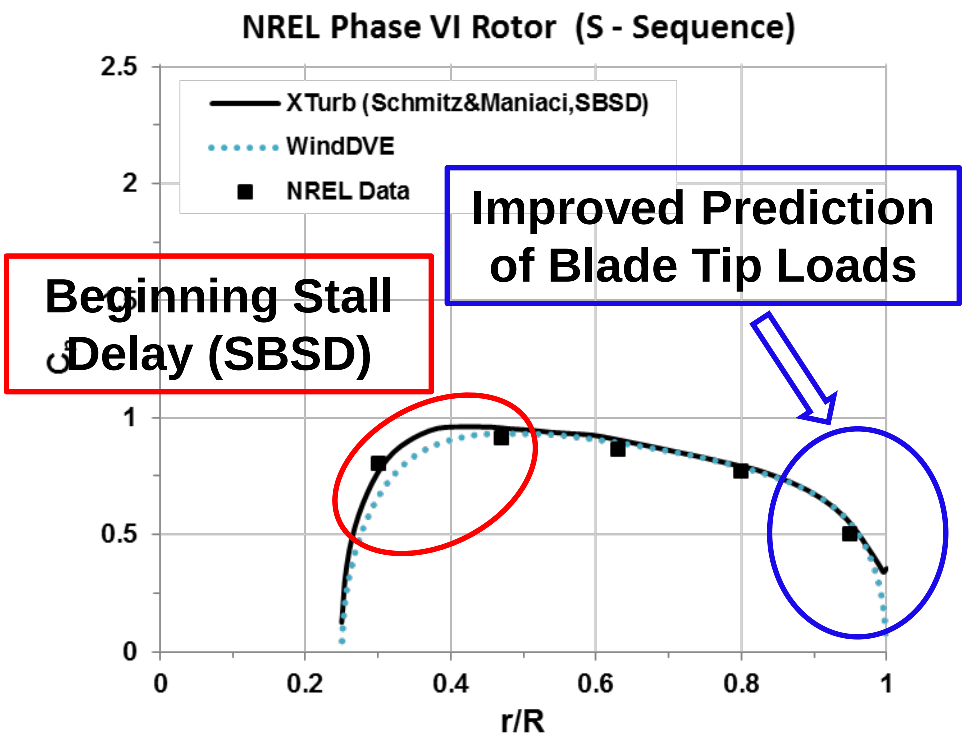


F_T, computed from Analytical Method

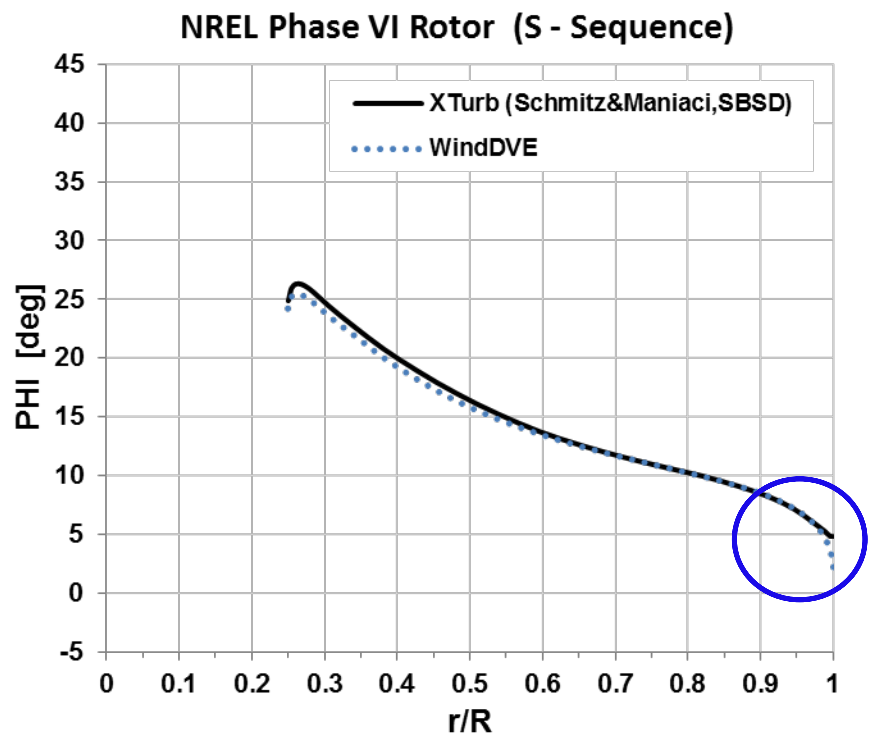
Computed g function and tip loss factor F_T, - TSR = 5.42, V_0 = 7m/s.

Free Wake vs. BEMT

- BEMT (XTurb) including ‘Approximated’ *g* function



Normal force coefficient, Cn

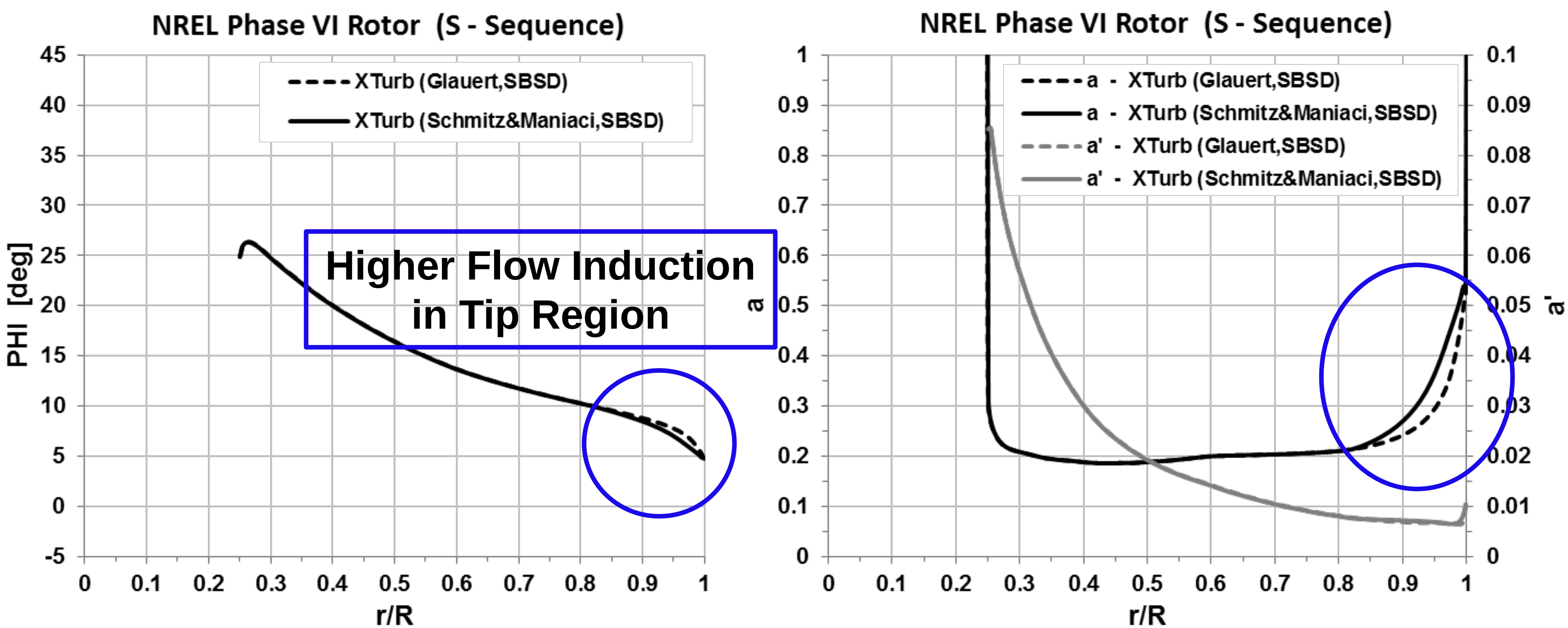


Blade flow angle, ϕ

Results of implementing g function into XTurb, - TSR = 5.42, V_0 = 7m/s.

XTurb + g function

- Glauert (*g*=1) vs. Schmitz & Maniaci (*g* function)



Blade flow angle, ϕ **Induction factors, a & a'**

Effects of g function on Φ, a, and a' (TSR = 5.42, V_0 = 7m/s).

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Results – NREL Phase VI

- Sequences T, U, J, S, X

Effect of Tip Pitch, β_0

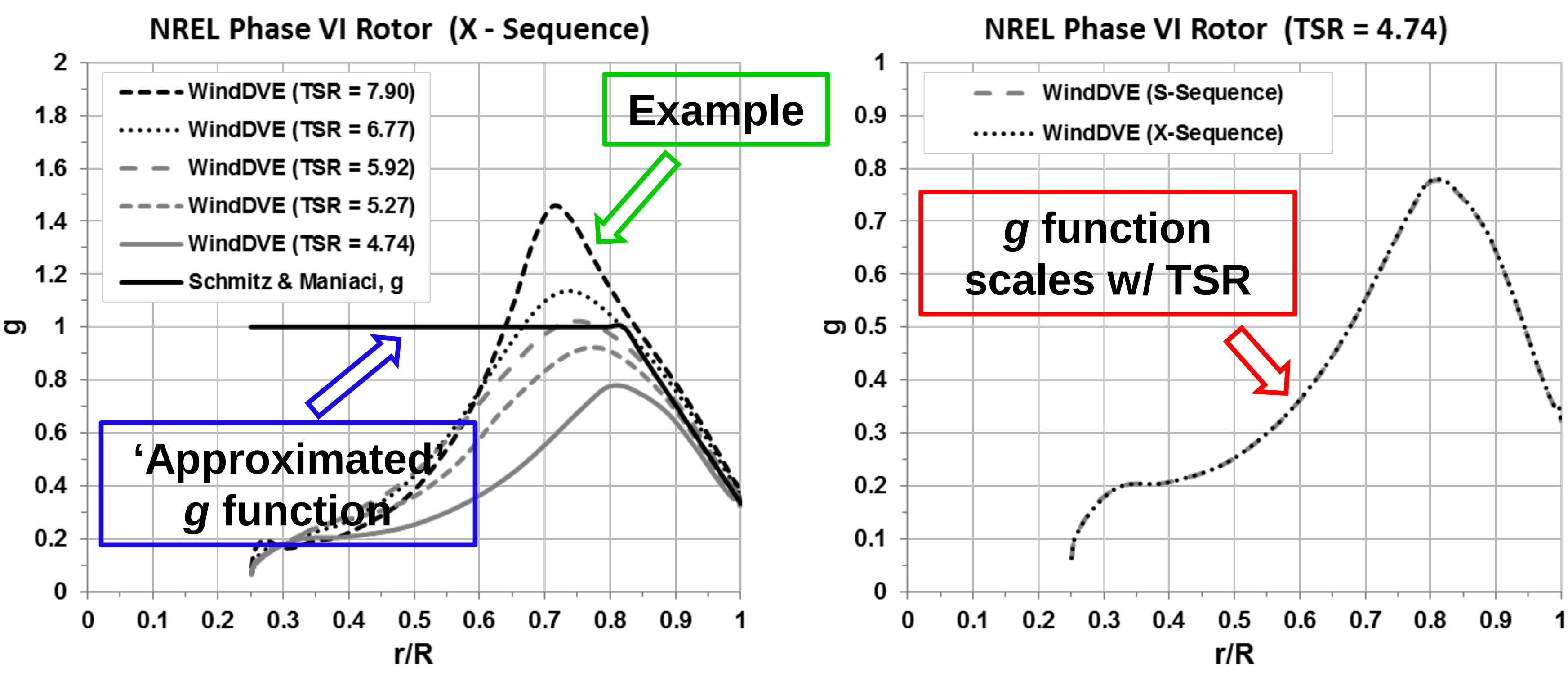
Effect of Rotor Speed, *RPM*

NREL UAE VI Data	T-Sequence	U-Sequence	J-Sequence	S-Sequence	X-Sequence
β_0 [deg]	2	4	6	3	3
Rotor RPM	72	72	72	72	90
Wind Speed [m/s]	5 – 10	5 – 10	5 – 10	5 – 10	5 - 10

NREL Phase VI rotor test sequences considered in *XTurb* and *WindDVE*.

Results – NREL Phase VI

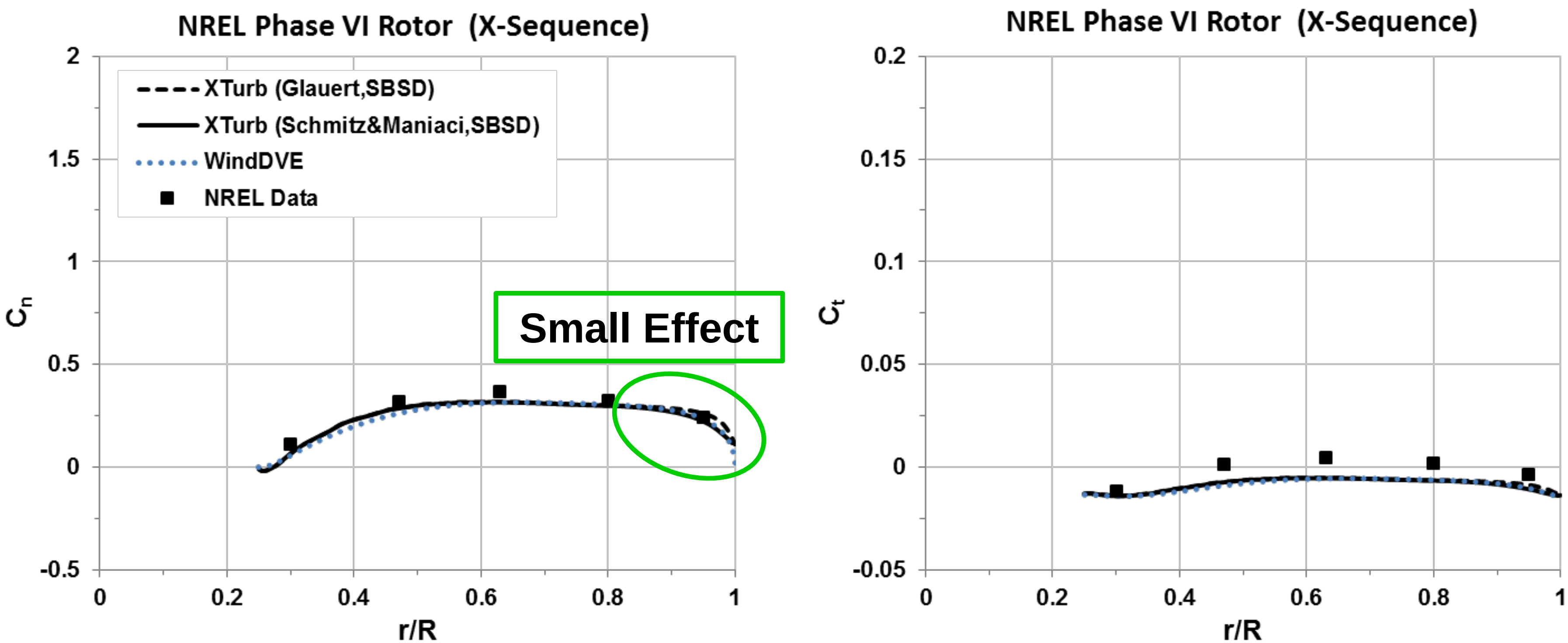
- Effect of Tip-Speed Ratio (TSR), β_0 = 3deg



Computed *g* functions (*WindDVE*) and modeled *g* (Schmitz & Maniaci) for use in *XTurb*.

Results – NREL Phase VI

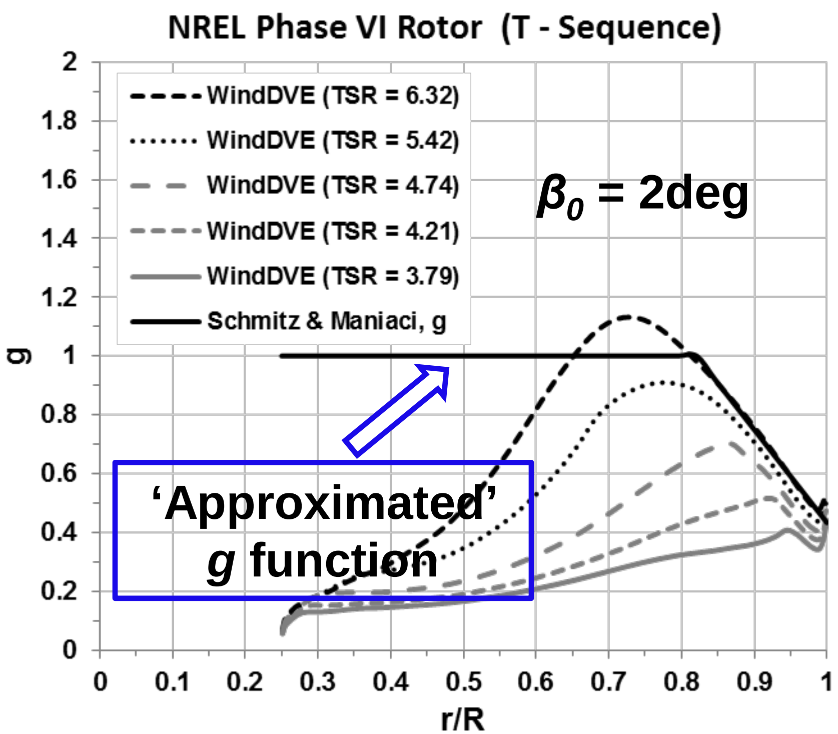
- Data Comparison - High TSR (Lower Loading)

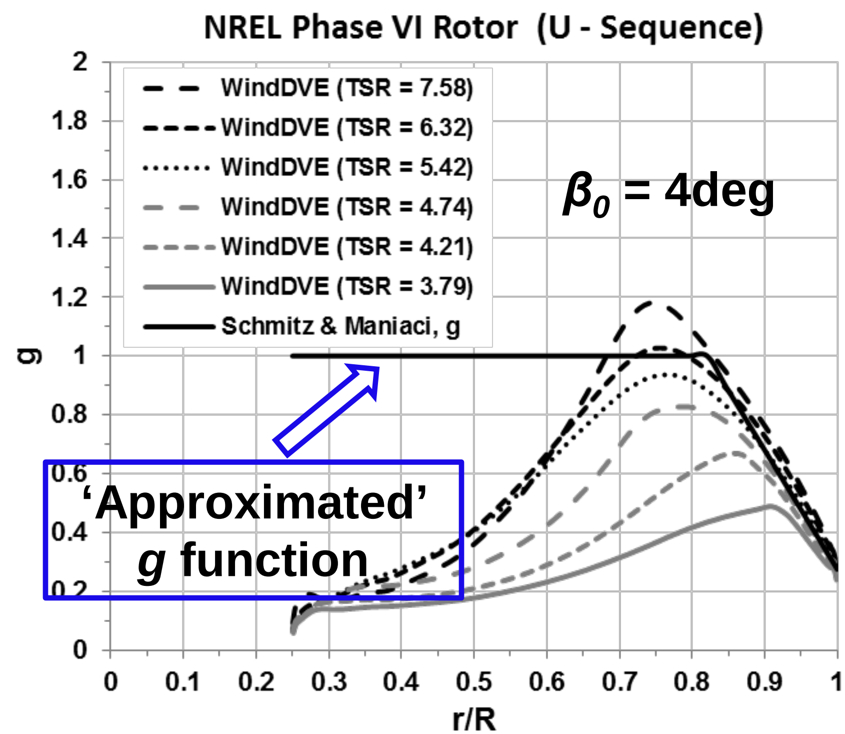


Comparisons of normal and tangential force coefficients.
(NREL Phase VI rotor, X-Sequence) TSR = 7.90 (V_0 = 5m/s), β_0 = 3deg

Results – NREL Phase VI

- Effect of TSR & β_0





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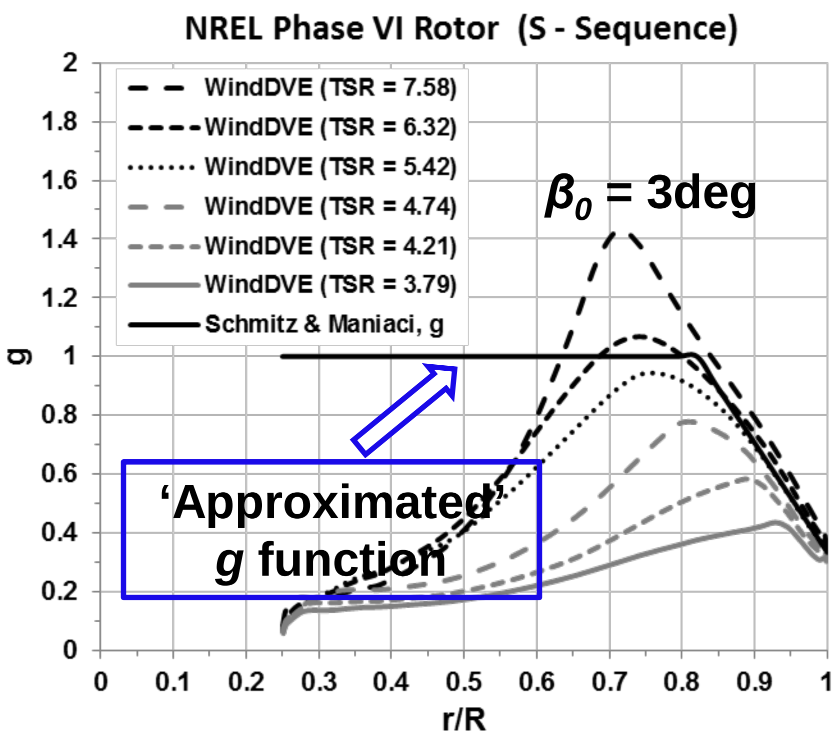
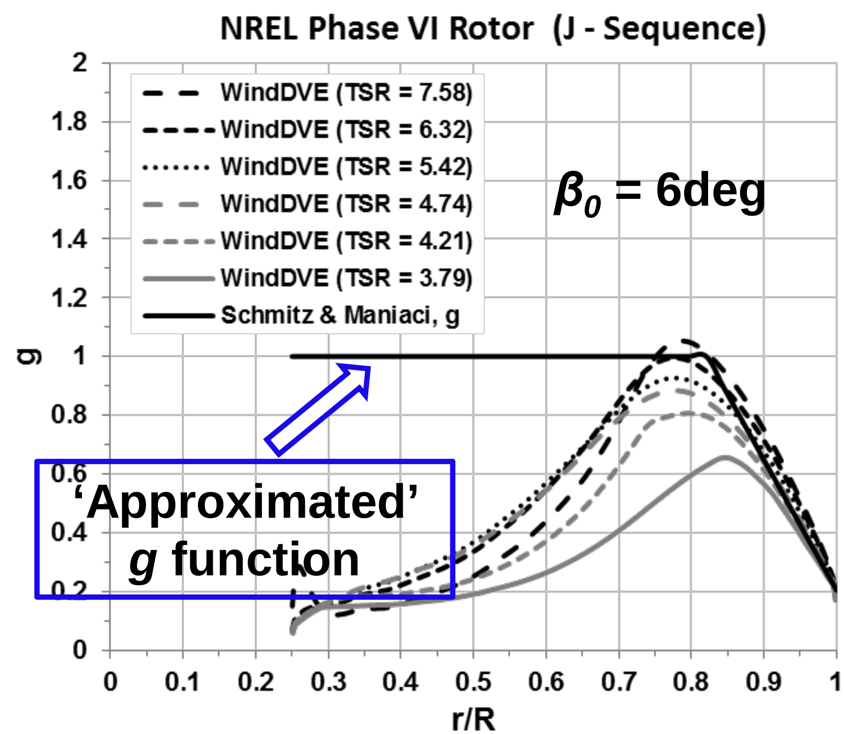
Results – NREL Phase VI

- Effect of TSR & β_0

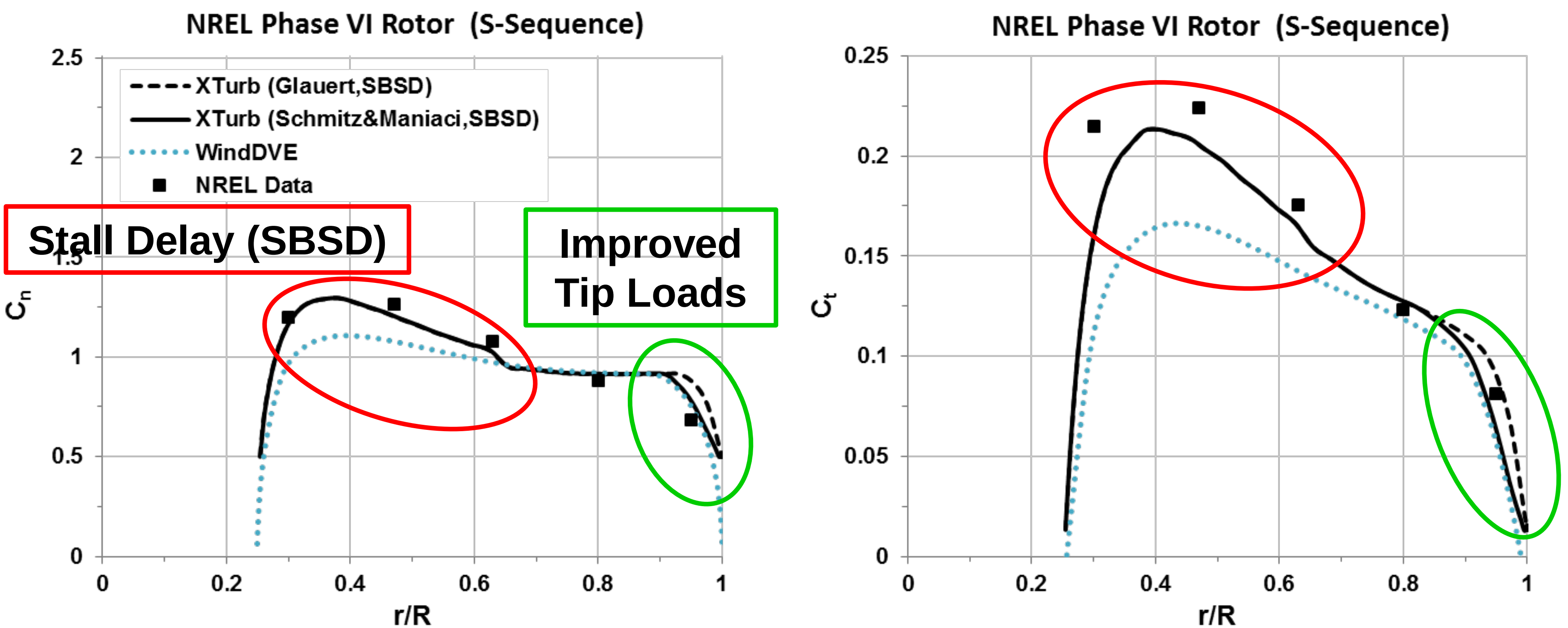




Computed *g* functions (*WindDVE*) and modeled *g* (Schmitz & Maniaci) for use in *XTurb.*

Results – NREL Phase VI

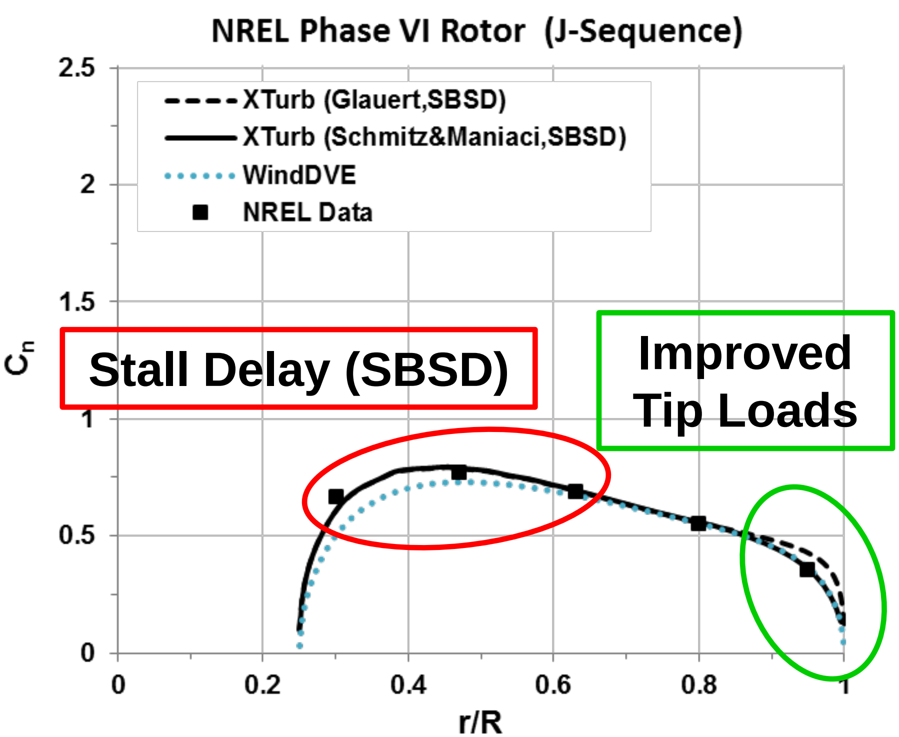
- Example 1 (Mean TSR & β_0)

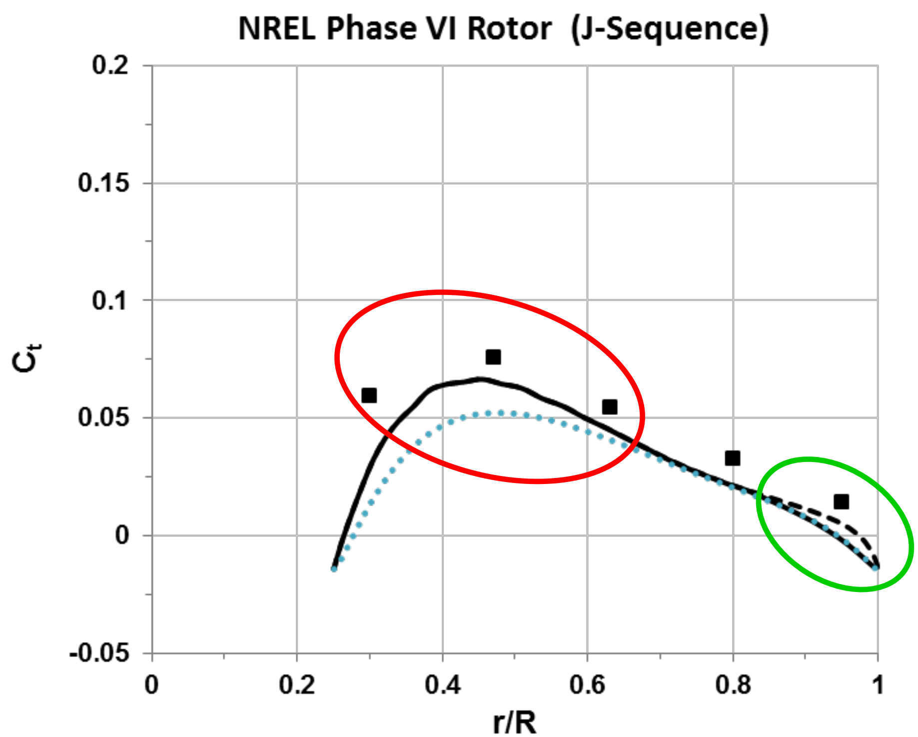


Comparisons of normal and tangential force coefficients.
(NREL Phase VI rotor, S-Sequence) TSR = 4.21 (V_0 = 9m/s), β_0 = 3deg

Results – NREL Phase VI

- <u>Example 2</u> (Higher TSR & β_0)





Comparisons of normal and tangential force coefficients.
(NREL Phase VI rotor, J-Sequence) TSR = 5.42 (V_0 = 7m/s), β_0 = 6deg

Conclusions

- An 'Analytical Method' has been derived to include tip vortex rollup in BEMT analysis
- Results for the NREL Phase VI test sequences suggest that an approximated *g* function can be used in BEMT analysis to account for the effects of tip vortex rollup.
- Rotational augmentation effects in the inboard blade region are predicted quite well by the solution based stall delay model (SBSD) in XTurb.
- The SBSD model is not affected by a *g* function, which is due to the fact that $g = 1$ inboard of tip vortex rollup.
- A general *g* function has a weaker dependence on the tip-speed ratio (TSR) than on the blade tip pitch angle, β_0, and spanwise location, r/R.

Future Work

- Development of a ‘universal’ g function
  - Find **g** = **g**(*r/R*, *TSR*, β_0, *σ’*) to implement into an adjusted tip loss factor in BEMT methods
  - Consider additional wind turbine rotors such as the MEXICO and Krogstad rotors as well as Glauert rotors

- Acknowledgements

  - The authors would like to thank Dr. Scott Schreck from NREL for providing the measured data of the NREL Phase VI rotor.
  - The DOE Wind and Water Power Technologies Office supported attendance at this conference.

Wake Roll-up and Expansion