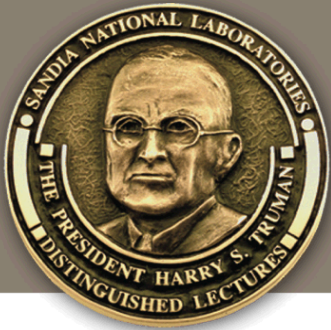


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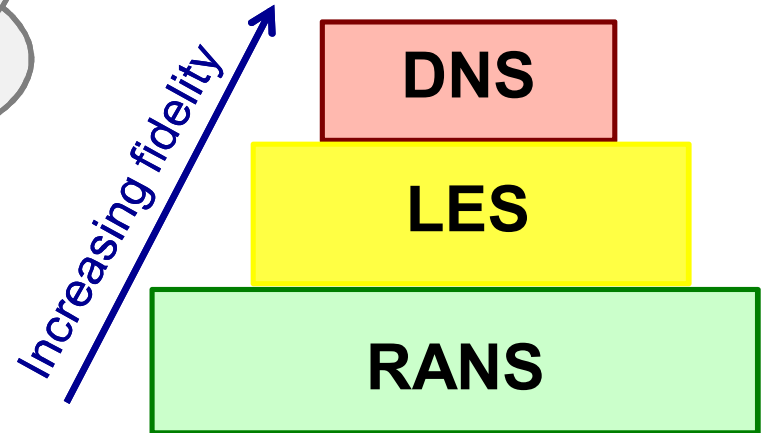
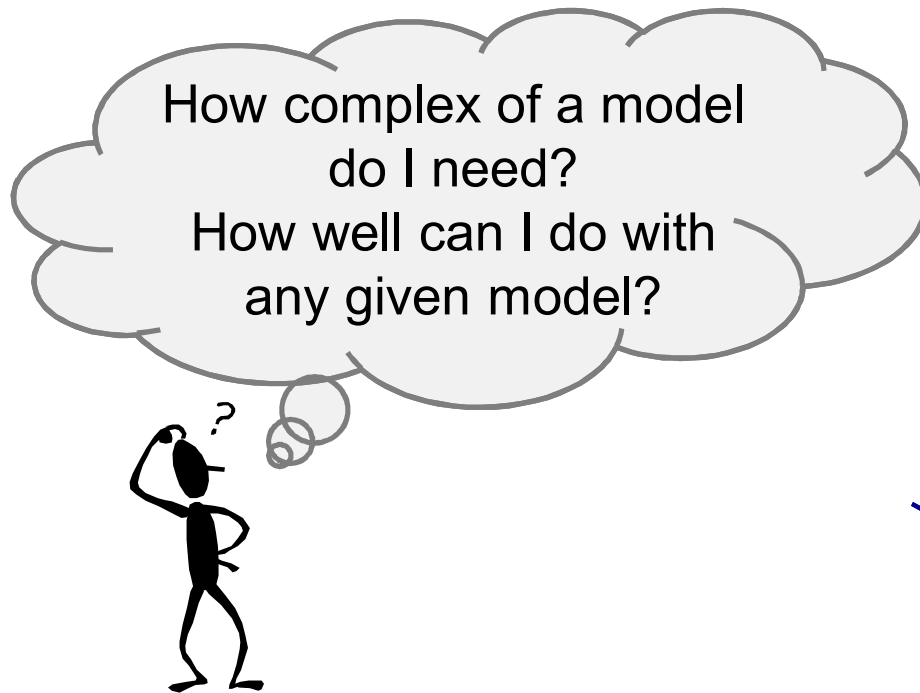
President Harry S. Truman Fellowship in  
National Security Science and Engineering

# Machine Learning for Uncertainty Quantification in Turbulent Flow Simulations

Julia Ling, Jeremy Templeton

Apr 2016

# Turbulence Simulations

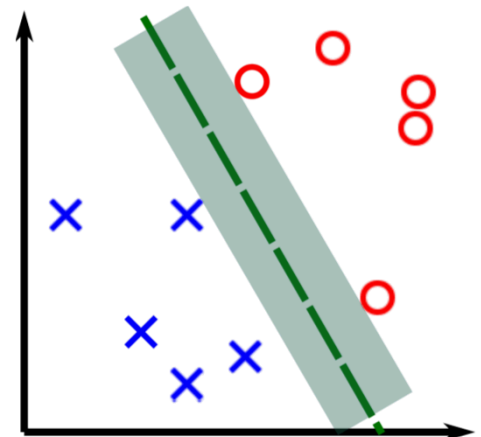
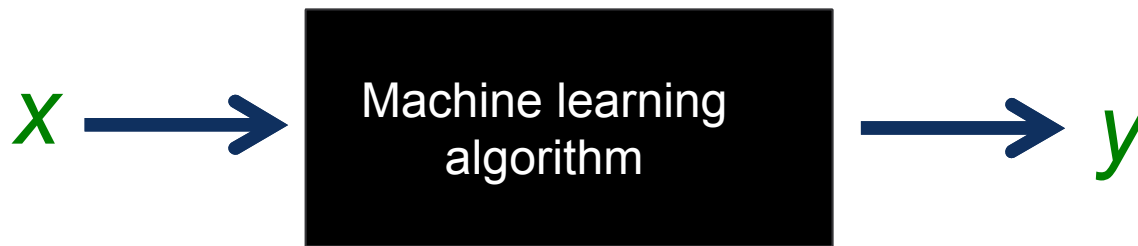


## RANS

- Most widely used turbulence model
- Relies on modeling assumptions → Model form uncertainty
- Sometimes accurate, sometimes inaccurate
  - Very difficult to assess model form uncertainty
- **Idea: Use machine learning to detect regions of high uncertainty based on when specific model assumptions are violated**

# Machine Learning

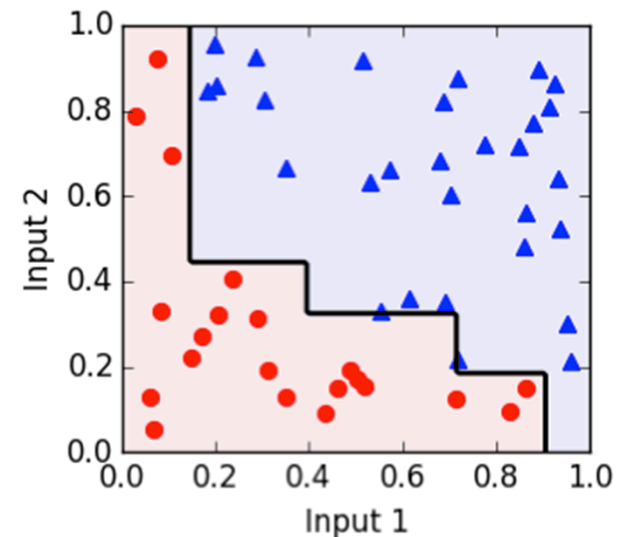
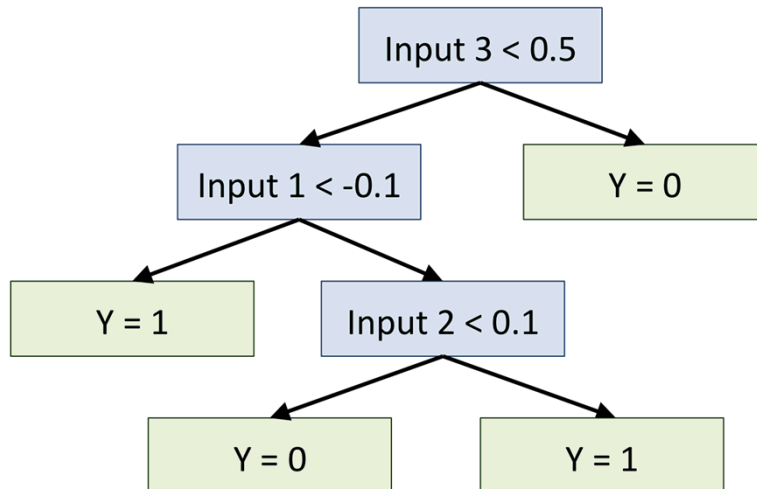
- Set of data-driven algorithms for regression, classification, clustering
- *E.g.*: linear regression, support vector machines, neural networks
- Have been broadly applied in finance, software engineering, retail
- Challenge: how to incorporate domain knowledge into machine learning algorithms
  - These techniques have a range of physics applications
- **For this application: use binary classifier to flag regions of high RANS uncertainty on a point-by-point basis**



# Random Forests

## ■ Binary Decision Trees:

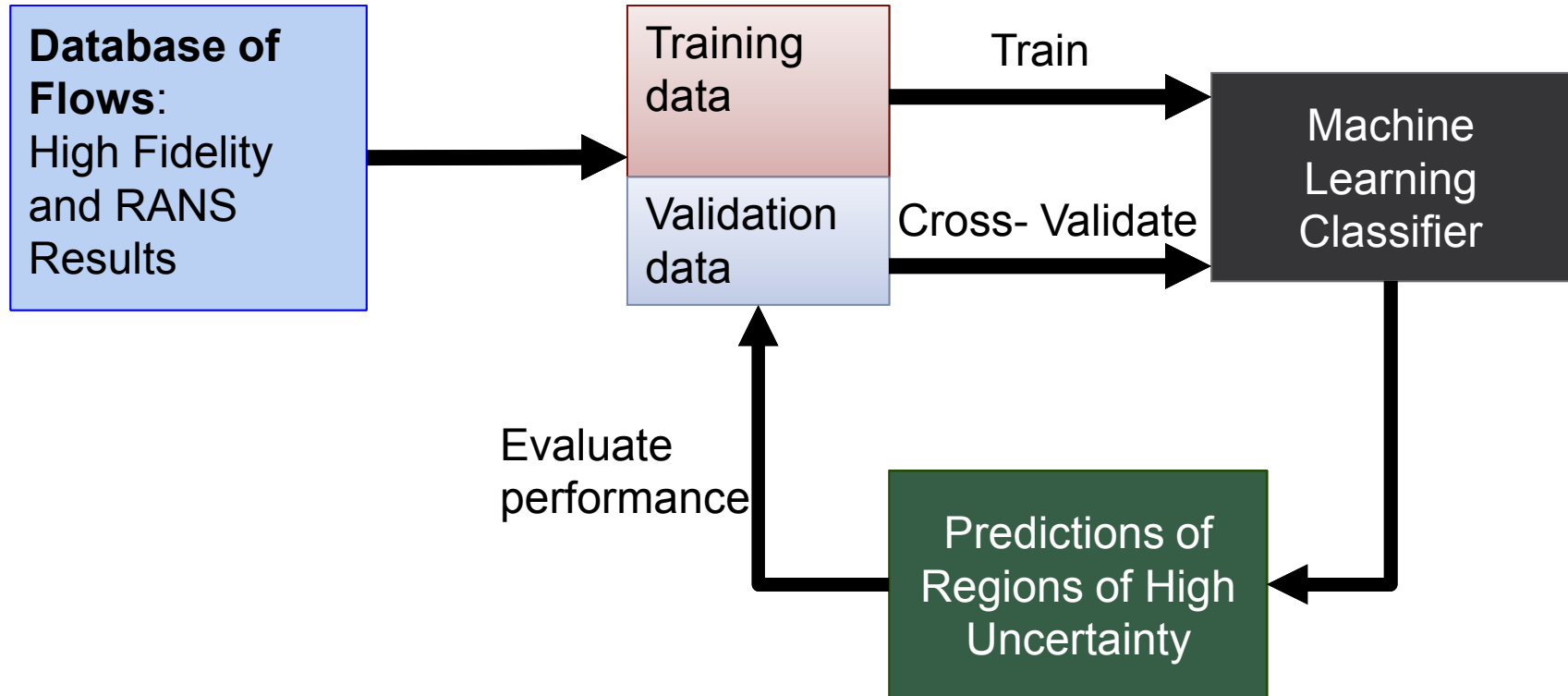
- Simple, easy to understand and use
- Tendency to overfit, poor performance with non-linear behavior



## ■ Ensembles of Decision Trees:

- Much more robust
- Random Forests are a type of ensemble of decision trees

# Classifier Development



# Classifier Development

## Database of Flows:

High Fidelity  
and RANS  
Results

Contours of velocity magnitude

Angled jet in crossflow

Flow over wavy wall

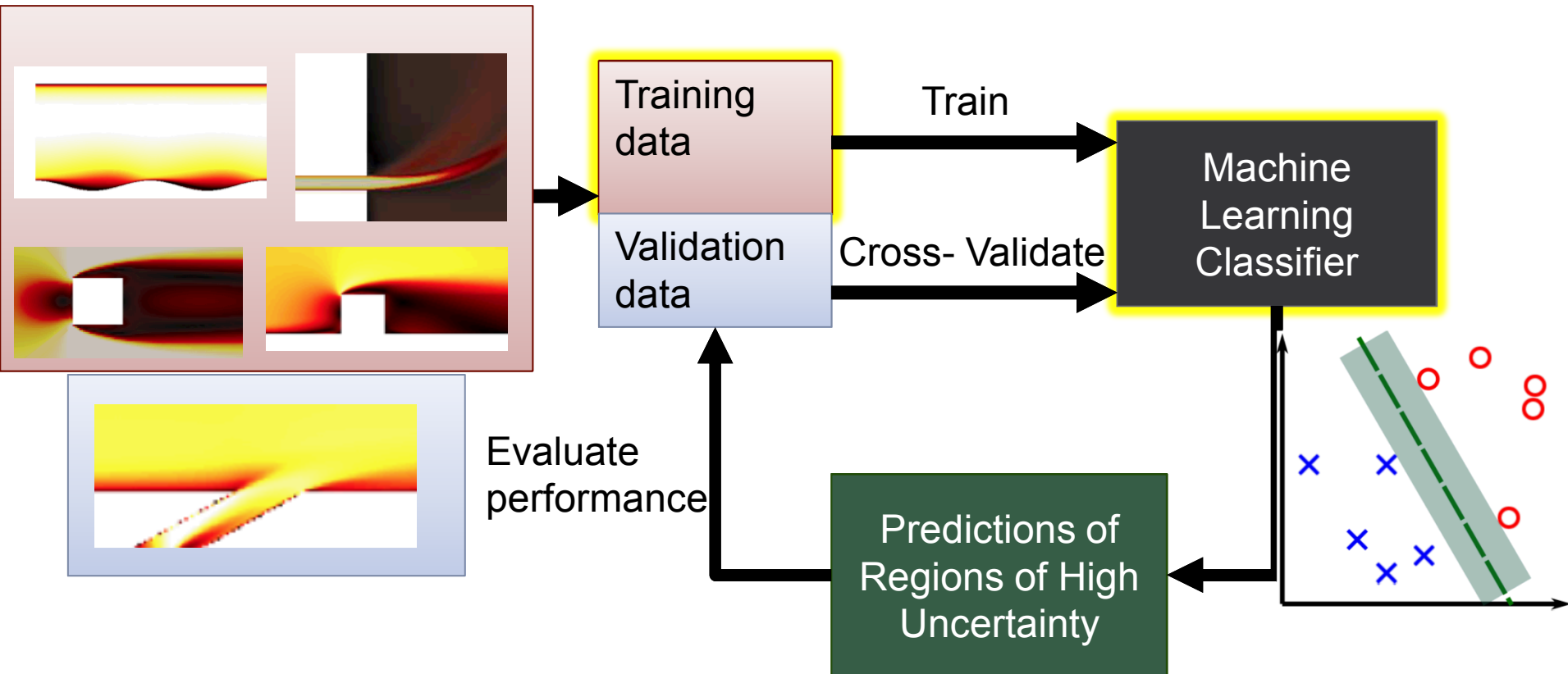
Flow around square

Flow around cube

Machine  
Learning  
Classifier

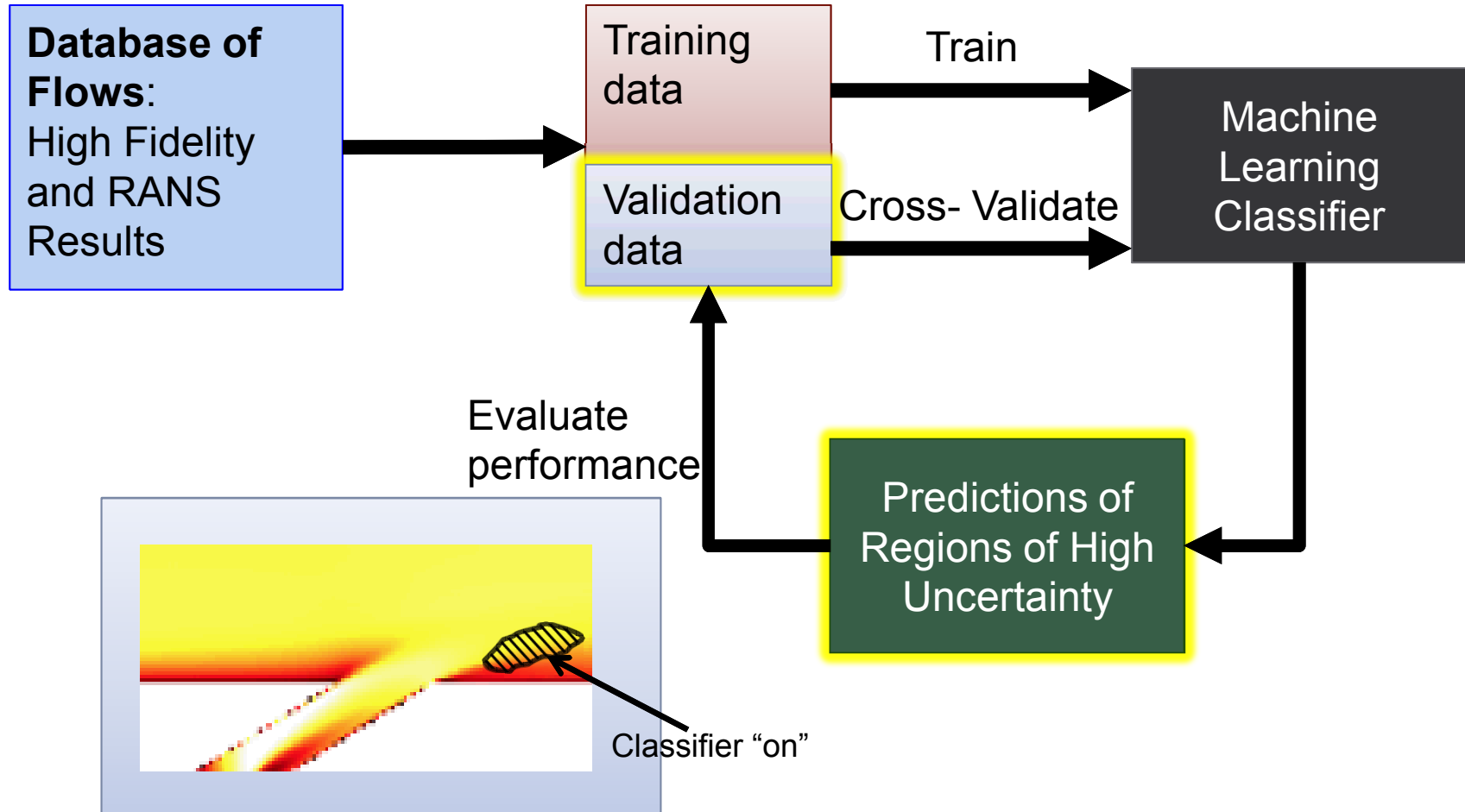
- Have database of canonical “building block” flows

# Classifier Development



- Split data base into training and validation sets
- Train classifier
  - Input: Local flow variables from RANS
  - Output: Binary flag– “on” if RANS assumption violated, “off” otherwise

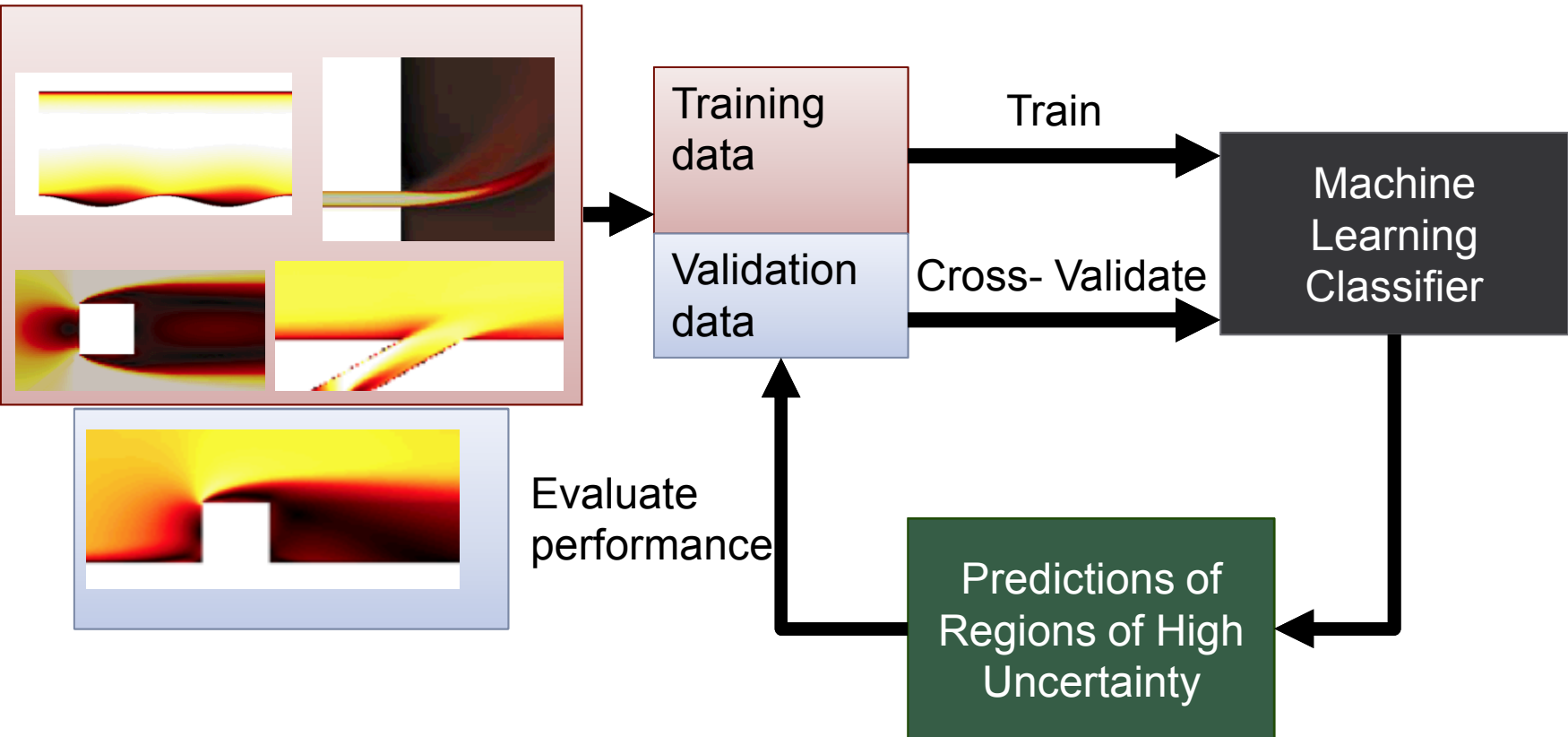
# Classifier Development



- Use classifier to make predictions on validation set
- Evaluate classifier by comparing to high fidelity results



# Classifier Development



- Cross-validate to ensure generalization

# Assumptions Tested

$$\overline{u'_i u'_j} = \frac{2}{3} k \delta_{ij} - 2\nu_t S_{ij}$$

## 1. Non-negativity of eddy viscosity

- Can extract eddy viscosity from LES/DNS
- Classifier should be “on” when LES/DNS eddy viscosity goes negative

$$\nu_t = \frac{-\overline{u'_i u'_j} S_{ij} + \frac{2}{3} k \delta_{ij} S_{ij}}{2 S_{kl} S_{kl}}$$

## 2. Isotropy of Reynolds stresses

- Classifier should be “on” when second invariant of anisotropy tensor exceeds a set threshold

## 3. Linearity of Boussinesq hypothesis

- Extract linear and cubic eddy viscosity from LES/DNS data
- If these values differ significantly, then uncertainty associated with linearity assumption is high

## Inputs:

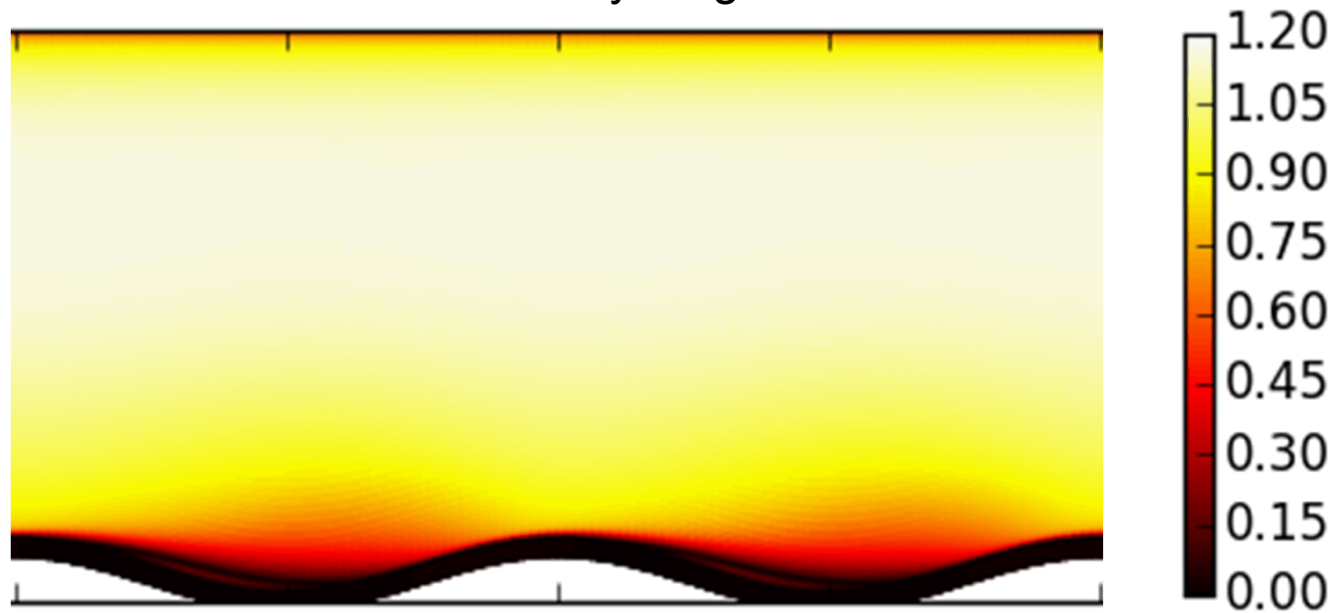
- Non-dimensional, rotationally invariant local flow variables from RANS

# Applications of Classifiers

- Can quickly post-process RANS simulation to determine whether it's reliable in region of interest
  - Don't have to wait around for validation data set
  - Can determine what corrections to implement
- Can enable adaptive corrections during run time
- Experimental design
  - Design experiments to provide the strongest validation
- LES-RANS hybrids
  - Use classifiers to inform switching function

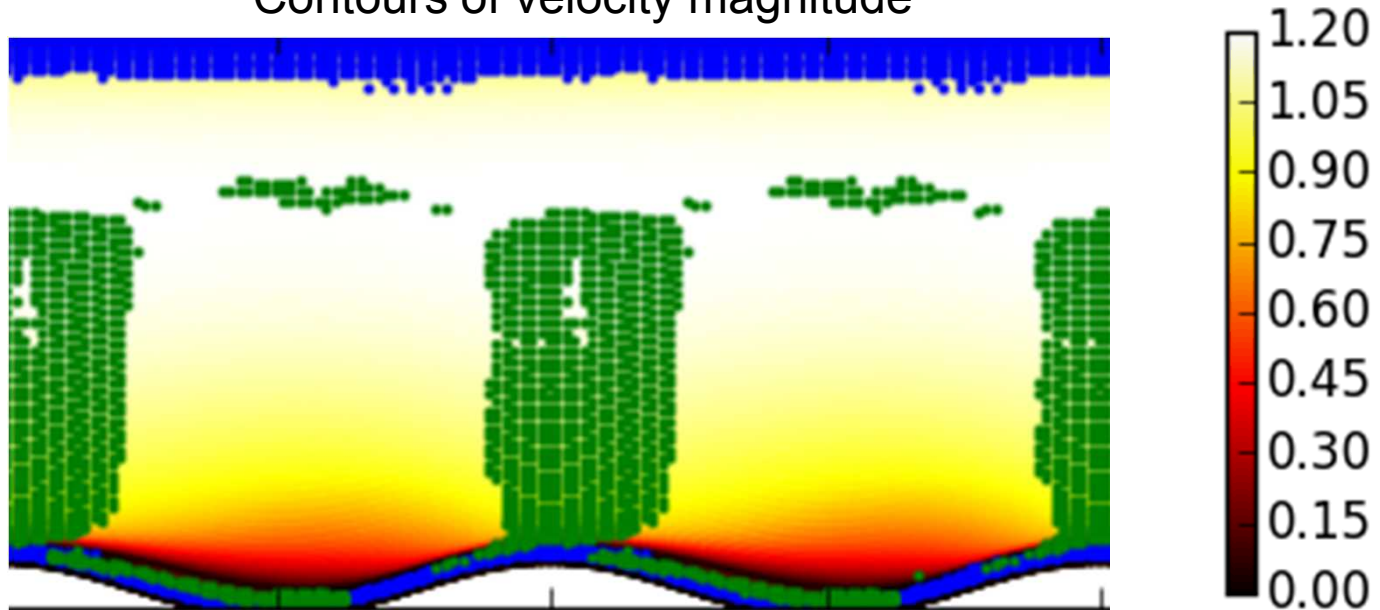
# The Status Quo

Contours of velocity magnitude



# A Better Option

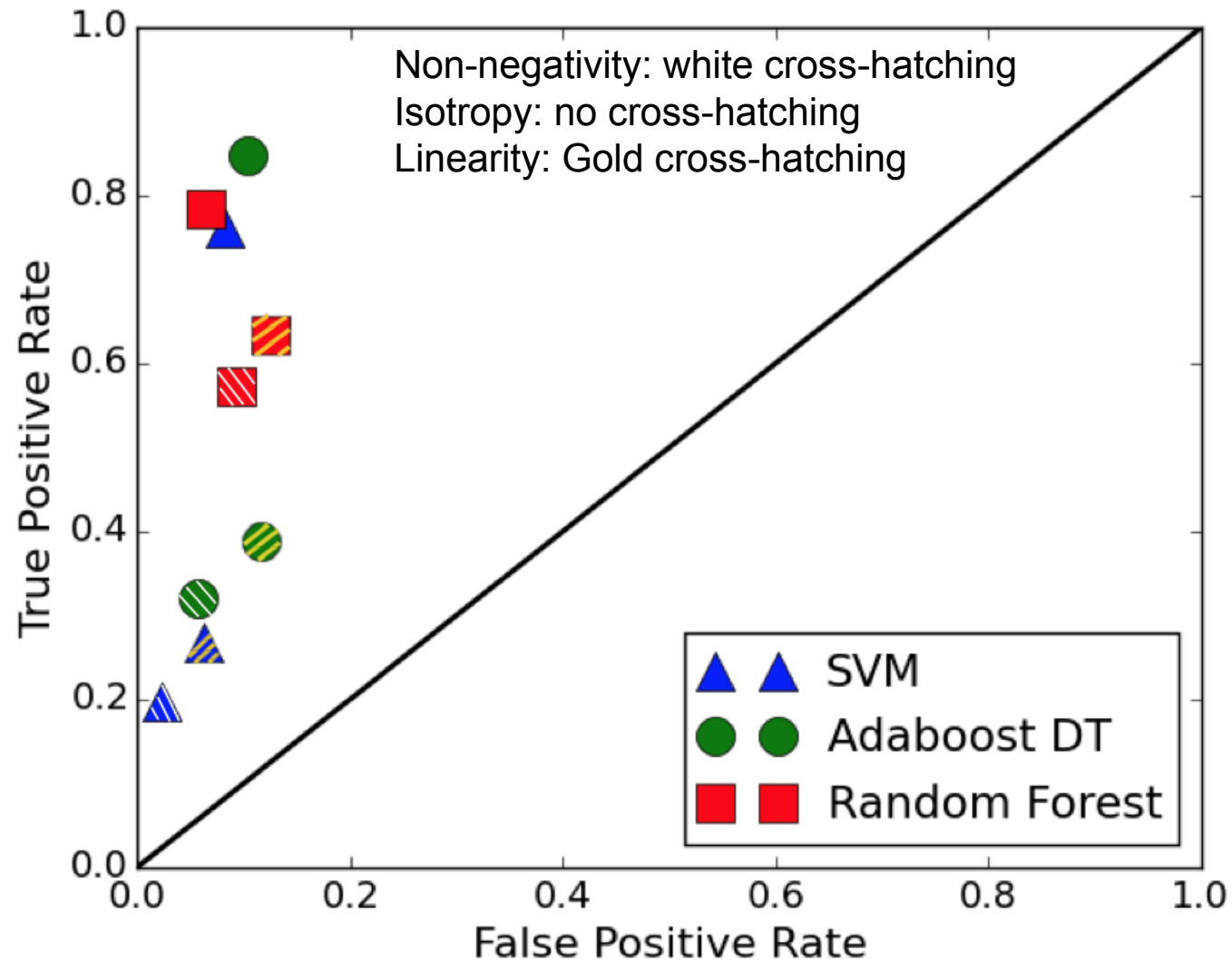
Contours of velocity magnitude



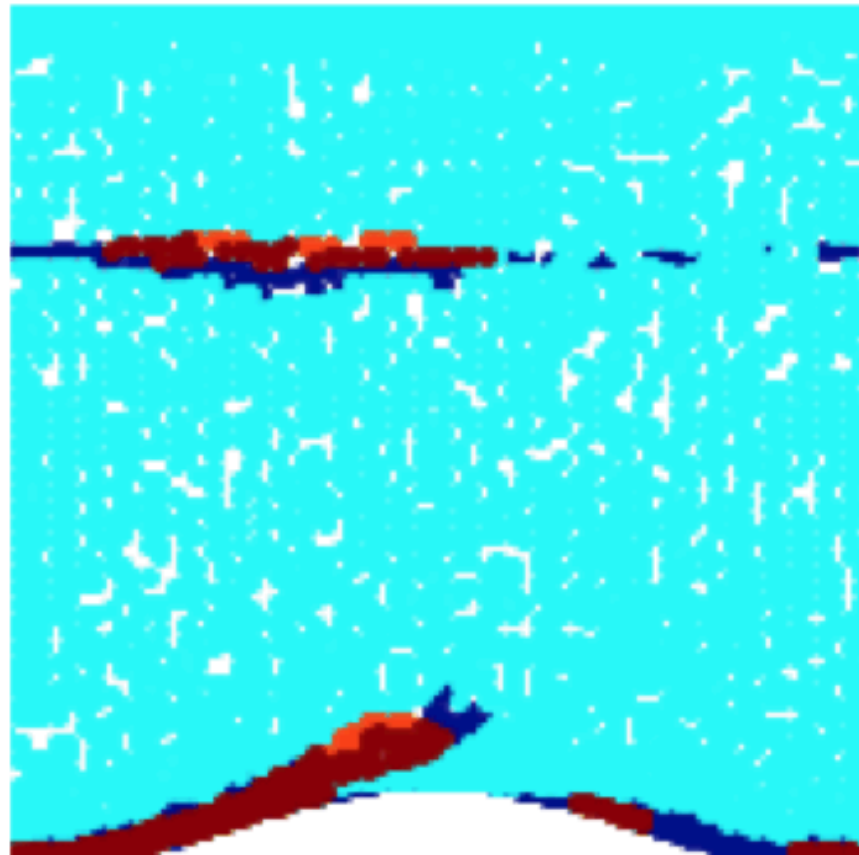
Blue: Regions where classifier predicts isotropy assumption violated

Green: Regions where classifier predicts linearity assumption violated

# Classifier Performance



# Classifier Performance

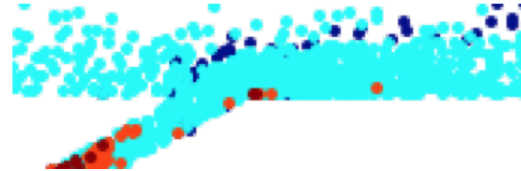
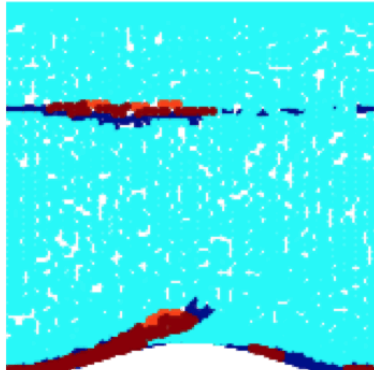


- True Negative
- False Negative
- True Positive
- False Positive

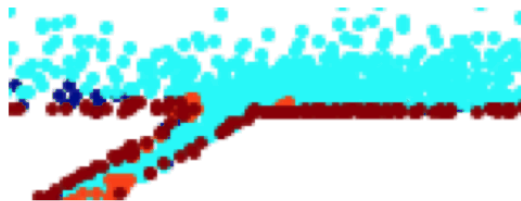
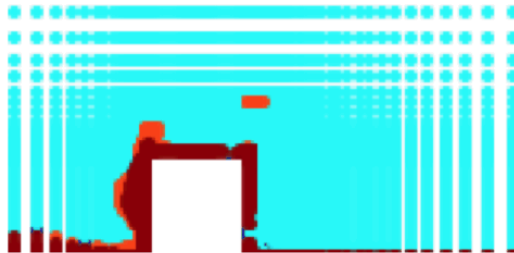
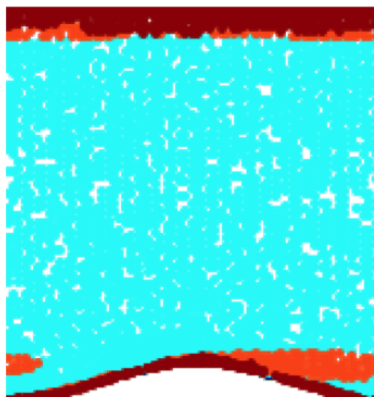
(a) Case 1, Marker 1:  
Negative  $\nu_t$

# Classifier Performance

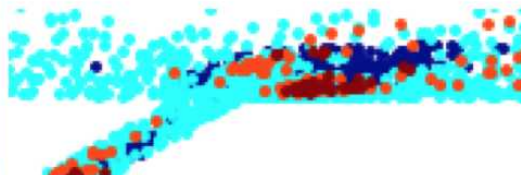
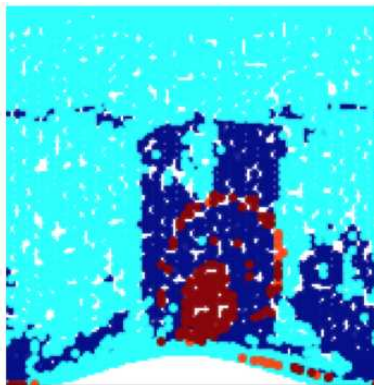
Non-negativity  
assumption



Isotropy  
assumption



Linearity  
assumption

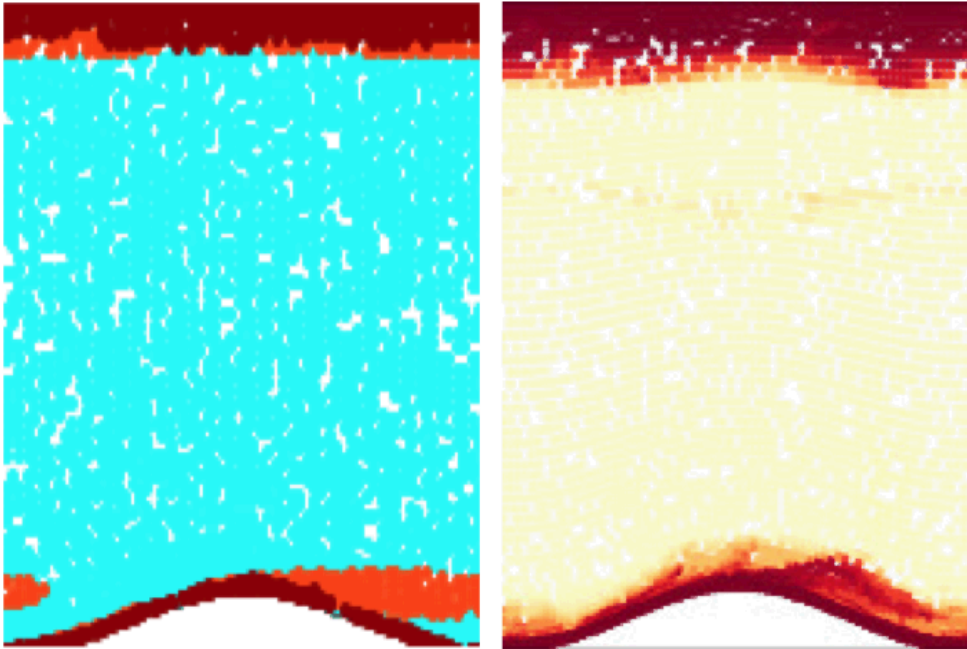


True Negative  
False Negative  
True Positive  
False Positive

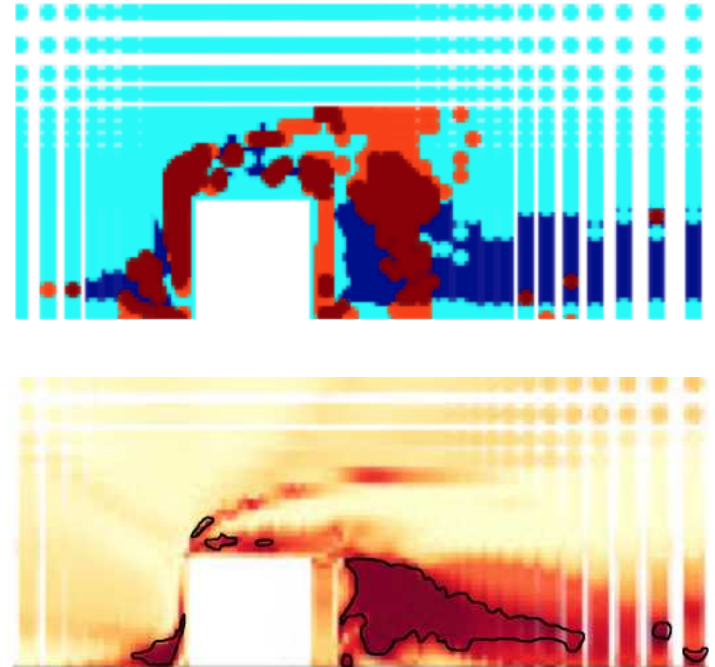


# Classifier Confidence

Uncertain classifications



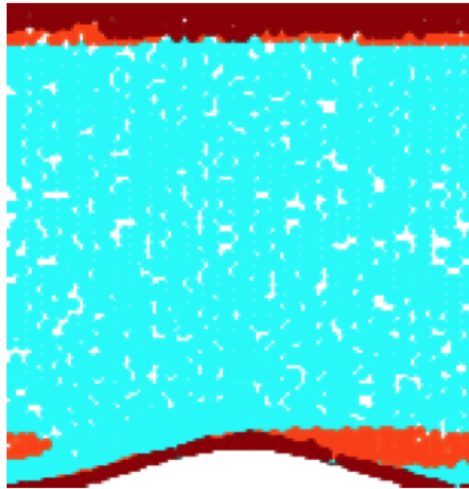
Extrapolation Detection



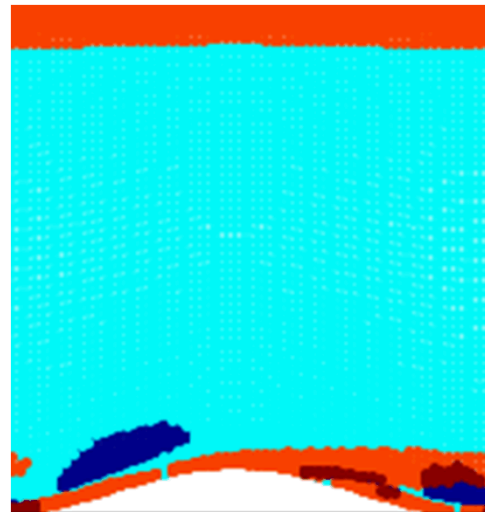
# Comparison against State of the Art

Machine Learned Classifier

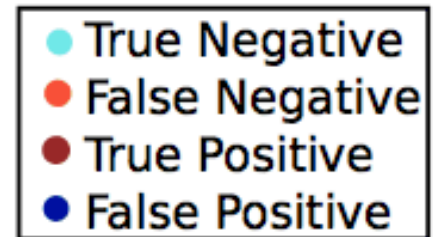
Physics-based Classifier of Gorle et al.



Cross-validation  
Error rate: 11%



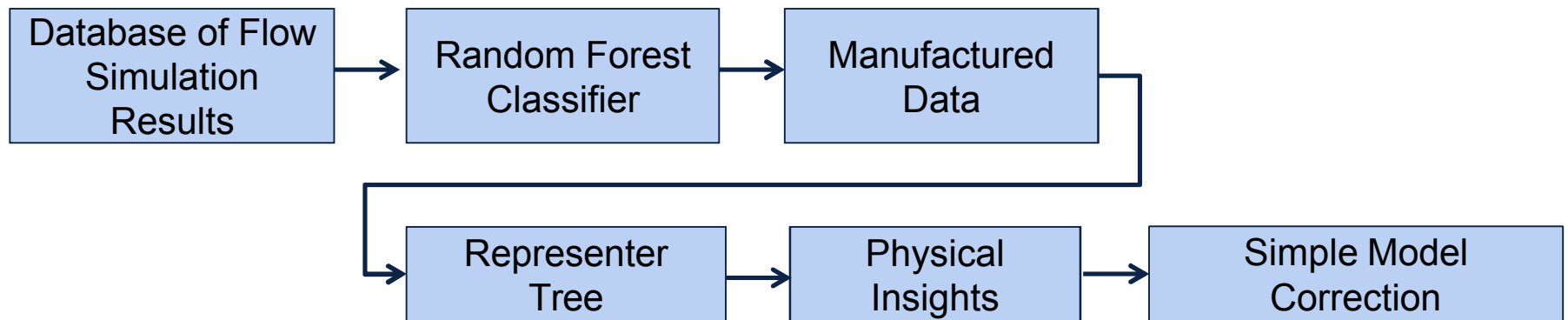
Cross-validation  
Error rate: 33%



- Cross-validated classifier error rate: 11%
- 3 X more accurate than current state of the art physics-driven classifier of Gorle et al.
  - Gorle et al.'s classifier is used as an input to the ML classifier

- Classifiers for RANS model uncertainty can transform the way RANS results are post-processed and understood
  - Clarify when RANS simulations are predictive
  - Enable adaptive modeling corrections
  
- Develop techniques for using machine learning algorithms on physical systems
  - Leverage domain knowledge and physical constraints to develop smarter models
  - Use data-driven models to learn about the physical system

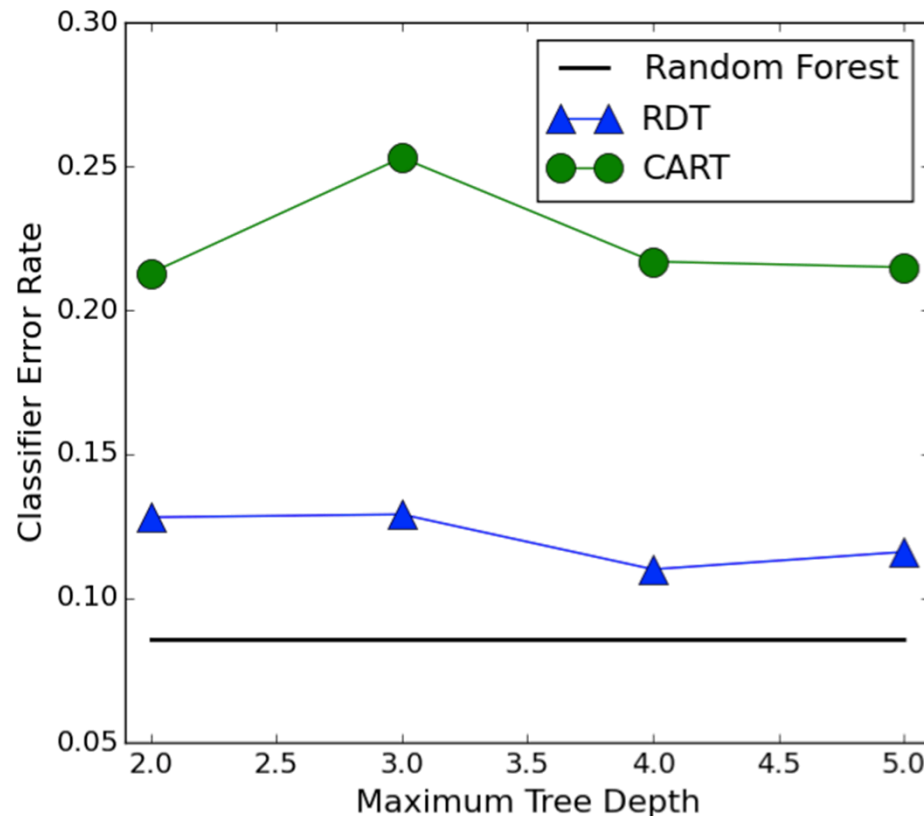
- Random Forests are much more robust and high-performance than single decision trees, but what have we lost?
  - Clarity—how can we understand these machine learned models?
- Representer Trees



- Trained a representer decision tree based on Random Forest that predicted when the Boussinesq isotropy assumption was invalid

# Representer Decision Trees

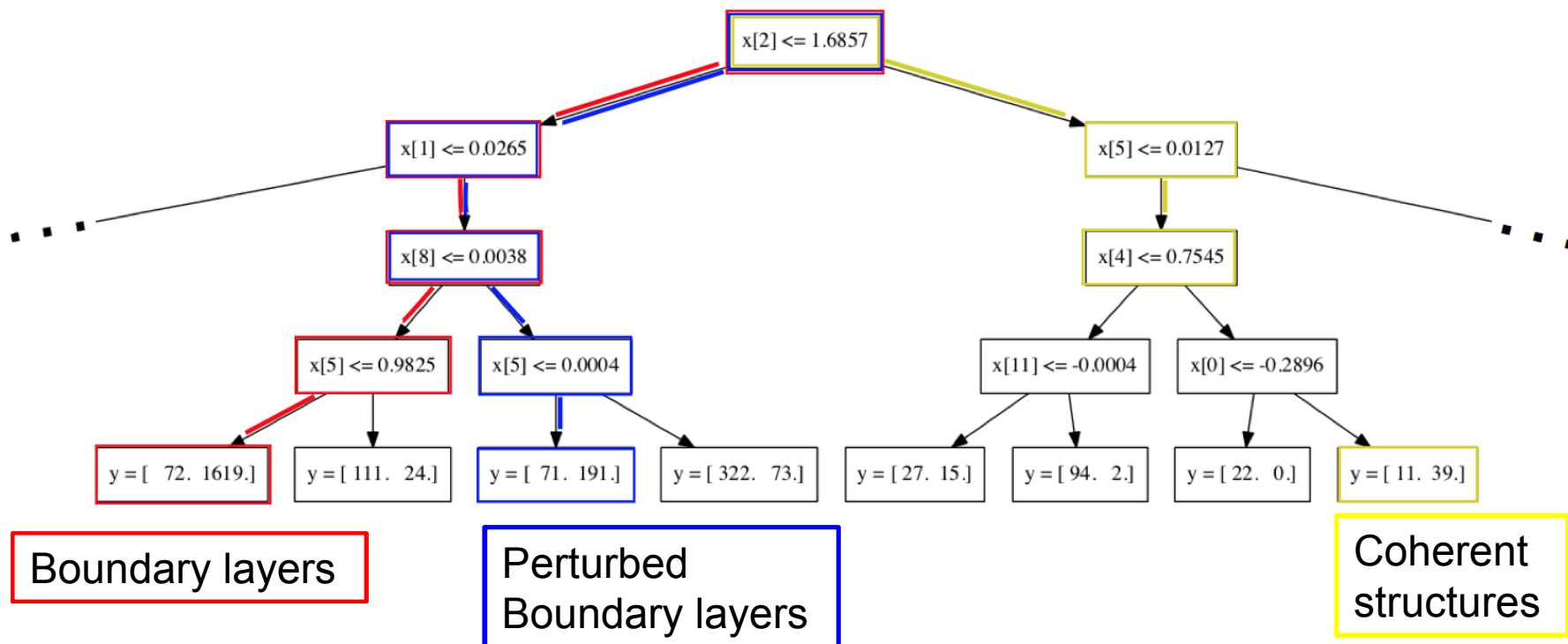
- Surprising result: the representer tree has better performance and is more stable than a tree trained on the original tra



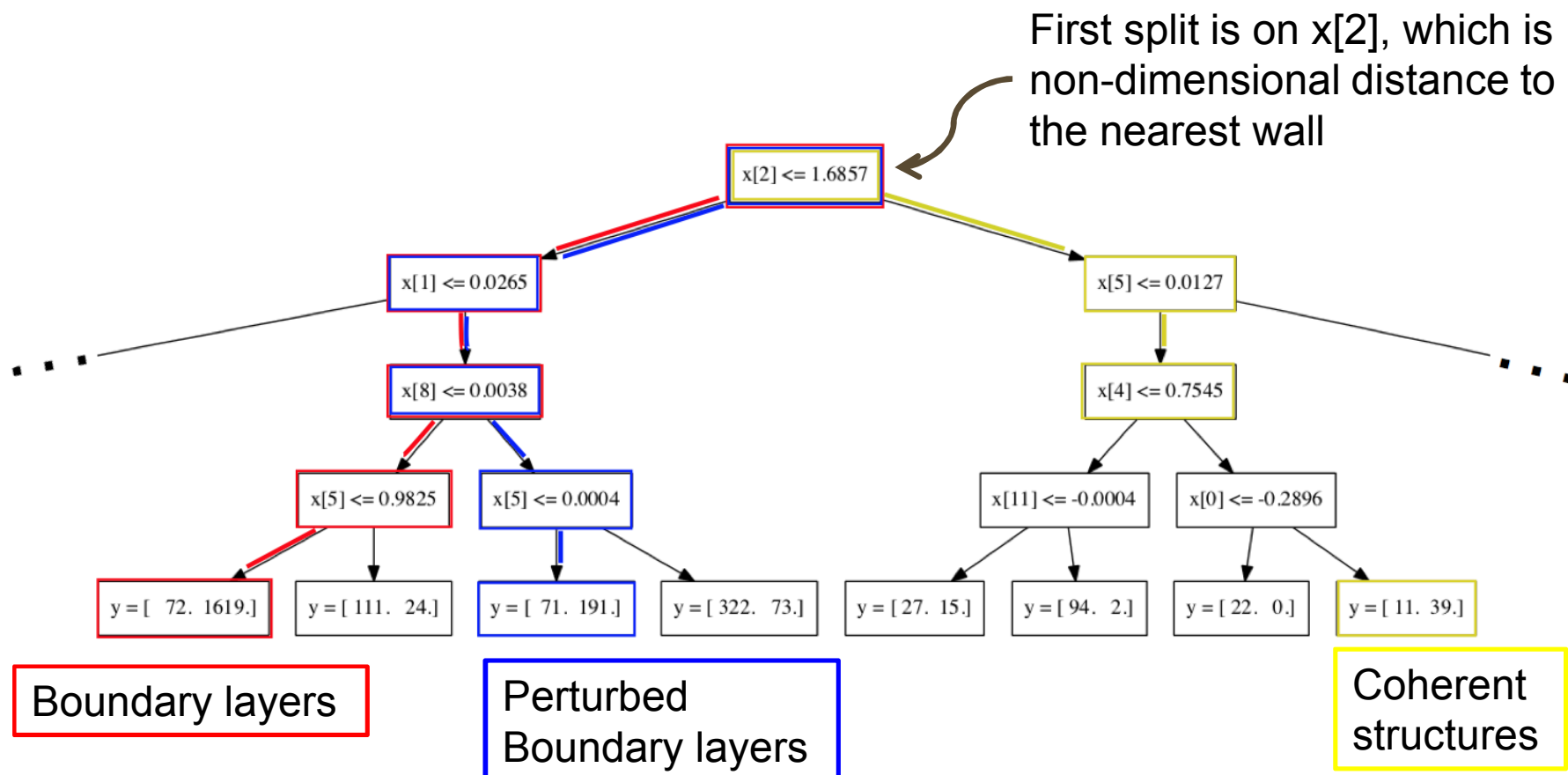
- Trained a representer decision tree based on Random Forest that predicted when the Boussinesq isotropy assumption was invalid

# Analyzing the Representer Tree

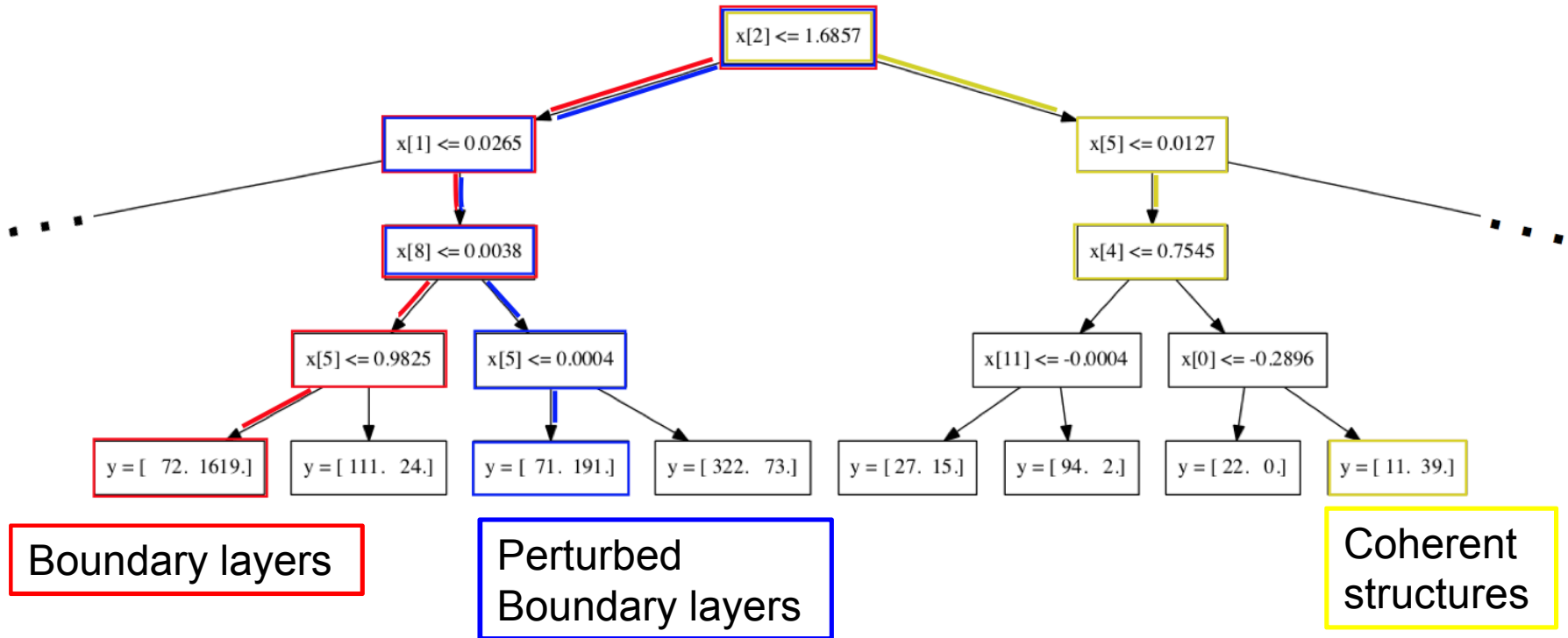
- Look for consistent branches



# Analyzing the Representer Tree



# Analyzing the Representer Tree



- Can determine physical regimes where assumptions are violated
- Can see that different mechanisms cause assumption to break down in near wall region than in free stream



# Conclusions

- Machine learning was used to detect when RANS assumptions break down
- These data-driven methods achieved significantly improved classification accuracy by leveraging the high-dimensional data
- Rule extraction techniques were then used to regain physical intuition from the machine learning classifiers

## Acknowledgments

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- J. Ling, "Using Machine Learning to Understand and Mitigate Model Form Uncertainty in Turbulence Models," *ICMLA*, (2015).
- C. Gorle, J. Larsson, M. Emory, G. Iaccarino, "The deviation from parallel shear flow as an indicator of linear eddy viscosity model inaccuracy," *Physics of Fluids*, (2014).