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Implementation and Analysis of a Robust Return Mapping Algorithm for Anisotropic Yield Surfaces

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Motivation

- Ductile failure in large deformation plasticity
  - Mechanics of failure
  - Numerical methods
  - Boundary value problem
  - Load paths

- Improve models for plastic deformation

Yield Function

- The general form we will use for the yield function is as follows

$$f = \phi\left(\boldsymbol{\sigma}\right) - \bar{\sigma}\left(\bar{\varepsilon}^{p}\right) = 0$$

  - This defines a surface in stress space – the yield surface
  - Assume associated flow

Isotropic Plasticity Models

von Mises – 1 parameter

$$\phi = \sqrt{\frac{3}{2}\mathbf{s} : \mathbf{s}} \qquad \mathbf{s} = \boldsymbol{\sigma} - \frac{1}{3}\left(\mathrm{tr}\boldsymbol{\sigma}\right)\,\mathbf{I}$$

Hosford – 2 parameters

$$\phi = \left\{\frac{1}{2}\left[|\sigma_1 - \sigma_2|^a + |\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a\right]\right\}^{1/a}$$

Orthotropic Plasticity Models

Hill – 7 parameters

$$\phi^2\left(\boldsymbol{\sigma}\right) = F\left(\hat{\sigma}_{22} - \hat{\sigma}_{33}\right)^2 + G\left(\hat{\sigma}_{33} - \hat{\sigma}_{11}\right)^2 + H\left(\hat{\sigma}_{11} - \hat{\sigma}_{22}\right)^2$$

$$+ 2L\hat{\sigma}_{23}^2 + 2M\hat{\sigma}_{31}^2 + 2N\hat{\sigma}_{12}^2$$

$$\phi = \sqrt{\frac{3}{2}\boldsymbol{\sigma} : \mathbf{P} : \boldsymbol{\sigma}}$$

depends on material orientation

Orthotropic Plasticity Models

Barlat (Yld2004-18p) – 20 parameters *

$$\mathbf{s}' = \mathbf{L}' : \boldsymbol{\sigma} \quad ; \quad \mathbf{s}'' = \mathbf{L}'' : \boldsymbol{\sigma}$$

$$\phi(\boldsymbol{\sigma}) = \left\{ \frac{1}{4} \Big[|s'_1 - s''_1|^a + |s'_1 - s''_2|^a + |s'_1 - s''_3|^a + |s'_2 - s''_1|^a + |s'_2 - s''_2|^a \right.$$

$$\left. + |s'_2 - s''_3|^a + |s'_3 - s''_1|^a + |s'_3 - s''_2|^a + |s'_3 - s''_3|^a \Big] \right\}^{1/a}$$

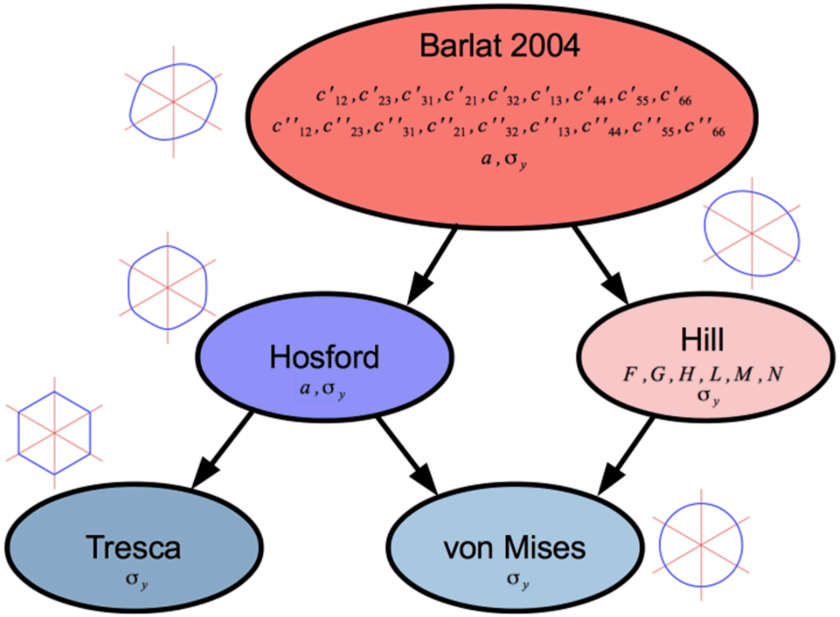
* Barlat et. al., “Linear transformation based anisotropic yield functions”, IJP, v. 21, 2005.

Orthotropic Plasticity Models

$$\mathbf{s}' = \underbrace{\mathbf{C}' : \mathbb{I}'}_{\mathbf{L}'} : \boldsymbol{\sigma} = \mathbf{C}' : \mathbf{s}$$

$$\begin{Bmatrix} s'_{11} \\ s'_{22} \\ s'_{33} \\ s'_{12} \\ s'_{23} \\ s'_{31} \end{Bmatrix} = \begin{bmatrix} 0 & -c'_{12} & -c'_{13} & 0 & 0 & 0 \\ -c'_{21} & 0 & -c'_{23} & 0 & 0 & 0 \\ -c'_{31} & -c'_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_{66} \end{bmatrix} \begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ s_{12} \\ s_{23} \\ s_{31} \end{Bmatrix}$$

Model Hierarchy



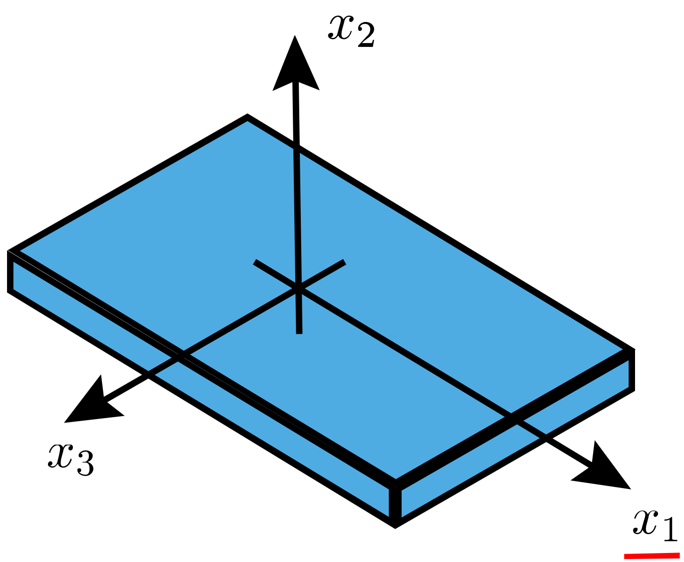
- Models are related to each other
- Other models to consider
  - Karafillis-Boyce
  - Cazacu

Barlat Model

2090-T3 Al *

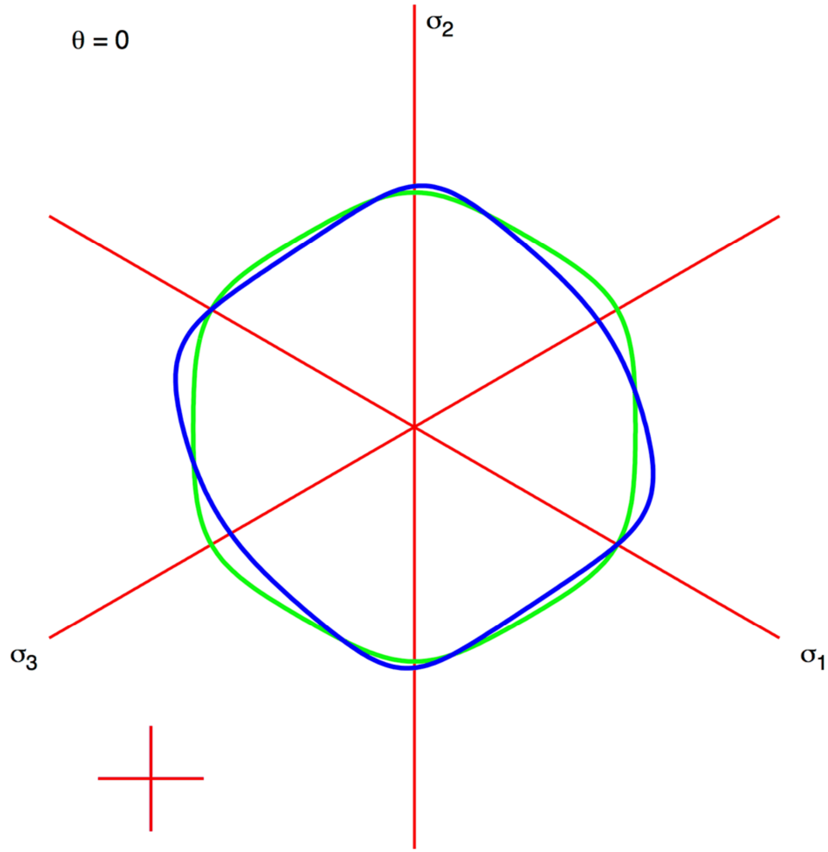
$$a = 8$$

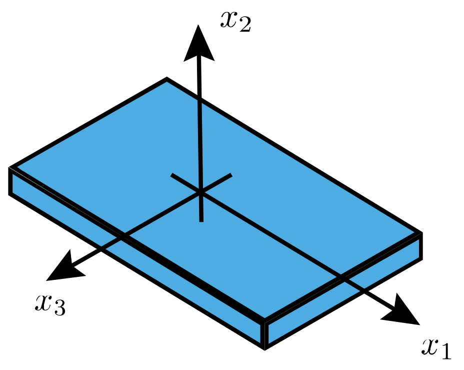
$$
\begin{aligned}
c'_{12} &= -0.069888 \quad ; \quad c''_{12} = 0.981171 \\
c'_{13} &= 0.936408 \quad ; \quad c''_{13} = 0.476741 \\
c'_{21} &= 0.079143 \quad ; \quad c''_{21} = 0.575316 \\
c'_{23} &= 1.003060 \quad ; \quad c''_{23} = 0.866827 \\
c'_{31} &= 0.524741 \quad ; \quad c''_{31} = 1.145010 \\
c'_{32} &= 1.363180 \quad ; \quad c''_{32} = -0.079294 \\
c'_{44} &= 1.023770 \quad ; \quad c''_{44} = 1.051660 \\
c'_{55} &= 1.069060 \quad ; \quad c''_{55} = 1.147100 \\
c'_{66} &= 0.954322 \quad ; \quad c''_{66} = 1.404620
\end{aligned}
$$



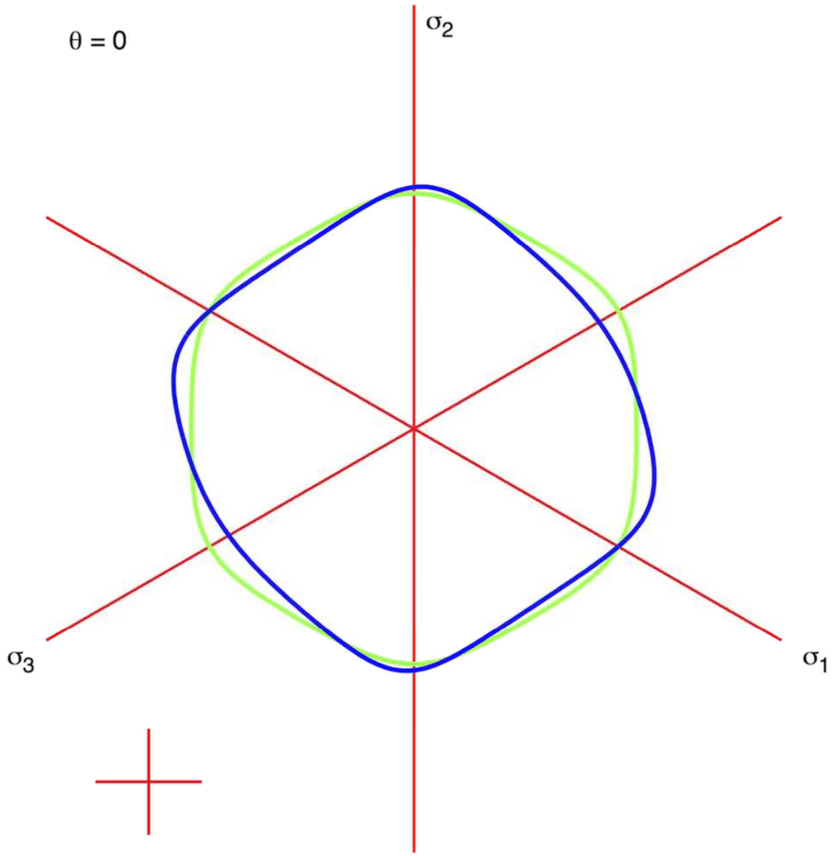
* Barlat et. al., "Linear transformation based anisotropic yield functions", IJP, v. 21, 2005.

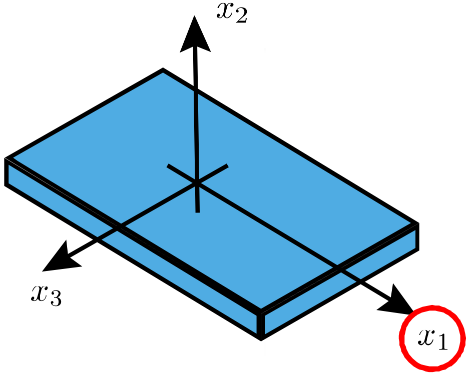
Hosford and Barlat



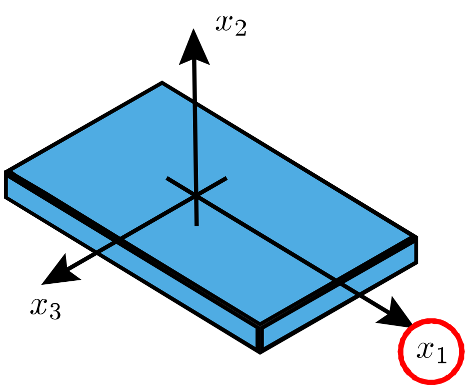


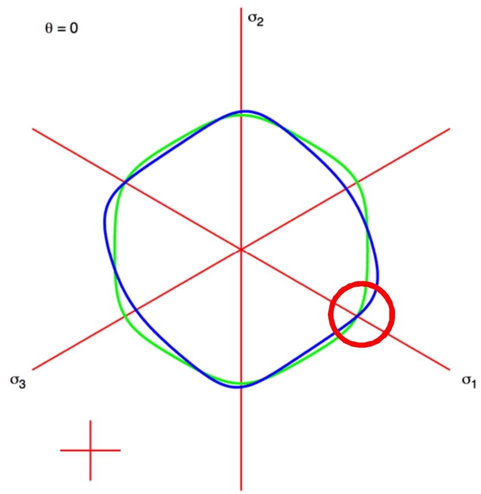
Hosford and Barlat Model

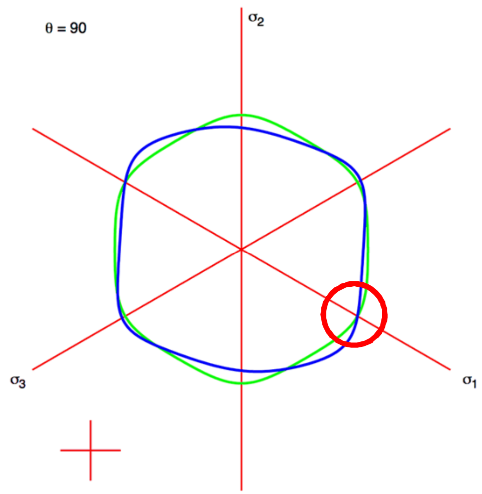




Hosford and Barlat Model







Hosford and Barlat Model

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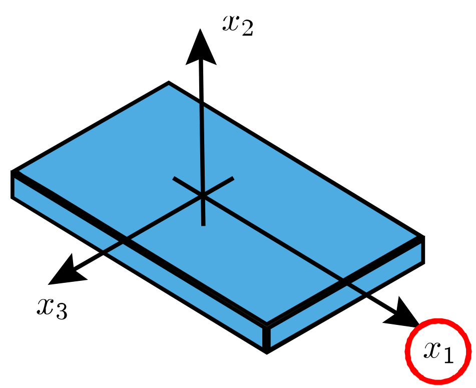
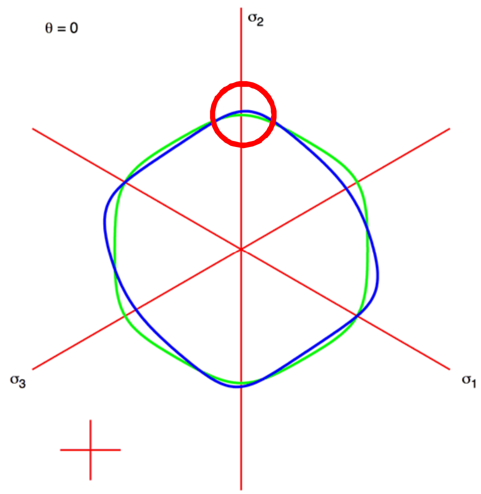
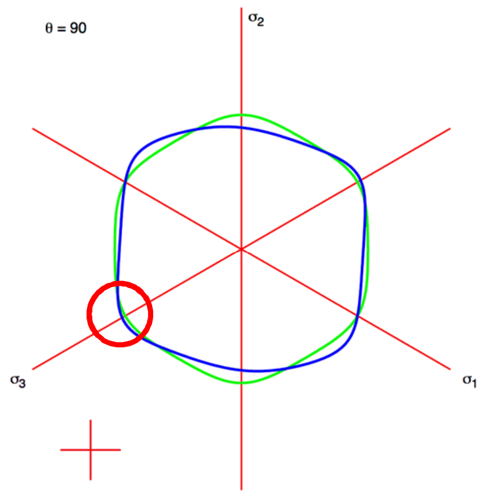
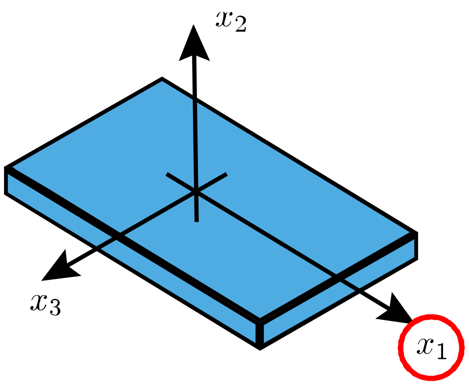
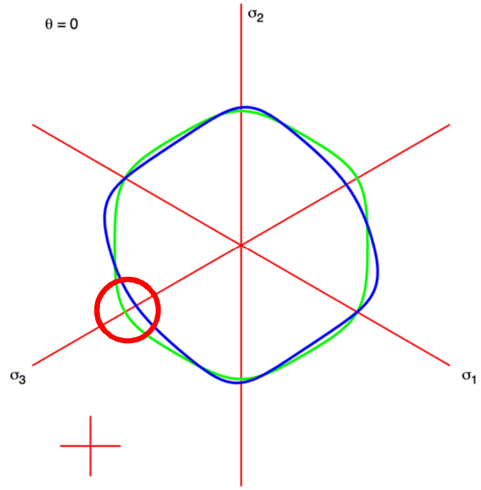


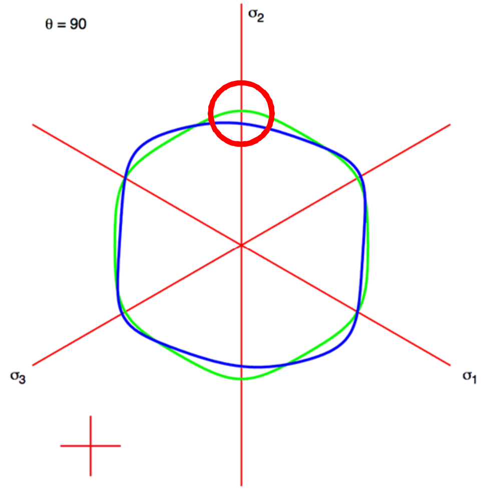




Hosford and Barlat Model



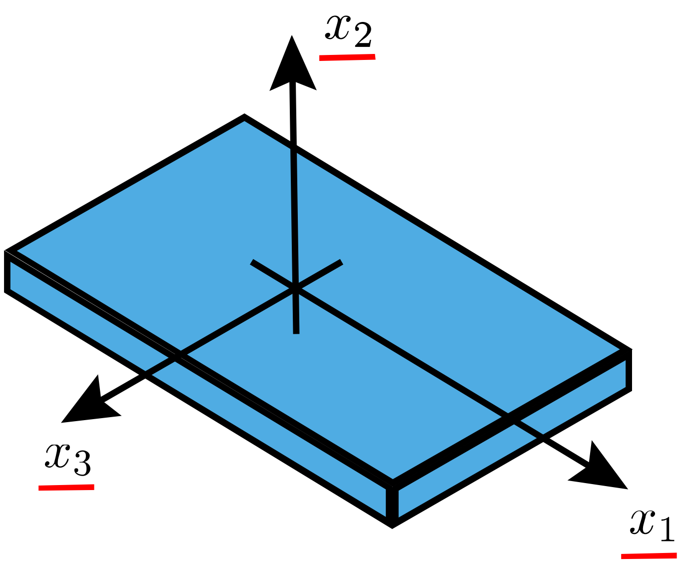




Hill Model

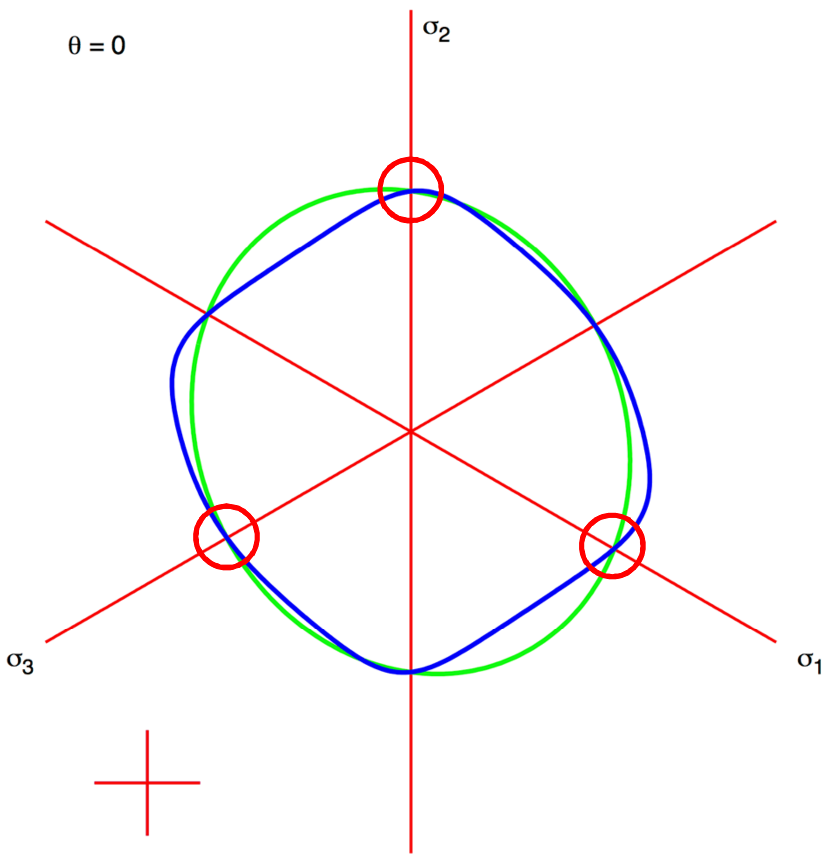
2090-T3 Al *

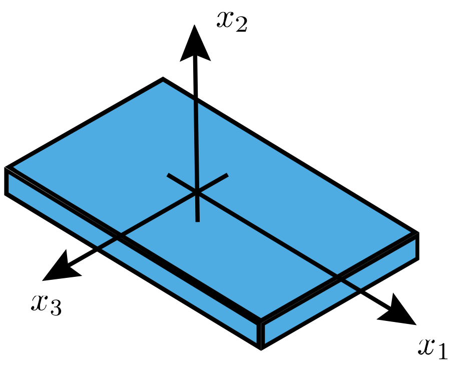
$$F = 0.690940 \quad , \quad L = 2.85422$$
$$G = 0.206643 \quad ; \quad M = 3.67927$$
$$H = 0.790646 \quad ; \quad N = 2.19515$$



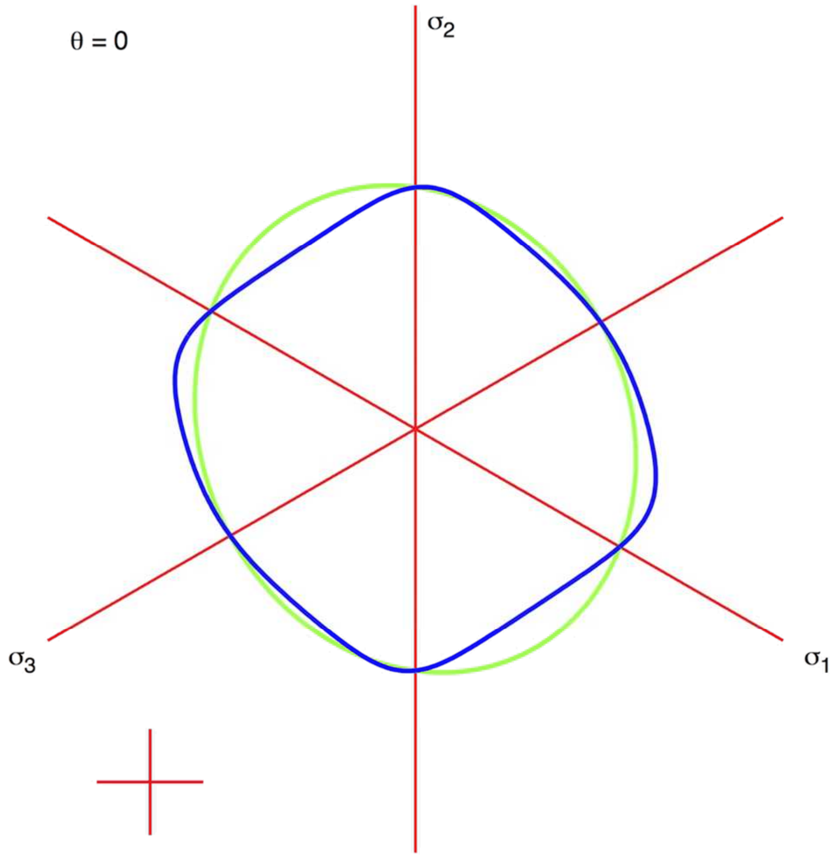
* Barlat et. al., "Linear transformation based anisotropic yield functions", IJP, v. 21, 2005.

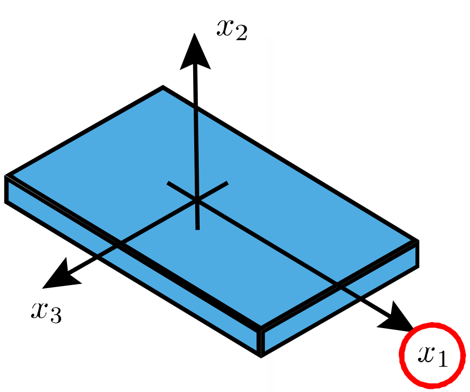
Hill and Barlat



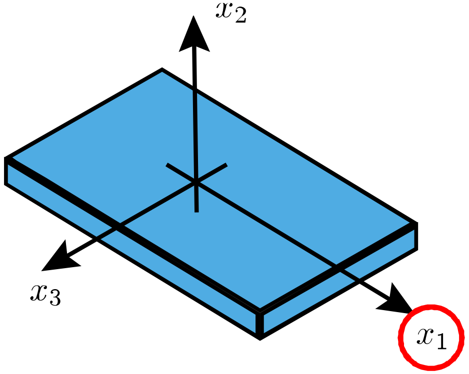


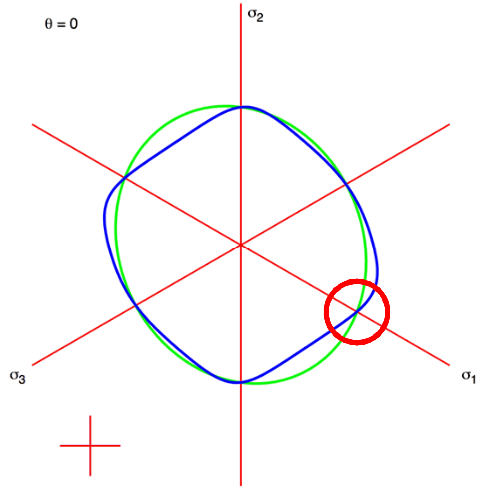
Hill and Barlat Model

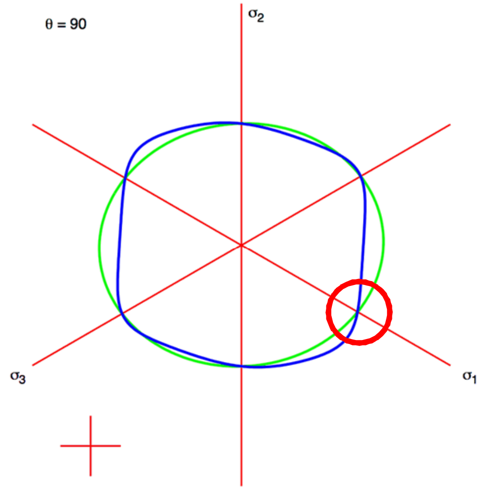




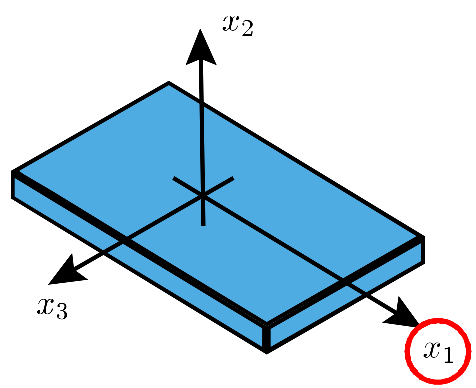
Hill and Barlat Model

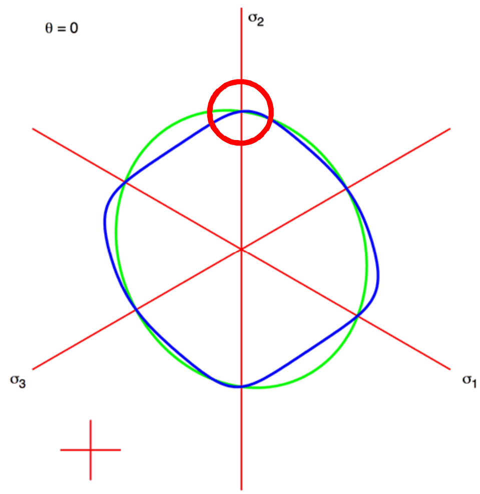


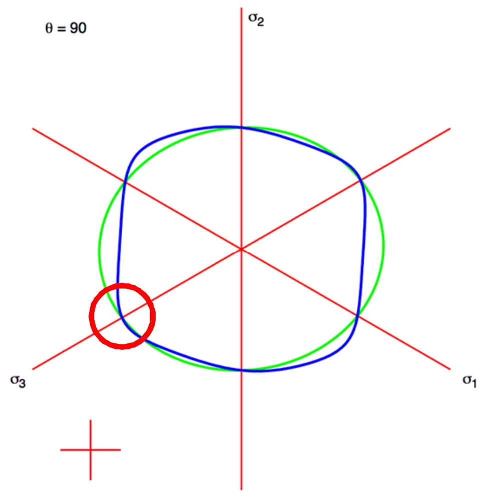




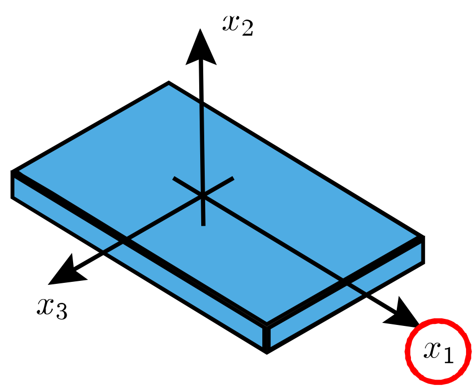
Hill and Barlat Model

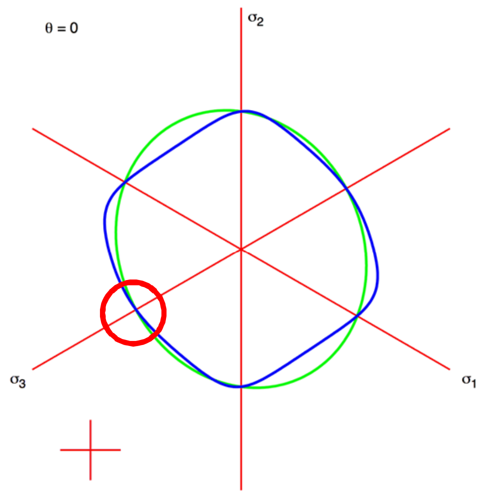


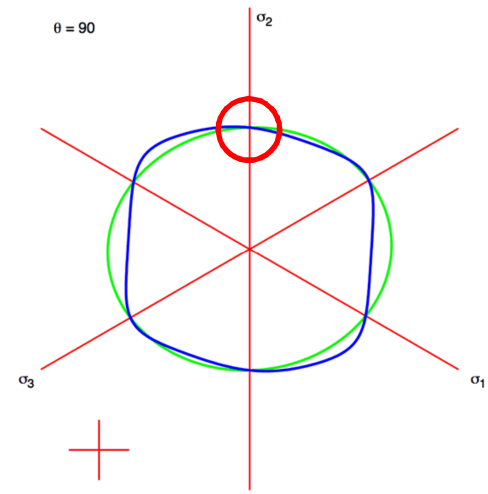




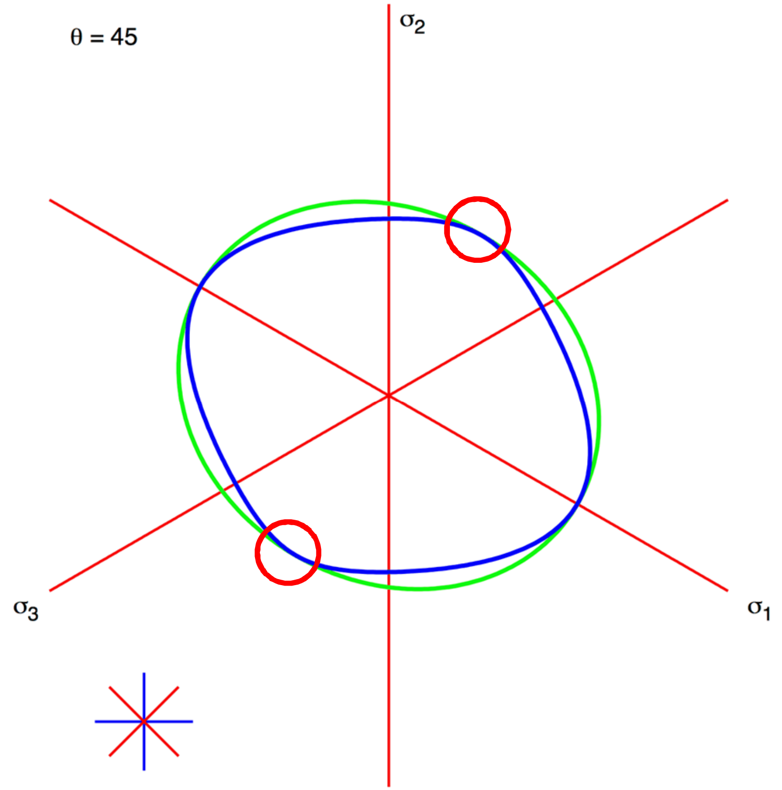
Hill and Barlat Model







Hill and Barlat Model



Rate Formulation

rate form of the model

$$\dot{\boldsymbol{\sigma}} = \mathbb{C} : \dot{\boldsymbol{\varepsilon}}^e$$

additive decomposition of strain rate

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p$$

associated flow

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\gamma} \frac{\partial \phi}{\partial \boldsymbol{\sigma}}$$

$$\boxed{\dot{\boldsymbol{\sigma}} = \mathbb{C} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\gamma} \frac{\partial \phi}{\partial \boldsymbol{\sigma}} \right)}$$

Return Mapping Algorithms

trial stress

$$\boldsymbol{\sigma}^{\mathrm{tr}} = \boldsymbol{\sigma}^{n} + \mathbb{C} : \Delta\boldsymbol{\varepsilon}$$

yield function

$$f = \phi\left(\boldsymbol{\sigma}\right) - \bar{\sigma}\left(\Delta\gamma\right)$$

plastic strain residual

$$\mathbf{R} = -\Delta\boldsymbol{\varepsilon}^{p} + \Delta\gamma\frac{\partial\phi}{\partial\boldsymbol{\sigma}}$$

plastic strain increment

$$\Delta\boldsymbol{\varepsilon}^{p} = \mathbb{C}^{-1} : \left(\boldsymbol{\sigma} - \boldsymbol{\sigma}^{tr}\right)$$

equations we want to solve

$$f = 0$$

$$\mathbf{R} = \mathbf{0}$$

unknowns

$$\Delta\gamma \quad , \quad \boldsymbol{\sigma}$$

Iterative Algorithm

create iterative solution for unknowns

$$\Delta\gamma^{(k+1)} = \Delta\gamma^{(k)} + \Delta\left(\Delta\gamma\right)$$

$$\boldsymbol{\sigma}^{(k+1)} = \boldsymbol{\sigma}^{(k)} + \Delta\boldsymbol{\sigma}$$

two algorithm to solve for increment in unknowns

- Newton
- line search based on Newton

Newton Algorithm

$$\Delta\left(\Delta\gamma\right) = \frac{f^{(k)} - \mathbf{R}^{(k)} : \mathscr{L}^{(k)} : \dfrac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}}}{\dfrac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}} : \mathscr{L}^{(k)} : \dfrac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}} + H'_{(k)}}$$

$$\Delta\boldsymbol{\sigma} = -\mathscr{L}^{(k)} : \left(\mathbf{R}^{(k)} + \Delta\left(\Delta\gamma\right)\frac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}}\right)$$

slope of hardening curve

$$H' = \frac{d\bar{\sigma}}{d\Delta\gamma}$$

Hessian

$$\mathscr{L}^{-1} = \mathbb{C}^{-1} + \Delta\gamma\,\frac{\partial^2\phi}{\partial\boldsymbol{\sigma}\partial\boldsymbol{\sigma}}$$

von Mises

$$\Delta\left(\Delta\gamma\right) = \frac{f^{(k)}}{3\mu + H'_{(k)}} \qquad \Delta\boldsymbol{\sigma} = -\Delta\left(\Delta\gamma\right)\frac{3}{2\phi}\mathscr{L}^{(k)} : \mathbf{s}$$

Line Search Algorithm

$$\Delta\gamma^{(k+1)} = \Delta\gamma^{(k)} + \alpha\,\Delta\left(\Delta\gamma\right)$$

search direction

$$\boldsymbol{\sigma}^{(k+1)} = \boldsymbol{\sigma}^{(k)} + \alpha\,\Delta\boldsymbol{\sigma}$$

search magnitude

$$\alpha \in [0,1]$$

search direction comes from Newton algorithm

need to determine "best" value for α

Line Search Algorithm *

residual

$$\mathbf{r} = \left(\frac{f}{2\mu} \; , \; \mathbf{R} \right)$$

merit function based on residual

$$\psi = \frac{1}{2} \mathbf{r} \cdot \mathbf{r}$$

…as a function of a

$$\psi\left(\alpha\right) = \frac{1}{2} \mathbf{r}\left(\alpha\right) \cdot \mathbf{r}\left(\alpha\right)$$

we want

$$\psi\left(\alpha\right) < \psi\left(0\right)$$

if we get this, then the solution is improving

* Perez-Foguet and Armero, "On the formulation of closest point projection algorithms in elastoplasticity – part II: globally convergent schemes", IJNME, v. 53, 2002.

Line Search Algorithm

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check

$$\psi(1) < \psi(0)$$

Yes

$$\alpha = 1$$

use Newton

No

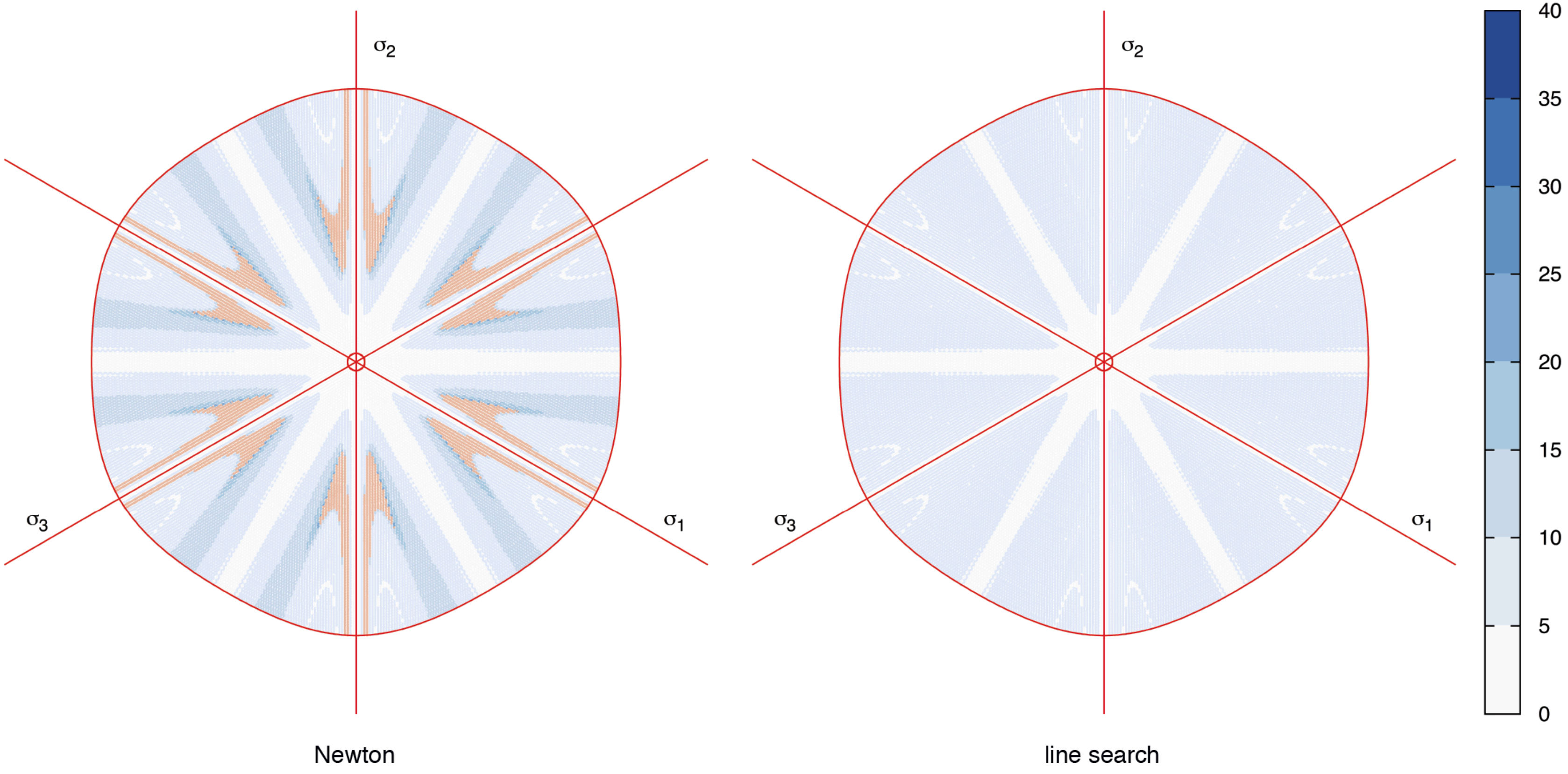
$$\hat{\psi} = \psi_0 + \alpha\psi_1 + \alpha^2\psi_2$$

$$\hat{\psi} = \left(1 - 2\alpha + \alpha^2\right)\psi(0) + \alpha^2\psi(1)$$

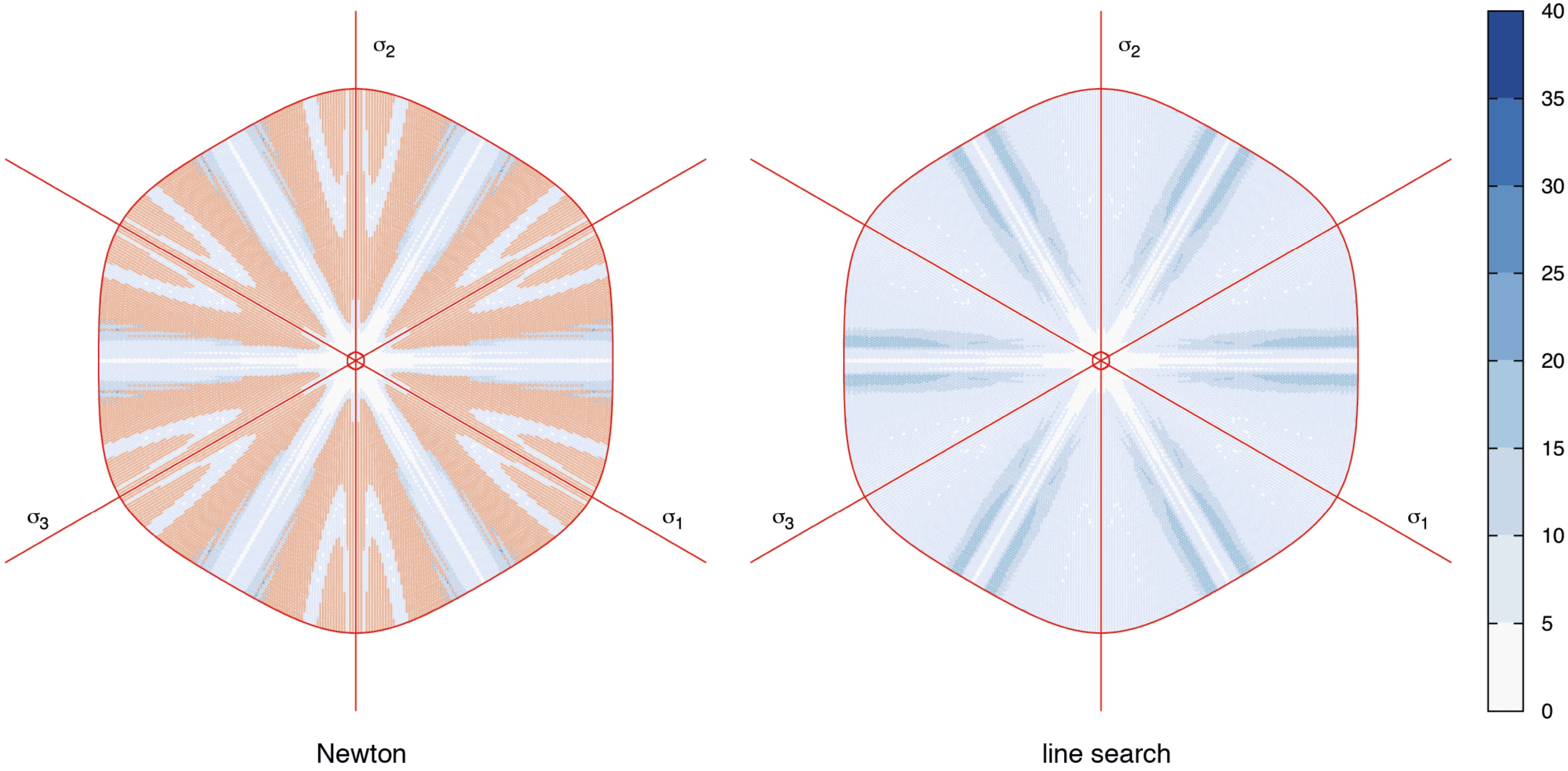
$$\alpha = \frac{\psi(0)}{\psi(1) + \psi(0)}$$

Hosford Model

Hosford (a=6)

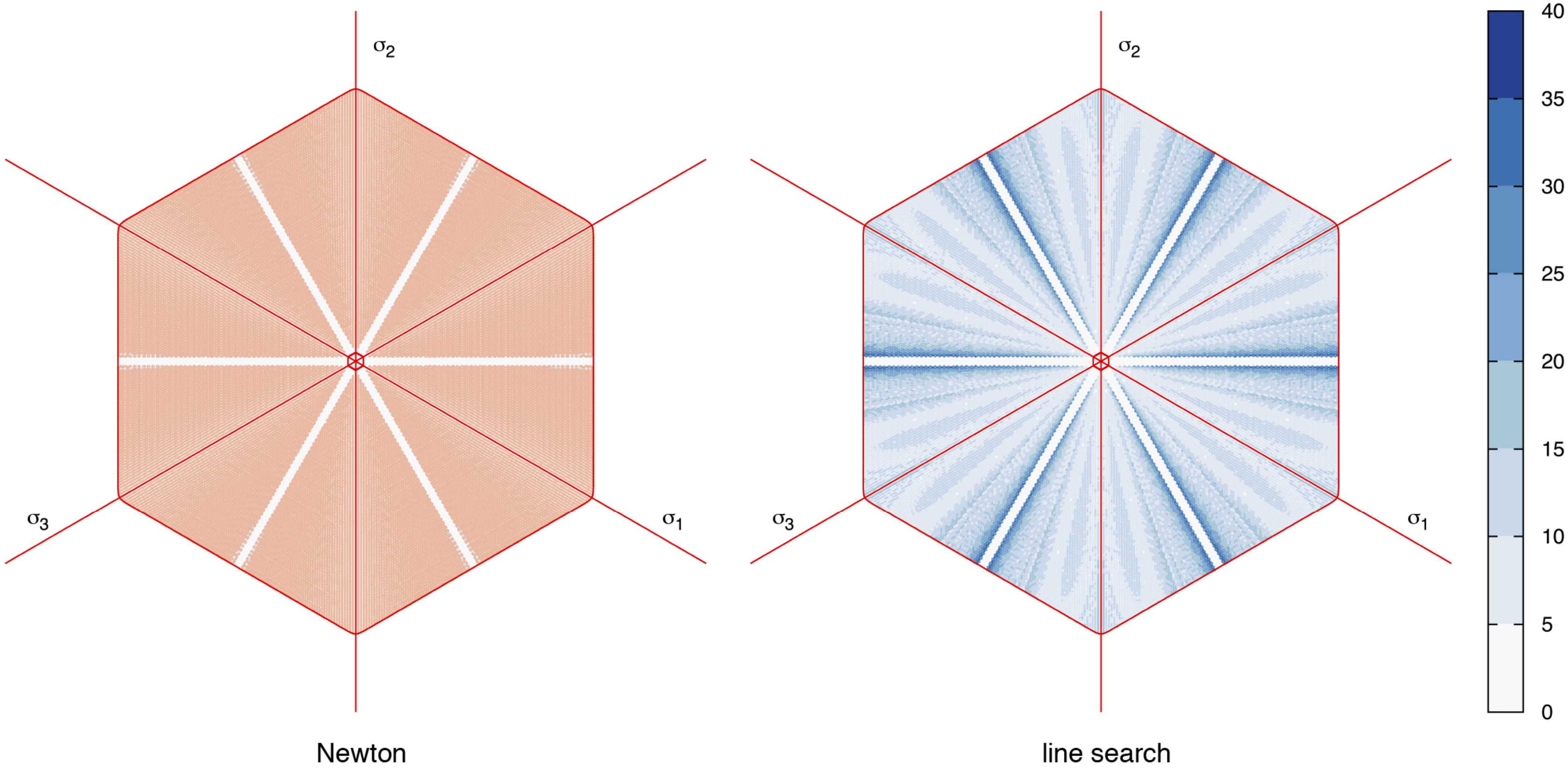


Hosford (a=8)



σ_2
σ_3
σ_1
Newton
σ_2
σ_3
σ_1
line search
40
35
30
25
20
15
10
5
0

Hosford (a=100)



Visualization

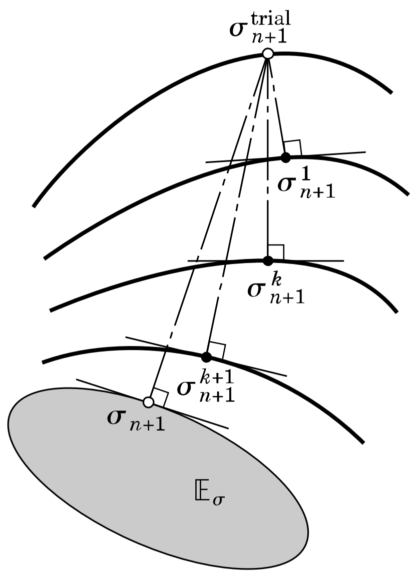
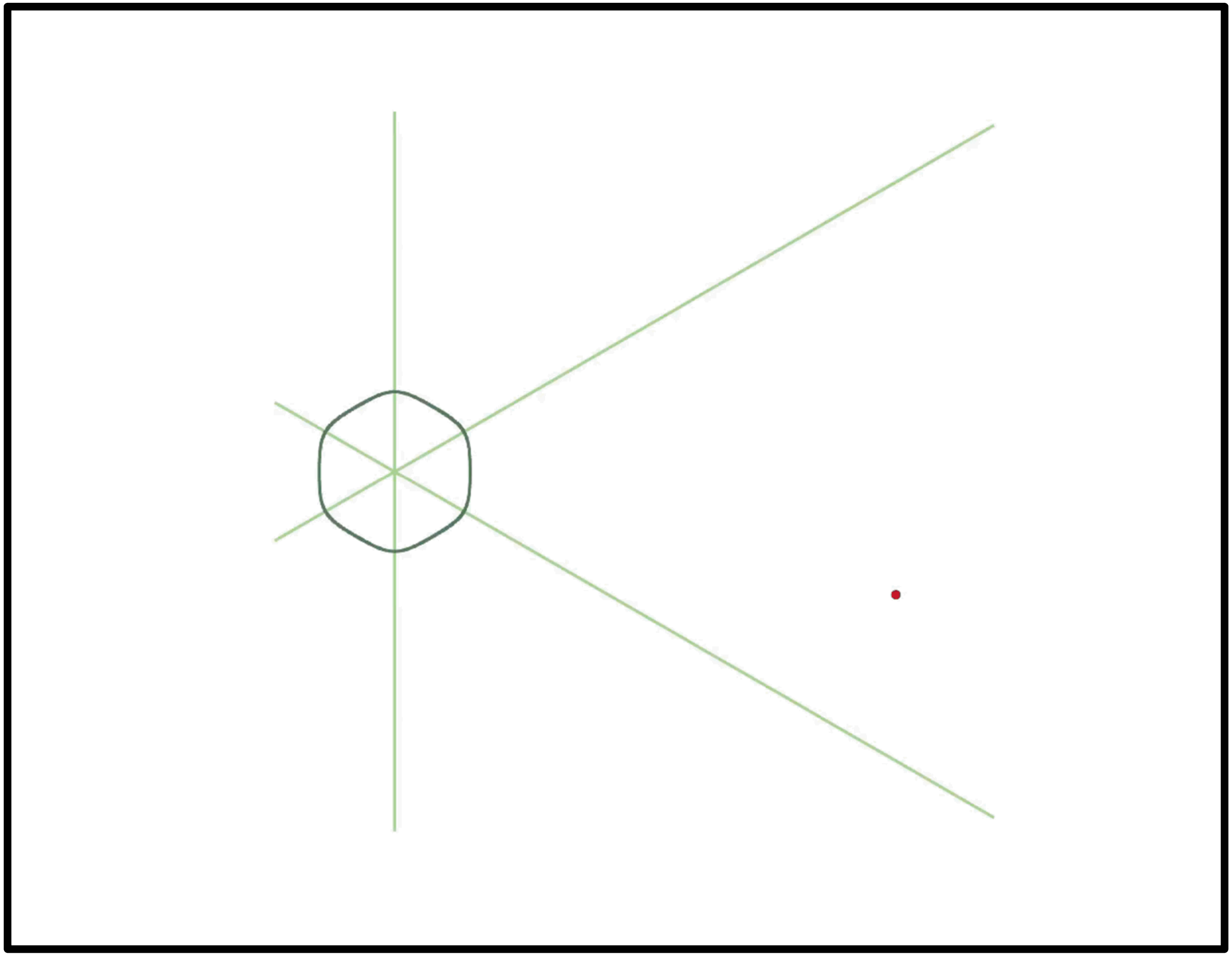


FIGURE 3-10. A geometric interpretation of the closest point projection algorithm in stress space. At each iterate $(\bullet)^{(k)}$, the constraint is linearized to find the intersection (cut) with $f = 0$. The next iterate $(\bullet)^{(k+1)}$, located on level set $f_{n+1}^{(k+1)} > 0$, is the closest point of that level set to the previous iterate $(\bullet)^{(k)}$ in the metric defined by the elasticities $\mathbf{C}$.

Simo and Hughes, Computational Inelasticity, 1998.

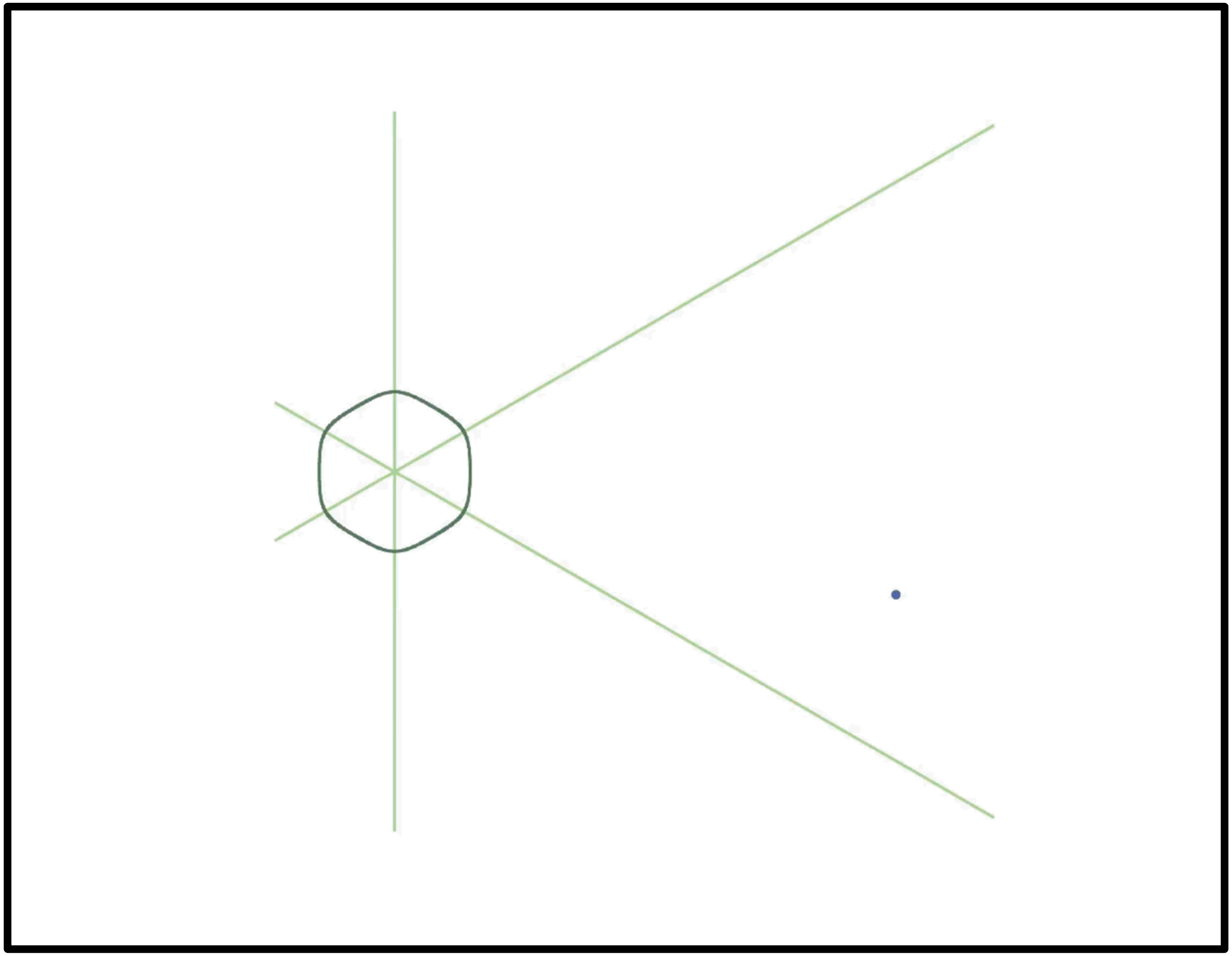
Hosford (a=8)

Newton algorithm



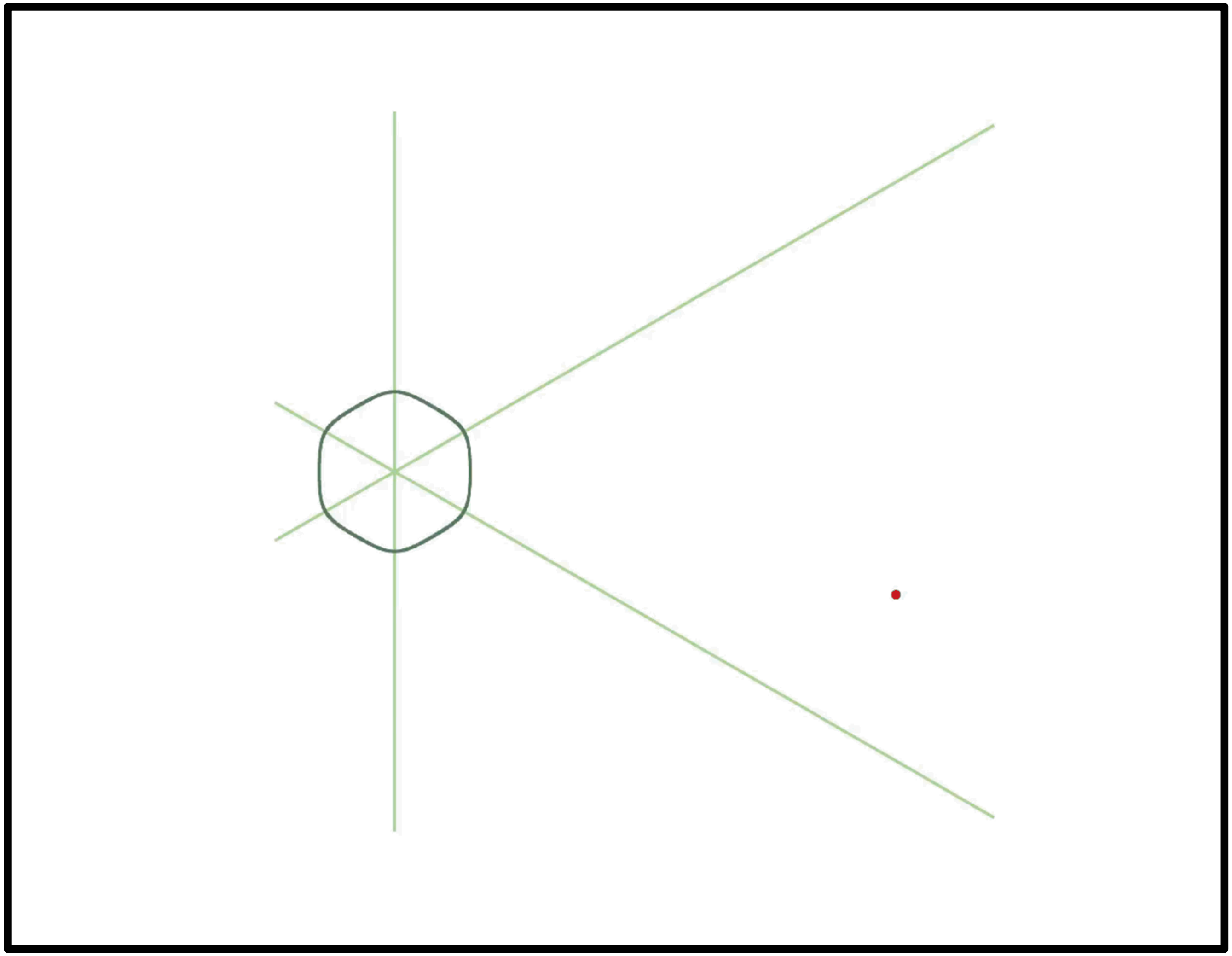
Hosford (a=8)

line search algorithm



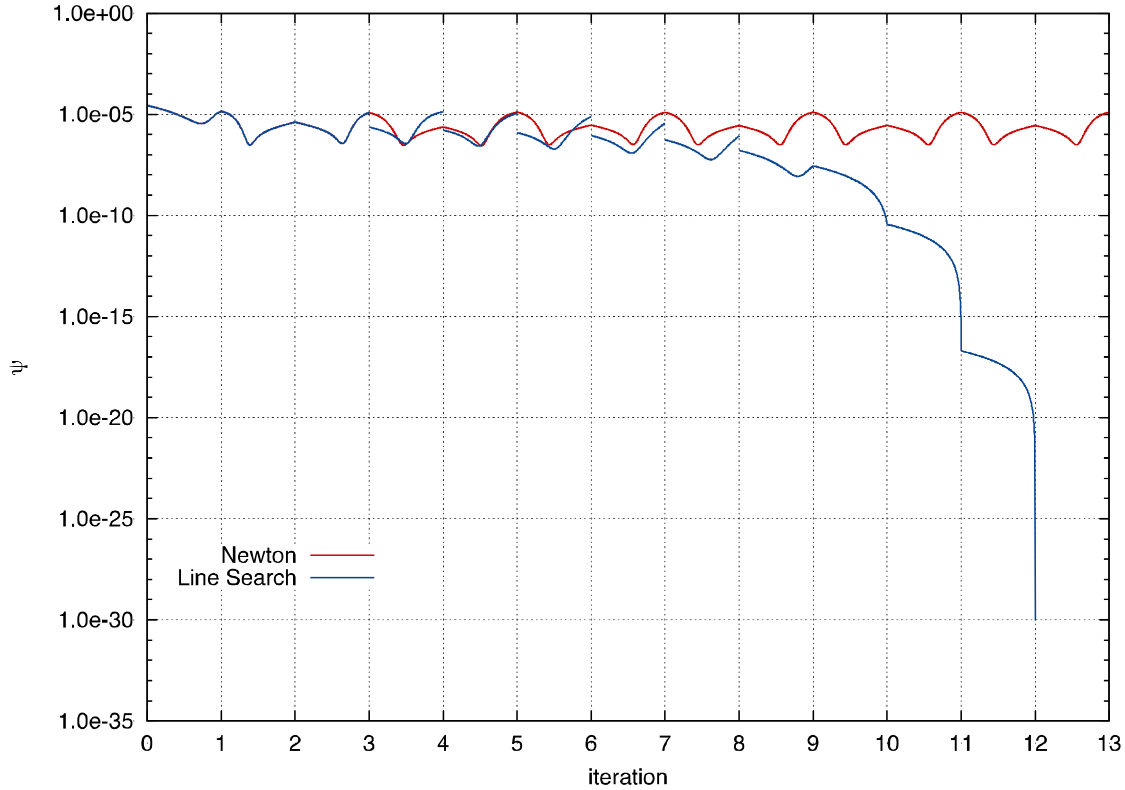
Hosford (a=8)

Newton and line search algorithms

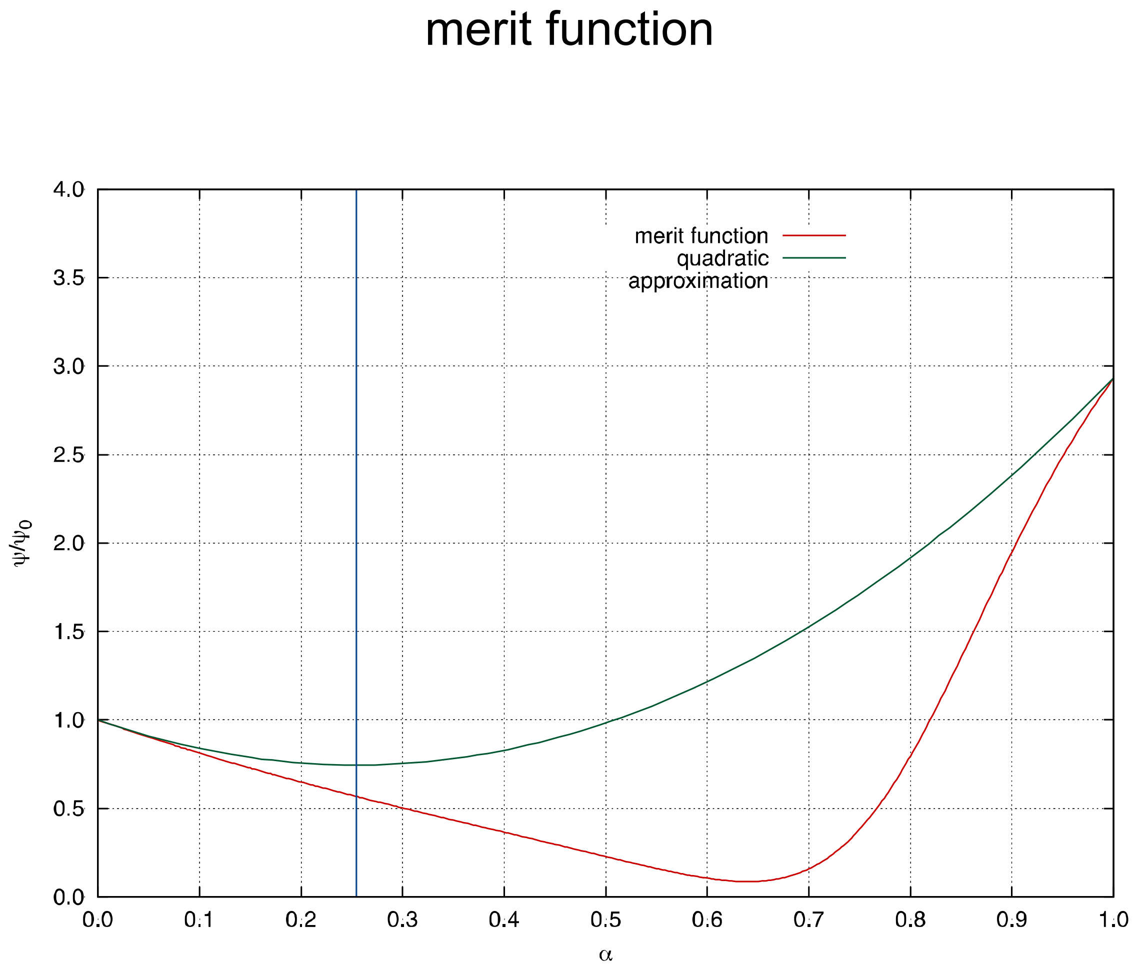


Hosford (a=8)

merit function



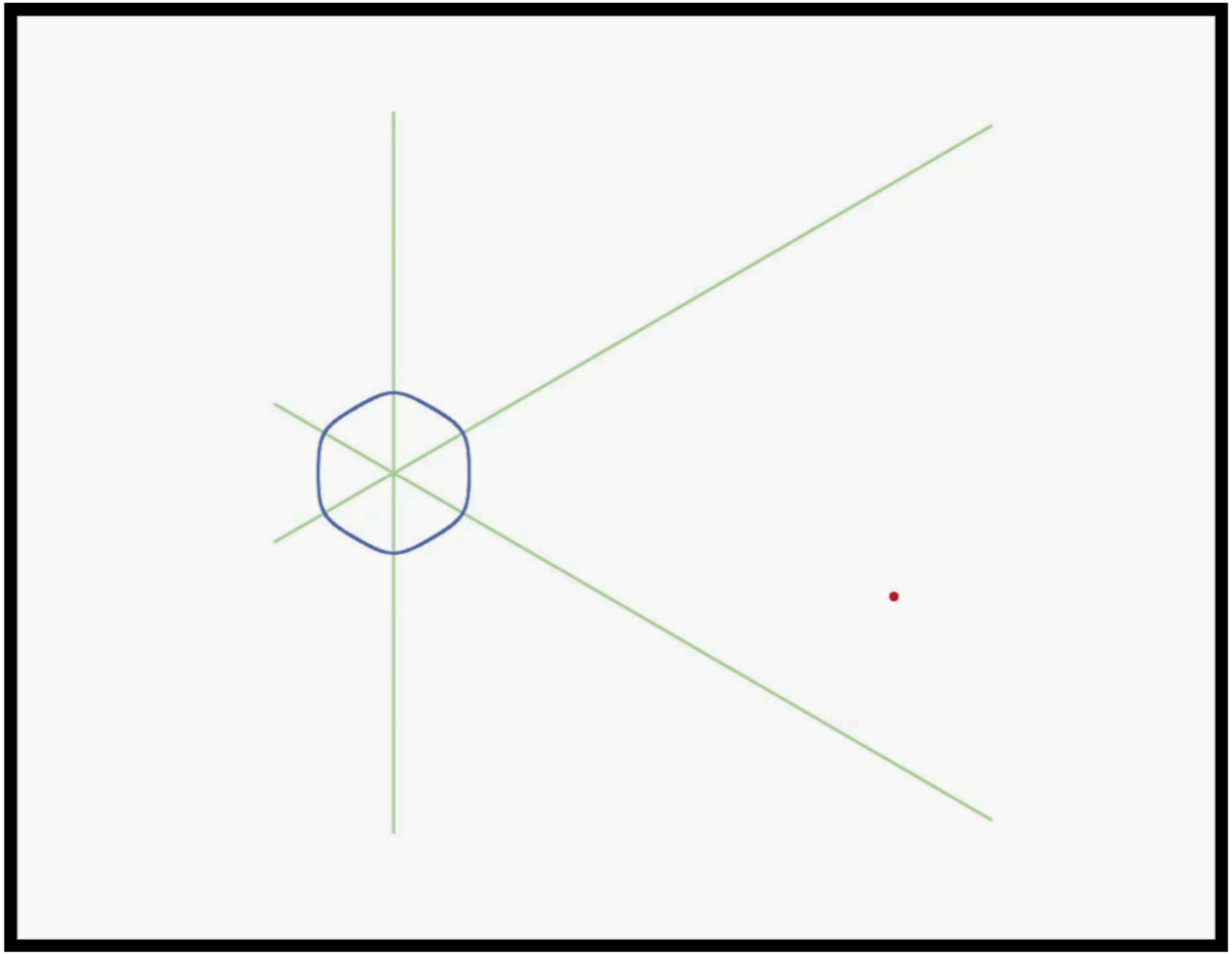
Hosford (a=8)



merit function
merit function
quadratic approximation
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
ψ/ψ0
α

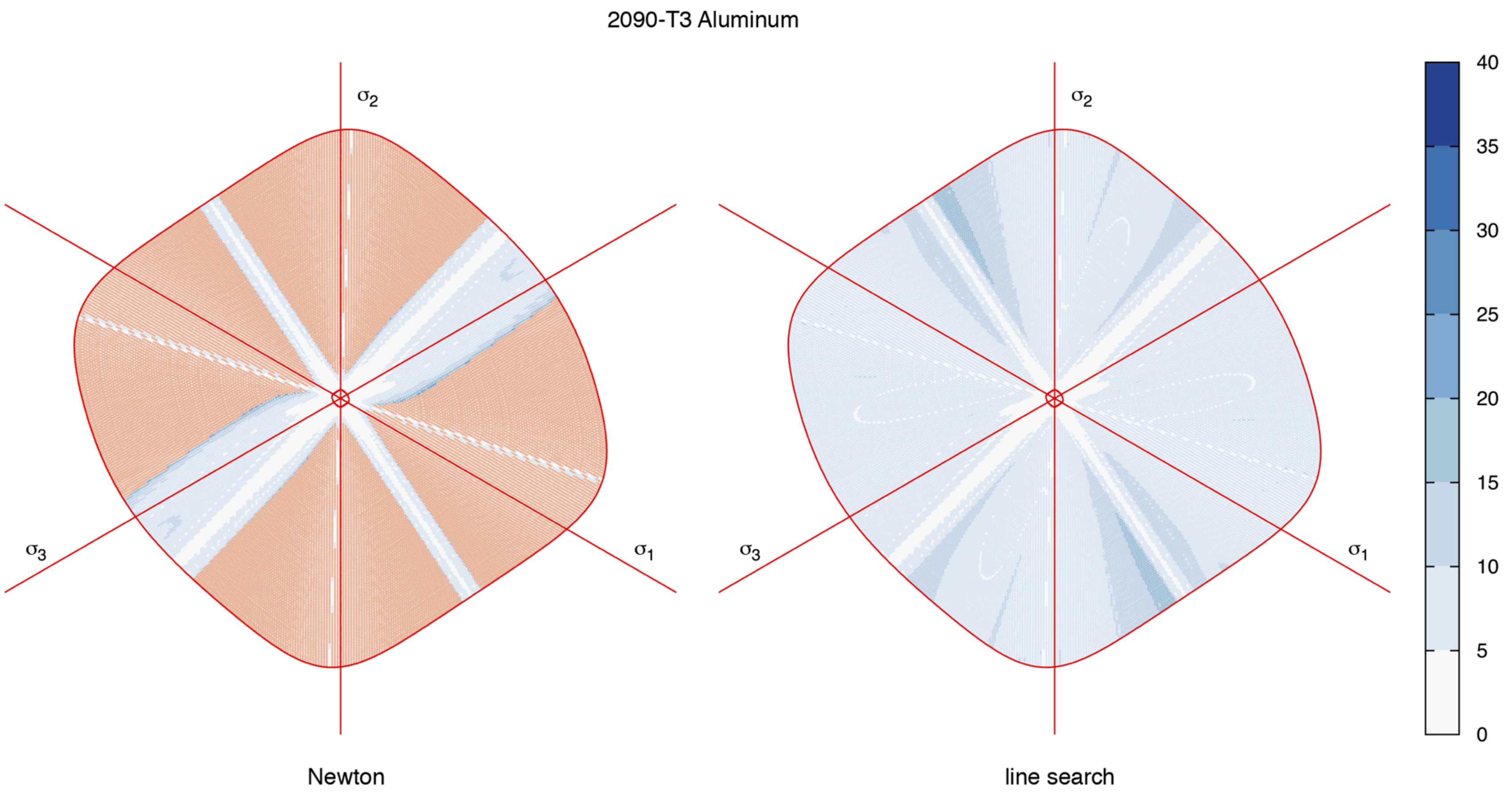
Hosford (a=8)

return map with hardening

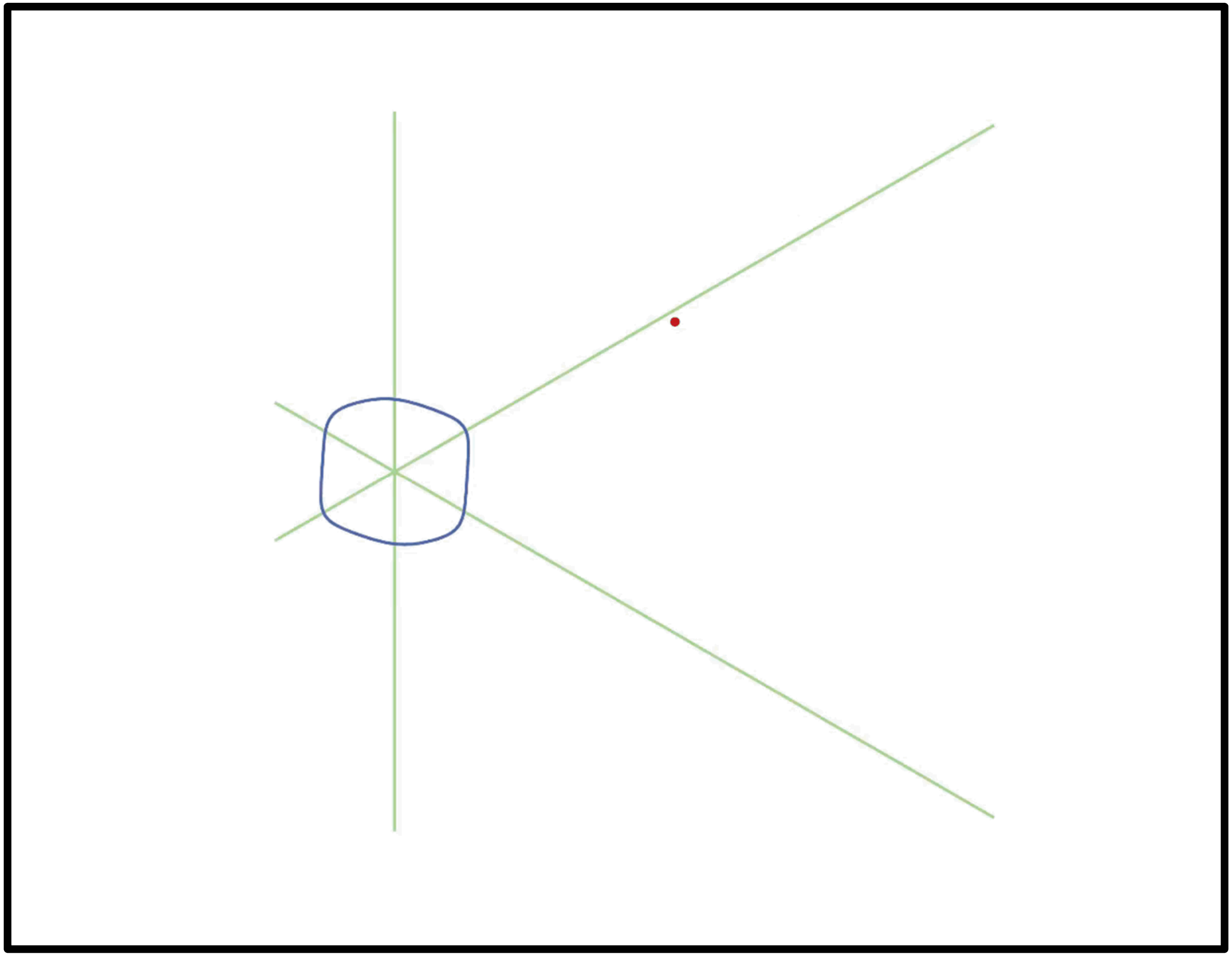


Barlat Model

Barlat Model



Aligned with Material Axes



Orthotropic Plasticity Models

$$\begin{Bmatrix} s'_{11} \\ s'_{22} \\ s'_{33} \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0 & -c'_{12} & -c'_{13} & 0 & 0 & 0 \\ -c'_{21} & 0 & -c'_{23} & 0 & 0 & 0 \\ -c'_{31} & -c'_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_{66} \end{bmatrix} \begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \longleftarrow$$

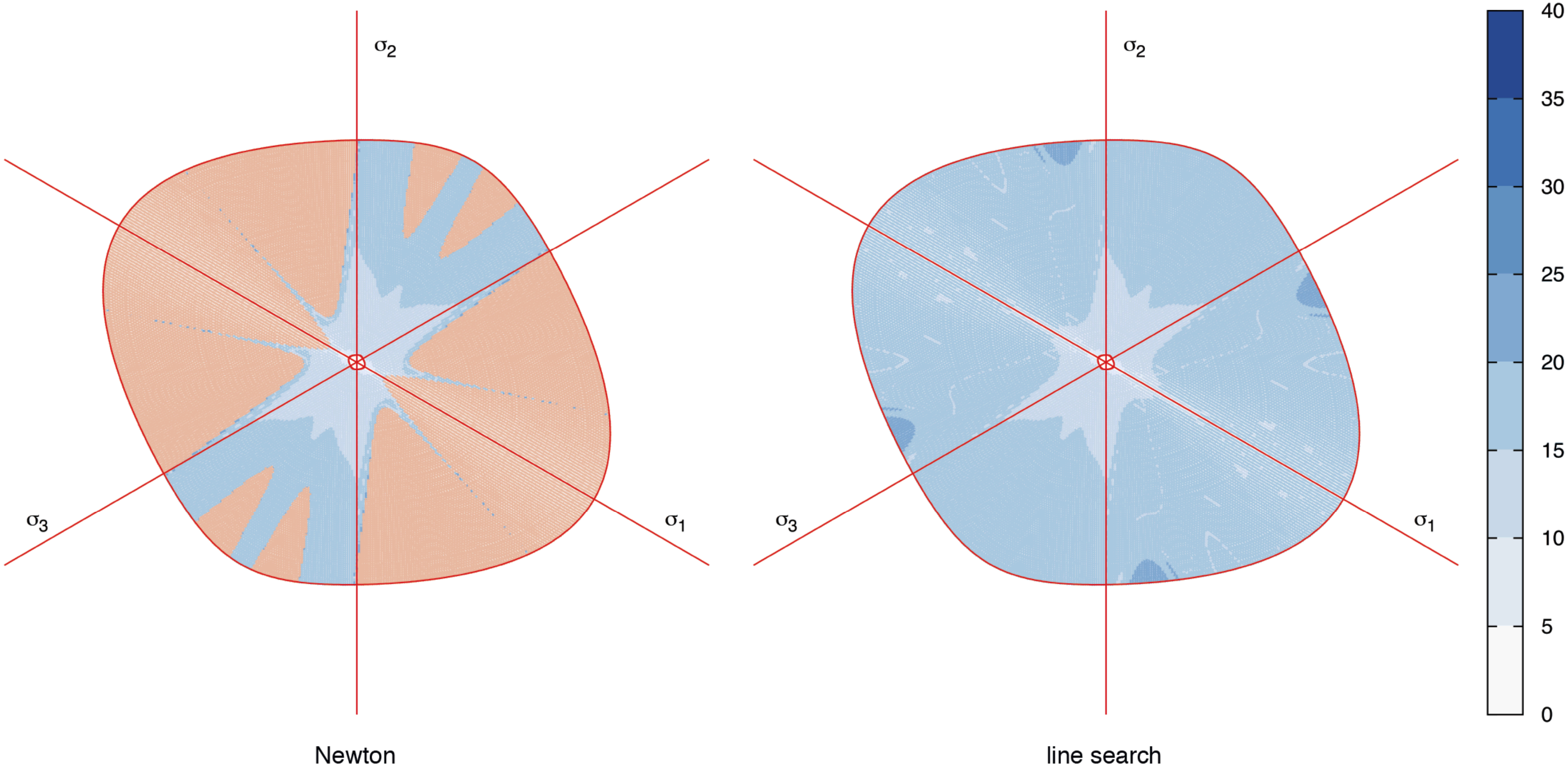
Components in material coordinate system

$$s'_1 = s'_{11} \quad ; \quad s'_2 = s'_{22} \quad ; \quad s'_3 = s'_{33}$$

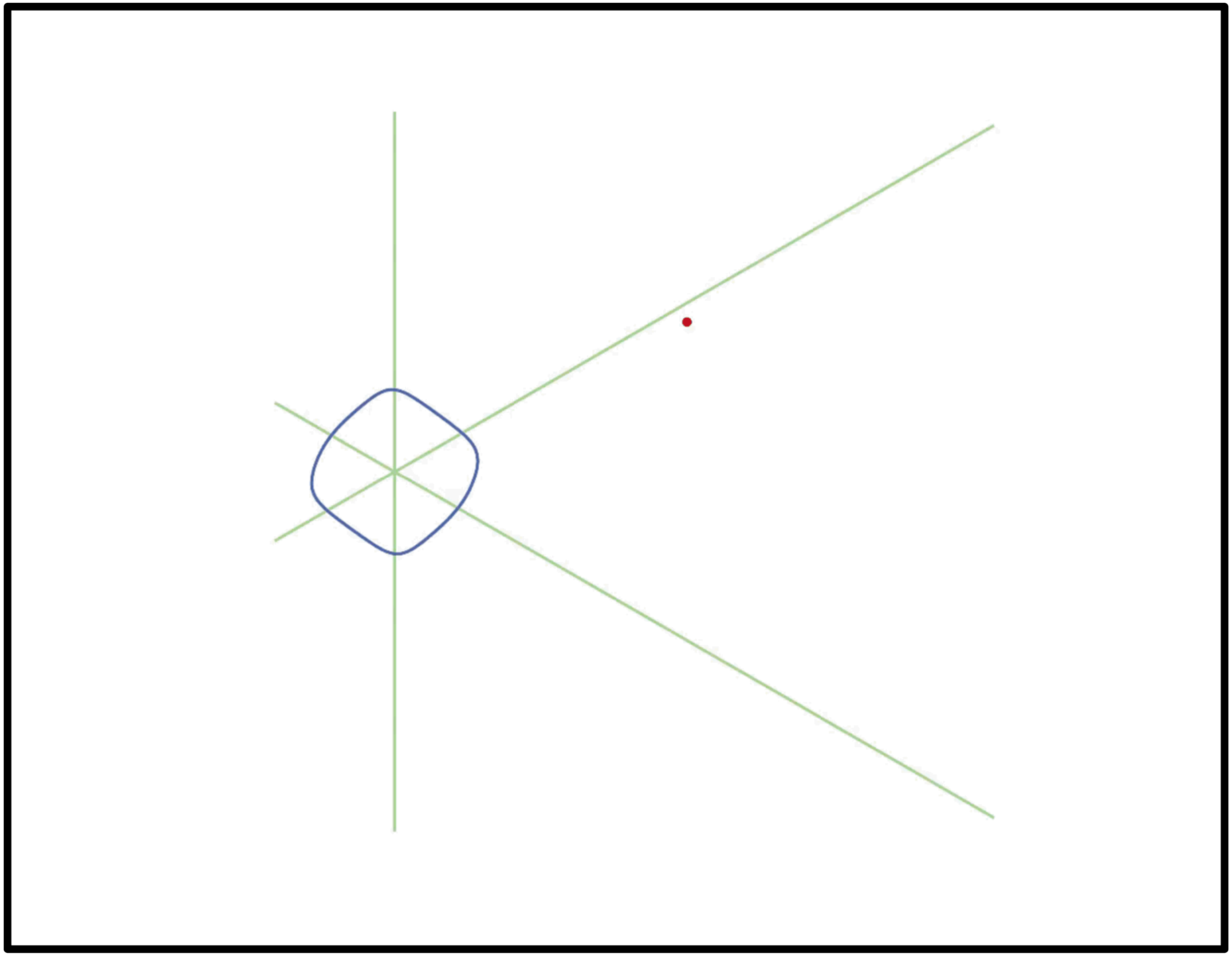
If trial stress is aligned with material coordinate system, then the solution stays aligned with the material coordinate system

Barlat Model

45 degrees about x_1 axis



Not Aligned with Material Axes



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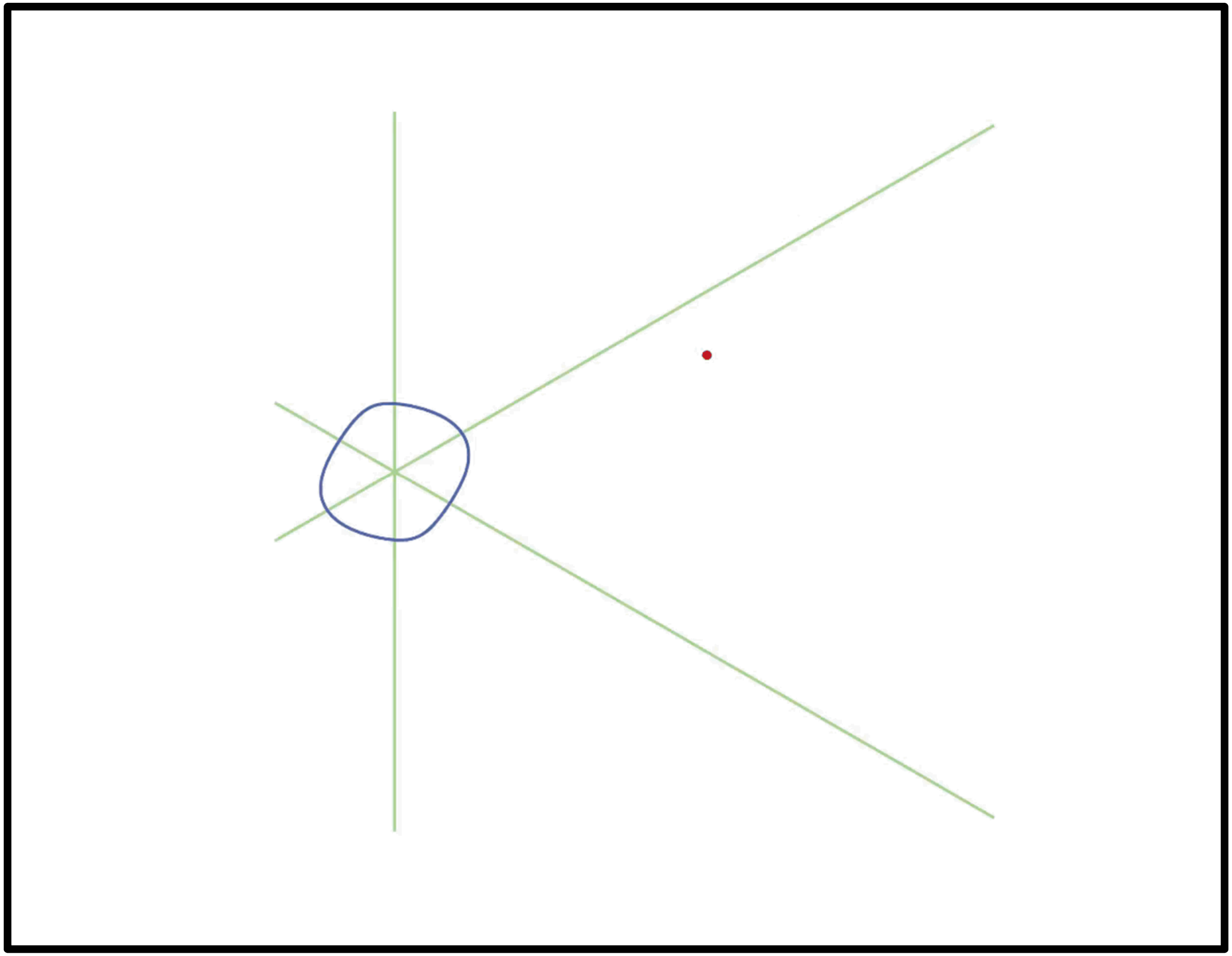

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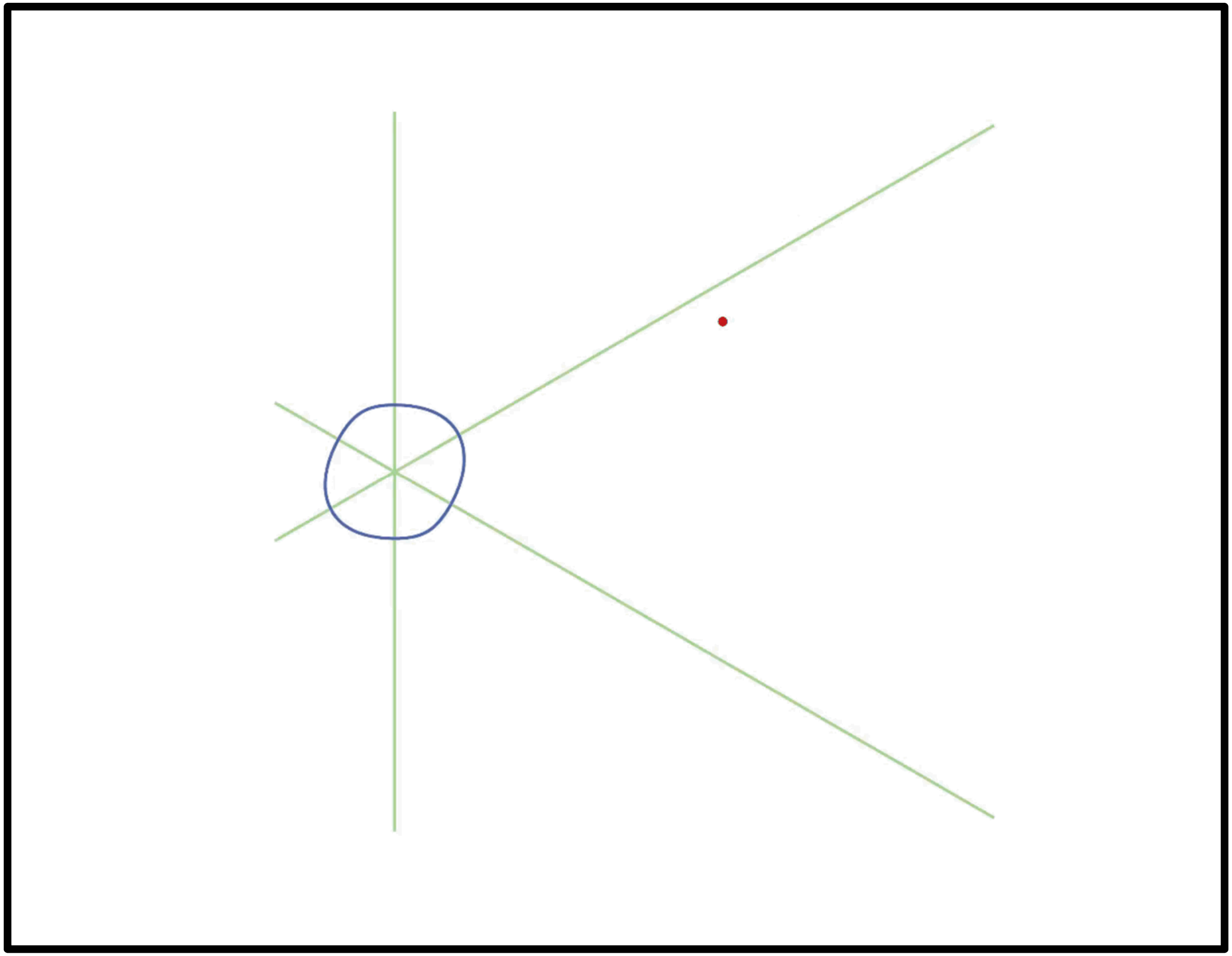

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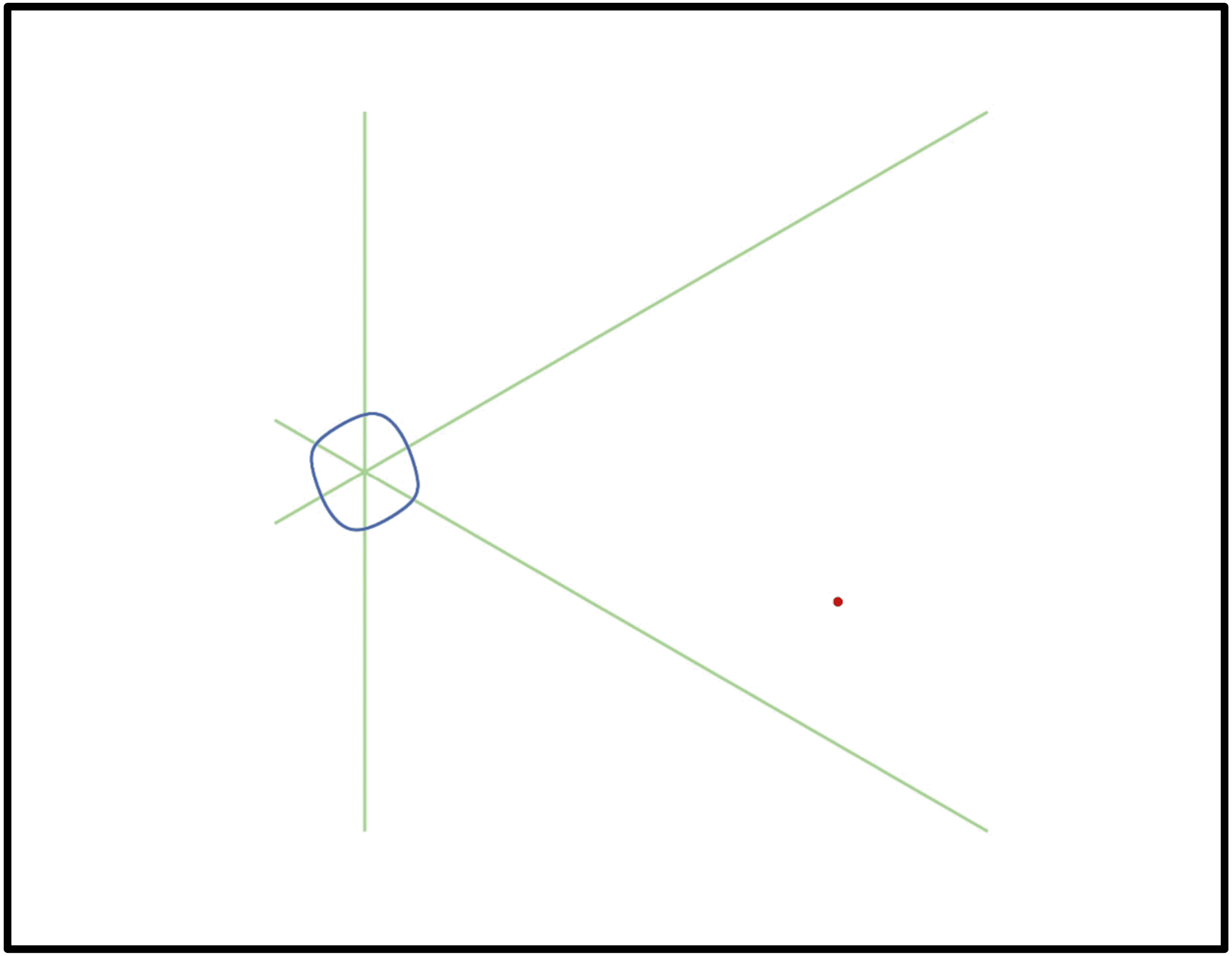
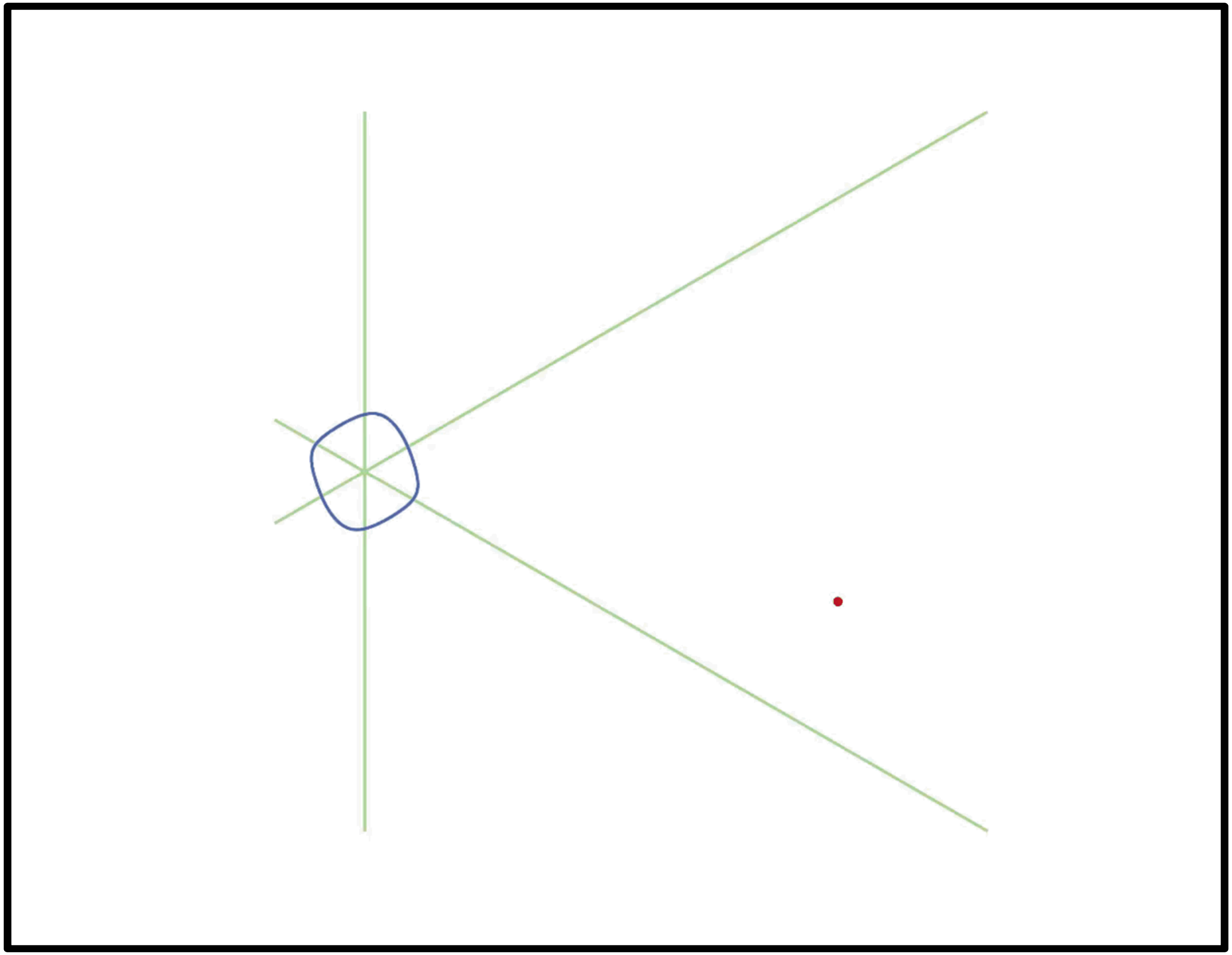


Not Aligned with Material Axes



Barlat Model

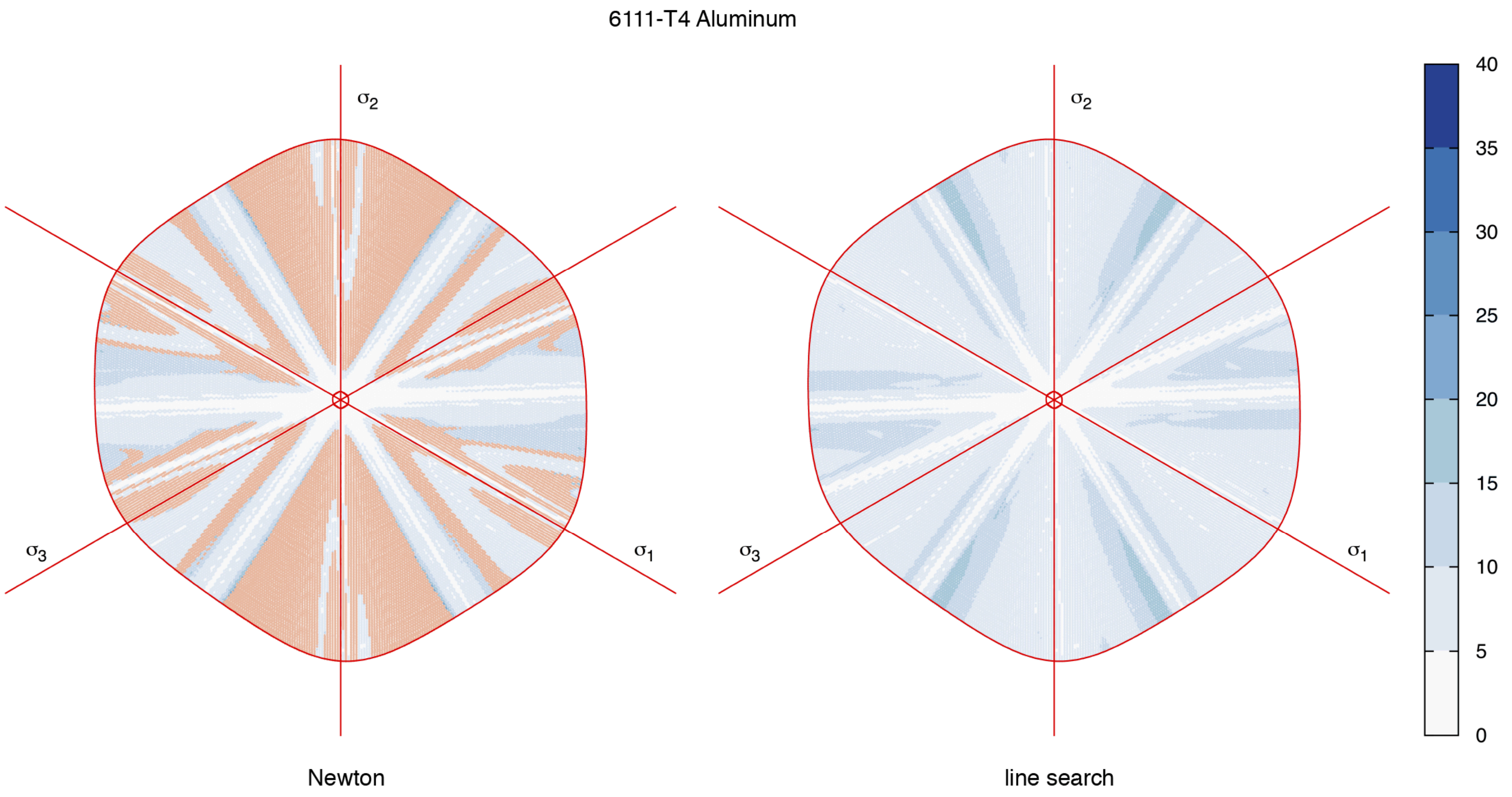
6111-T4 Al *

$$a = 8$$

$$\begin{aligned}
c'_{12} &= 1.241024 \quad ; \quad c''_{12} = 0.775366 \\
c'_{13} &= 1.078271 \quad ; \quad c''_{13} = 0.922743 \\
c'_{21} &= 1.216463 \quad ; \quad c''_{21} = 0.765487 \\
c'_{23} &= 1.223867 \quad ; \quad c''_{23} = 0.793356 \\
c'_{31} &= 1.093105 \quad ; \quad c''_{31} = 0.918689 \\
c'_{32} &= 0.889161 \quad ; \quad c''_{32} = 1.027625 \\
c'_{44} &= 0.501909 \quad ; \quad c''_{44} = 1.115833 \\
c'_{55} &= 0.557173 \quad ; \quad c''_{55} = 1.112273 \\
c'_{66} &= 1.349094 \quad ; \quad c''_{66} = 0.589787
\end{aligned}$$

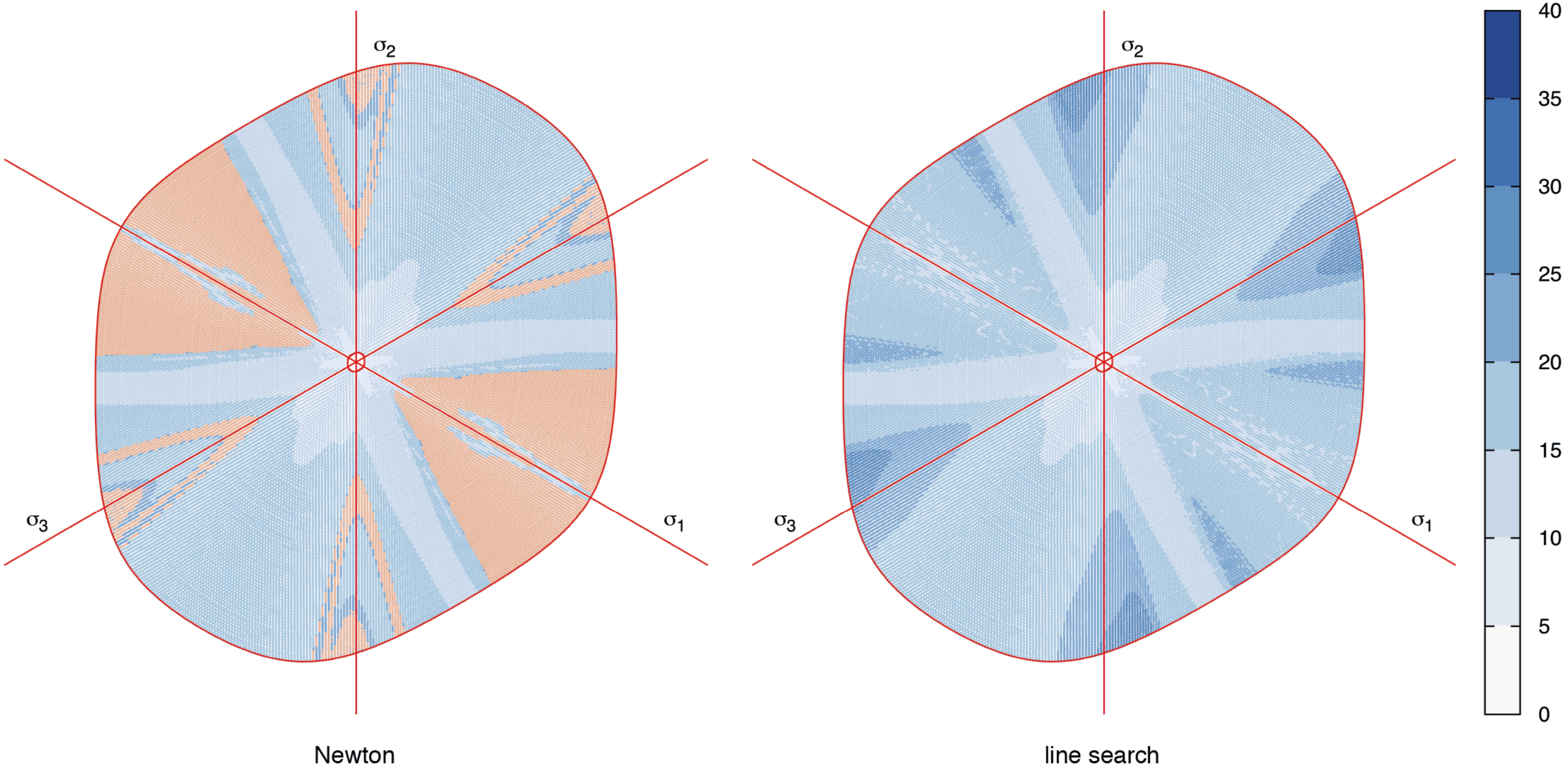
* Barlat et. al., “Linear transformation based anisotropic yield functions”, IJP, v. 21, 2005.

Barlat Model



Barlat Model

45 degrees about x_1 axis

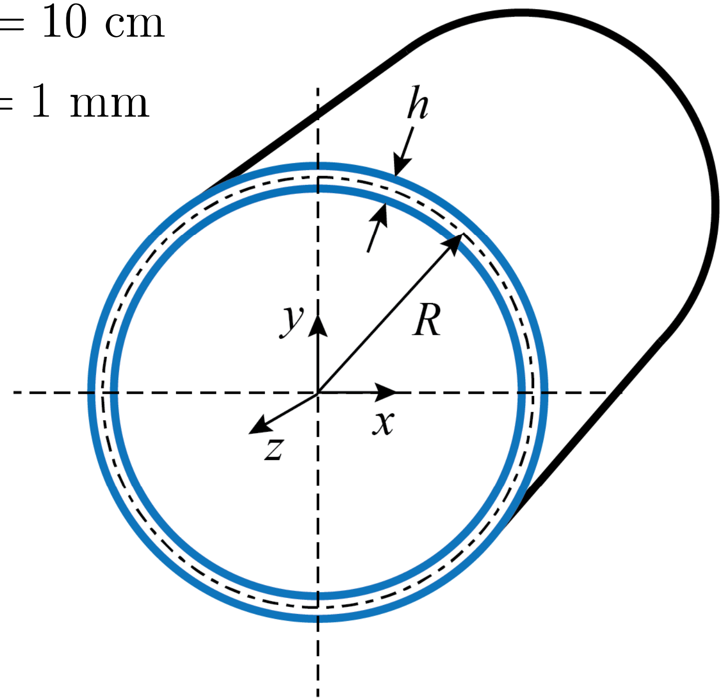


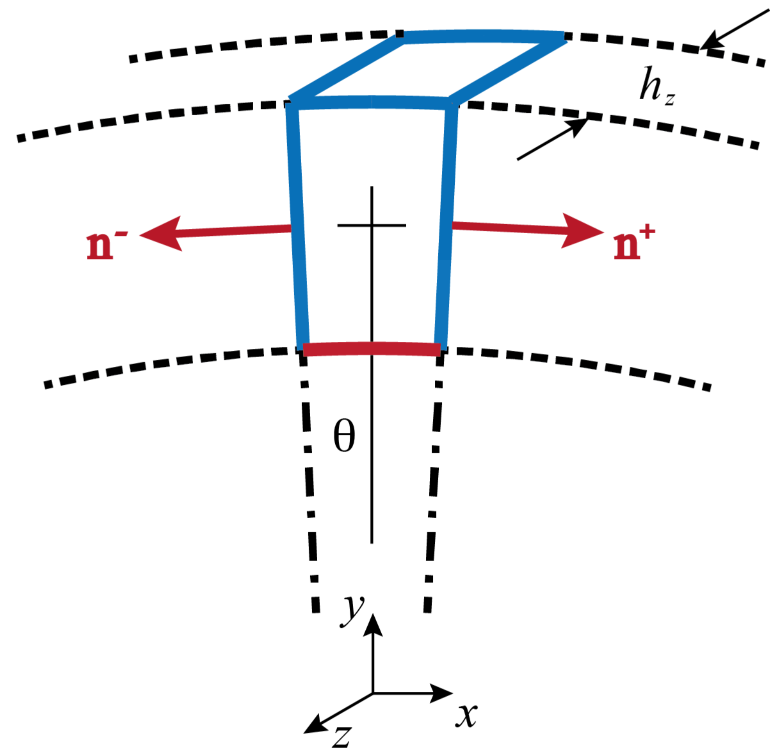
Example Problem

2090-T3 Aluminum

$R = 10$ cm

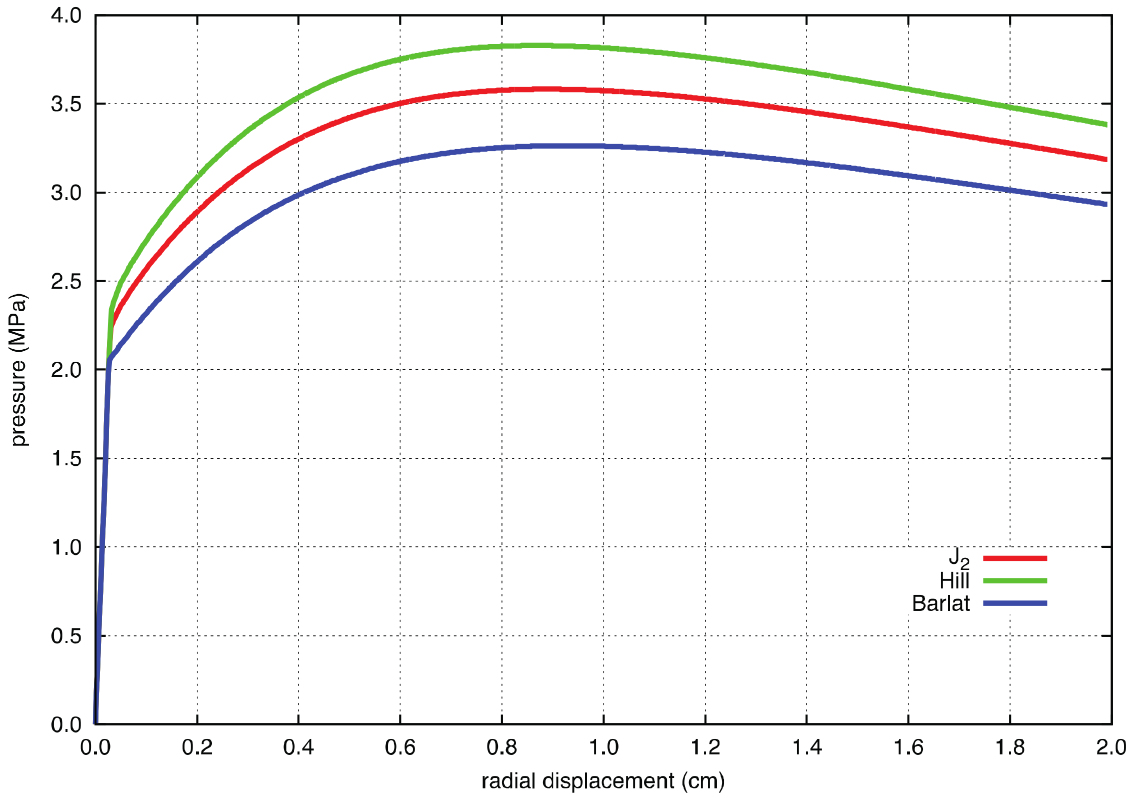
$h = 1$ mm





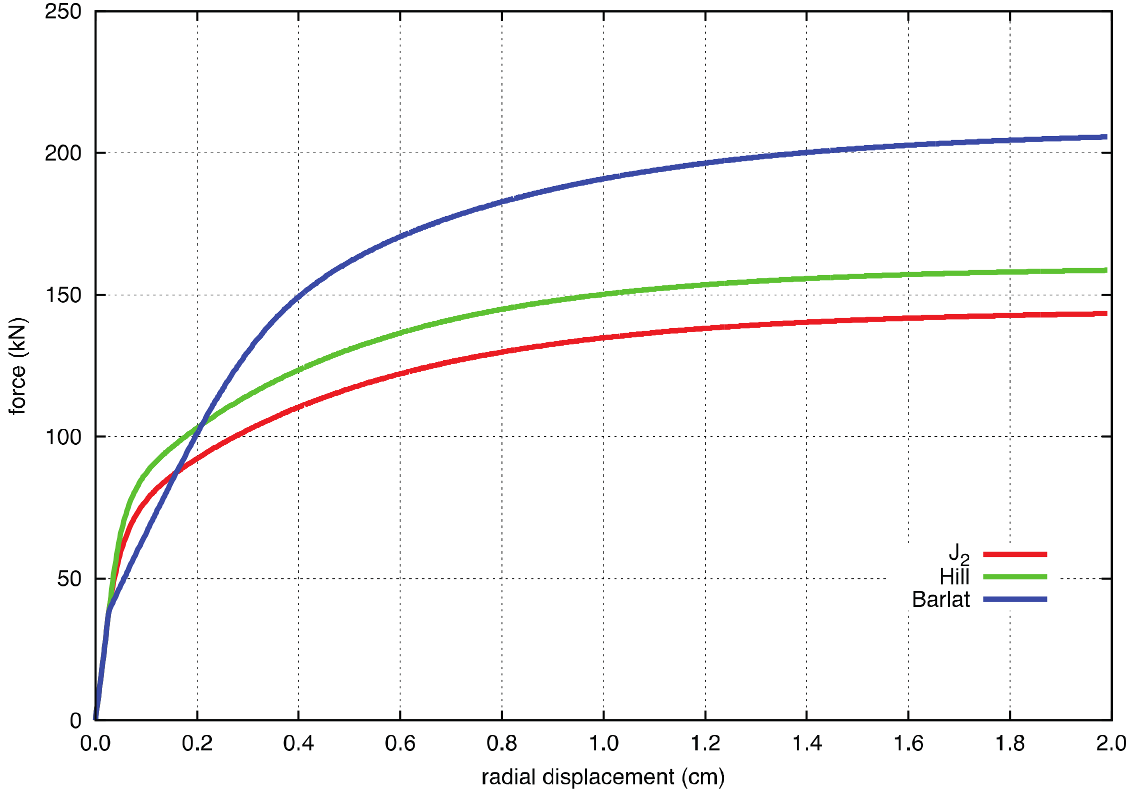
$$\bar{\sigma} = 200\,(1 - \exp(-20\,\bar{\varepsilon}^p))\ \text{MPa} \quad \rightarrow \quad \sigma_y = 200\ \text{MPa}$$

Pressure



4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
pressure (MPa)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
radial displacement (cm)
J_2
Hill
Barlat

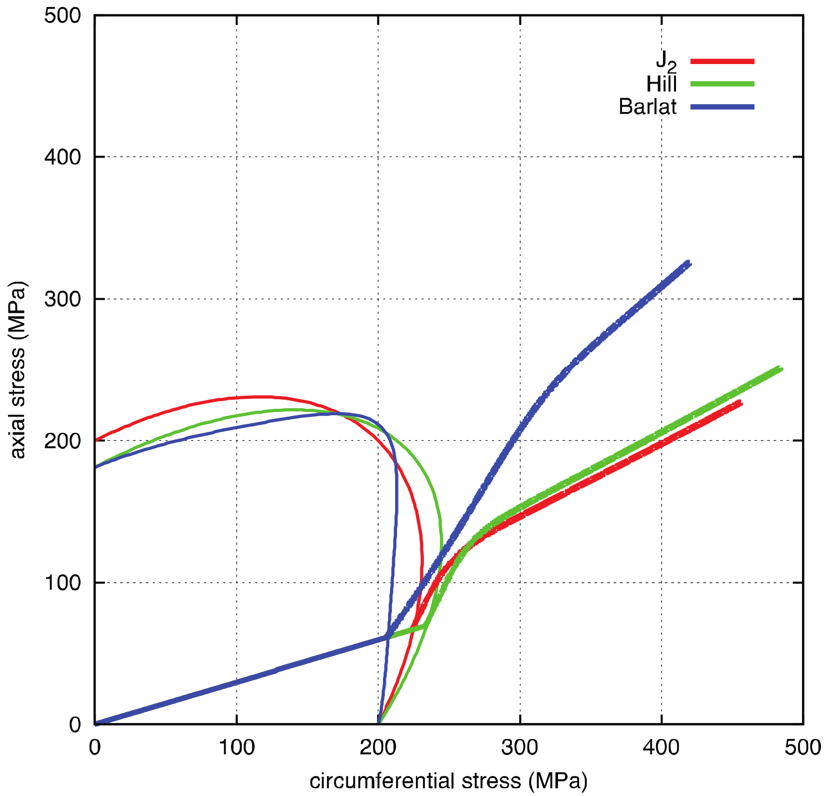
Axial Load



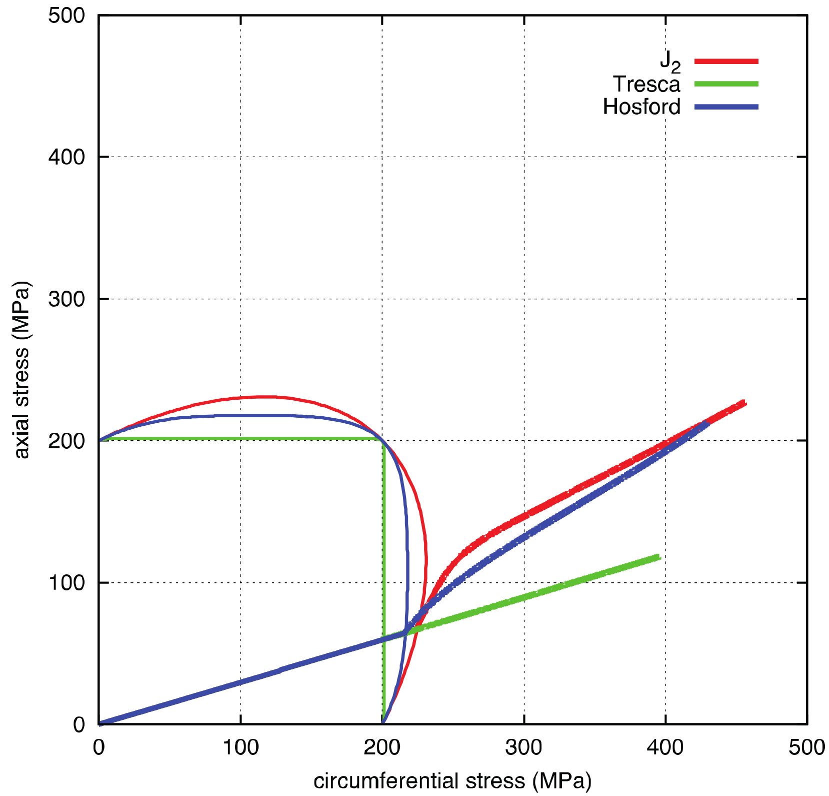
force (kN)
250
200
150
100
50
0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
radial displacement (cm)
J_2
Hill
Barlat

Stress Paths

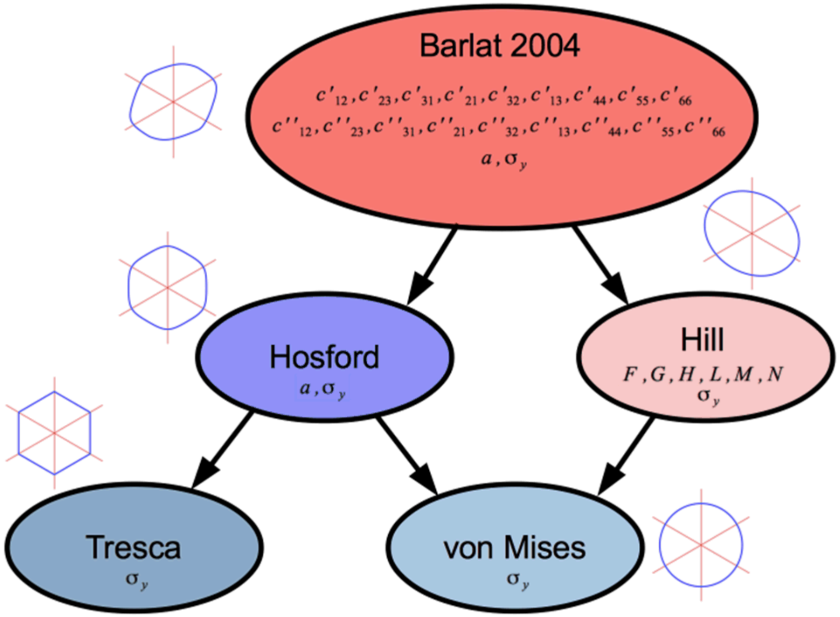
Sandia National Laboratories



Stress Paths



Uncertainty Quantification



- Can we use the Barlat model to same something **quantitative** about model form uncertainty?
- How does the choice of model affect our analysis?

Conclusions

- **isotropic/anisotropic yield descriptions**
- **robust integration algorithm**
    - capability can be used for other yield surfaces
- can we fit the models?
- can we extend to viscoplastic models?
- can we get quantitative model form error and UQ?
- can we model anisotropic hardening and failure?