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# Energy Based Representations of Mechanical Shock for Failure Characterization

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# Outline

- Project Motivation & Objectives
- Shock Response Spectra
  - Pseudo-Velocity Shock Response Spectrum
  - Energy Response
- Experiment Description
  - Test Structure
  - Shock Testing
- Test Results
  - Shock Failure
  - Low Cycle Fatigue Failure
- Conclusions and Future Work

# Motivation & Objectives

- Mechanical Shock Testing Margin Assessment
  - Sandia continually tests our systems to assess their structural integrity
    - Destructive and Evaluation testing
  - Some programs have adopted energy (dissipated and input) as a straightforward metric to relate the severity of mechanical insults to structural capacity
    - Margin assessment
  - The domain of applicability and implementation details are not fleshed out for our problems of interest
    - Energy dissipation models
    - Failure criteria
    - Localized failures
    - Relationship to design approaches

**Characterize the effectiveness of energy-based failure models for quantifying margins and uncertainties for shock environments**

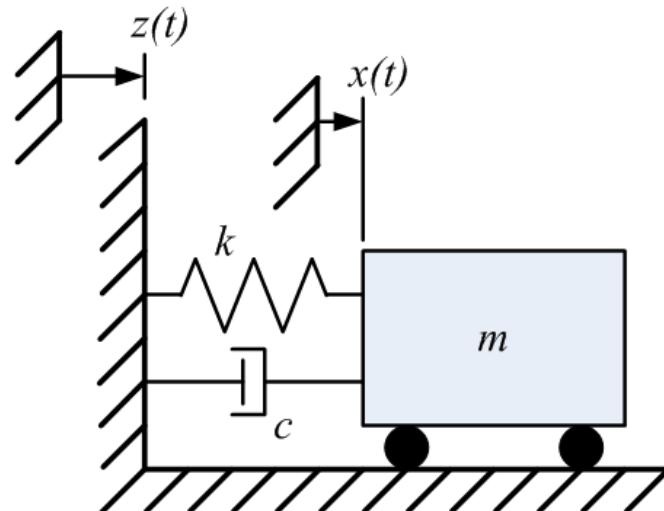
# Present Project Objectives

- Advance our understanding of best practices to quantify margins and uncertainties for shock environments
  - Compare the performance of energy-based metrics with published failure metrics
- Build on previous work on random vibration margin assessment
  - 2010 – 2014: theory, test development, and experimental work
- Develop a shock test structure
  - Want a simple and economical test system
  - Want components that are readily available or easily made
  - Want to conduct a statistically significant number of tests
- This is the beginning of a multi-year effort
  - Anticipate testing more component representative items in future efforts

# Introduction

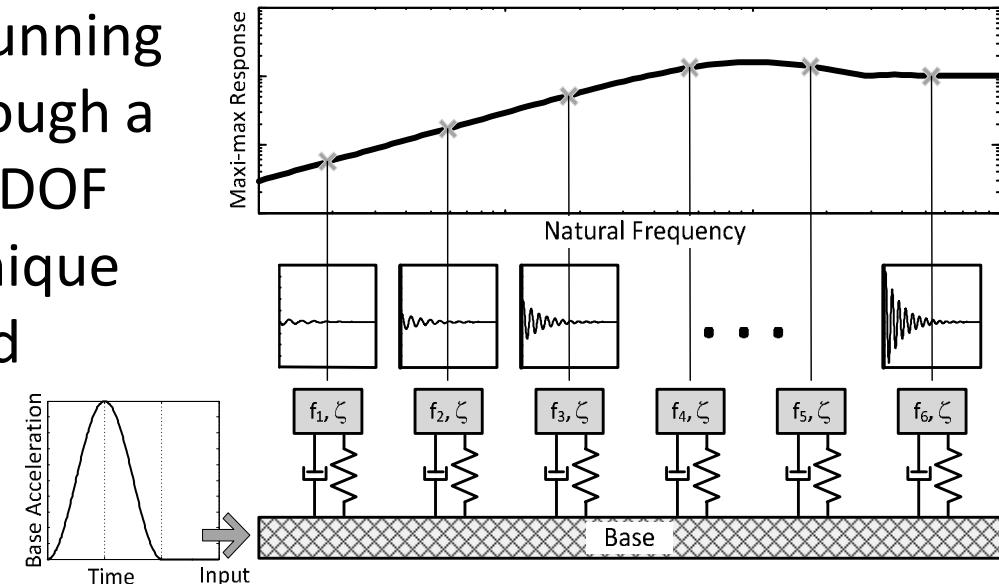
- Mechanical Shock Characterization
  - Three main ways to characterize a shock, Absolute Acceleration Shock Response Spectrum or AASRS, Pseudo-Velocity Shock Response Spectrum (PVRS), and Energy Response Spectrum (ERS).
- All three spectra are derived from the SDOF oscillator with base excitation equation of motion. The equation in terms of relative coordinates is:

$$m\ddot{w}(t) + c\dot{w}(t) + kw(t) = -m\ddot{z}(t)$$



# Shock Response Spectrum(SRS)

- The SRS is calculated by running the base acceleration through a number of independent SDOF systems, each having a unique natural frequency,  $\omega_n$ , and damping ratio,  $\zeta$
- The response quantity is then pulled from each system and plotted at that frequency



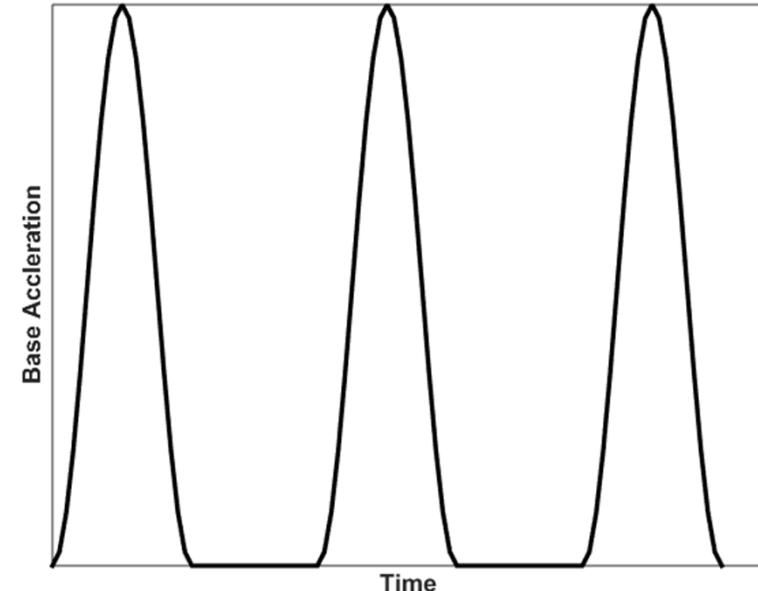
- Absolute Acceleration  $AA = \ddot{w}(t) + \ddot{z}(t)$
- Pseudo-velocity  $PV = \omega_n w(t)$
- Energy

**Relative Displacement**

$\ddot{w}(t) + 2\zeta\omega_n\dot{w}(t) + \omega_n^2 w(t) = -\ddot{z}(t)$

# SRS, continued

- The AASRS and PVRS of multiple shocks are the same as for a single shock
- Multiple time histories can produce the same SRS
  - Duration of the excitation is important



**The AASRS and PVRS are only applicable to a single shock**

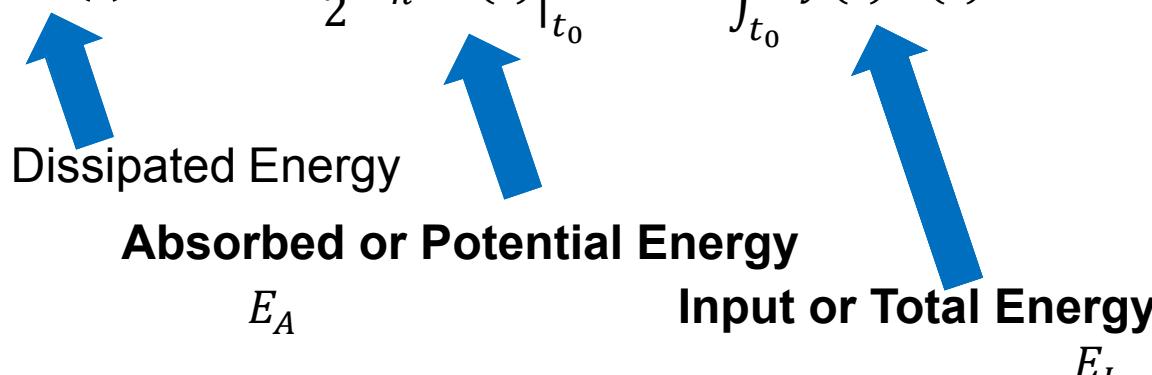
# Energy Response Spectra

- Multiply the SDOF equation by incremental displacement  $dw \doteq w(t)dt$  and integrate

$$\int_{t_0}^{t_f} \ddot{w}(t) \dot{w}(t) dt + 2\zeta\omega_n \int_{t_0}^{t_f} \dot{w}^2(t) dt + \omega_n^2 \int_{t_0}^{t_f} w(t) \dot{w}(t) dt = - \int_{t_0}^{t_f} \ddot{y}(t) \dot{w}(t) dt$$

- Energy Balance Equation

$$\frac{1}{2} \dot{w}^2(t) \Big|_{t_0}^{t_f} + 2\zeta\omega_n \int_{t_0}^{t_f} \dot{w}^2(t) dt + \frac{1}{2} \omega_n^2 w^2(t) \Big|_{t_0}^{t_f} = - \int_{t_0}^{t_f} \ddot{y}(t) \dot{w}(t) dt$$


  
 Relative Kinetic Energy  $E_K^R$       Viscously Dissipated Energy  $E_D$       Absorbed or Potential Energy  $E_A$       Input or Total Energy  $E_I$

**Dissipated and Total Energy increase with multiple applied shocks**

# Absorbed Energy and PVRS

- Just like the SRS, the energy response spectrum is a plot of the maximum response (energy) of SDOF systems to a specific (transient) input
- Looking at the Absorbed Energy

$$E_A = \frac{1}{2} \omega_n^2 w^2(t) \Big|_{t_0}^{t_f}$$

- And the Pseudo Velocity

$$PV = \omega_n w(t)$$

- Absorbed Energy is related to PV by:

$$E_A = \frac{1}{2} (PV)^2$$

**The Absorbed Energy Spectrum is  $\frac{1}{2}$  the PVRS squared**

# Recap

- The AASRS, PVSRS and ERS are all derived from the SDOF EOM

$$\ddot{w}(t) + 2\zeta\omega_n\dot{w}(t) + \omega_n^2 w(t) = -\ddot{z}(t)$$

- Shock Response Spectra characterize a single shock
  - Duration is important
- The total energy is

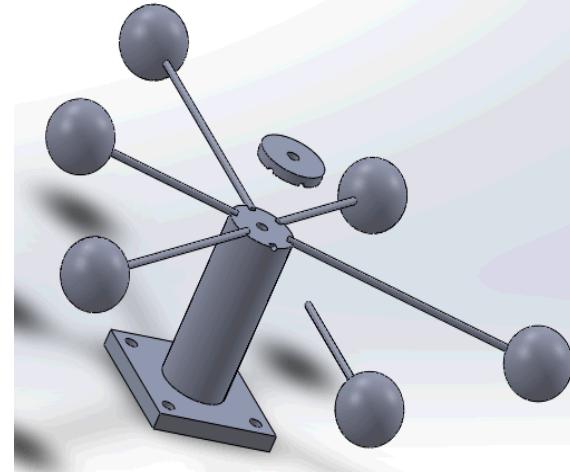
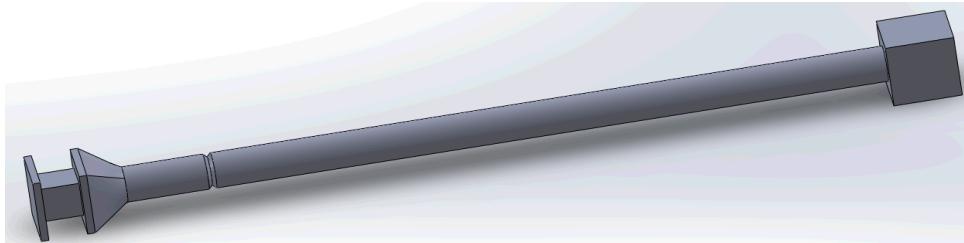
$$E_K^R + E_D + E_A = E_I$$

- Dissipated and Total Energy increase with multiple shocks
- Absorbed Energy and PV are closely related

**Project Objective: Relate low cycle fatigue shock failures to energy metrics**

# Test Structure

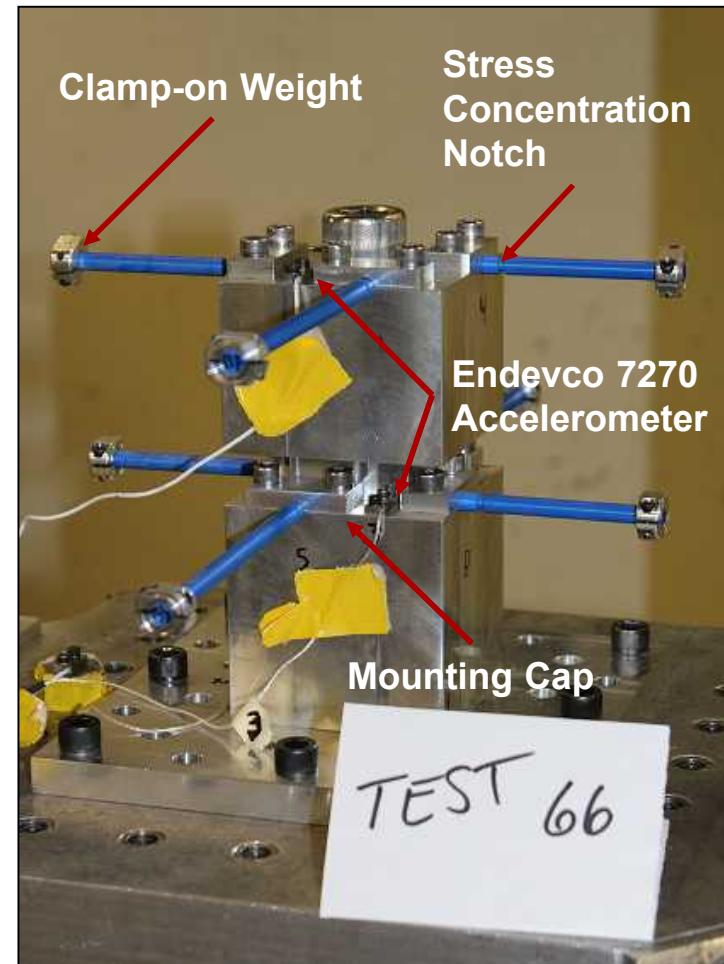
- Early Concept of Test Structure and Beam



Test Fixture Design Requirements	Test Article Design Requirements
<ul style="list-style-type: none"><li>• Test multiple test articles simultaneously</li><li>• Control the test article failure location</li><li>• Tailor the dynamic response of the articles under test</li><li>• Do not unduly influence the dynamic response of the test articles</li></ul>	<ul style="list-style-type: none"><li>• Easy and economical to manufacture</li><li>• Includes a stress concentration zone to control failure mode and location</li><li>• Tailorable fundamental natural frequency</li><li>• Tailorable failure load</li></ul>

# Test Fixture

- Aluminum fixture compatible with drop table and shaker
- Each base section accommodates 4 beams (up to 8 total)
- 3D printed cantilever beams with stress concentration notches
  - Easily replaceable
  - Held in using caps that are bolted down to the structure
  - Clamp-on weights at beam ends
    - Used to tailor natural frequency and beam stress under shock
- Instrumentation
  - Endevco 7270 accelerometers on the base, middle, and upper tower levels
  - No instrumentation on beams



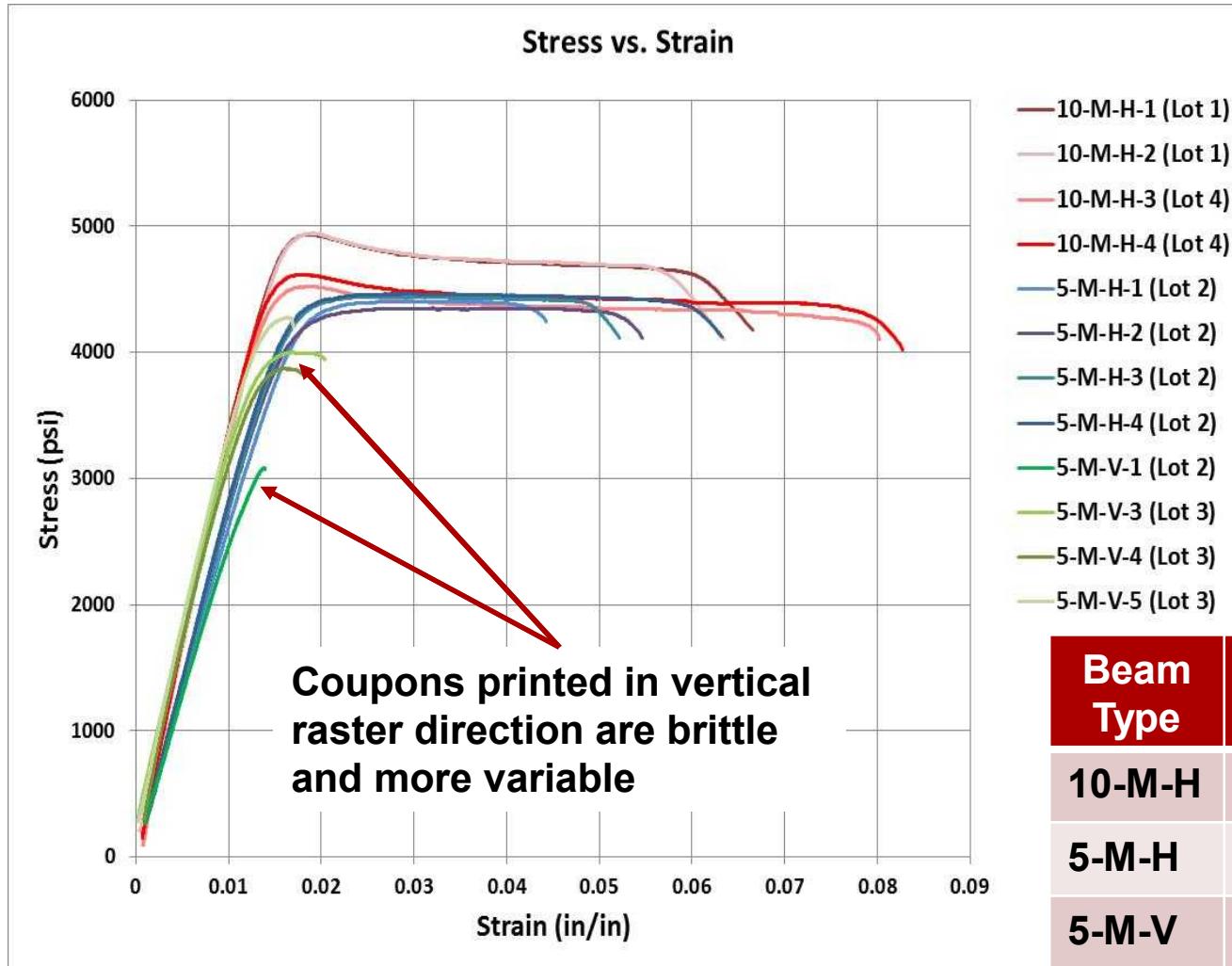
After several iterations a fixture that met all requirements was designed

# Cantilever Beam Description

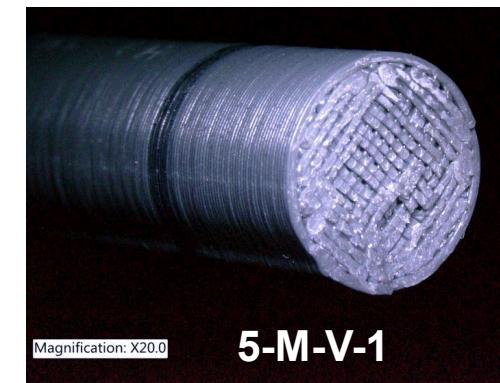
- All beams were made from ABS plastic
  - Cantilever beams were printed with layers oriented perpendicular to the beam axis
- Notched and Un-notched beams
  - Un-notched beams
  - 0.025 inch notch
  - 0.050 inch notch
- 3 inch and 5 inch lengths
  - 3 inch length needed to fit between uprights on the drop table
- SNL 3D Printing and Additive Manufacturing group made all beams



# Elastic Properties of 3D Printed Beams



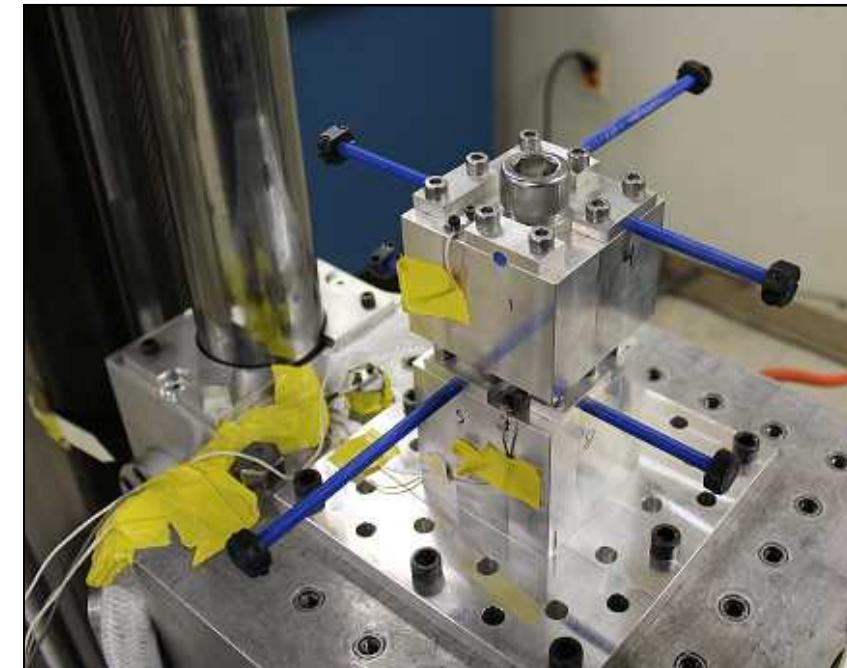
- Static pull tests were performed on 3D printed coupons



Beam Type	Avg Modulus (MPa)	Modulus CoV (%)
10-M-H	2342	1.9
5-M-H	1899	2.63
5-M-V	2026	8.94

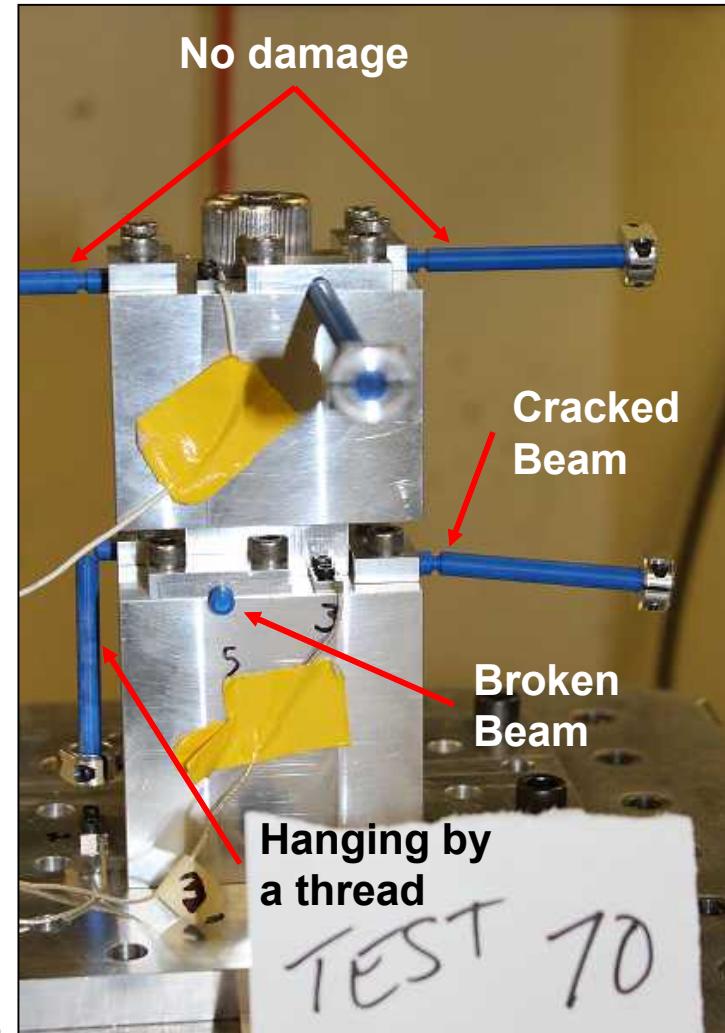
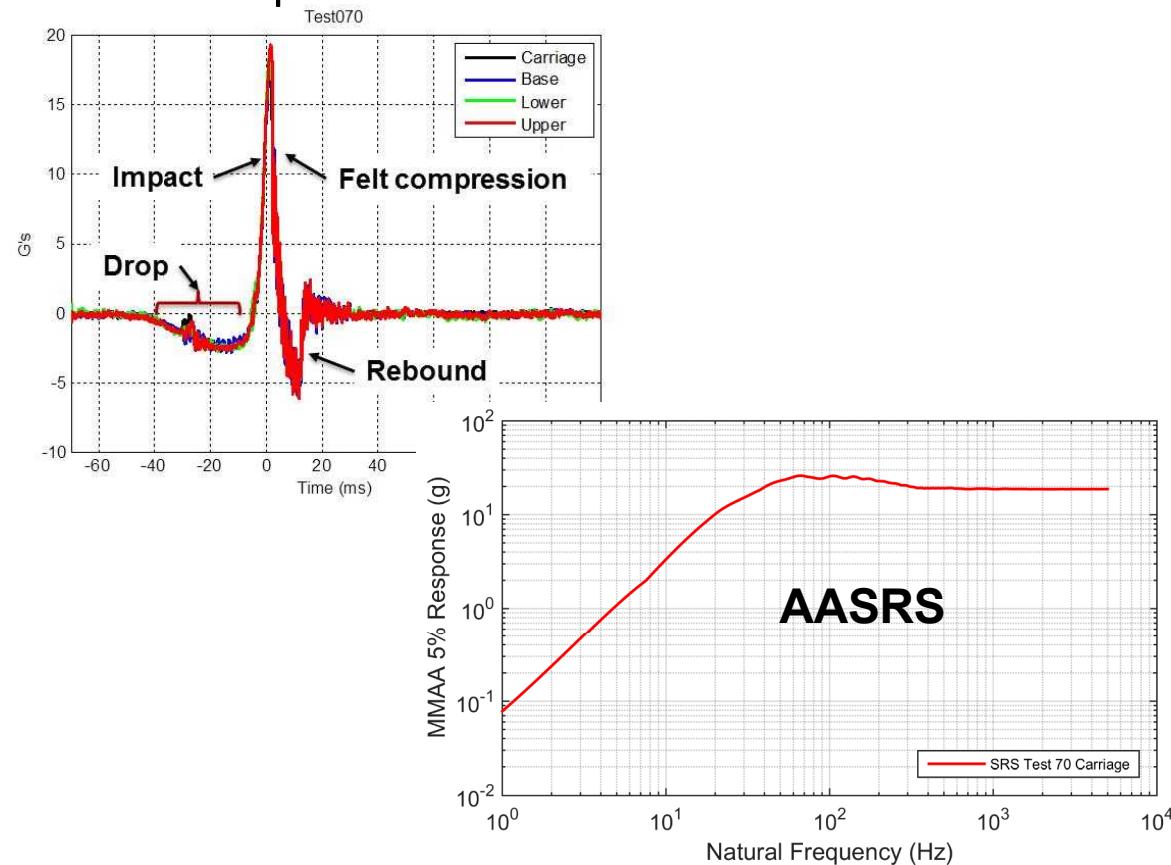
# Shock Testing

- Tested 72 cantilever beams on a drop table
  - Sets of 8 beams per test; four 5 inch and four 3 inch beams
- First passage failures
  - Stepped up input load incrementally until all beams failed
- Low-cycle fatigue failures
  - Repeated tests at an input level below failure level until all beams failed
- Approximately 148 shocks



# Shock Testing Failures

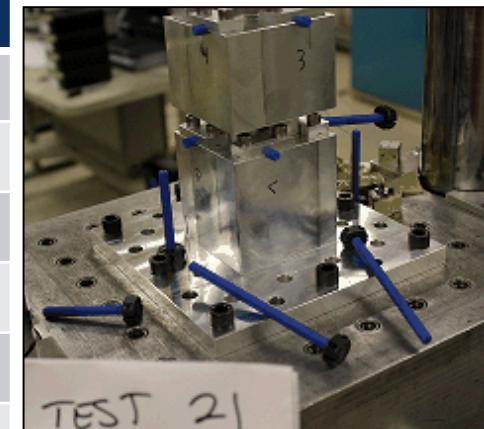
- All failures were brittle failures
  - Cantilever beams were intentionally printed to ensure brittle failure



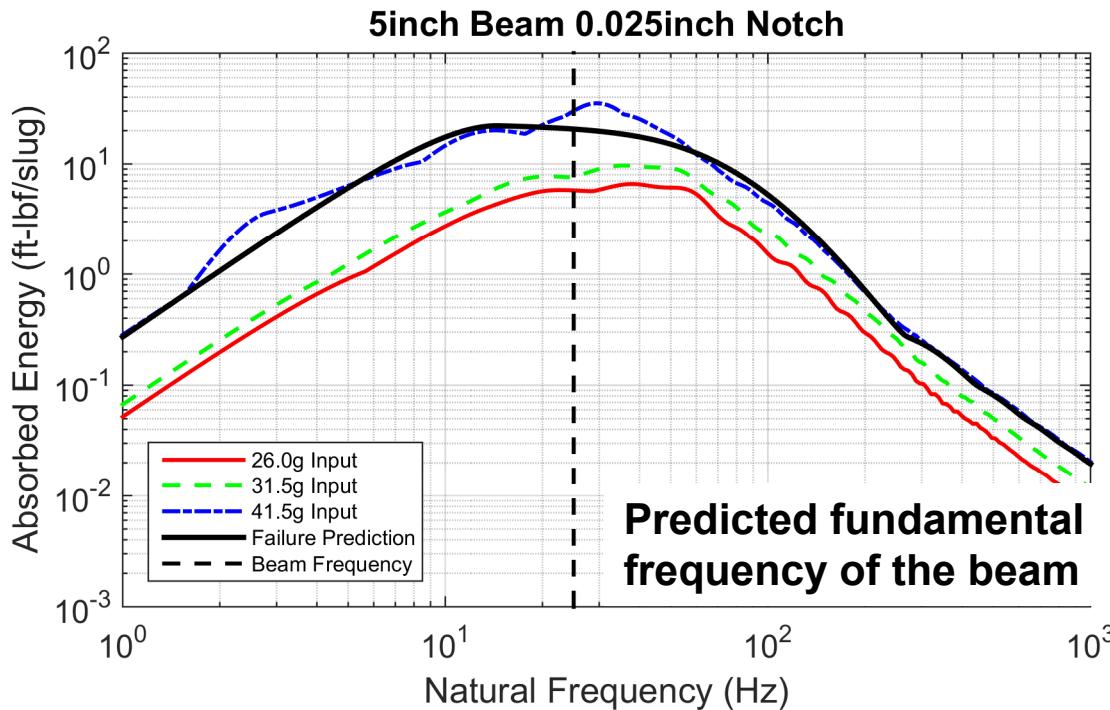
# Shock Testing Results

- First tests were conducted to determine failure points of all beams and compare to FEA beam model predictions
  - Incremental nature of testing does not reveal the exact failure point
  - All tests were designed to achieve a nominal 90Hz Haversine pulse
  - Drop table was low end limited—unable to hit with less than ~21g
- Results show good agreement between test and FEA
  - Predictions for 5in un-notched beams were high

Length	Notch	Observed Test Failure	Predicted Failure
5 in	None	32.0g → 62.5g	64.5g
5 in	0.025 in	30.5g → 41.5g	38.5g
5 in	0.050 in	< 27.0g	18.8g
3 in	None	42.5g → 98.0g	58.8g
3 in	0.025 in	30.5g → 41.5g	37.4g
3 in	0.050 in	< 27g	20.4g



# Overstress Shock Failure



- Test Series #3
- Pre-test prediction was pretty good for failure from a single shock

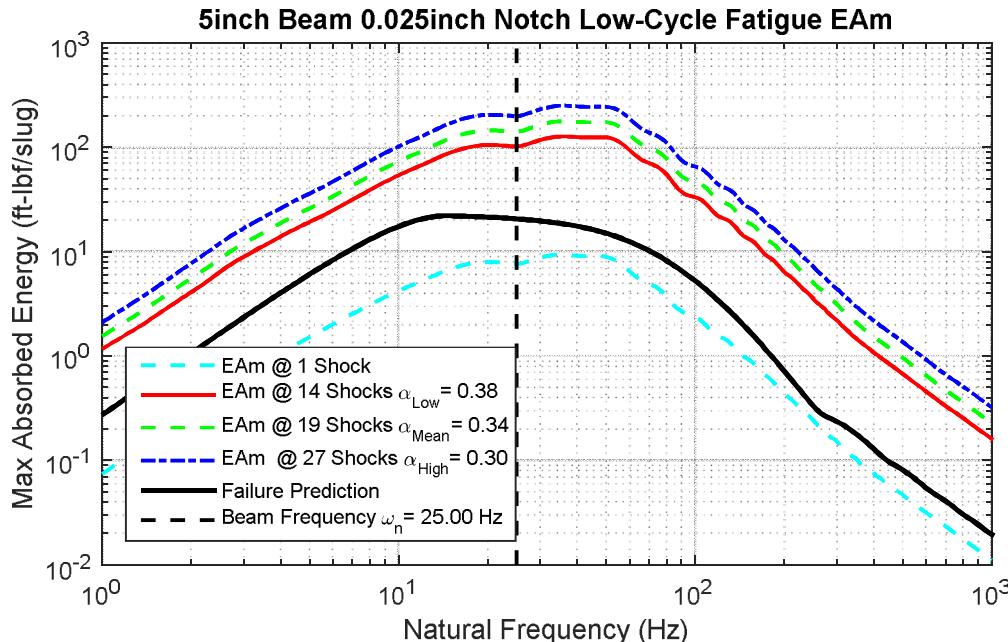
- Tested 4 beams to determine the input level for a single shock failure
- Found it on the 3<sup>rd</sup> try at 41.5 G
  - This might be conservative if the lower level inputs created some latent damage

# Fatigue Testing Results

- Simple low-cycle fatigue testing performed
  - Tested shock level was around 10g's below first passage failure
  - Repeated shocks at nominally the same level until all beams failed

Beam Length	Notch	Tip Weight	Tested Shock Level	% of Strain Allowable	Average Hits to Fail	Range of Hits to Fail
5 in	None	0.028lbf	44.8g	61%	36	27 → 47
5 in	0.025 in	0.028lbf	31.7g	77%	19	14 → 27
5 in	0.050 in	0.010lbf	22.0g	61%	5	2 → 10
3 in	None	0.057lbf	38.8g	53%	3	1 → 6
3 in	0.025 in	0.028lbf	31.5g	78%	12	1 → 18
3 in	0.050 in	0.010lbf	21.7g	44%	13	4 → 33

# Low Cycle Fatigue Experiment

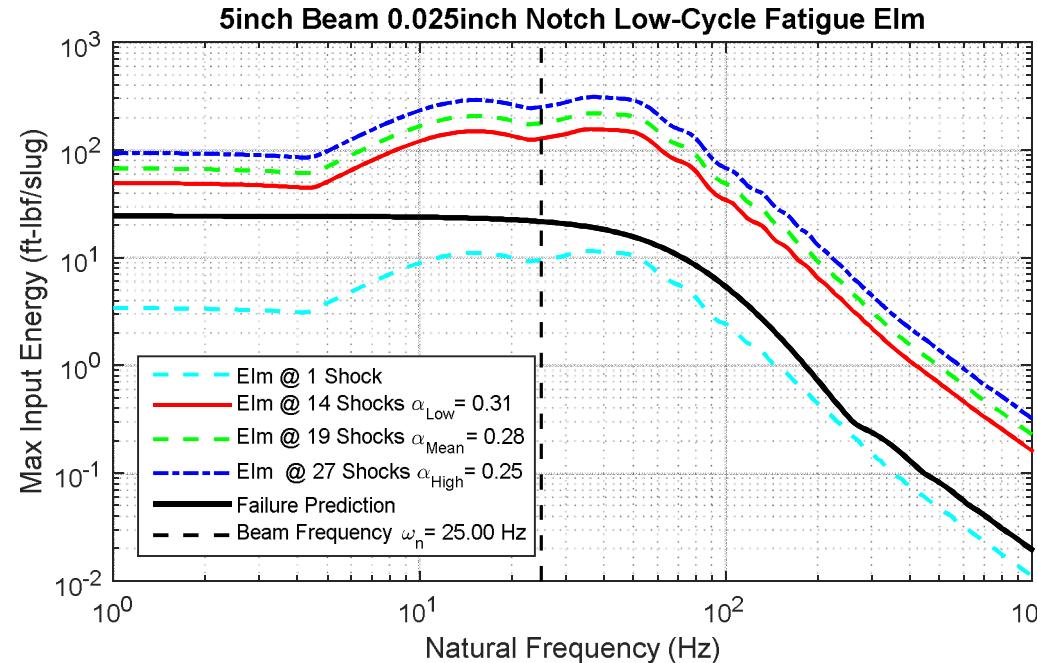


- Results suggest the beam can absorb more energy from a series of small shocks than from one large amplitude shock
- We postulated that Basquin's equation, used for high cycle fatigue, applies

$$E_{X1} = E_X N^\alpha$$

- Tested 4 beams with 31 g +/- 3 g 11 ms impacts to failure
- Beams had 1 steel collar
- Failure prediction is for a single impact
  - 41 G
- Absorbed energy lines are summed plots

# Low Cycle Fatigue Experiment



- The same characteristic applies to Input Energy
  - Input Energy shows that the beam can absorb more energy from a series of small shocks than from one large amplitude shock
  - The power law (Basquin Equation) exponents are similar

# Low Cycle Fatigue Experiment

## ■ Summary

- For The maximum absorbed, kinetic, and input energy the  $\alpha$  values ranged from 0.17 to 0.41
- Values with NA are because the low failure value was 1

Beam Configuration	$E_{AM}$			$E_{IM}$		
	$\alpha_{Low}$	$\alpha_{Mean}$	$\alpha_{High}$	$\alpha_{Low}$	$\alpha_{Mean}$	$\alpha_{High}$
5 in / No Notch	0.41	0.38	0.35	0.37	0.34	0.31
5 in / 0.025 in	0.38	0.34	0.30	0.31	0.28	0.25
3 in / 0.025 in	NA	0.23	0.20	NA	0.19	0.17

$$E_{X1} = E_X N^\alpha$$

The alpha values are comparable to those of high cycle fatigue

# Conclusions and Future Work

- Conclusions
  - 3D printed ABS beams had remarkably consistent moduli (best within a lot)
    - Will print & test tensile coupons with each batch of beams
  - Test fixture functions as we expected
  - Predicted single shock failure levels reasonably well
    - Absorbed energy seems to be a reasonable metric
  - Relationship of failure from accumulated shocks (low cycle fatigue) to single shock failure with absorbed energy may follow a power law
- Future Work
  - Perform shaker shock tests
    - Richer dynamic environment
  - Evolve to more representative structures

# Thank You

# Questions?