



SAND2015-10661C

## JOINT DoD/DOE MUNITIONS PROGRAM

# Modeling techniques for localization and failure



*J. Foulk, K. Karlson, A. Mota, J.T. Ostien,  
M. Veilleux, J. Emery*



**2015 TCG-I/XI Fall Meeting**  
**December 8, 2015**  
**Sandia National Laboratory**  
**Albuquerque, NM**



Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

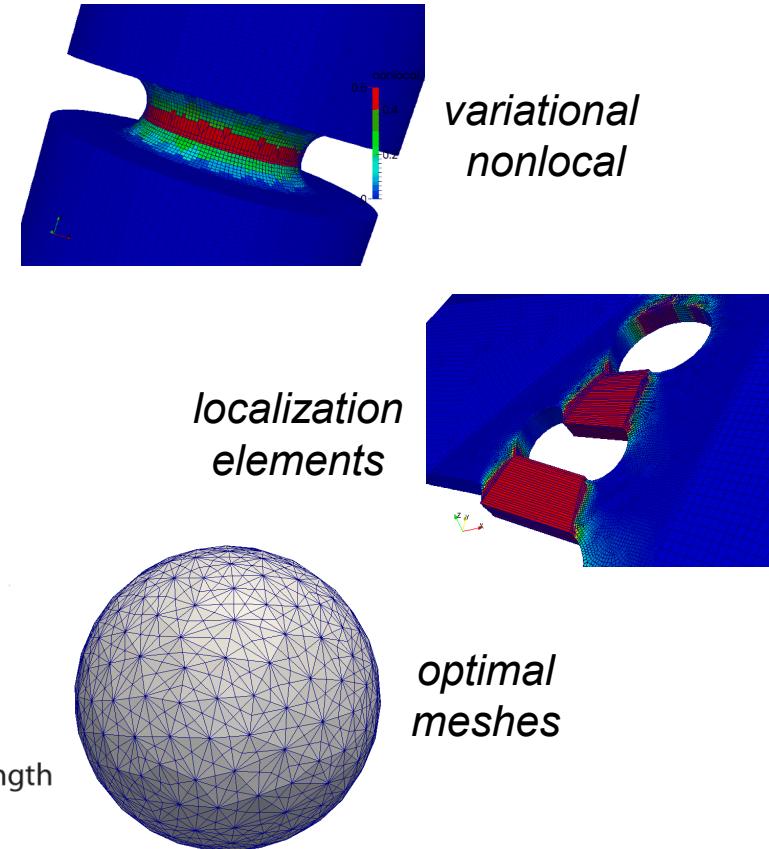
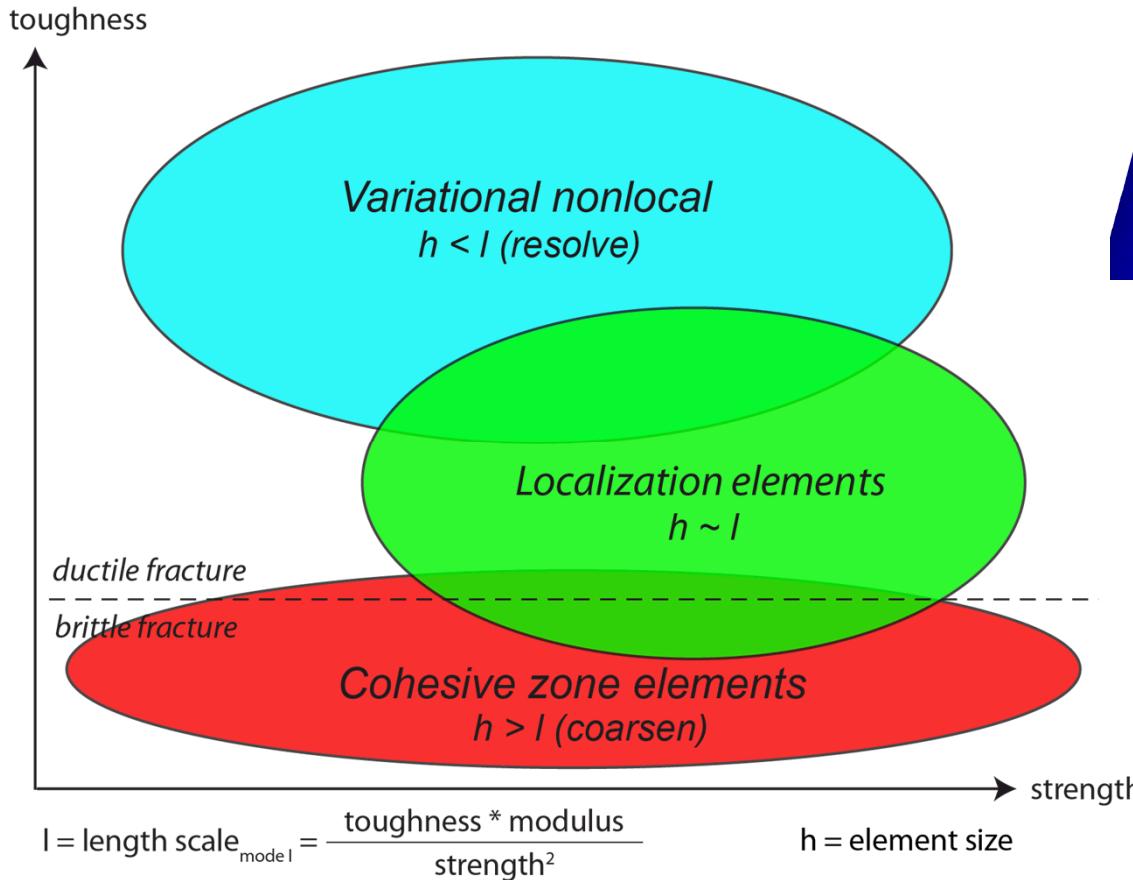


**Sandia  
National  
Laboratories**



# Methods broadly applicable

Goal: Provide techniques for modeling localization and failure that are not mesh dependent to enable predictive simulations of munitions behavior

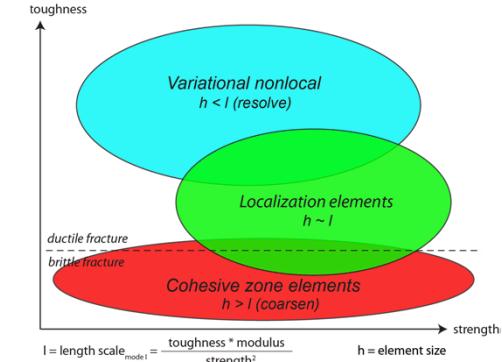




# Changing scope, broadening impact

## Task 5, Modeling techniques for localization and failure

- Variational nonlocal method (1.1)
- Localization elements (1.2)
  - Optimal meshes (1.2.1)
  - Adaptive insertion (1.2.2)
  - Linkage to XFEM (1.2.3)



*FY16 Deliverable.* Demonstration of the adaptive insertion of localization elements on an optimized mesh for mixed-mode fracture. Insertion will be dictated by the bifurcation condition applied to a general class of material models.

*FY15 Issue.* With the Sierra Toolkit (STK) in flux, we were not able to accommodate the churn in our research environment. We have paused efforts in adaptive insertion to *leverage new work*.

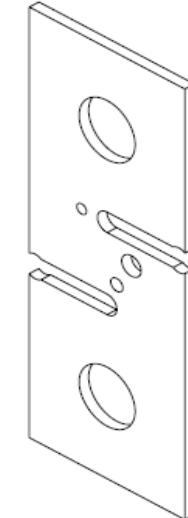
## New focus for Task 5, Modeling techniques for localization and failure

- Harden variational nonlocal method and localization elements
- Document work on optimal meshes and the bifurcation condition
- Provide linkage to SierraSM XFEM w/localization elements + insertion criteria
- Leverage efforts to include anisotropy, temperature/rate dependence, damage evolution



# Case study - Sandia Fracture Challenge

- Sandia Fracture Challenge (SFC) is a computational challenge for predicting failure open to internal/external competitors
- The second challenge focused on variable rate, mixed-mode crack initiation and propagation in Ti-6Al-4V sheet.
- The challenge announcement included two data sets for material model calibration, geometry information, and test procedures



## *Our approach(s)*

- Because Ti-6Al-4V has low thermal conductivity and high strength, one must include thermo-mechanical coupling for the rates of interest.
- Both the provided experimental data and the literature illustrate the need for rate dependence, temperature dependence, anisotropy, and void evolution.
- We leverage a local model with the appropriate phenomenology (micromechanics). We learn about the BVPs through local models.
- We seek to regularize the solution through multiple technologies and understand the space of applicability.



# Initial approach to SFC geometry

- Production codes (Sierra) employed for all calculations
- Simulations employ segregated coupling (Adagio/Aria)
- Implicit solution for long time scales (statics & dynamics)
- Isotropic poro-thermo-viscoplasticity model
- Hexahedral elements (SD, constant pressure)
- Pins are fixed
- Element death was employed when the first integration point reached the coalescence criterion  $\phi_{coal}$  (0.15)
- Learn with local damage and coarse meshes
- Employ techniques for regularizing the solution
  - Variational nonlocal method
  - Localization elements





# Initial model calibration (tension)

$$\beta = 0.8$$

$$H = 3084 \times 10^6 \text{ (Pa)}$$

$$R_d = 13$$

$$f = 1 \times 10^{-6}$$

$$n = 26$$

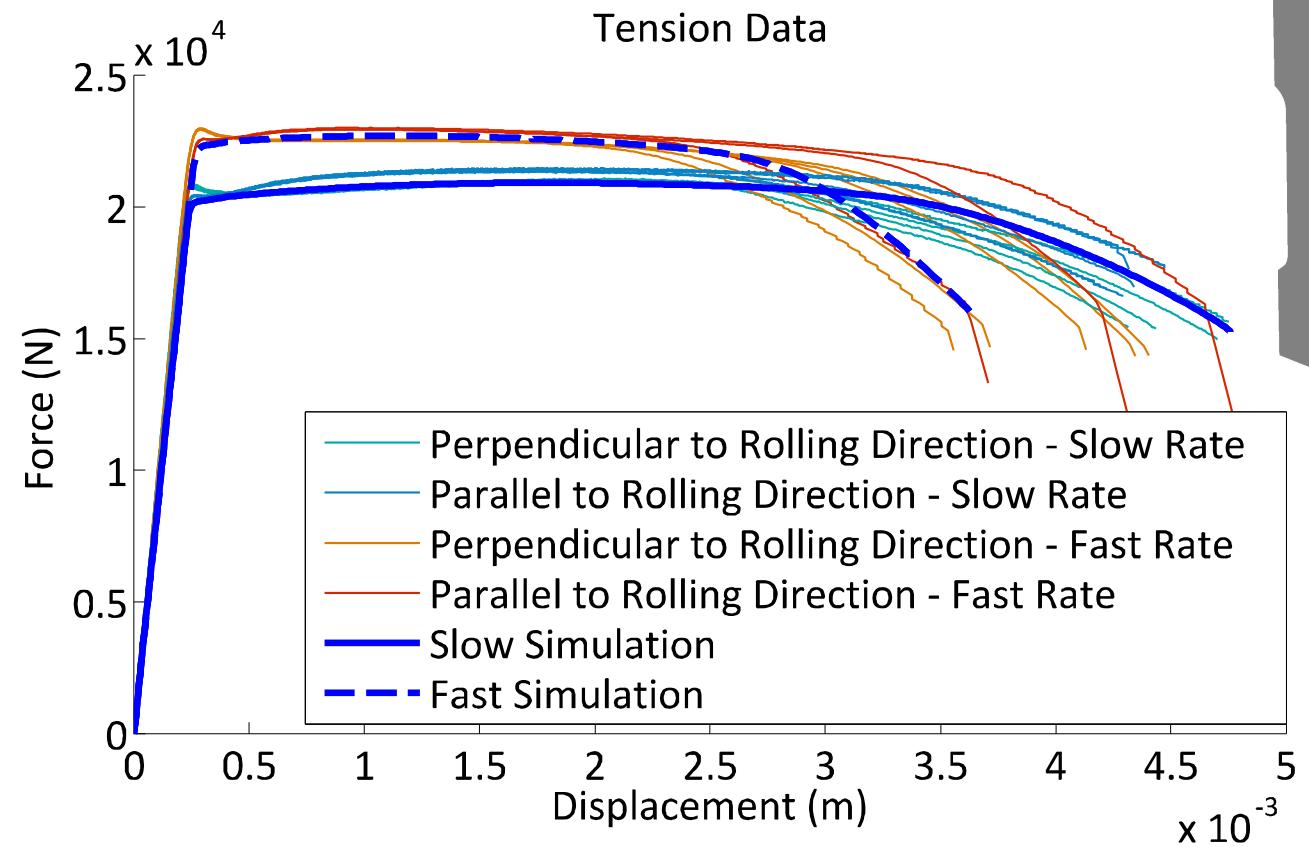
$$Y_{RT} = 493 \times 10^6 \text{ (Pa)}$$

$$\phi_0 = 1 \times 10^{-4}$$

$$\phi_{coal} = 0.15$$

$$m = 6$$

Thermo-mechanical simulations employed for model calibration



$$\sigma_y = (1 - \phi) \left[ Y(\theta) + \frac{H}{R_d} (1 - e^{-R_d \epsilon_p}) \right] \left\{ 1 + \sinh^{-1} \left[ \left( \frac{\dot{\epsilon}_p}{f} \right)^{1/\textcolor{red}{n}} \right] \right\}$$

$$\dot{\phi} = \sqrt{\frac{3}{2}} \dot{\epsilon}_p \frac{1 - (1 - \phi)^{\textcolor{red}{m}+1}}{(1 - \phi)^{\textcolor{red}{m}}} \sinh \left[ \frac{2(2\textcolor{red}{m} - 1)}{2\textcolor{red}{m} + 1} \frac{\langle \frac{I_1}{3} \rangle}{\sqrt{3J_2}} \right]$$

isotropic damage  $\phi$  taken from Cocks and Ashby (1972)

$$\dot{q} = \beta \bar{\sigma} : \mathbf{D}^p$$

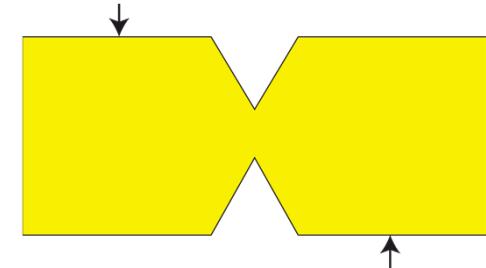
uncertainty in conversion of plastic work to heat  $\beta$

NOTE: Temperature-dependent thermal conductivity and specific heat also taken from MMPDS-08.



# Incorporating shear data

- Calibrated model did not predict the shear behavior
- Anisotropy evident in yield, hardening and damage evolution
- Focused on orientations relevant (// to RD) to the SFC
- Reduced the initial yield  $Y_{RT}$  and the recovery  $R_d$
- Incorporated void nucleation through  $J_3$  (n is the evolving void density)



$$\frac{\dot{n}}{n} = N_1 \dot{\epsilon}_p \left( \frac{4}{27} - \frac{J_3^2}{J_2^3} \right)$$

(Horstemeyer, Gokhale, 1999)

$$\frac{\dot{\phi}_n}{\phi} = k_\omega \dot{\epsilon}_p \left( 1 - \frac{27 J_3^2}{4 J_2^3} \right)$$

(Nahshon, Hutchinson, 2008)

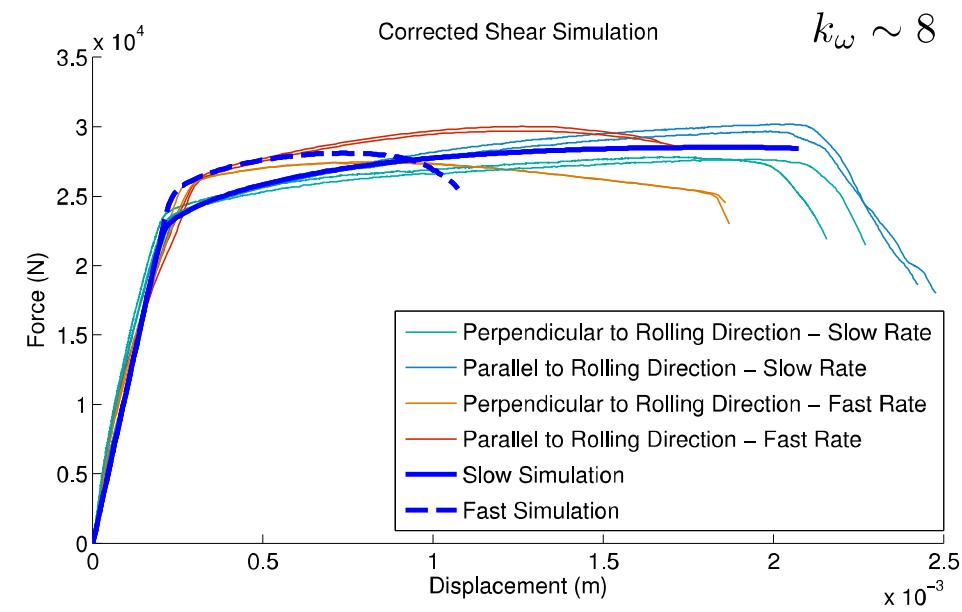
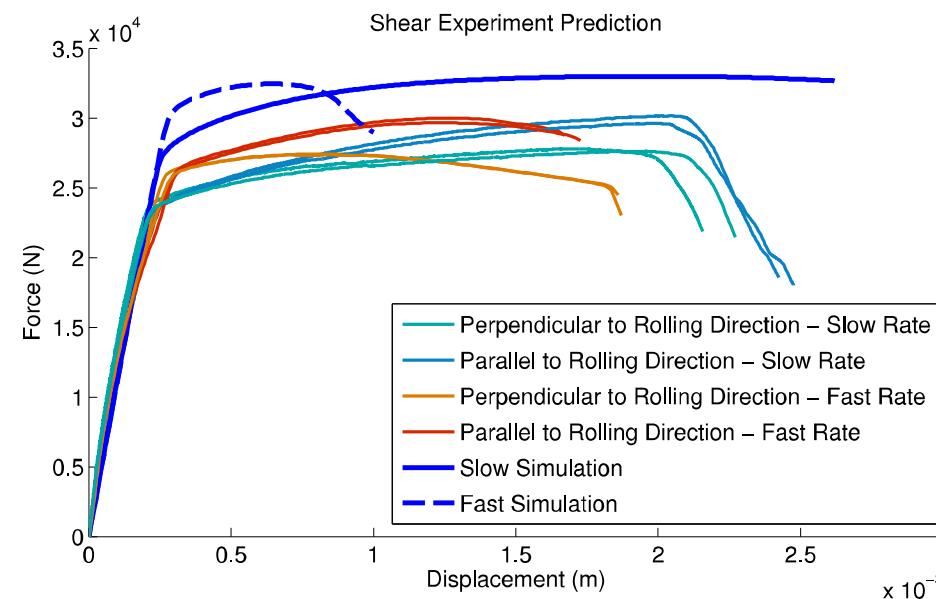
$$N_1 = \frac{27}{4} \left( \frac{1}{1 - \phi} \right) k_\omega$$

$$Y_{RT}^s = 0.87 Y_{RT}$$

$$R_d^s = 0.92 R_d$$

$$N_1 = 54$$

$$k_\omega \sim 8$$





# Revised approach to SFC geometry

- Calibrated a Hill, anisotropic yield surface to the shear and tensile data
- Although rate and temperature independent, modest agreement at lower rates
- Anisotropic yield predicted SFC would localize in the lower notch  $Y_{RT}^s = 0.8$

*Idealization.* Keep poro-thermo-viscoplasticity.

Accept isotropy. Assign different isotropic material parameters to regions being sheared.

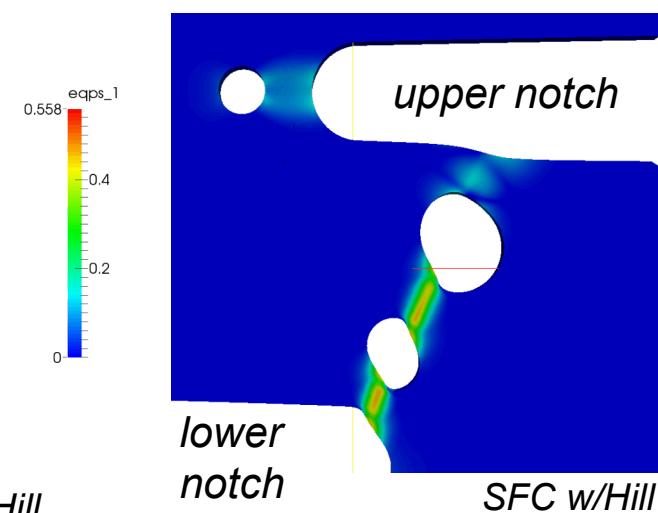
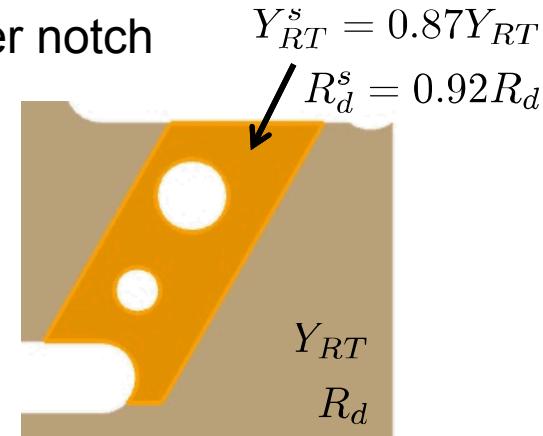
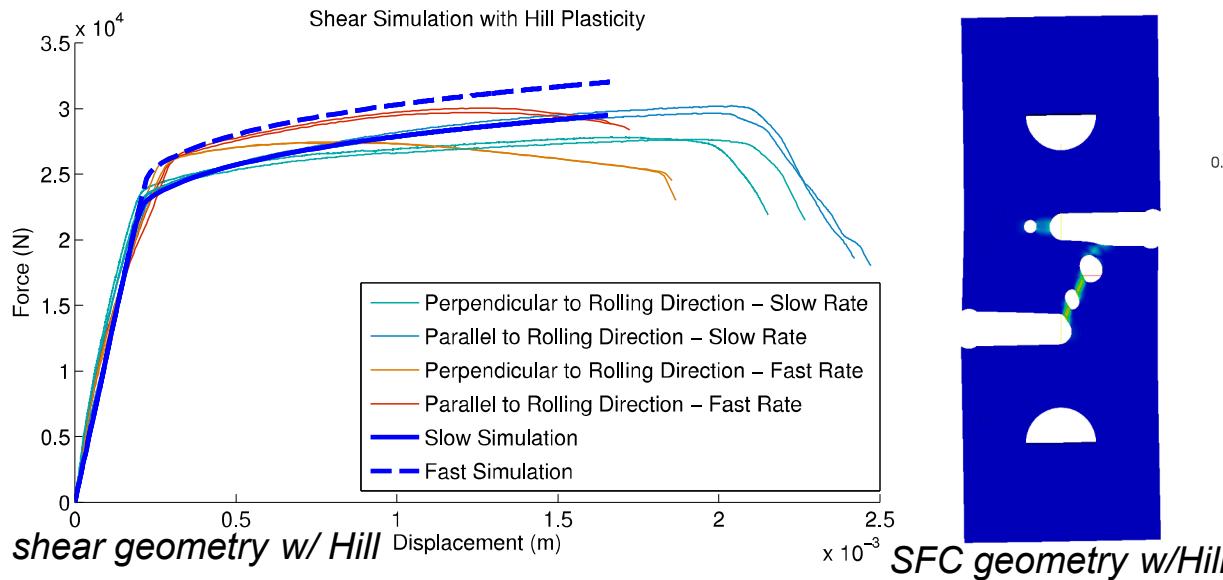
**Goal.** Mimic Hill at lower notch, add physics



$$Y_{RT}^s = 0.87Y_{RT}$$

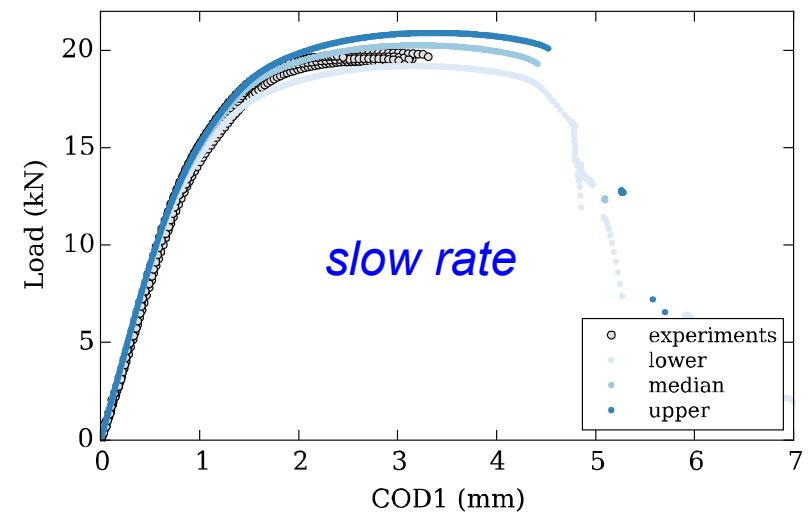
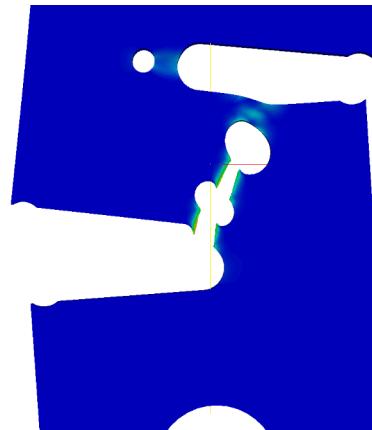
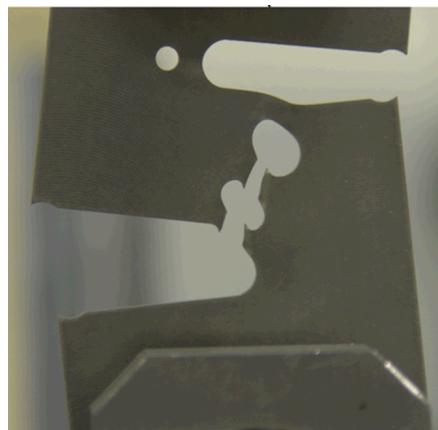
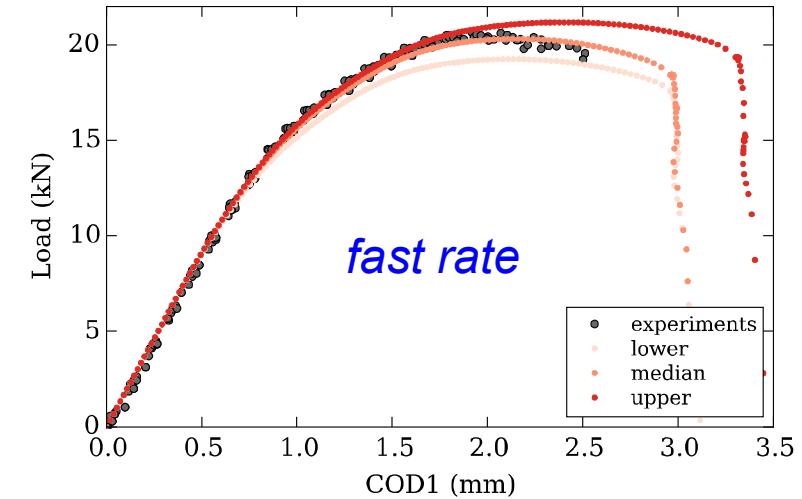
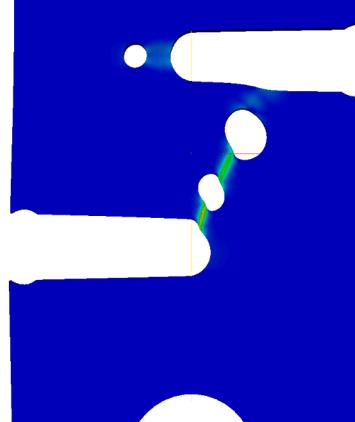
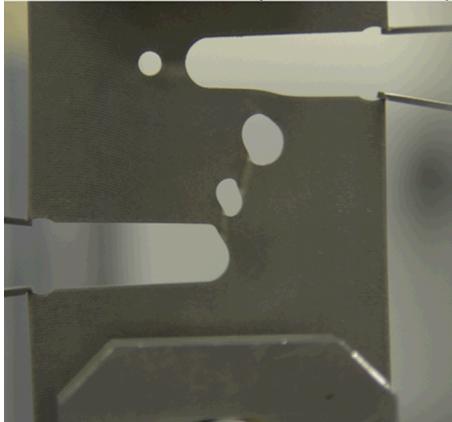
$$R_d^s = 0.92R_d$$

$$Y_{RT}$$





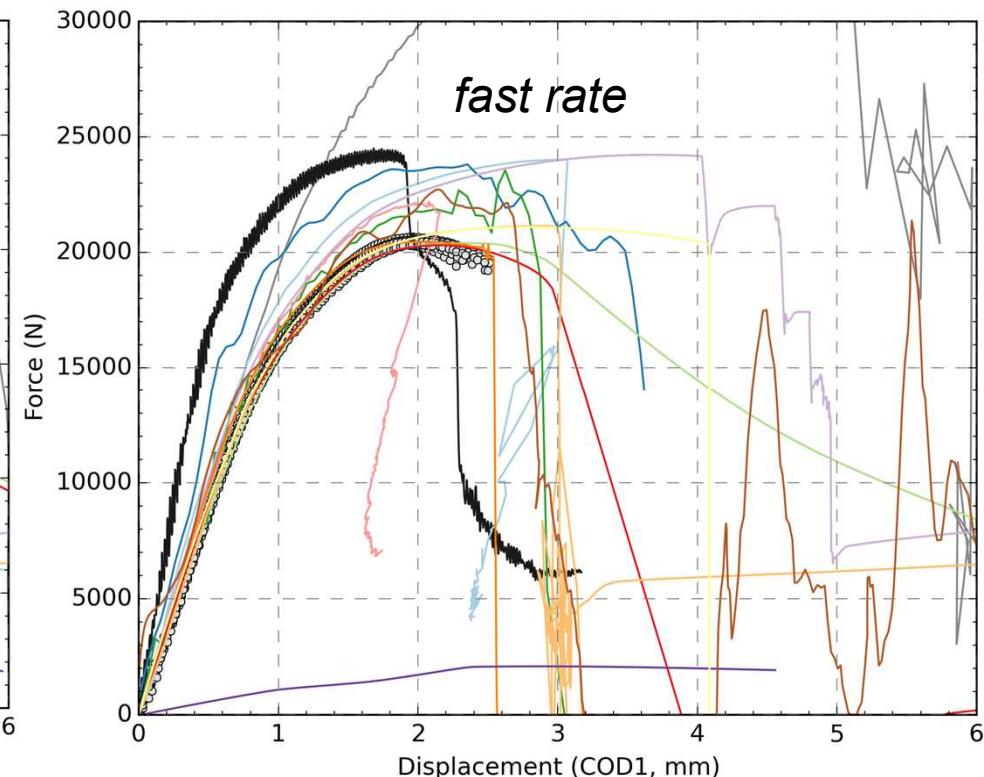
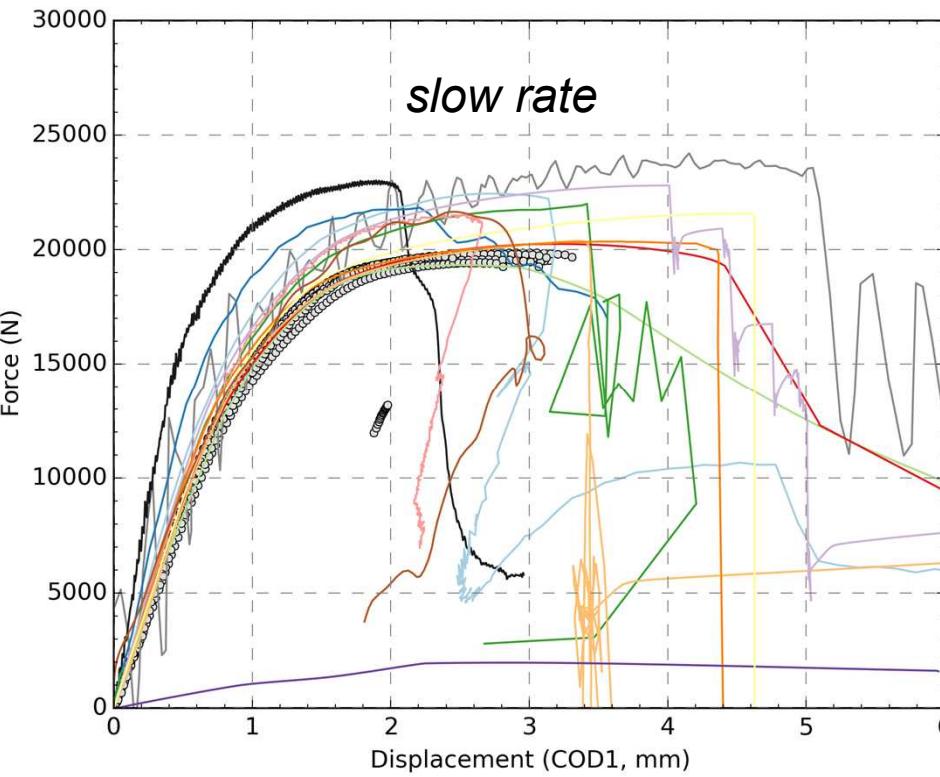
# Blind predictions





# In good company...

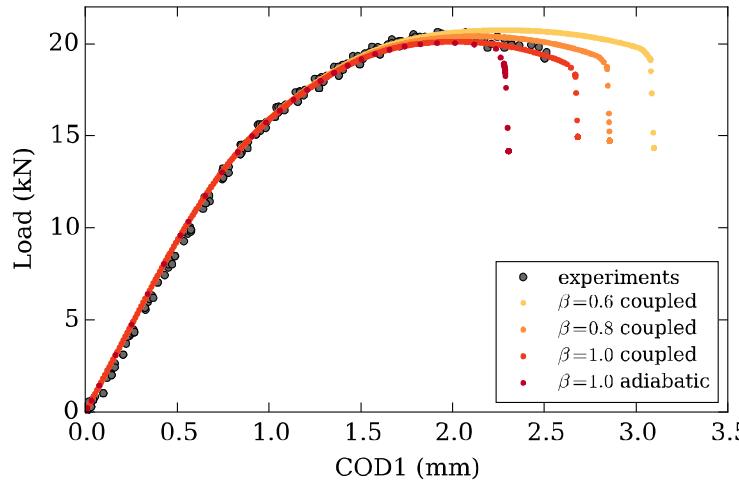
- Majority of teams predicted the correct crack path w/error in load-displacement
- Majority of teams over-predicted both the loads and displacements to failure
- We believed that the role of plastic anisotropy would improve our predictions



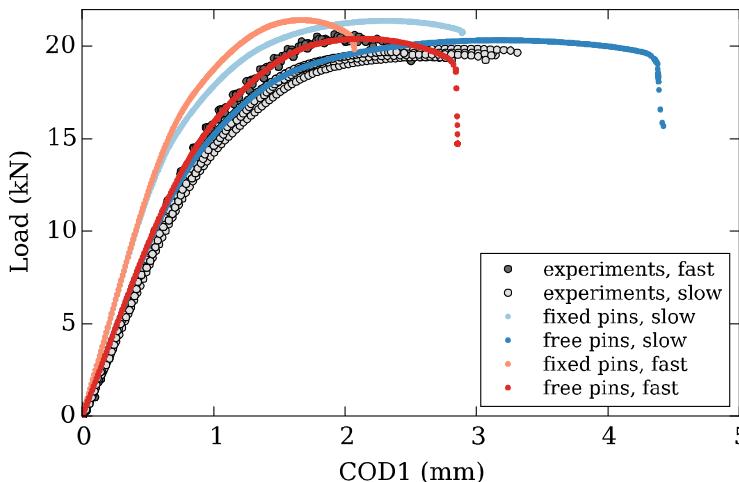
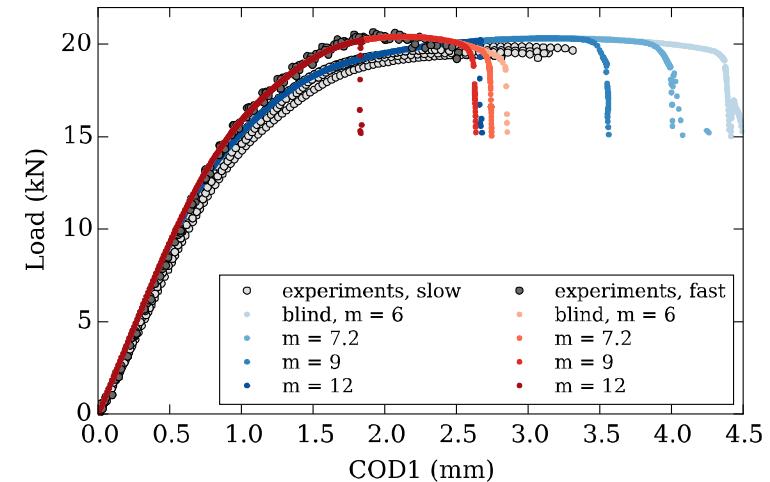


# Sensitivity studies

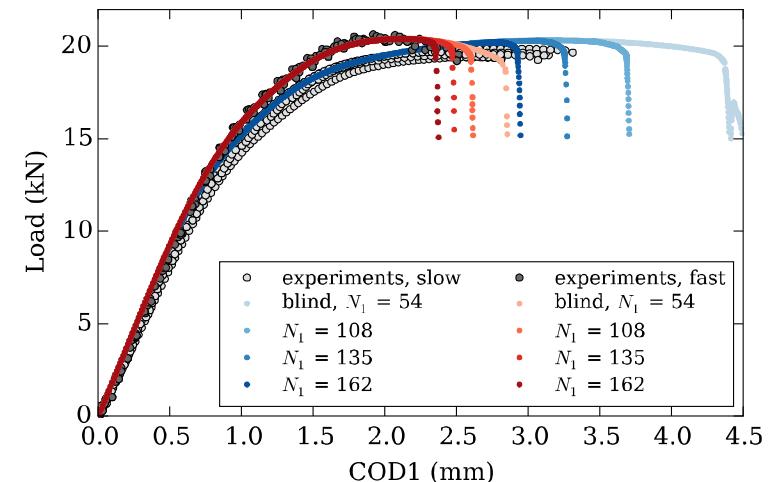
thermal parameter,  $\beta$



void growth damage,  $m$



pin boundary condition



Void nucleation damage,  $N_1$

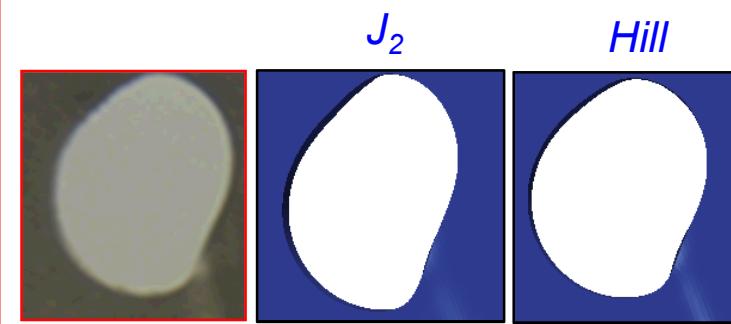
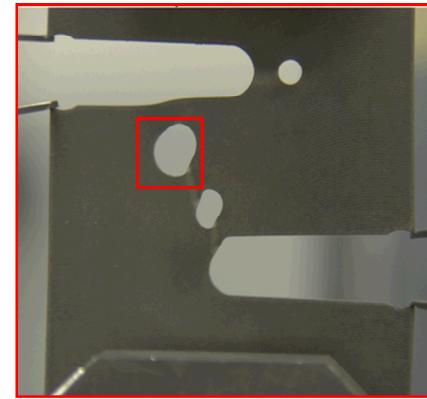


# Revisiting anisotropy

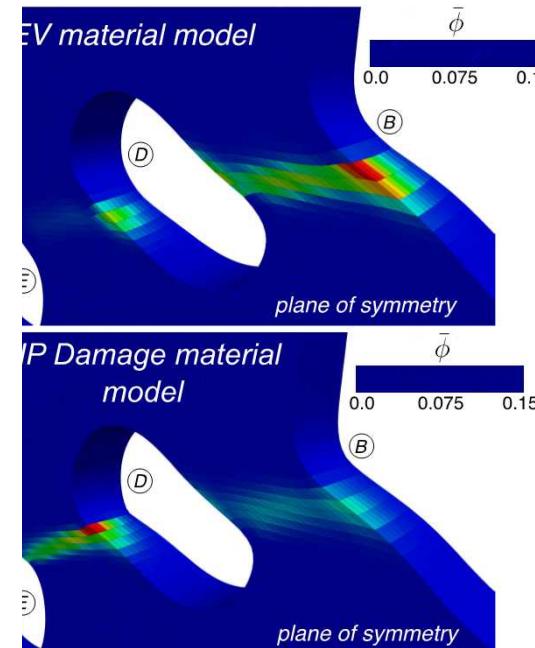
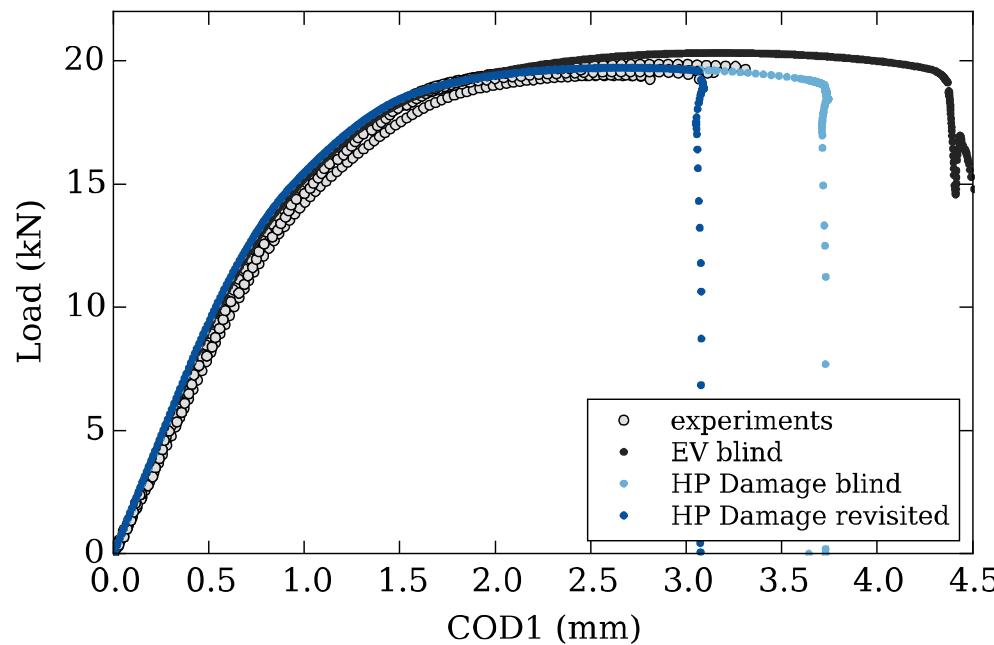
- Keep micromechanics (damage)
- Add Hill yield surface
- Aids understanding

$$\dot{\phi} = \sqrt{\frac{3}{2}} \dot{\epsilon}_p \frac{1 - (1 - \phi)^{m+1}}{(1 - \phi)^m} \sinh \left[ \frac{2(2m - 1)}{2m + 1} \frac{\langle \frac{I_1}{3} \rangle}{\sqrt{3J_2}} \right]$$
$$\dot{\eta} = \eta \dot{\epsilon}_p \mathbf{N}_1 \left[ \frac{4}{27} - \frac{J_3^2}{J_2^3} \right]$$

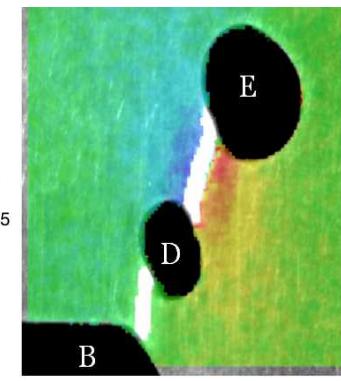
$$f_Y^2(\sigma_{ij}) \equiv F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 = \bar{\sigma}_c^2(\epsilon_p)$$



hole elongation reflects anisotropy

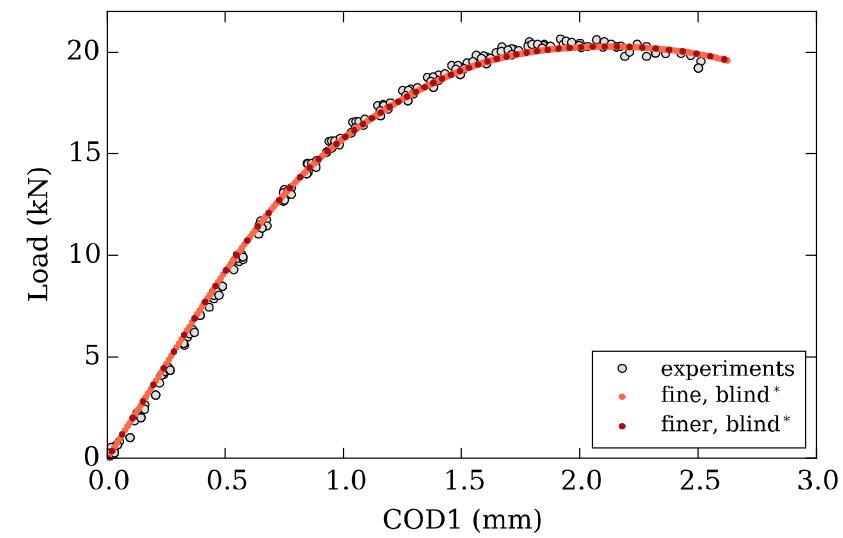
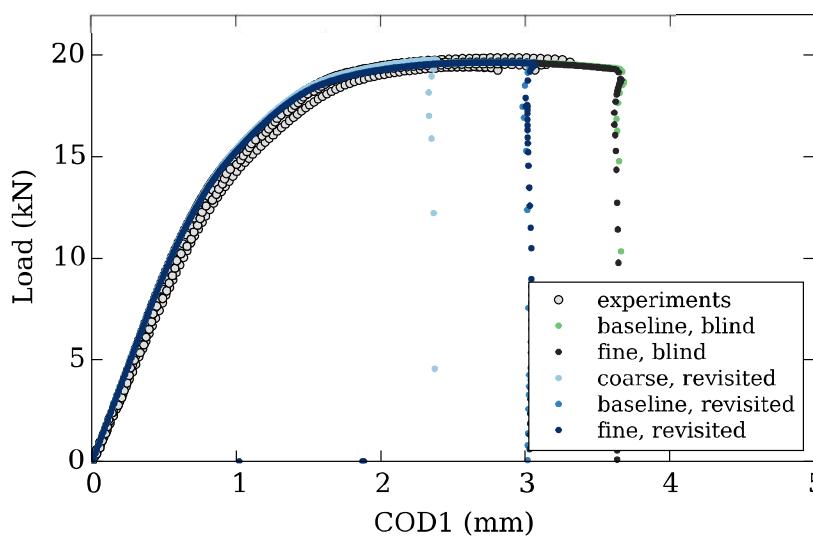
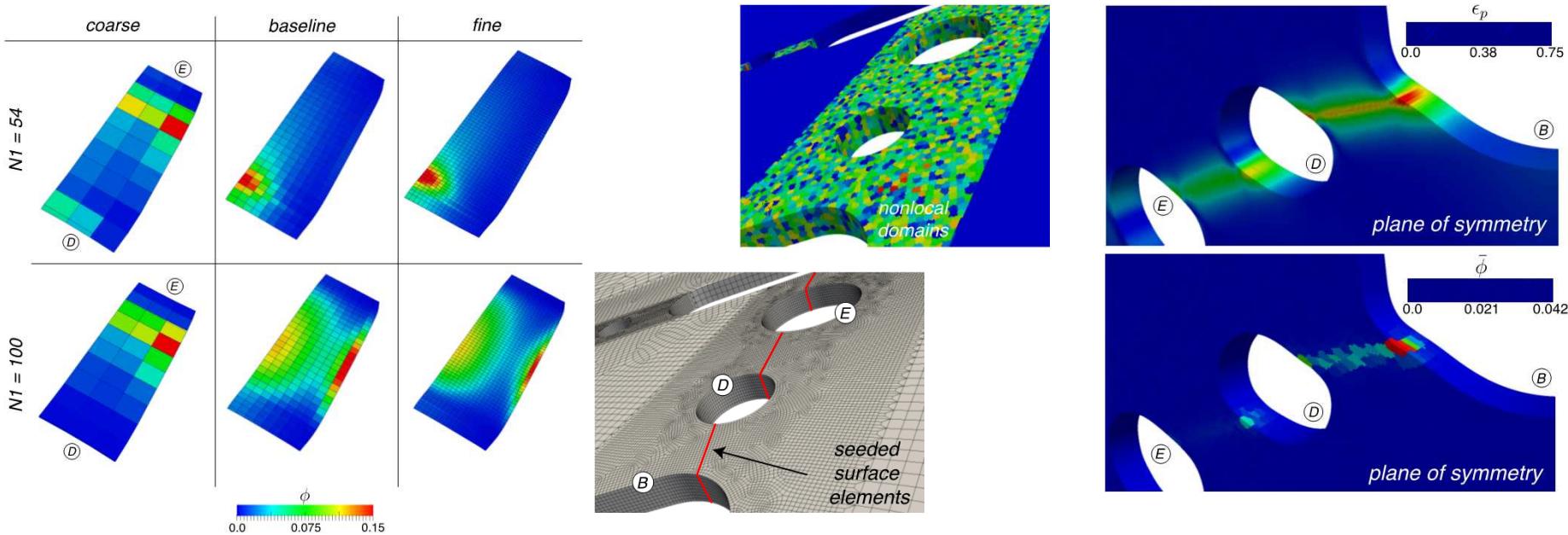


Ravi-Chandar  
Gross (UT)





# Revisiting regularization



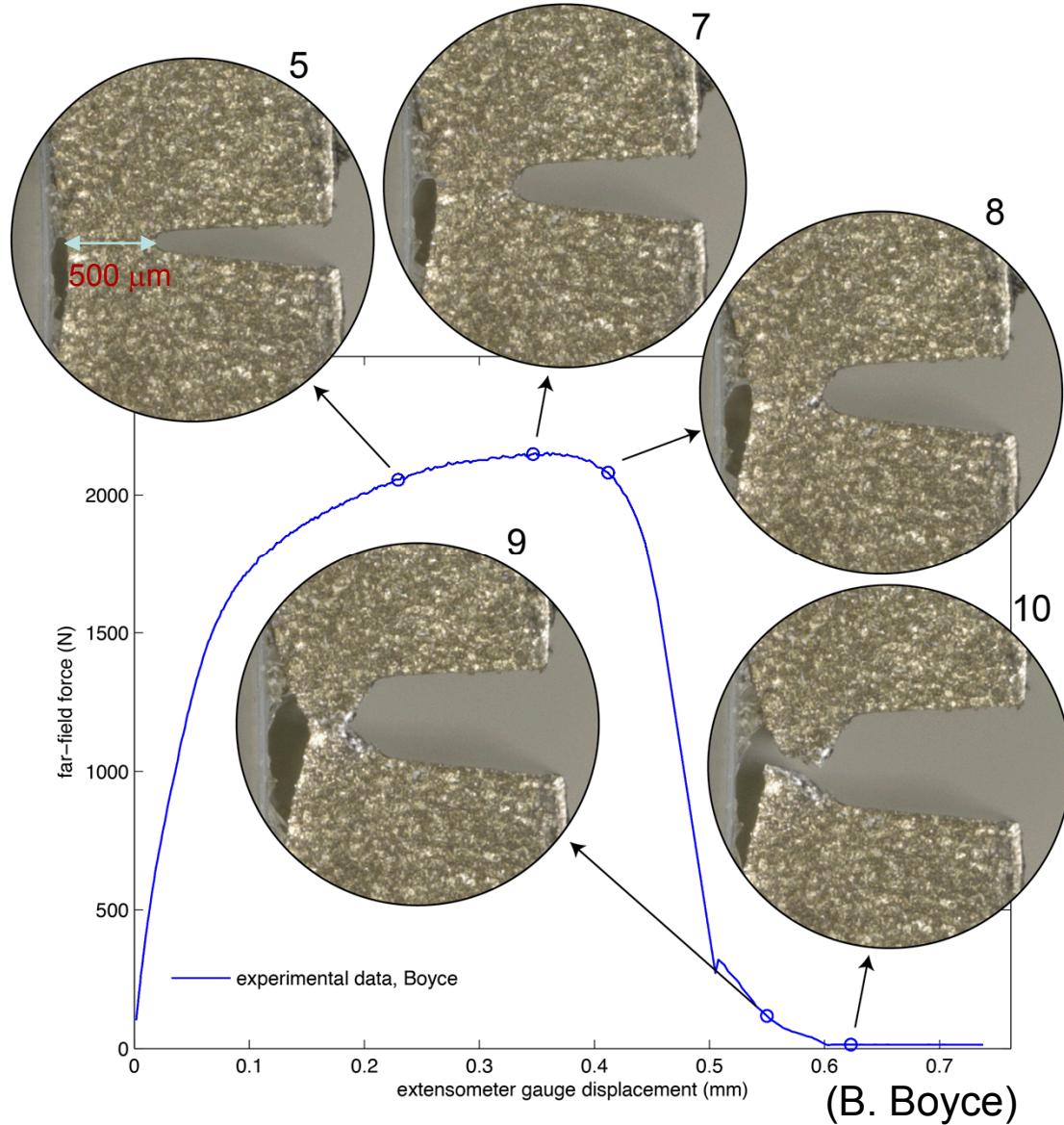
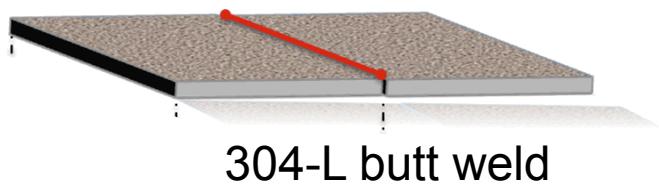


# 304L laser welds provide motivation

Surface observations indicate that the failure of 304-L is a primarily a necking process. Interrupted testing will determine the role of crack initiation.

Hypothesis: Pore size and distribution can aid the necking process and crack initiation

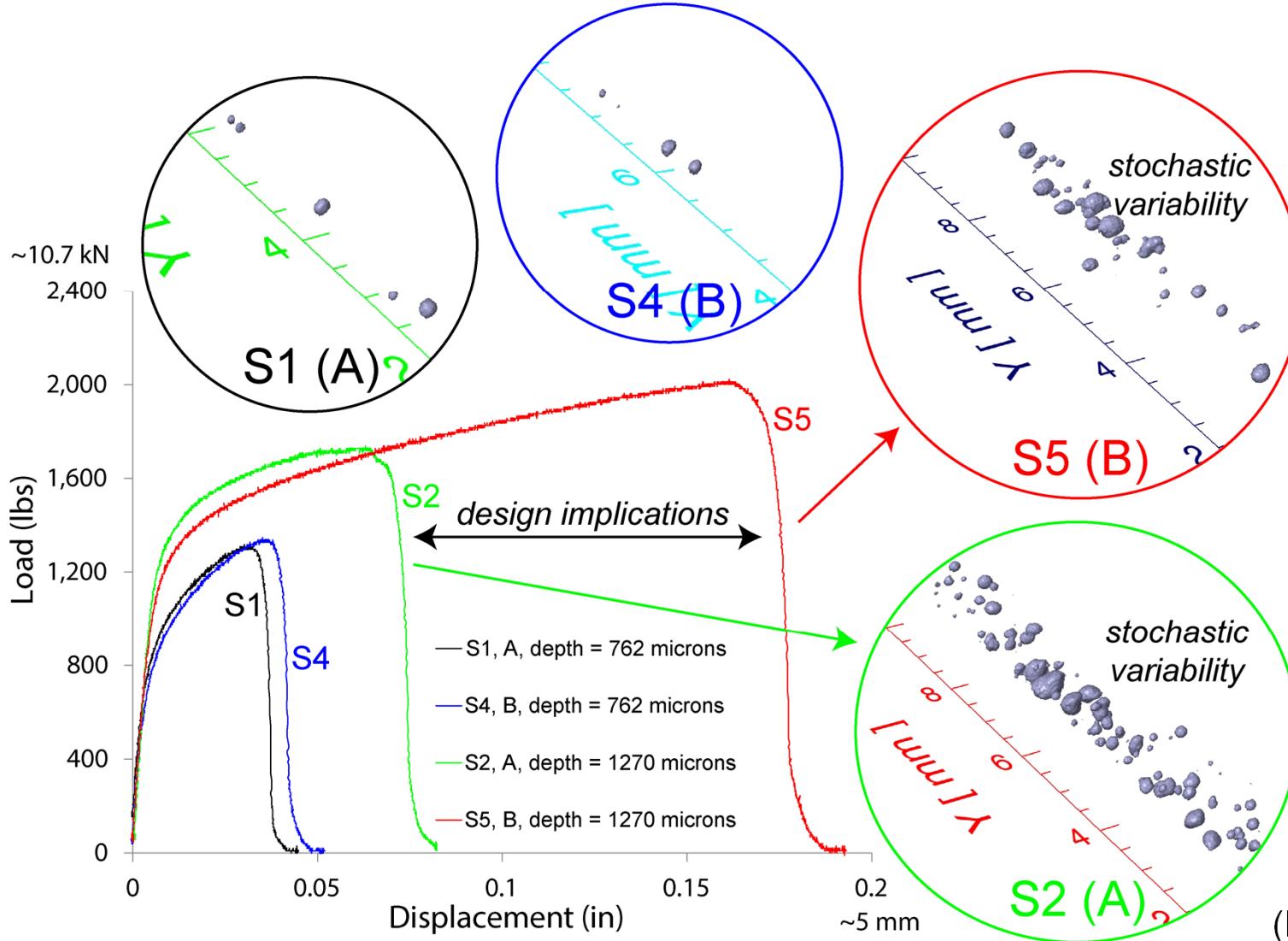
- $\mu$ -CT needed to probe initial and interrupted pore structures
- Remeshing/mapping needed to resolve the evolution of pore structure
- Homogenization not applicable





# Deeper welds galvanize efforts

*Weld schedule impacts porosity. Porosity impacts performance.*





# 10-Node Composite Tetrahedral Element

Motivated by prior work of Thoutireddy, et. al., IJNME (2002)

$$\Phi[\varphi, \bar{\mathbf{F}}, \bar{\mathbf{P}}] := \int_B A(\bar{\mathbf{F}}) \, dV + \int_B \bar{\mathbf{P}} : (\mathbf{F} - \bar{\mathbf{F}}) \, dV - \int_B \mathbf{R}\mathbf{B} \cdot \varphi \, dV - \int_{\partial_T B} \mathbf{T} \cdot \varphi \, dS$$

$$\bar{\mathbf{P}} = \lambda_\alpha \left( \int_\Omega \lambda_\alpha \lambda_\beta \mathbf{I} \, dV \right)^{-1} \int_\Omega \lambda_\beta \mathbf{P} \, dV,$$

$$\bar{\mathbf{F}} = \lambda_\alpha \left( \int_\Omega \lambda_\alpha \lambda_\beta \mathbf{I} \, dV \right)^{-1} \int_\Omega \lambda_\beta \mathbf{F} \, dV$$

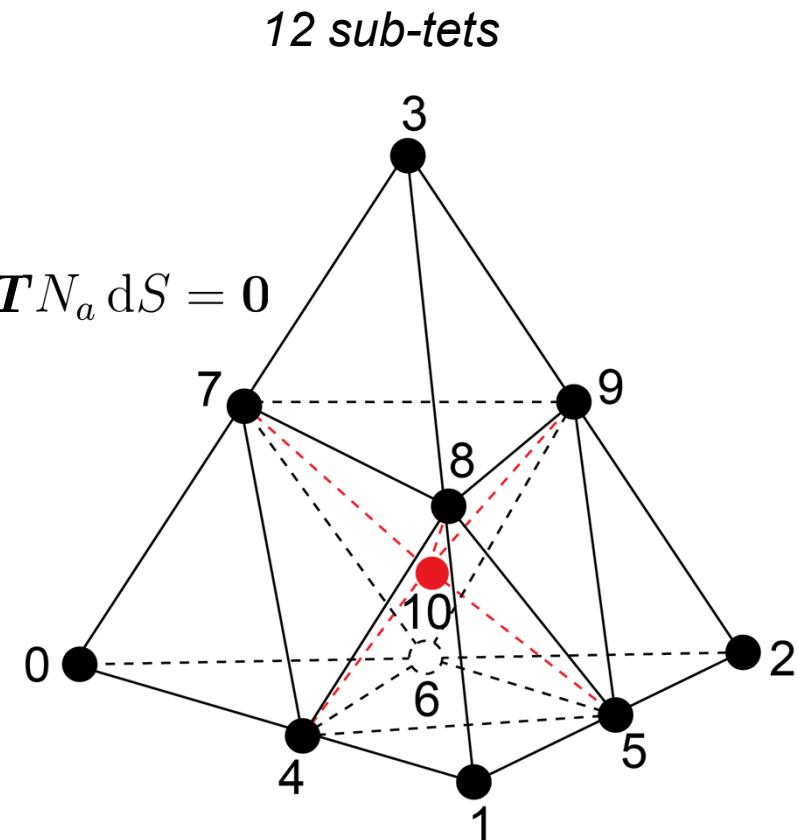
$$\mathbf{R}_a(\varphi) := \int_\Omega \bar{\mathbf{P}} \cdot \mathbf{B}_a \, dV - \int_\Omega \mathbf{R}\mathbf{B} N_a \, dV - \int_{\partial_T \Omega} \mathbf{T} N_a \, dS = \mathbf{0}$$

$$\mathbf{B}_a(\mathbf{X}) := \delta_{ik} \frac{\partial N_a(\mathbf{X})}{\partial X_J} \mathbf{e}_i \otimes \mathbf{E}_J \otimes \mathbf{e}_k$$

$\varphi$  C<sup>0</sup> piecewise linear

$\mathbf{F}$  C<sup>-1</sup> linear over parent element

$\bar{\mathbf{P}}$  C<sup>-1</sup> linear over parent element





# Analytical gradient operator

Develop an exact gradient operator that projects and interpolates sub-tet gradients

$$\bar{\mathbf{F}}(\mathbf{X}) := \bar{\mathcal{B}}_a(\mathbf{X}) \mathbf{x}_a$$

$$\bar{\mathcal{B}}_a(\mathbf{X}) := \lambda_\alpha(\mathbf{X}) \left[ \int_{\Omega} \delta_{ik} \lambda_\alpha(\mathbf{X}) \lambda_\beta(\mathbf{X}) \, dV \right]^{-1} \int_{\Omega} \lambda_\beta(\mathbf{X}) \frac{\partial N_a(\mathbf{X})}{\partial X_J} \, dV \mathbf{e}_i \otimes \mathbf{E}_J \otimes \mathbf{e}_k$$

$$\bar{\mathcal{B}}_a(\boldsymbol{\xi}) = \lambda_\alpha(\boldsymbol{\xi}) \left[ \int_{\Omega_{\boldsymbol{\xi}}} \delta_{ik} \lambda_\alpha(\boldsymbol{\xi}) \lambda_\beta(\boldsymbol{\xi}) \, dV_{\boldsymbol{\xi}} \right]^{-1} \int_{\Omega_{\boldsymbol{\xi}}} \lambda_\beta(\boldsymbol{\xi}) \frac{\partial N_a(\boldsymbol{\xi})}{\partial \boldsymbol{\xi}} \, dV_{\boldsymbol{\xi}} \left( \frac{\partial \boldsymbol{\xi}}{\partial X_J} \right) \mathbf{e}_i \otimes \mathbf{E}_J \otimes \mathbf{e}_k$$

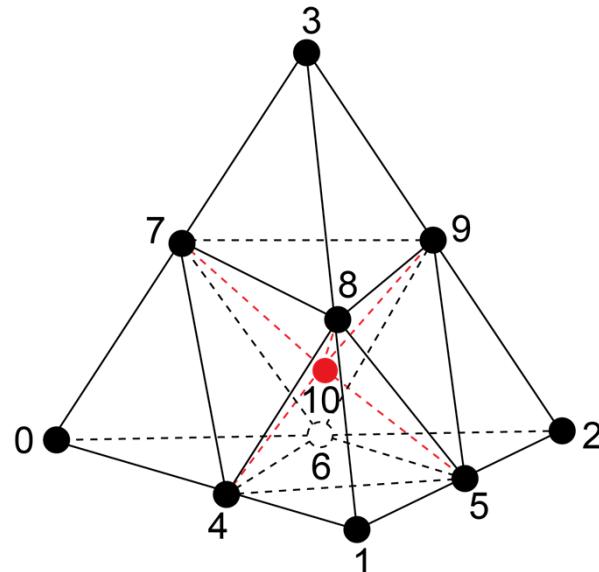
$$\bar{\mathcal{B}}_a(\boldsymbol{\xi}) = \bar{\mathcal{L}}_{a;ilk}(\boldsymbol{\xi}) \left( \frac{\partial \boldsymbol{\xi}_l}{\partial X_J} \right) \mathbf{e}_i \otimes \mathbf{E}_J \otimes \mathbf{e}_k$$

$$\bar{\mathcal{L}}_a(\boldsymbol{\xi}) = \lambda_\alpha(\boldsymbol{\xi}) \delta_{ik} (M_{\alpha\beta})^{-1} \sum_{S=0}^{11} \frac{\partial N_a}{\partial \boldsymbol{\xi}_l} \int_{E_S} \lambda_\beta(\boldsymbol{\xi}) \, dV_{\boldsymbol{\xi}} \mathbf{e}_i \otimes \mathbf{a}_l \otimes \mathbf{e}_k$$

$$\bar{\mathcal{B}}_a(\boldsymbol{\xi}) = \bar{\mathcal{L}}_{a;ilk}(\boldsymbol{\xi}) \left[ \bar{\mathcal{L}}_{b;JlM}(\boldsymbol{\xi}) X_{b;M} \right]^{-1} \mathbf{e}_i \otimes \mathbf{E}_J \otimes \mathbf{e}_k$$

$$\bar{B}_{aJ}(\boldsymbol{\xi}) = \bar{L}_{al}(\boldsymbol{\xi}) \left[ X_{Jb} \bar{L}_{bl}(\boldsymbol{\xi}) \right]^{-1}$$

$$\bar{F}_{iJ}(\boldsymbol{\xi}) = x_{ia} \bar{B}_{aJ}(\boldsymbol{\xi})$$



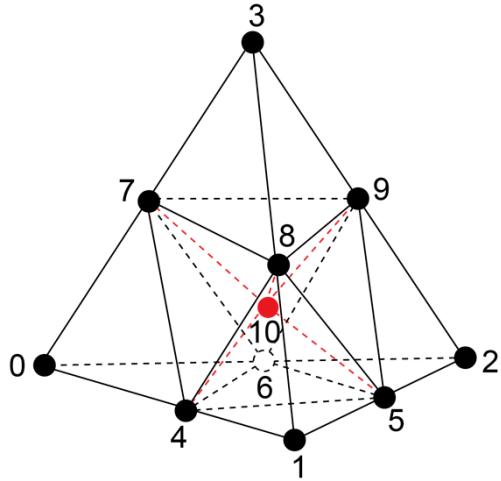
$$\bar{L}_{al}(\boldsymbol{\xi}) \equiv \bar{L}_{10 \times 3} = \frac{1}{24} \left( \begin{array}{ccc} \end{array} \right)$$

$$\left( \begin{array}{ccc} 9-60\xi_0 & 9-60\xi_0 & 9-60\xi_0 \\ -9+60\xi_1 & 0 & 0 \\ 0 & -9+60\xi_2 & 0 \\ 0 & 0 & -9+60\xi_3 \\ 70(\xi_0-\xi_1) & 2(-4-35\xi_1+5\xi_2+10\xi_3) & 2(-4-35\xi_1+10\xi_2+5\xi_3) \\ 2(-1+5\xi_1+40\xi_2-5\xi_3) & 2(-1+40\xi_1+5\xi_2-5\xi_3) & 10(\xi_0-\xi_3) \\ 2(-4+5\xi_1-35\xi_2+10\xi_3) & 70(\xi_0-\xi_2) & 2(-4+10\xi_1-35\xi_2+5\xi_3) \\ 2(-4+5\xi_1+10\xi_2-35\xi_3) & 2(-4+10\xi_1+5\xi_2-35\xi_3) & 70(\xi_0-\xi_3) \\ 2(-1+5\xi_1-5\xi_2+40\xi_3) & 10(\xi_0-\xi_2) & 2(-1+40\xi_1-5\xi_2+5\xi_3) \\ 10(\xi_0-\xi_1) & 2(-1-5\xi_1+5\xi_2+40\xi_3) & 2(-1-5\xi_1+40\xi_2+5\xi_3) \end{array} \right)$$

*Evaluate for your  
integration scheme*



# Suitable for isochoric motions

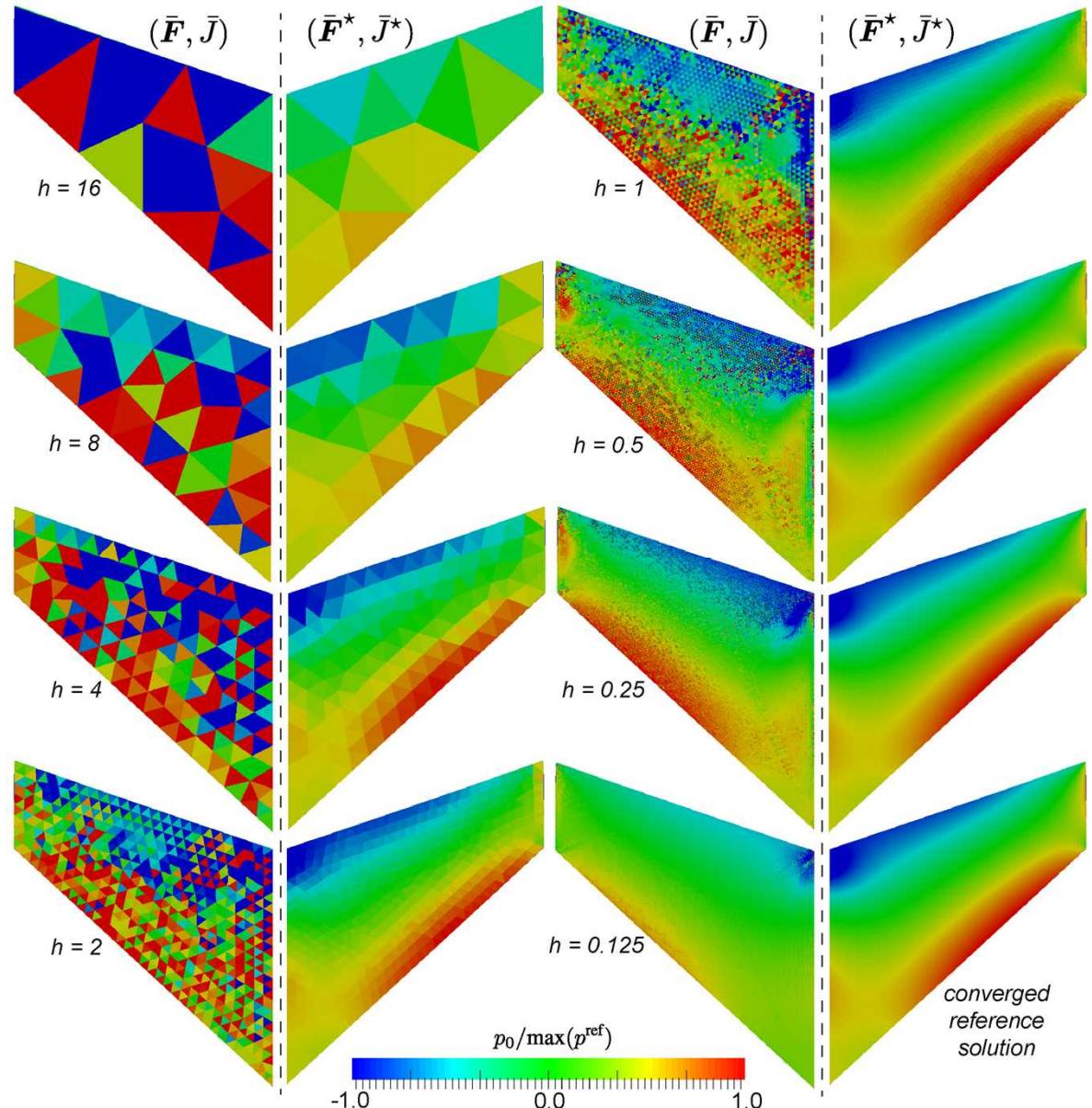


*Volume-averaged formulation  
does not exhibit spurious  
pressure oscillations.*

$$\bar{\mathbf{F}}^*(\xi) := \left( \frac{\bar{J}^*}{\bar{J}(\xi)} \right)^{\frac{1}{3}} \bar{\mathbf{F}}(\xi)$$

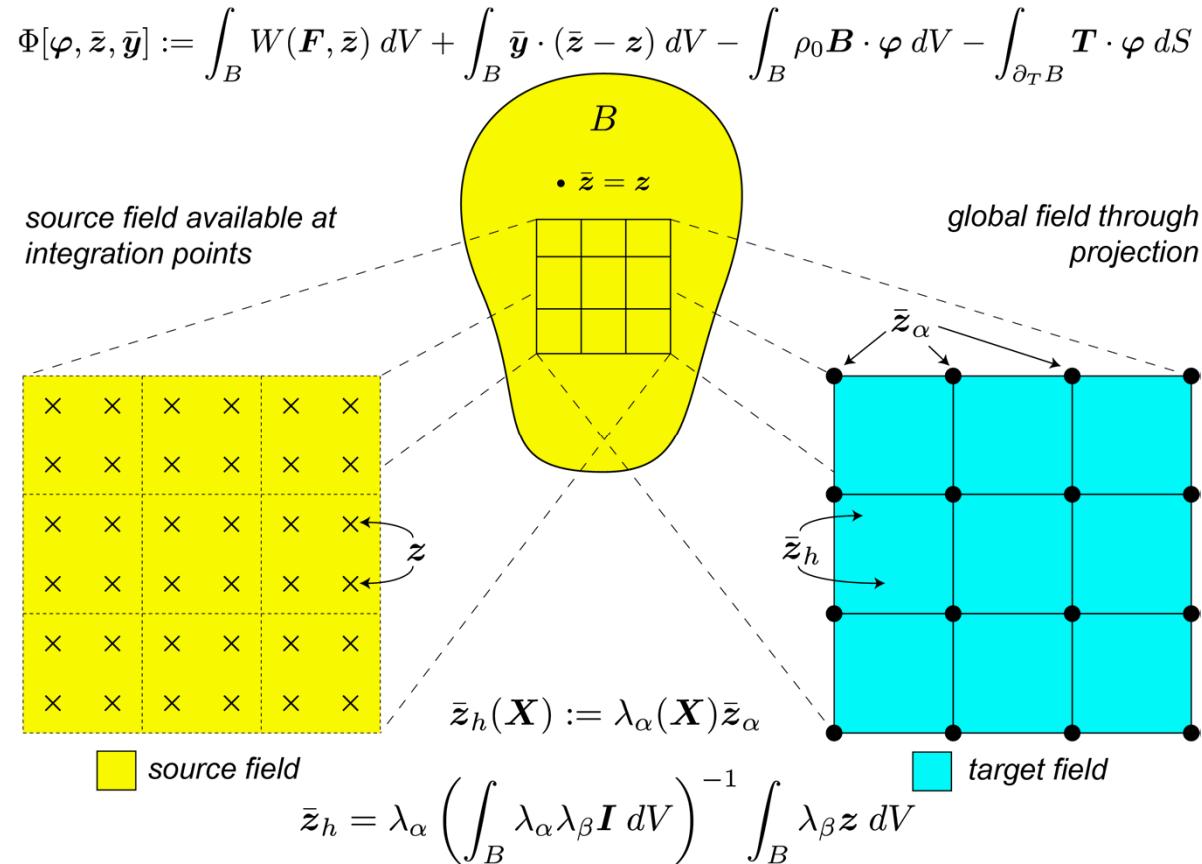
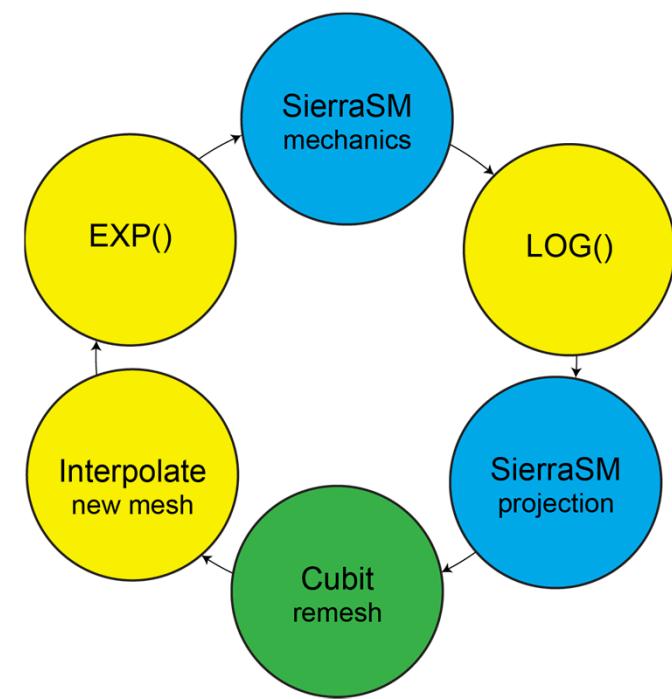
$$\bar{J}^* := \frac{\int_{\Omega} \bar{J} \, dV}{\int_{\Omega} dV}$$

$$\bar{p}^* := \frac{1}{V_{\Omega}} \int_{\Omega} \text{tr} \frac{\partial A(\mathbf{F}^*)}{\partial \bar{\mathbf{b}}^*} \, dV$$





# mapLL ( $L_2$ + Lie Group/Algebra)

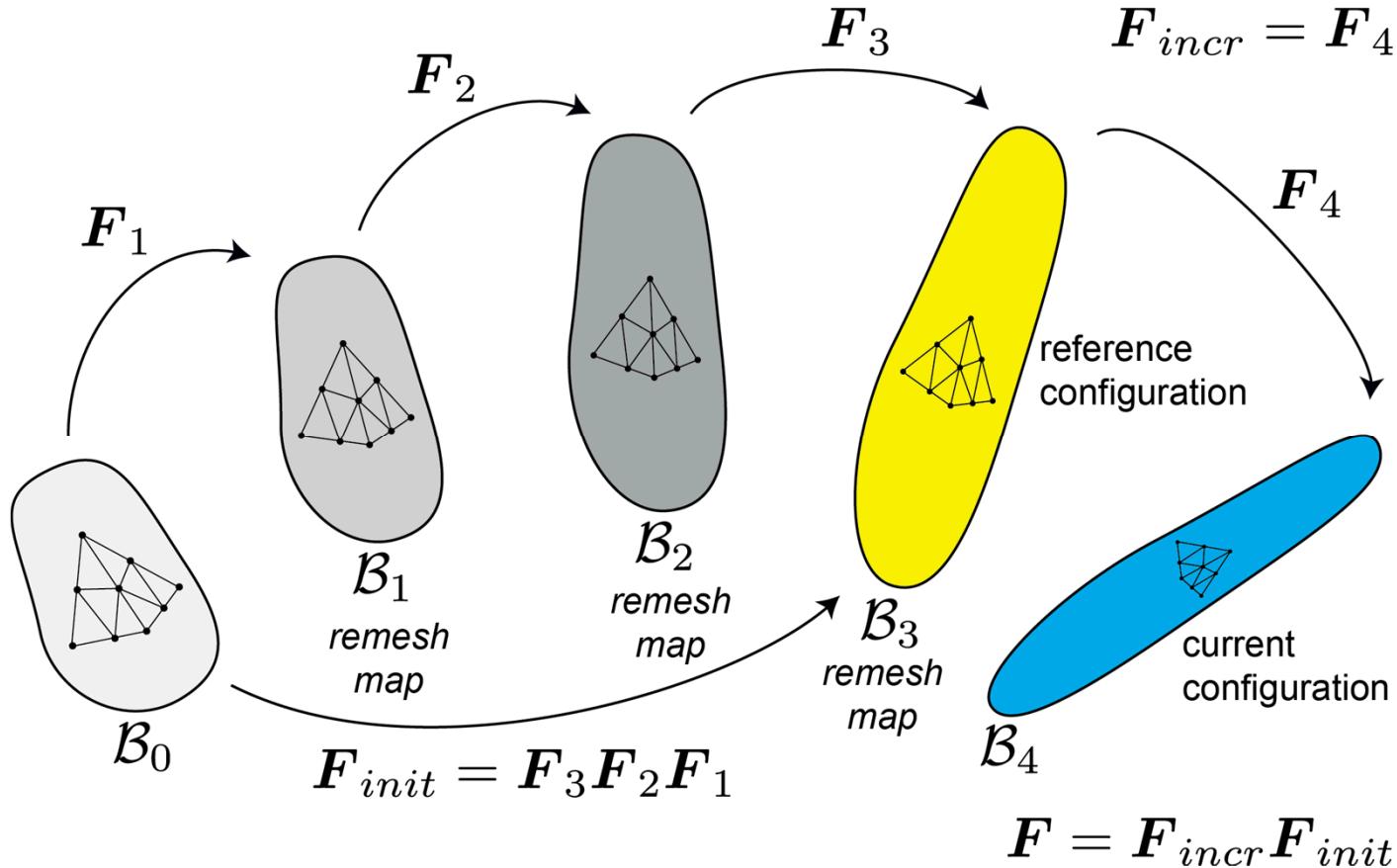


- The variational principle naturally yields an optimal,  $L_2$  projection
- The spaces of variables (Lie algebra, Lie Group) are honored through  $\log()$  and  $\exp()$
- Advocated by Mota, et. al., Computational Mechanics, 2013

Past works: Ortiz and Quigley (1991), Camacho and Ortiz (1997), Radovitzky and Ortiz (1999), Rashid (2002), Jiao and Heath (2004)



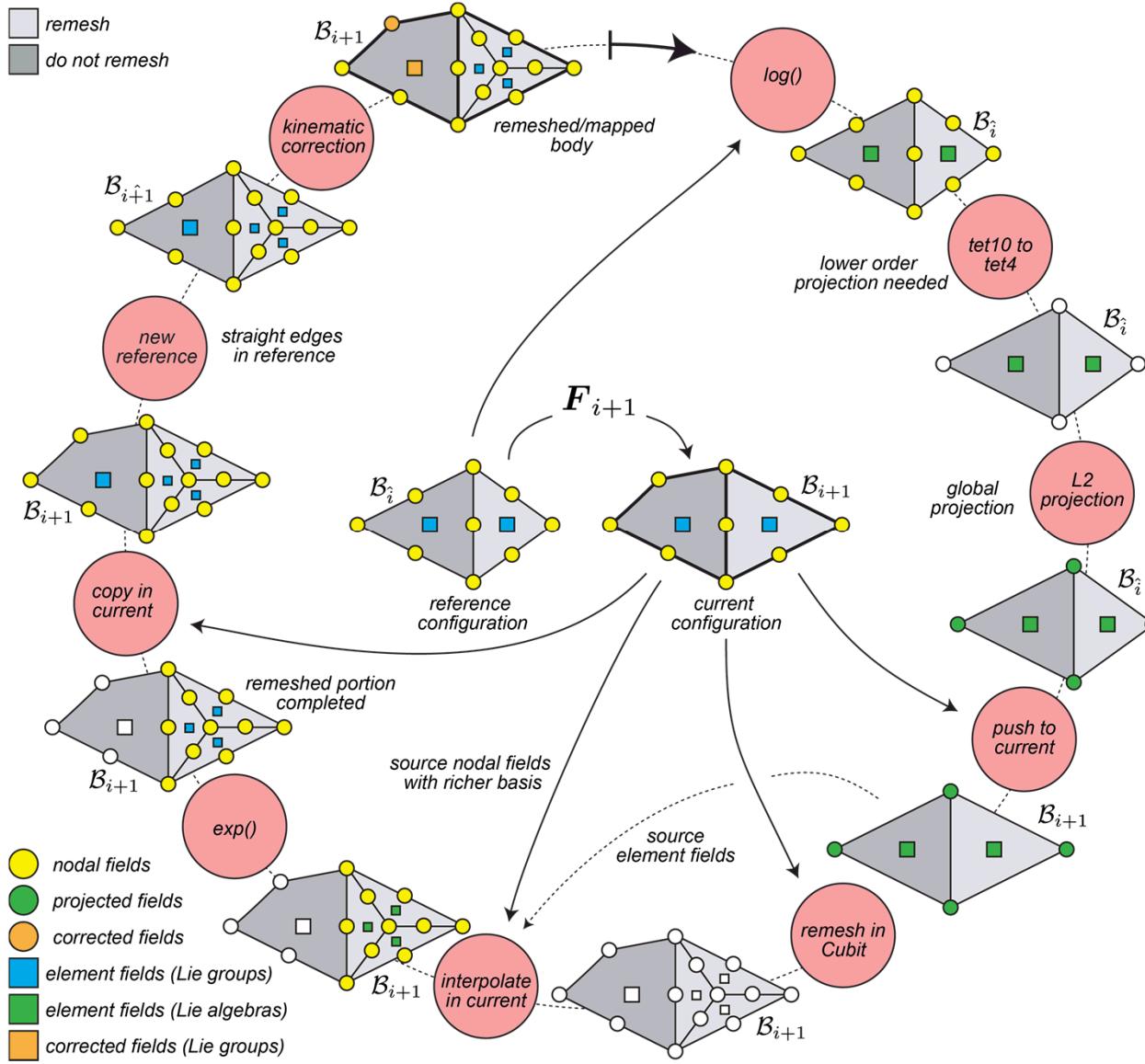
# Adopting a new reference



- Prior work on hexahedral elements maintained the reference configuration
- Elements degrade in the reference configuration - T-L element integrate in reference
- We now adopt a new reference configuration and map  $F_{init}$  (which lives in a Lie Group)

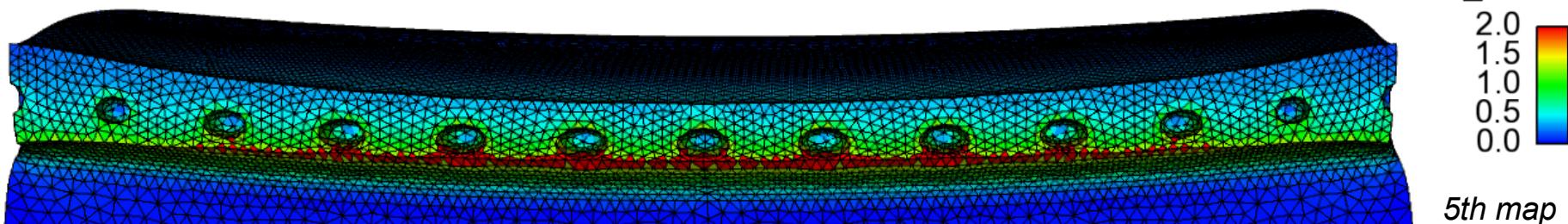
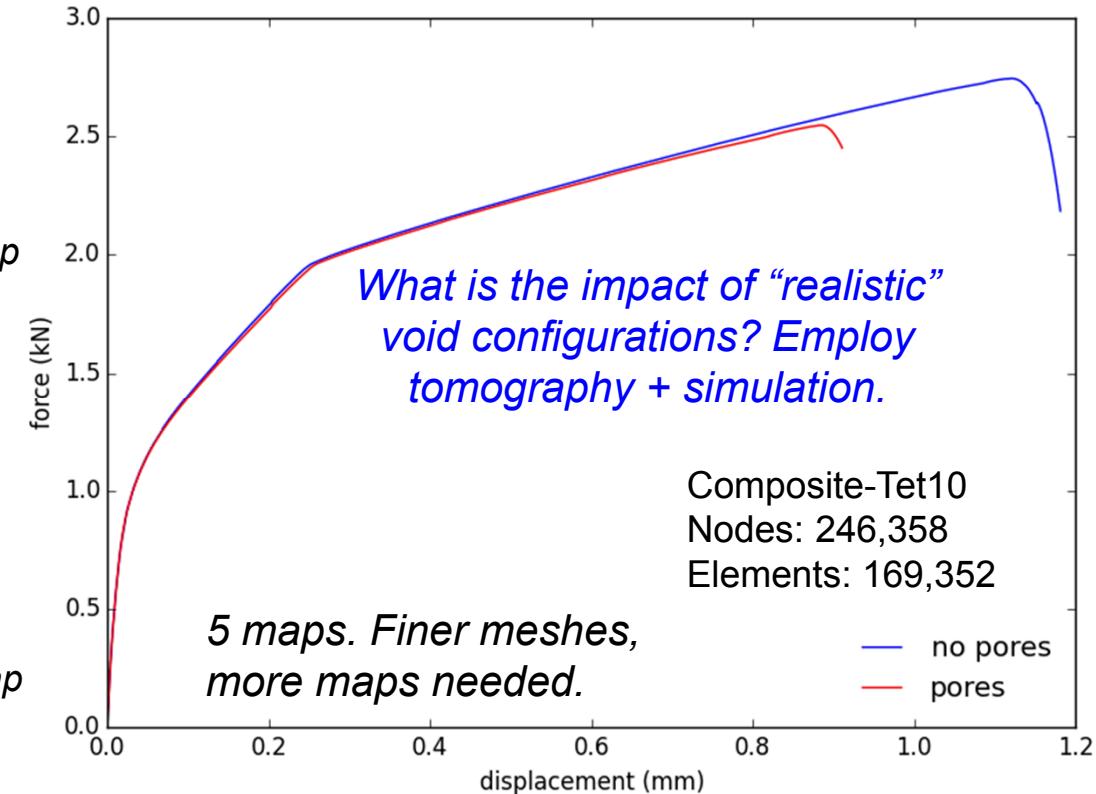
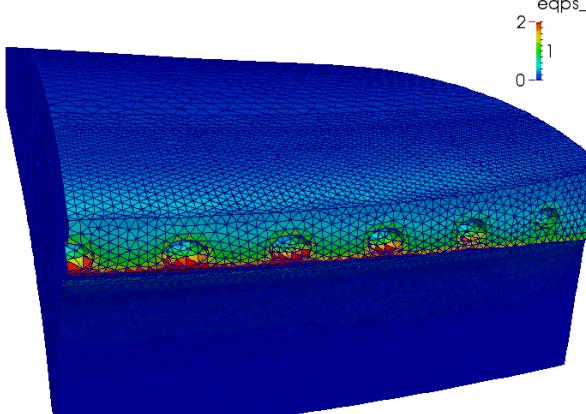
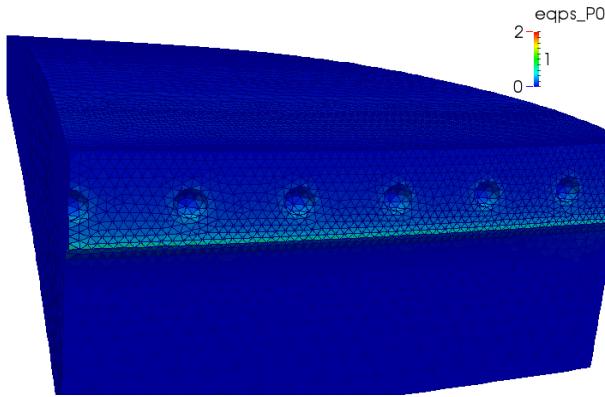


# Mapping procedure



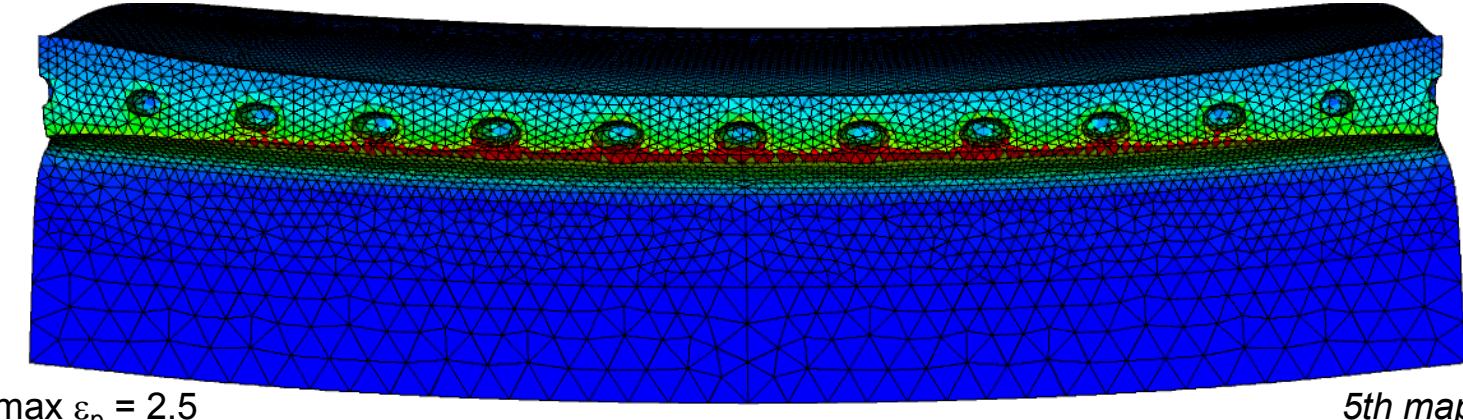
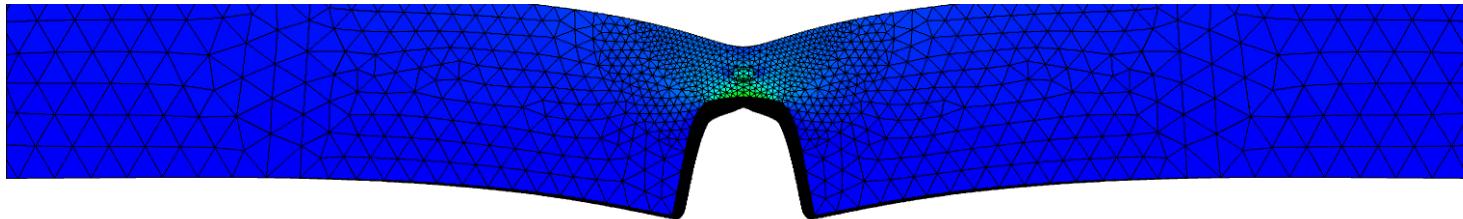
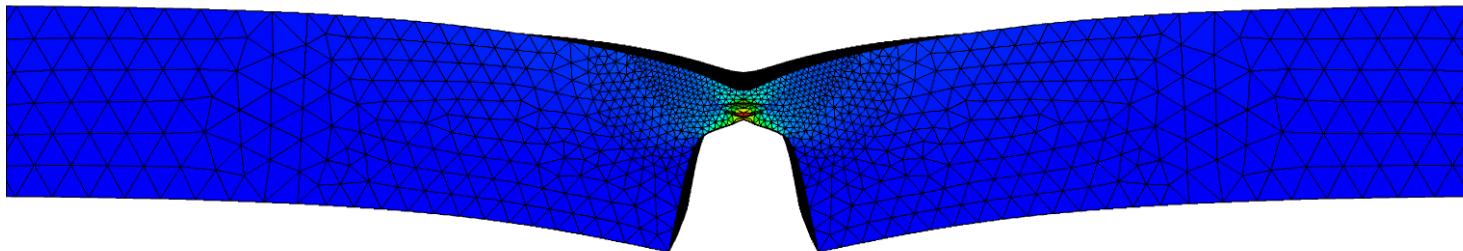
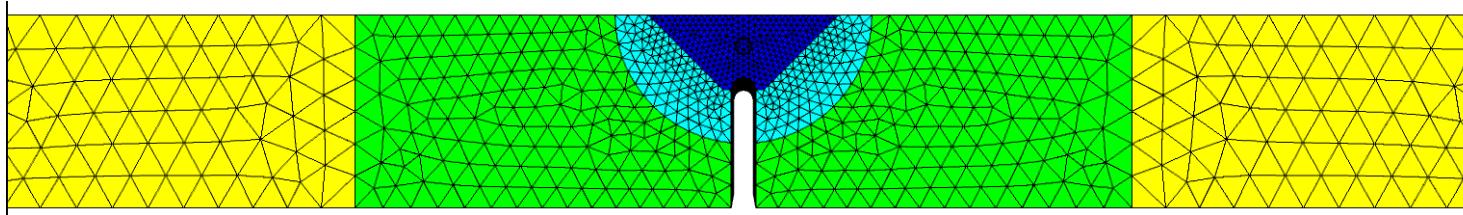


# Remeshing/mapping discrete pores



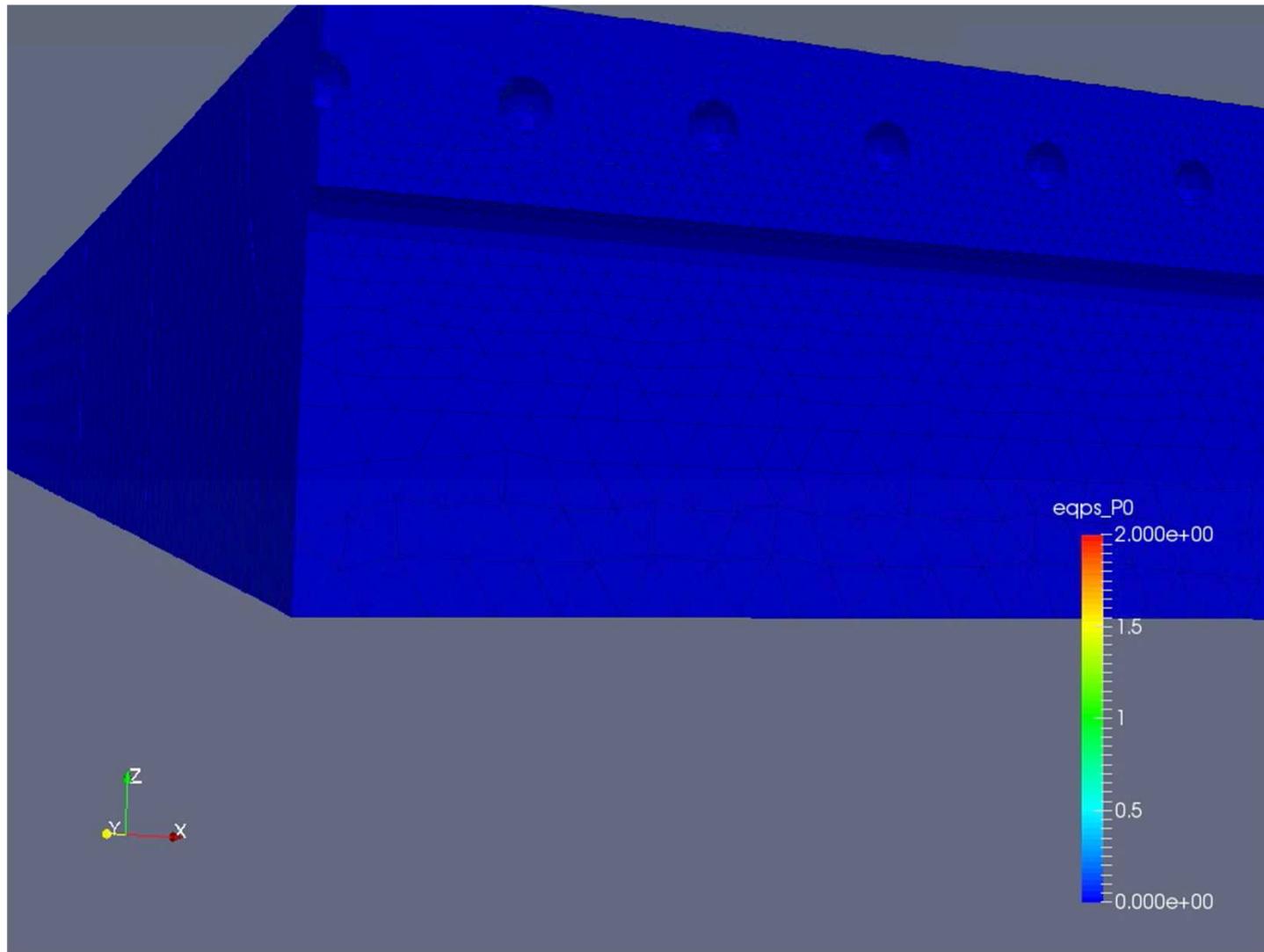


# Additional views of necking process





# Increasing the number of mappings





# Reflections and path forward

- Refocused task
  - Hardening nonlocality and localization elements
  - Physics drive localization – investing in constitutive modeling
  - Documenting work on optimal meshes/bifurcation
  - Emphasizing linkage to SierraSM and X-FEM
- Illustrated blind and revisited SFC2 predictions
  - Importance of coupling, rate dependence, anisotropy, and nucleation
  - Regularized physics with multiple methodologies
- Highlighted work in remeshing/mapping of internal state variables
  - Massive deformations often accompany the localization process
  - Proposed methodology resolves inelasticity to strains in excess of 6
  - Composite tetrahedral element technology complements adaptivity