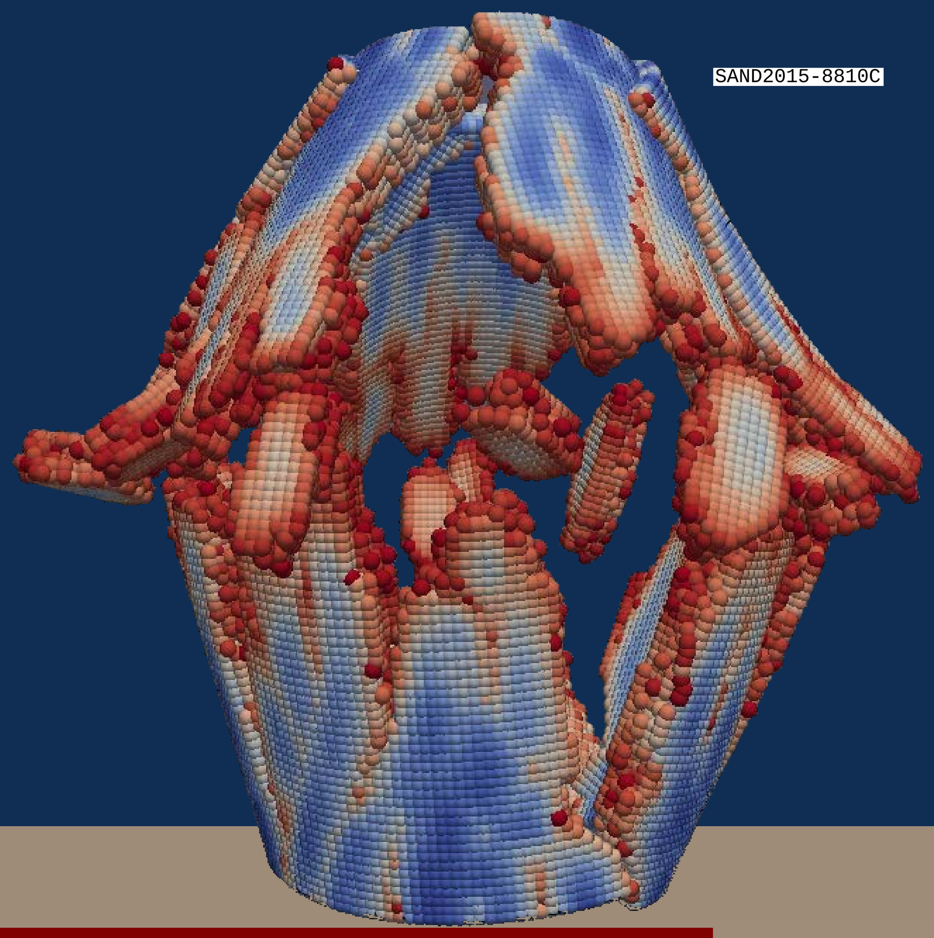

A Multi-Time-Step Method for Partitioned Time Integration of Peridynamics

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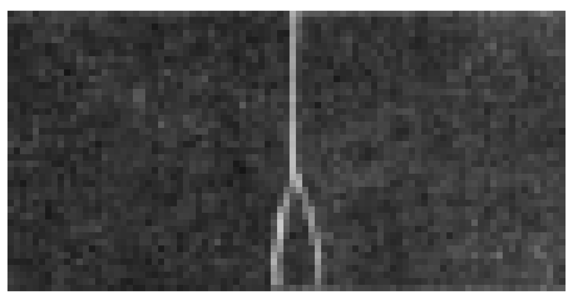
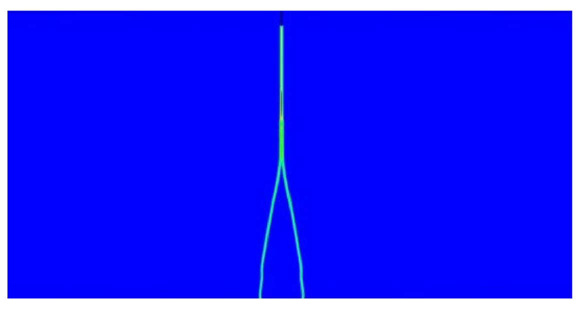
Problem

Peridynamics[1] is a nonlocal continuum model useful for modeling fracture.

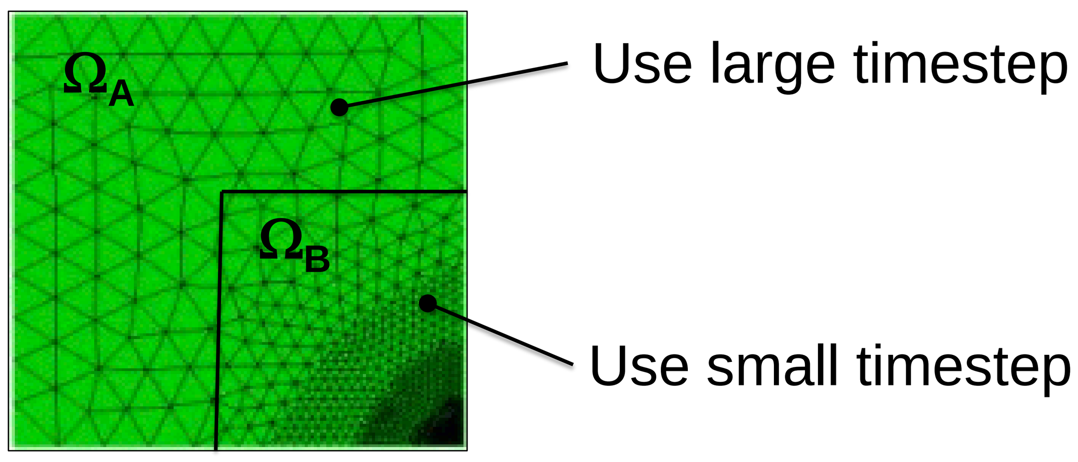


Fracture in Glass Plate[2]



Peridynamic Simulation of Fracture in Glass Plate

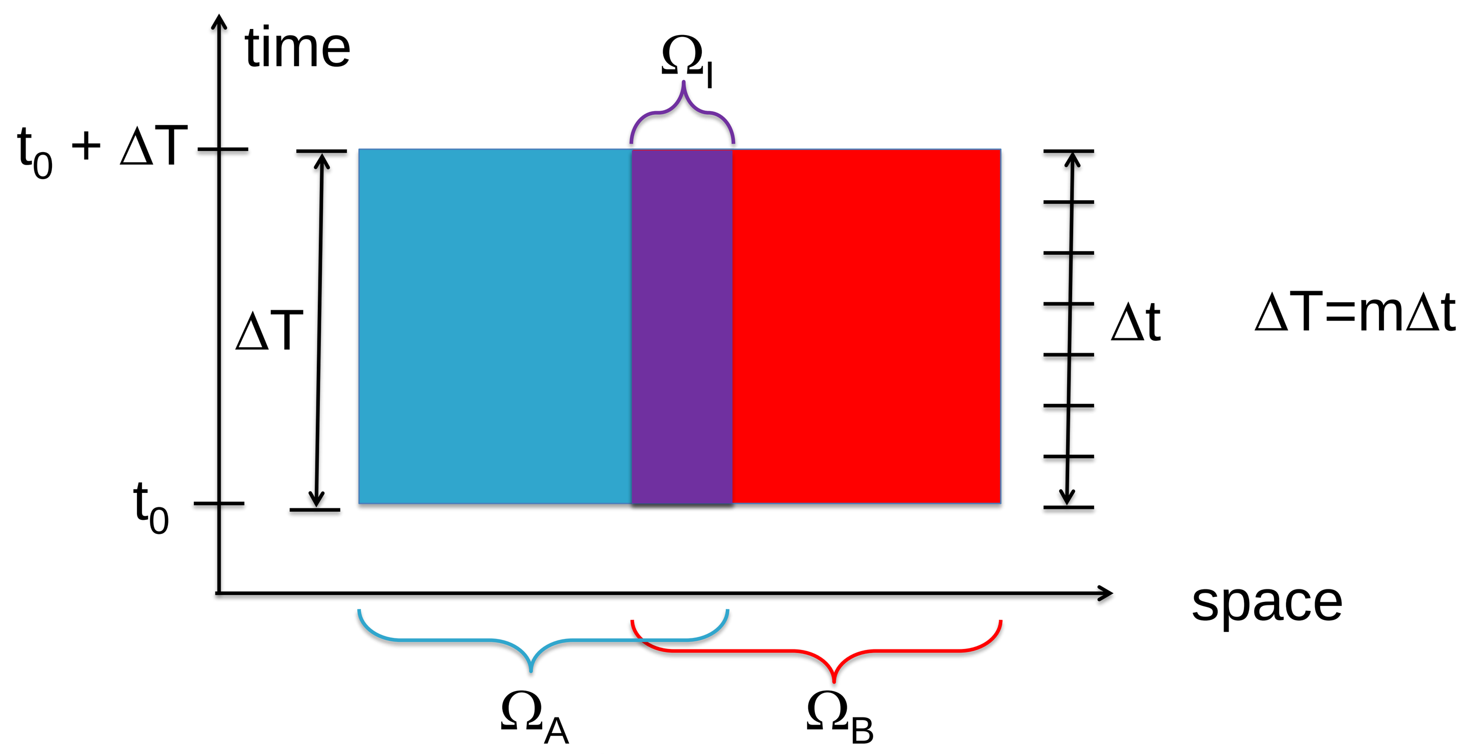
Current computational peridynamics methods enforce a single time-step for the whole domain, even if a small timestep is not required everywhere.



Approach

Nonlocal Multi-Time-Stepping

- Perform time integration with different timestep for each subdomain.
- Adapt multi-time-stepping[3] to nonlocal domain decomposition[4]. Couple subdomains with Lagrange multipliers and enforce constraint that velocity be consistent across subdomain overlap Ω_I. Solve resulting coupled system of equations.



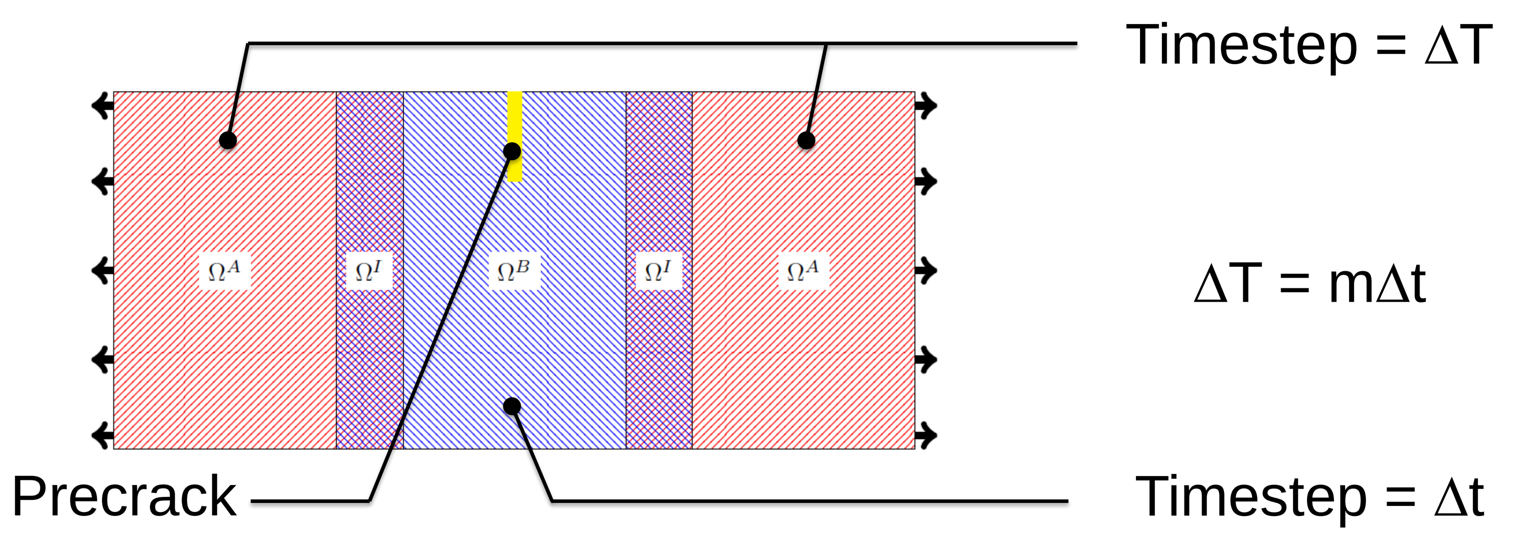
$$
\left[\begin{array}{cccc|c|c}
\mathbb{M}^B & & & & & \frac{1}{m}\mathbb{C}^B \\
\mathbb{N}^B & \mathbb{M}^B & & & & \frac{2}{m}\mathbb{C}^B \\
 & \ddots & \ddots & & & \vdots \\
 & & \mathbb{N}^B & \mathbb{M}^B & & \frac{m}{m}\mathbb{C}^B \\
\hline
 & & & & \mathbb{M}^A & \mathbb{C}^A \\
\hline
 & & & \mathbb{B}^A & \mathbb{B}^B & \mathbf{0}
\end{array}\right]
\left[\begin{array}{c}
\mathbb{U}_1^B \\ \mathbb{U}_2^B \\ \vdots \\ \mathbb{U}_m^B \\ \hline \mathbb{U}_m^A \\ \hline \Lambda_m
\end{array}\right]
=
\left[\begin{array}{c}
\mathbb{F}_1^B - \mathbb{N}^B\mathbb{U}_0^B - \mathbb{C}^B\mathbf{S}_1 \\
\mathbb{F}_2^B - \mathbb{C}^B\mathbf{S}_2 \\
\vdots \\
\mathbb{F}_m^B - \mathbb{C}^B\mathbf{S}_m \\
\hline
\mathbb{F}_m^A - \mathbb{N}^A\mathbb{U}_0^A \\
\hline
\mathbf{0}
\end{array}\right]
$$

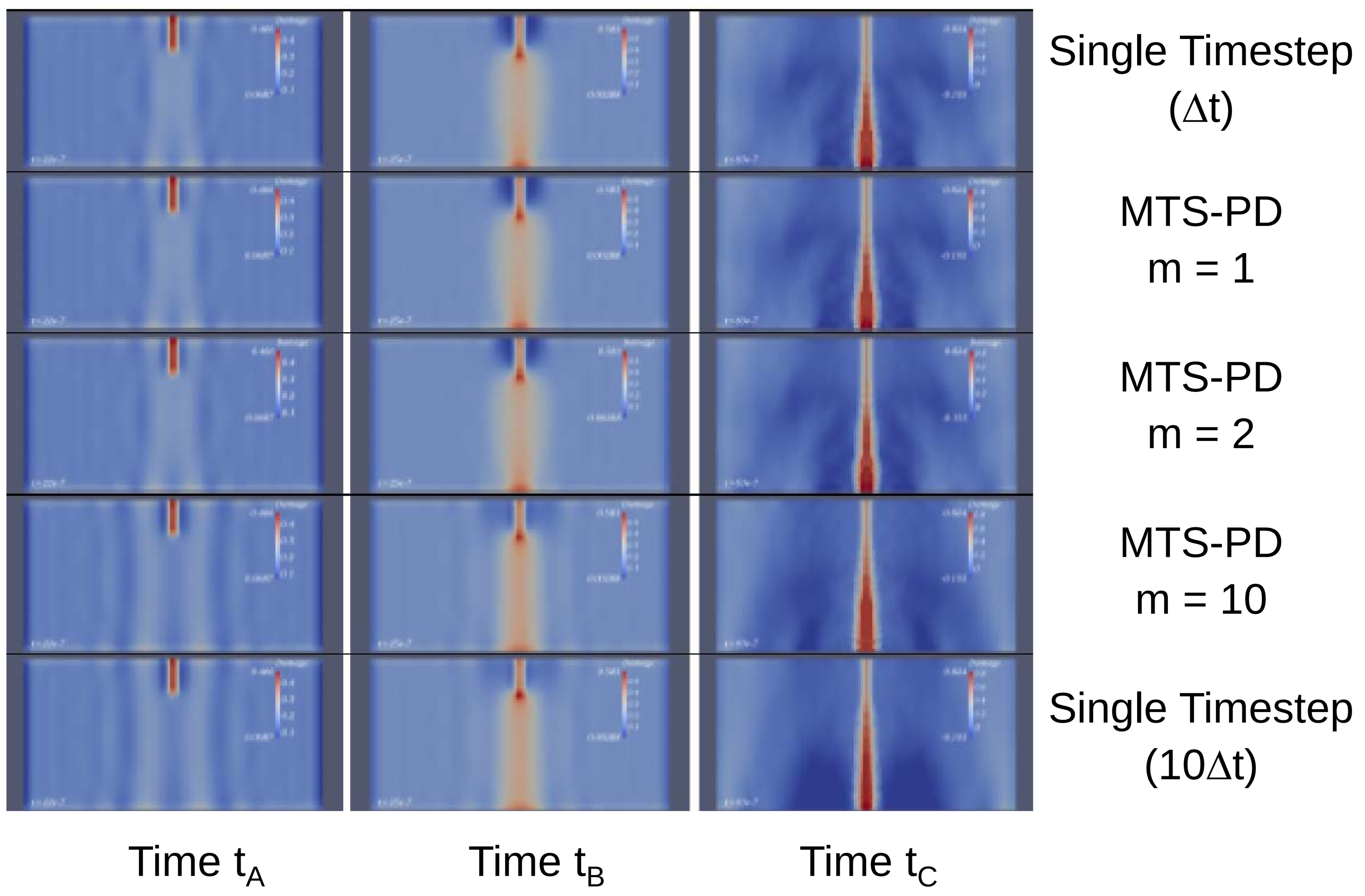
Results

Peridynamic Multi-Time-Stepping (MTS-PD)[5]

- Coupling method neither adds nor removes energy from coupled system (i.e., non-dissipative and stable)
- Coupling method as accurate as subdomain integrators (2^{nd} order for Newmark-Beta).
- 2x – 10x speedups or more possible

Fractured Plate Numerical Example





Case	Single Timestep (Δt)	MPS-PD m=1	MPS-PD m=2	MPS-PD m=10	Single Timestep ($10\Delta t$)
% Error	0% (Reference Soln)	0%	9.0×10^{-4}	9.0×10^{-3}	2.0×10^{-2}
Speedup	N/A	0%	25%	250%	N/A

Significance

- First-ever multi-time-step method for a nonlocal model.
- Relax timestep restrictions to improve efficiency while preserving solution accuracy.

References:
[1] S.A. Silling, R.B. Lehoucq, **Peridynamic theory of solid mechanics**, Adv. Appl. Mech., 44, pp. 73–166, 2010.
[2] F. Bowden, J. Brunton, J. Field, A. Heyes, **Controlled fracture of brittle solids and interruption of electrical current**, Nature, 216, pp. 38–42, 1967.
[3] A. Prakash, E. Taciroglu, K.D. Hjelmstad, **Computationally efficient multi-time-step method for partitioned time integration of highly nonlinear structural dynamics**, Comp. Struct., 133, pp. 51–63, 2014.
[4] B. Aksoylu, M.L. Parks, **Variational theory and domain decomposition for nonlocal problems**, Appl. Math. Comput., 217, pp. 6498–6515, 2011.
[5] P. Lindsay, M.L. Parks, and A. Prakash, **Enabling fast, stable and accurate peridynamics computations using multi-time-step integration**, Submitted, 2015.

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