SAND2015-8845C

# *Is a Steel-Cased Borehole an Electrical Transmission Line?*

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**85th Annual Meeting and International Exhibition of the Society of Exploration Geophysicists**

**New Orleans, Louisiana**
**18 - 23 October 2015**

## The Question:

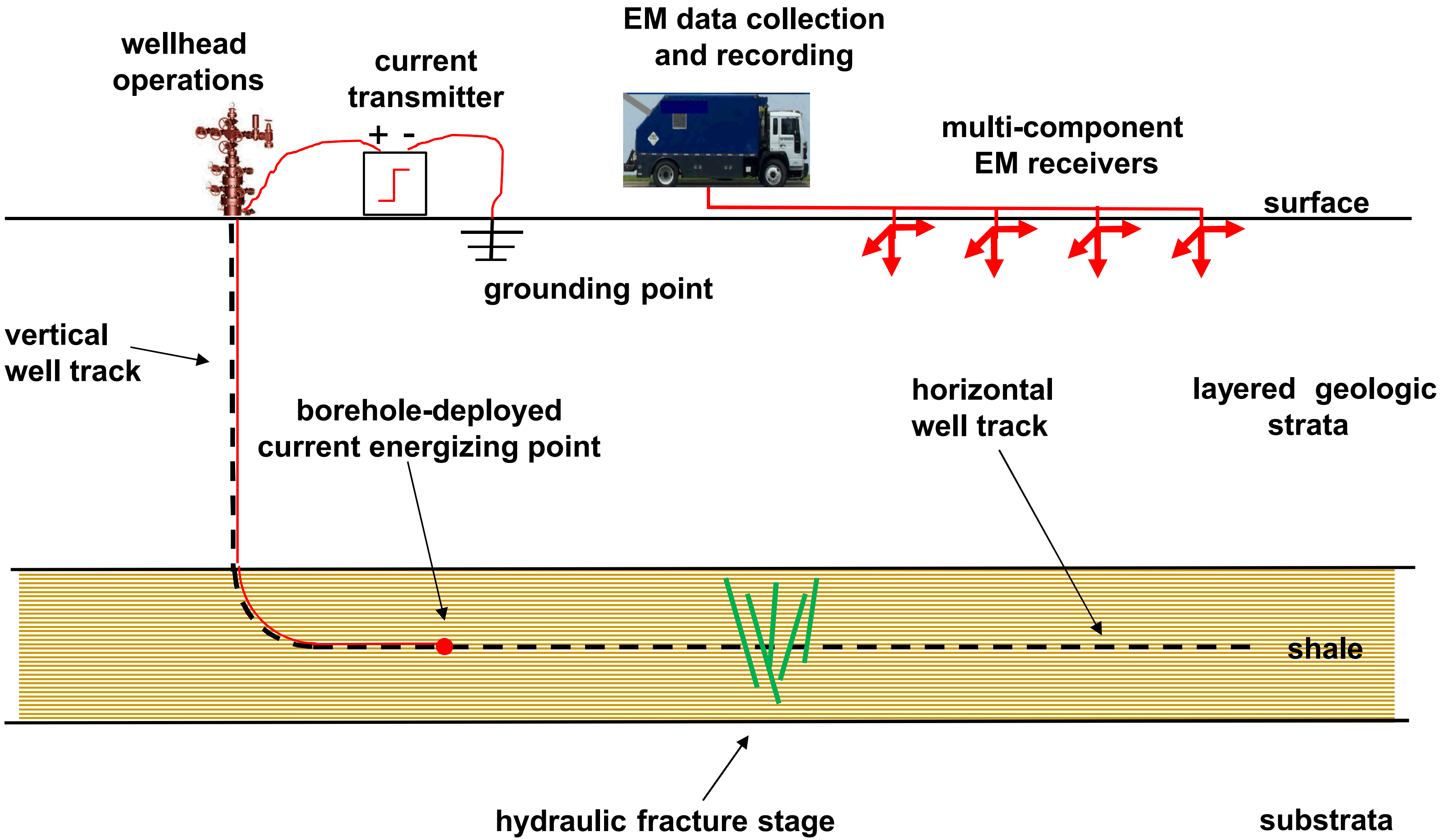
***Is a Steel-Cased Borehole an Electrical Transmission Line?***

## An Answer:

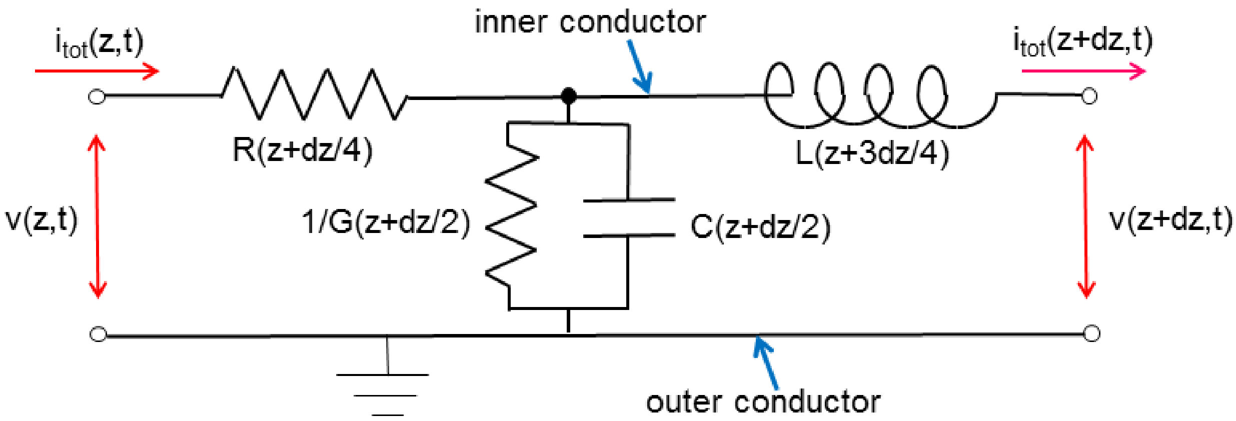
***Yes, but under certain restricting geophysical circumstances***:

- All medium (borehole and earth) material parameters are azimuthally symmetric around the wellbore (hard to achieve).
- All electric current sourcing and grounding conditions are also azimuthally symmetric around the borehole (somewhat hard).
- Frequencies are low (easy).
- At least one “free parameter” enters the physics / mathematics.

# EM Data Acquisition Experiment / Modeling Concept



# An Electrical Transmission Line (i.e., a Coaxial Cable)




R(z) = series resistance / length (Ω/m),

L(z) = series inductance / length (H/m),

C(z) = shunt capacitance / length (F/m),

G(z) = shunt conductance / length (S/m).

Apply Kirchhoff's two circuit theory rules (and let dz → 0) to obtain:

$$\frac{\partial i(z,t)}{\partial t} + \frac{R(z)}{L(z)} i(z,t) + \frac{1}{L(z)} \frac{\partial v(z,t)}{\partial z} = -\frac{\partial i_s(z,t)}{\partial t} - \frac{R(z)}{L(z)} i_s(z,t),$$

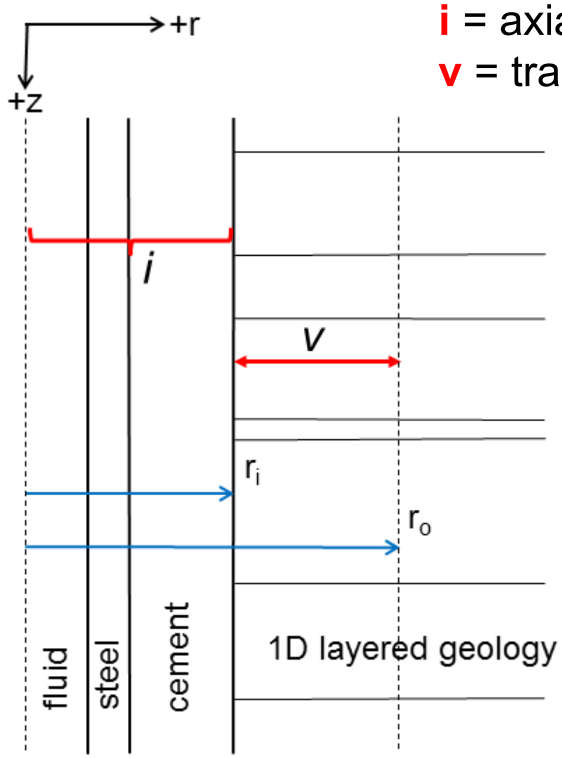
$$\frac{\partial v(z,t)}{\partial t} + \frac{G(z)}{C(z)} v(z,t) + \frac{1}{C(z)} \frac{\partial i(z,t)}{\partial z} = -\frac{1}{C(z)} \frac{\partial i_s(z,t)}{\partial z}.$$

The ***Transmission Line Equations***: a pair of coupled, linear, first-order PDEs

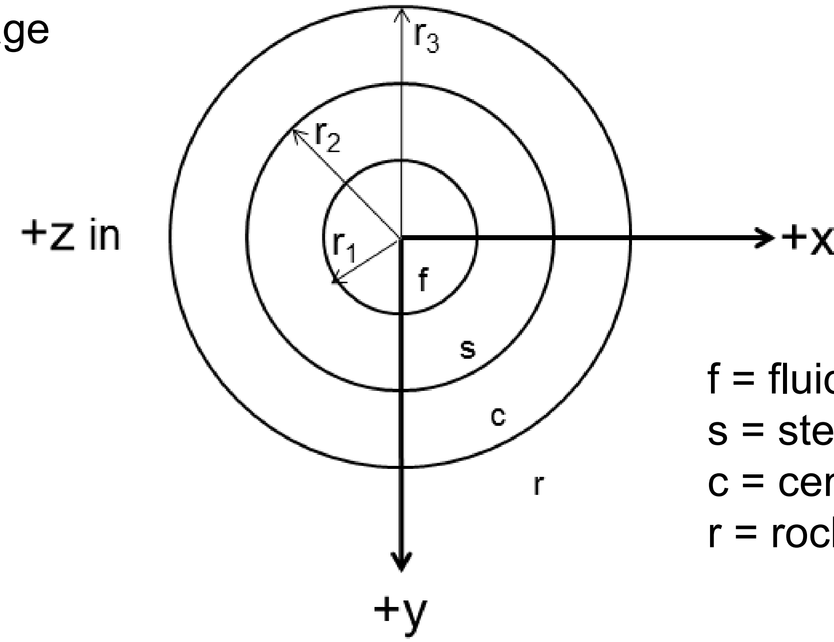
# Cased-Borehole Transmission Line Geometry and Parameters

side view

**i** = axial current
**v** = transverse voltage




end-on view



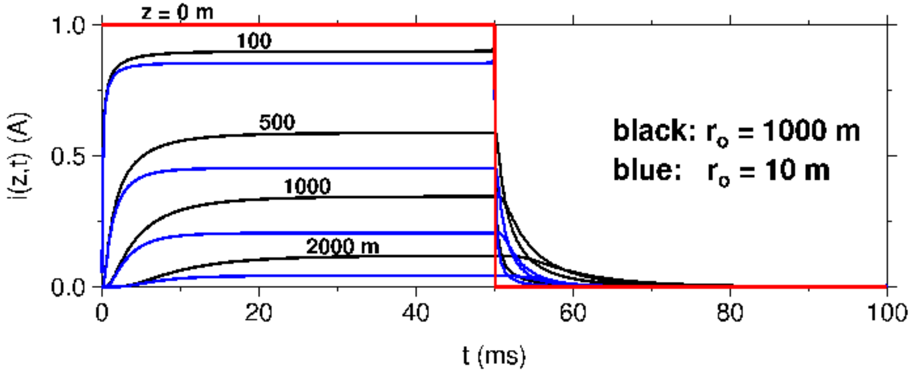

f = fluid
s = steel
c = cement
r = rock

$$C = \frac{2\pi\varepsilon_r}{\ln(r_o/r_i)}, \quad L = \frac{\mu_r \ln(r_o/r_i)}{2\pi}, \quad G = \frac{2\pi\sigma_r}{\ln(r_o/r_i)}, \quad \frac{1}{R} \approx 2\pi\bar{r}_S\, \delta r_s \sigma_s$$

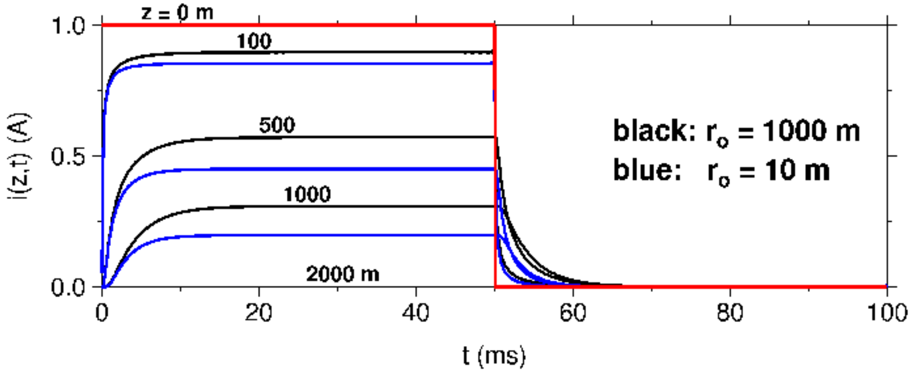
~$10^6$ S/m

$r_i$ , $r_o$ = "inner" and "outer" radii of borehole transmission line

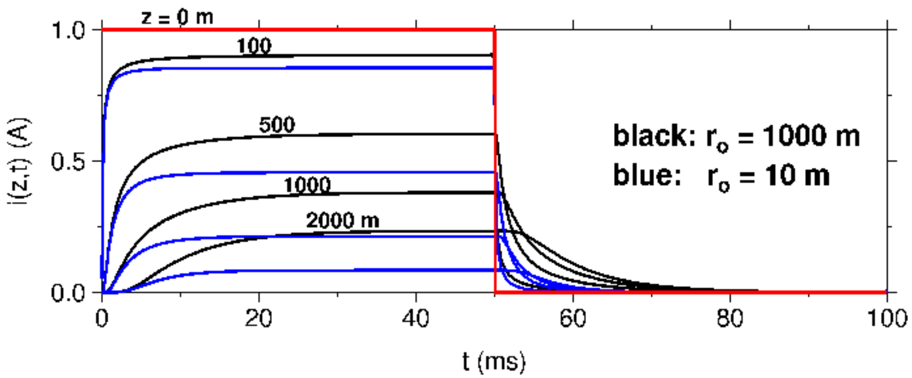
# Current Waveforms Along a *Homogeneous* Steel-Cased Borehole




Infinite-length transmission line:
one-way pulse propagation toward +z.




Vanishing *current* BC imposed at z = 2000 m:
two-way pulse propagation;
strong current leakage into formation.




Vanishing *voltage* BC imposed at z = 2000 m:
two-way pulse propagation.

Note sensitivity to outer radius $r_o$ !

# O(2,4) Staggered-Grid Finite-Difference Solution of the Coupled Transmission Line Equations

**Voltage PDE:**

$$\frac{\partial i(z,t)}{\partial t} + \omega_I(z) i(z,t) + \frac{\omega_I(z)}{R(z)} \frac{\partial v(z,t)}{\partial z} = 0$$

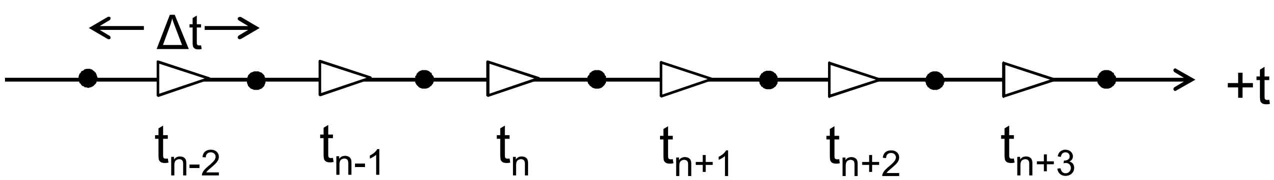
**Current PDE:**

$$\frac{\partial v(z,t)}{\partial t} + \omega_V(z) v(z,t) + \frac{\omega_V(z)}{G(z)} \frac{\partial i(z,t)}{\partial z} = 0$$

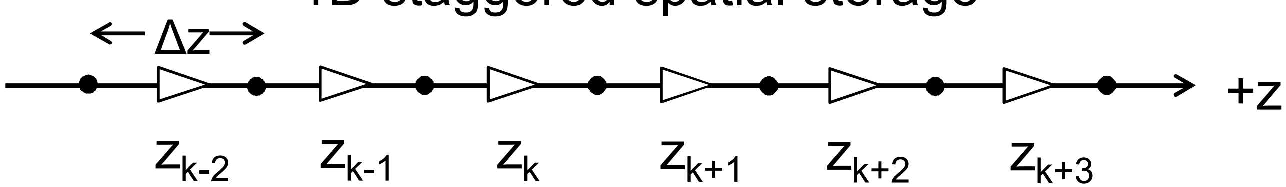
$\omega_I(z) \equiv R(z)/L(z)$ ← characteristic frequencies → $\omega_V(z) \equiv G(z)/C(z)$

Discretize on a 1D version of the famous 3D Yee (1966) staggered spatial grid:

Staggered temporal storage



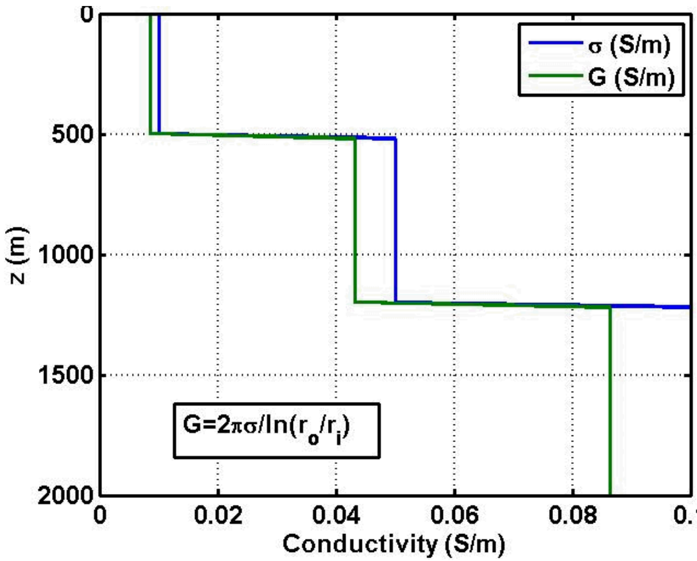

1D staggered spatial storage




• = voltage storage node

▷ = current storage node

# Current Magnitude Along a Borehole in a 1D Layered Earth




**Three-layer conductivity model**:

$\sigma_1$ = 0.01, 0.05, 0.10 S/m.

$\varepsilon_r$ = 150, $\mu_r$ = 1.

outer / inner radius $r_o$ / $r_i$ = 200 m / ~7 cm.

**Two current source locations:**

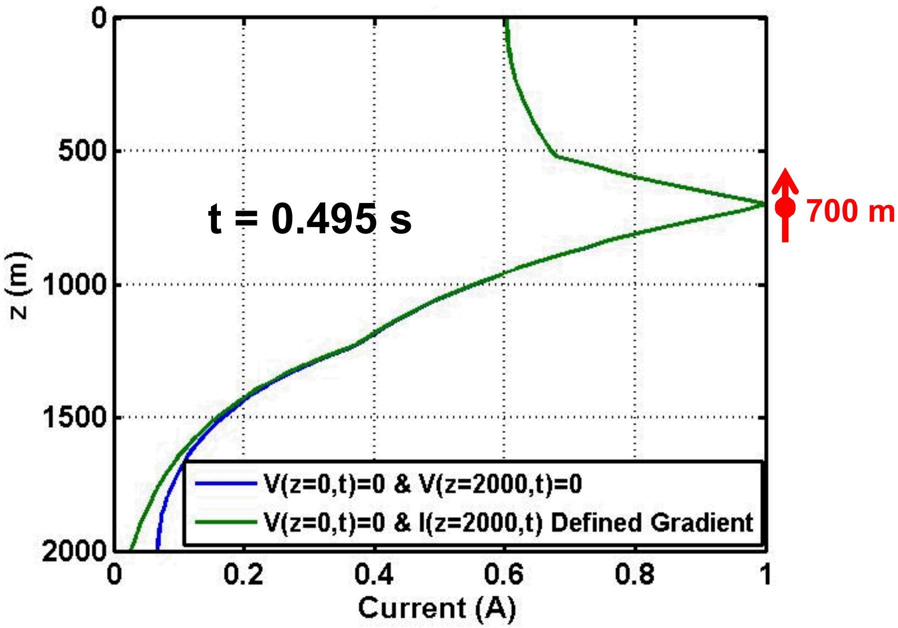
amplitude = 1 A, duration = 0.5 s.

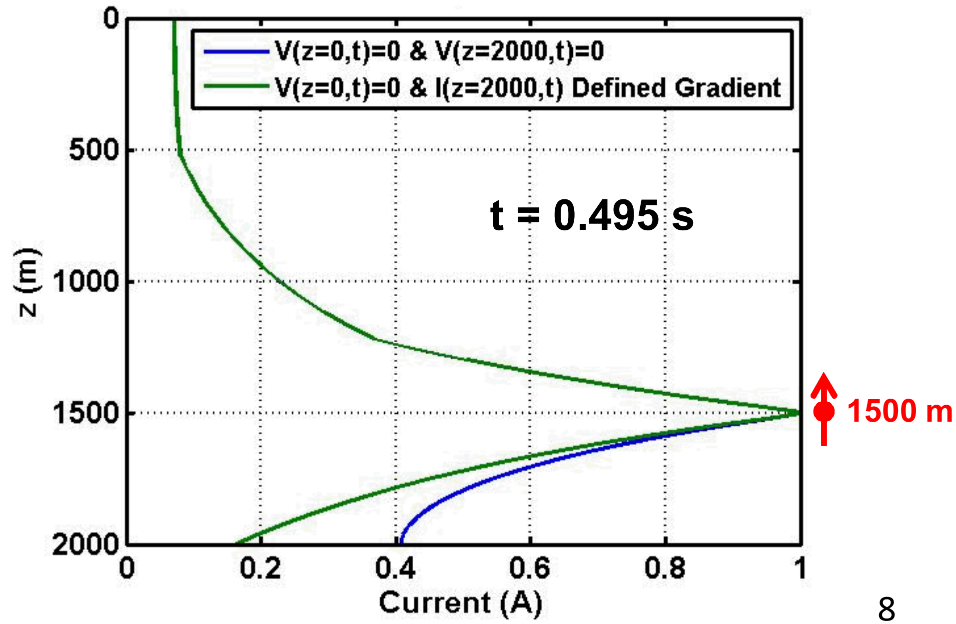
**Two boundary conditions:**

vanishing voltage, current flow.

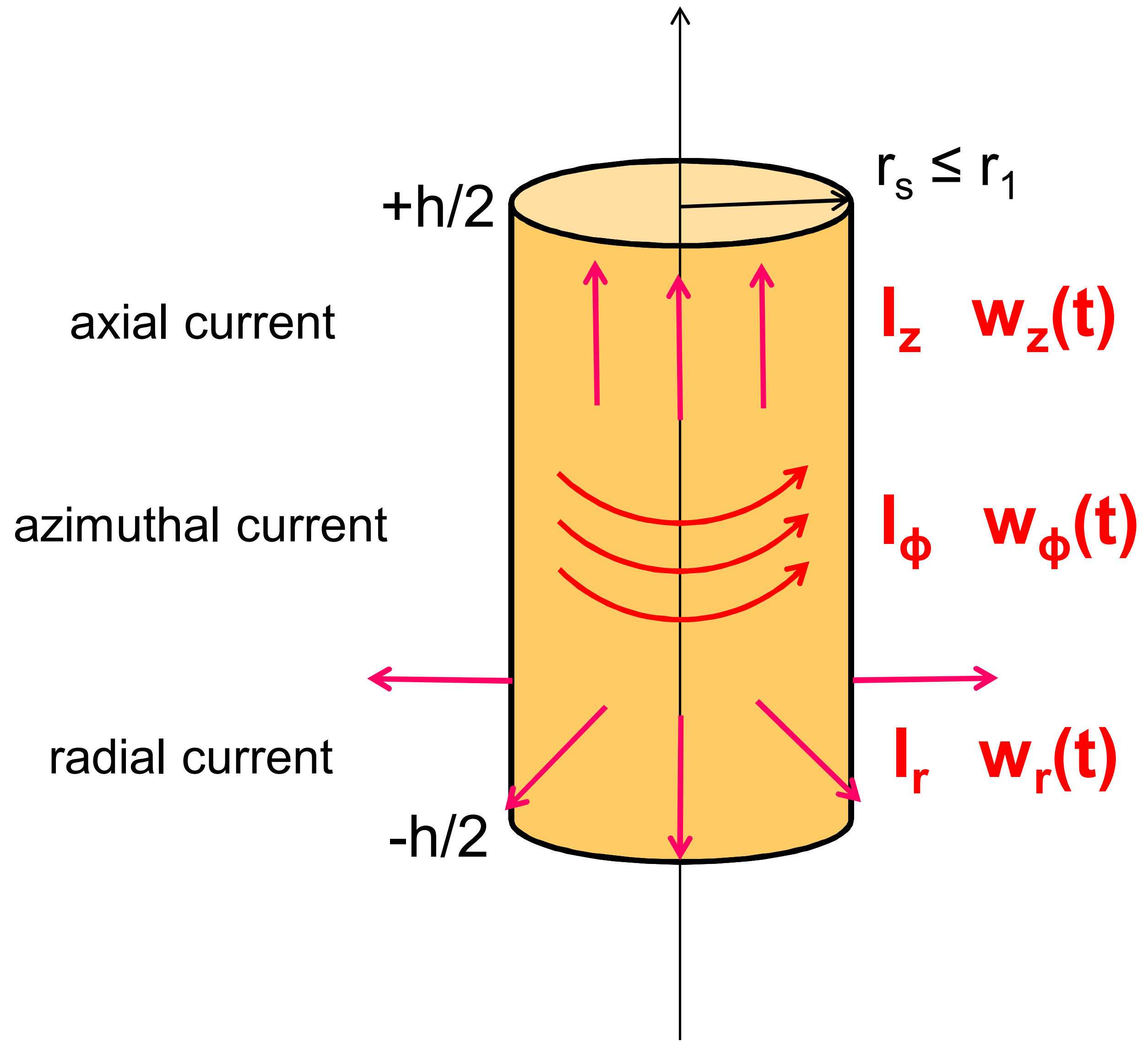
**Discretization:**

$\Delta z$ = 20 m, $\Delta t$ = 0.5 μs

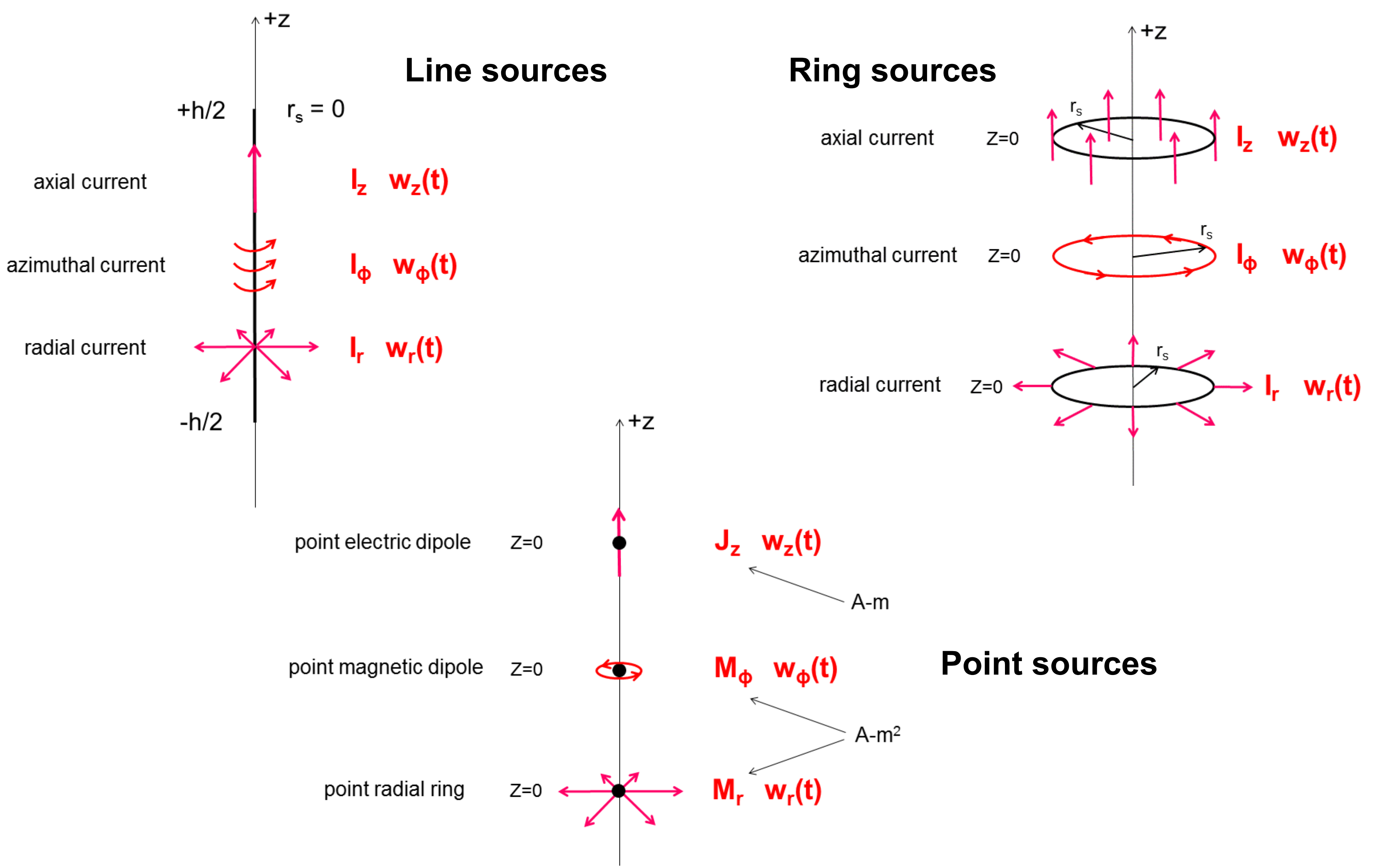


# Generalized Borehole Source Electrode



# Borehole Current Sources:  Three Limiting Cases



# Mathematical Solution Strategy

Start with two of the four *Maxwell equations* of electromagnetism:

*Faraday Law*

$$\frac{\partial \mathbf{b}(\mathbf{x},t)}{\partial t} + \mathbf{curl}\ \mathbf{e}(\mathbf{x},t) = \mathbf{0}$$

*Ampere-Maxwell Law*

$$\frac{\partial \mathbf{d}(\mathbf{x},t)}{\partial t} + \mathbf{j}(\mathbf{x},t) - \mathbf{curl}\ \mathbf{h}(\mathbf{x},t) = \mathbf{0}$$

Combine with three *constitutive relations* for isotropic media with *body sources:*

$$\mathbf{b}(\mathbf{x},t) = \mu(\mathbf{x})\mathbf{h}(\mathbf{x},t) + \mathbf{b}_s(\mathbf{x},t),$$

$$\mathbf{d}(\mathbf{x},t) = \varepsilon(\mathbf{x})\mathbf{e}(\mathbf{x},t) + \mathbf{d}_s(\mathbf{x},t),$$

$$\mathbf{j}(\mathbf{x},t) = \sigma(\mathbf{x})\mathbf{e}(\mathbf{x},t) + \mathbf{j}_s(\mathbf{x},t),$$

$\mu(\mathbf{x})$ - magnetic permeability (H/m)

$\varepsilon(\mathbf{x})$ - electric permittivity (F/m)

$\sigma(\mathbf{x})$ - current conductivity (S/m)

⇨ ***The EH Partial Differential System***:

$$\varepsilon(\mathbf{x})\frac{\partial \mathbf{e}(\mathbf{x},t)}{\partial t} + \sigma(\mathbf{x})\mathbf{e}(\mathbf{x},t) - \mathbf{curl}\ \mathbf{h}(\mathbf{x},t) = -\mathbf{j}_s(\mathbf{x},t)$$

$$\mu(\mathbf{x})\frac{\partial \mathbf{h}(\mathbf{x},t)}{\partial t} + \mathbf{curl}\ \mathbf{e}(\mathbf{x},t) = \mathbf{0}$$

only current sources!

Two, coupled, first-order, inhomogeneous PDEs:

$\mathbf{e}(\mathbf{x},t)$ - electric vector (V/m)

$\mathbf{h}(\mathbf{x},t)$ - magnetic vector (A/m)

# Cylindrical Coordinates with *Azimuthal Symmetry*

## (medium parameters *and* body forces independent of angle $\phi$)

***Circular Magnetic System*** **(CMS) or** ***Transverse Magnetic*** **(TM) solution:**

$$\varepsilon(r,z)\frac{\partial e_r(r,z,t)}{\partial t}+\sigma(r,z)e_r(r,z,t)+\frac{\partial h_\phi(r,z,t)}{\partial z}=-j_{sr}(r,z,t)$$

$$\mu(r,z)\frac{\partial h_\phi(r,z,t)}{\partial t}+\frac{\partial e_r(r,z,t)}{\partial z}-\frac{\partial e_z(r,z,t)}{\partial r}=0$$

$$\varepsilon(r,z)\frac{\partial e_z(r,z,t)}{\partial t}+\sigma(r,z)e_z(r,z,t)-\frac{\partial h_\phi(r,z,t)}{\partial r}-\frac{h_\phi(r,z,t)}{r}=-j_{sz}(r,z,t)$$

($e_r$, $h_\phi$, $e_z$)

activated by r- and z-component current source.

***Circular Electric System*** **(CES) or** ***Transverse Electric*** **(TE) solution:**

$$\mu(r,z)\frac{\partial h_r(r,z,t)}{\partial t}-\frac{\partial e_\phi(r,z,t)}{\partial z}=0$$

$$\varepsilon(r,z)\frac{\partial e_\phi(r,z,t)}{\partial t}+\sigma(r,z)e_\phi(r,z,t)-\frac{\partial h_r(r,z,t)}{\partial z}+\frac{\partial h_z(r,z,t)}{\partial r}=-j_{s\phi}(r,z,t)$$

$$\mu(r,z)\frac{\partial h_z(r,z,t)}{\partial t}+\frac{\partial e_\phi(r,z,t)}{\partial r}+\frac{e_\phi(r,z,t)}{r}=0$$

($h_r$, $e_\phi$, $h_z$)

activated by $\phi$-component current source.

(after S.A. Schelkunoff, 1934)

# Mathematically Exact Solution (CMS and CES) in the Wavenumber-Frequency Domain

axial coordinate: $z \leftrightarrow k_z$: axial wavenumber

time: $t \leftrightarrow \omega$: angular frequency

For example, azimuthal $E_\phi$ of the CMS is given by a Hankel function superposition:

$$E_\phi(r, k_z, \omega)\Big|_{\text{zone n}} = A_n(k_z, \omega)\, H_0^{(1)}(\beta_n r) + B_n(k_z, \omega)\, H_0^{(2)}(\beta_n r)$$

radial wavenumber:

$$\beta_n^2 = \left(\omega^2 \varepsilon_n \mu_n - k_z^2\right) + i\left(\omega \sigma_n \mu_n\right)$$

Superposition coefficients determined by imposing finiteness at $r = 0$ ($B_1 = A_1$), radiation condition at $r \to$ infinity ($B_4 = 0$), and *boundary conditions* at the three cylindrical interfaces:

tangential **E** and **H** components *continuous* across interfaces.

Leads to a 6 x 6 linear algebraic system for the six unknown coefficients (source terms appear in right-hand-side column vector).

# The Borehole Filter Concept
# (extension of work by Cuevas, *Geophysics*, 2014)

External electromagnetic field due to a current source placed within a borehole is related to EM field due to a current source within a homogeneous wholespace.

In spatial z domain:

$$E_z(r,z,\omega)\Big|_{\text{source in borehole}} = \mathrm{BF}(z,\omega) * E_z(r,z,\omega)\Big|_{\text{source in wholespace}}$$

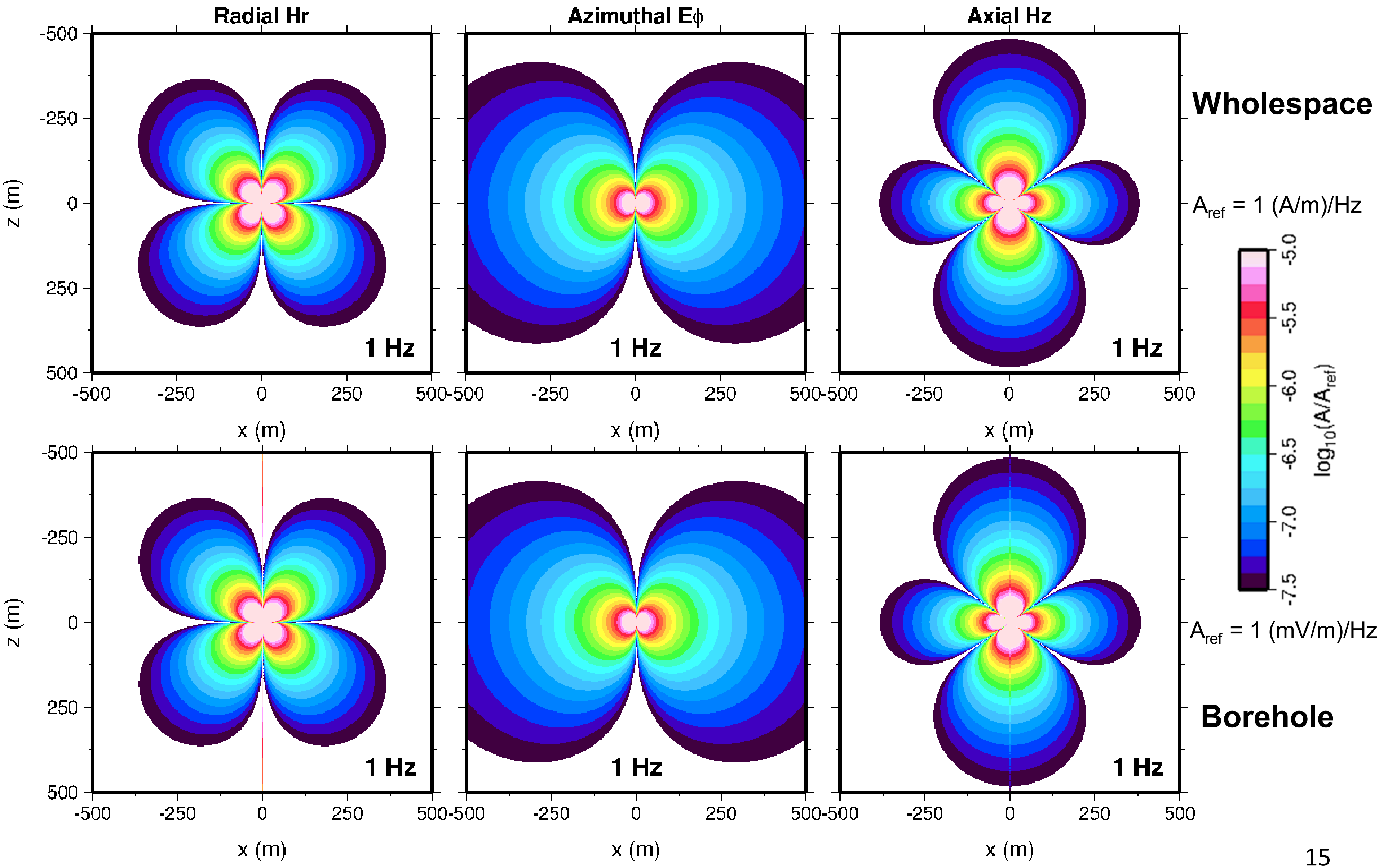
In wavenumber $k_z$ domain:

easy to calculate!

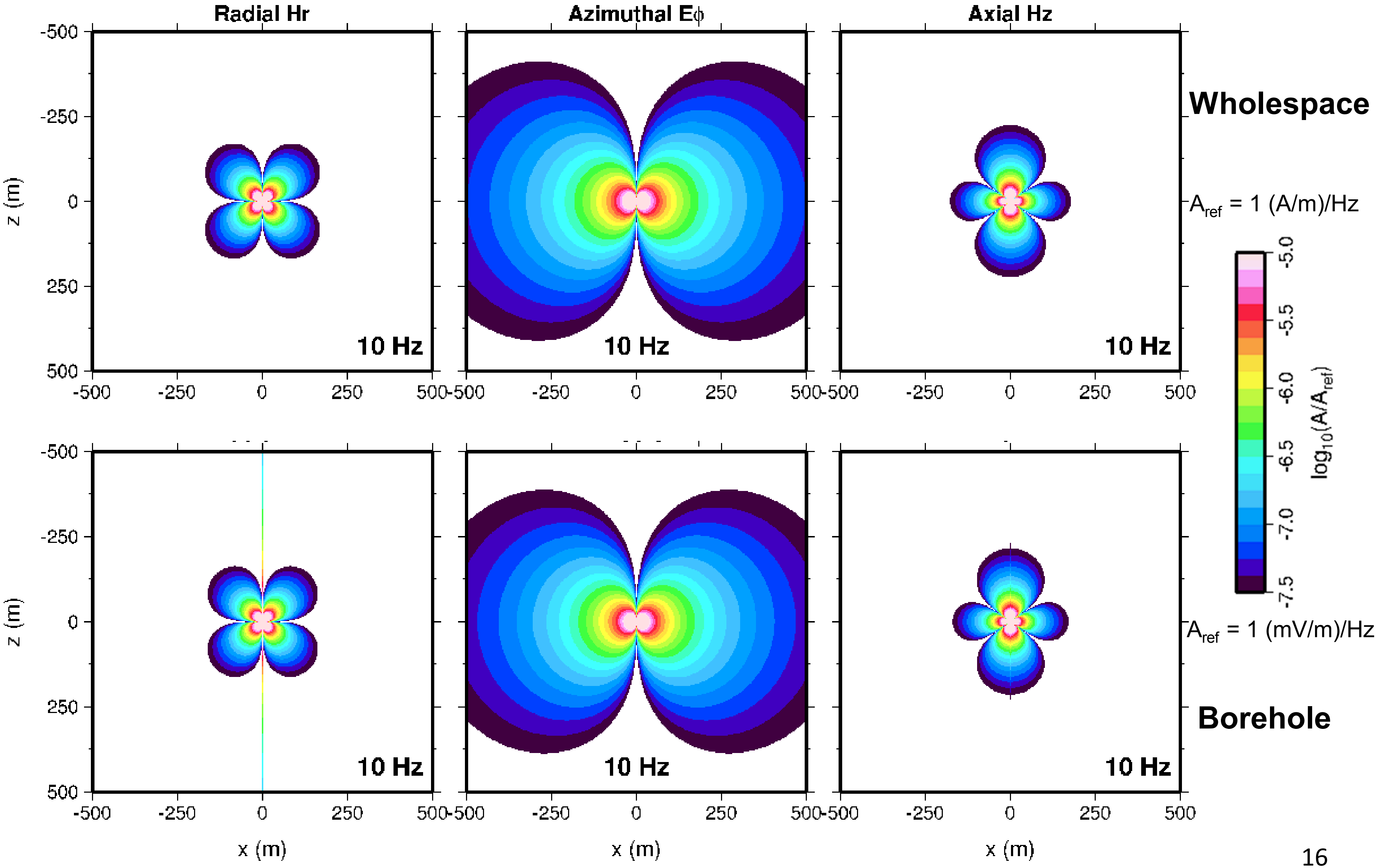
$$E_z(r,k_z,\omega)\Big|_{\text{source in borehole}} = \mathrm{BF}(k_z,\omega) \times E_z(r,k_z,\omega)\Big|_{\text{source in wholespace}}$$

Mathematical form of borehole filter $\mathrm{BF}(k_z,\omega)$ in wavenumber domain exists (albeit rather complicated)!

# Point Vertical Magnetic Dipole ($M_z$ = 1 A-m$^2$)



# Point Vertical Magnetic Dipole ($M_z$ = 1 A-$m^2$)



# Point Vertical Magnetic Dipole ($M_z$ = 1 A-m$^2$)

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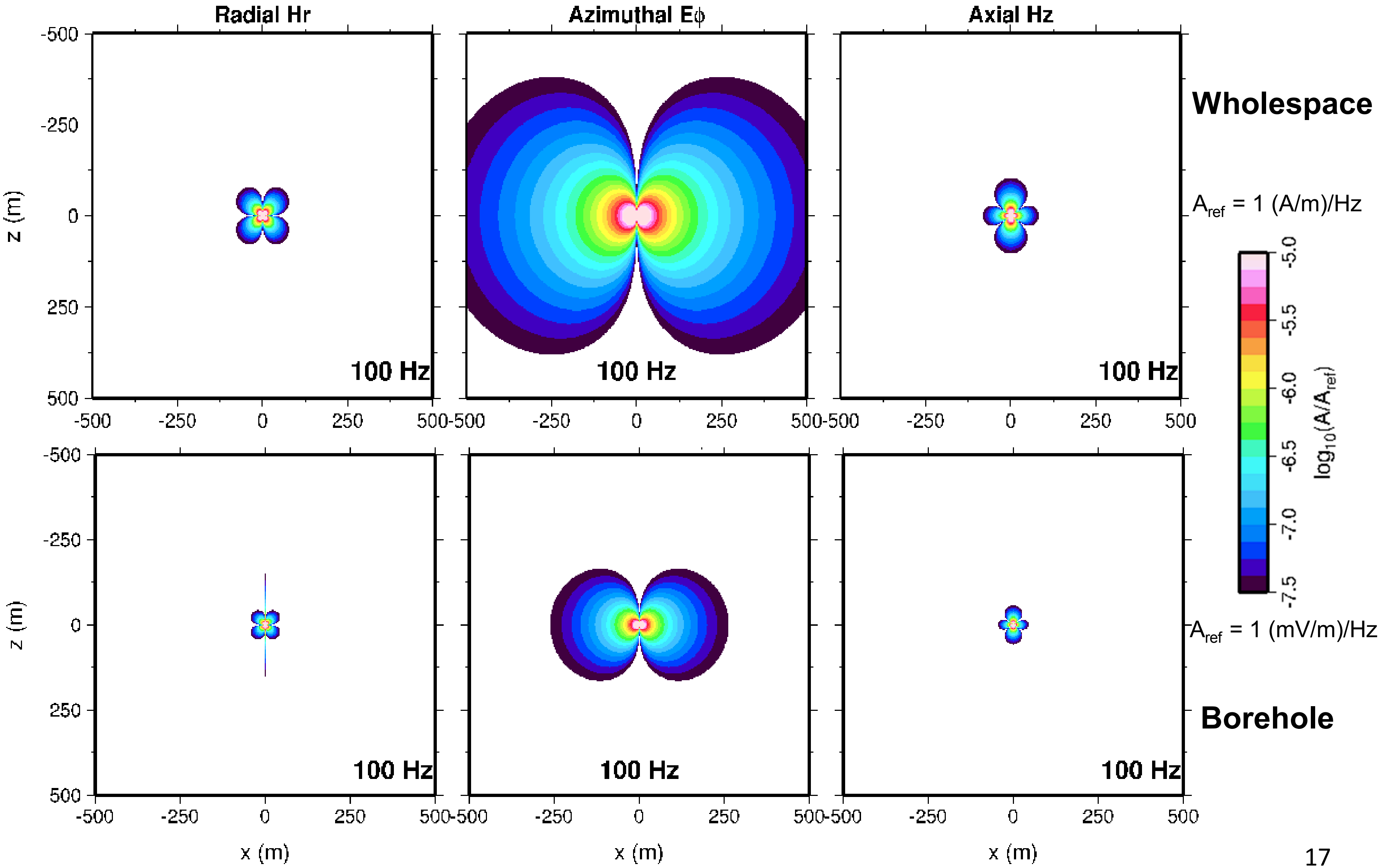
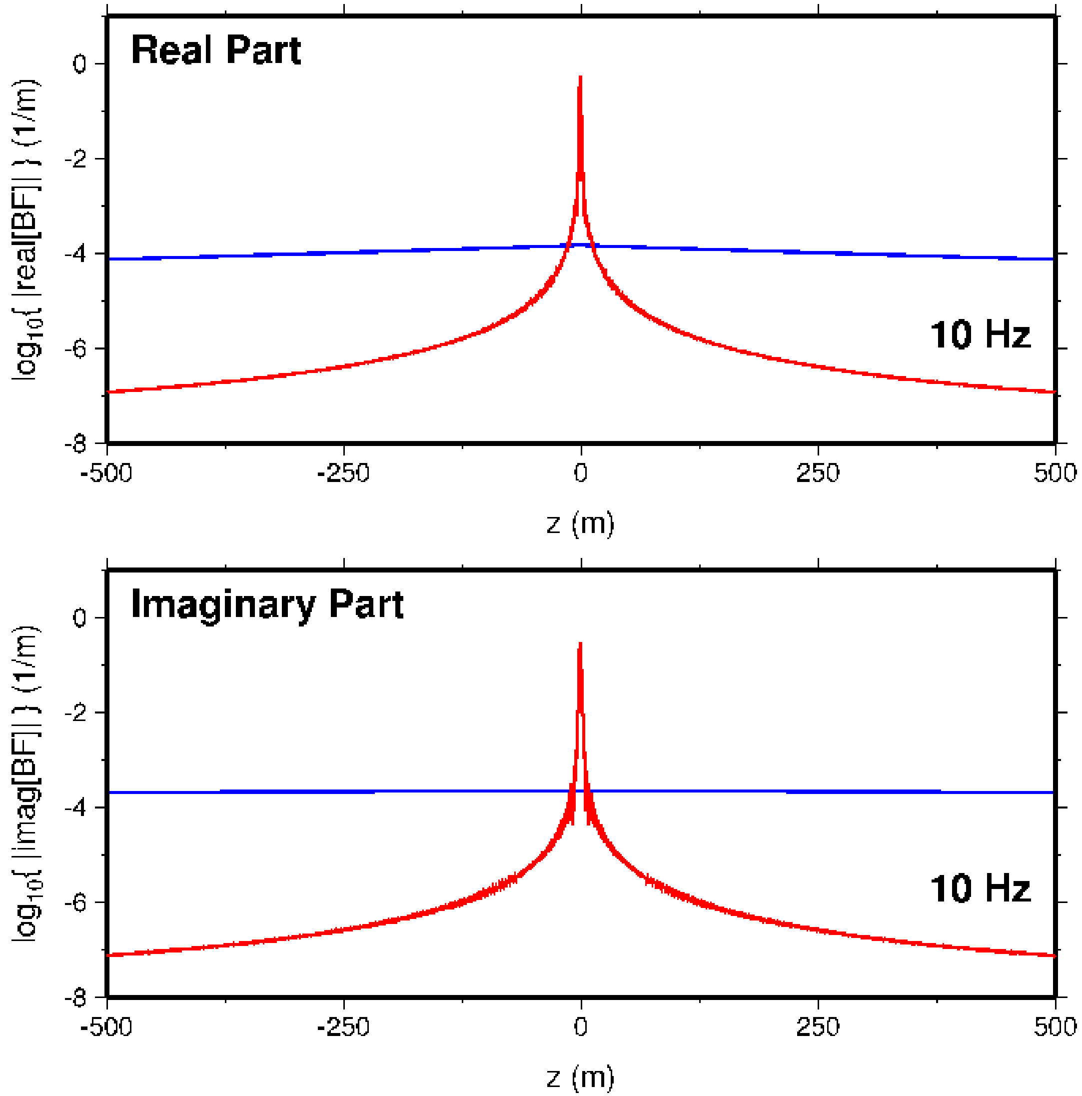


# Two Borehole Filters



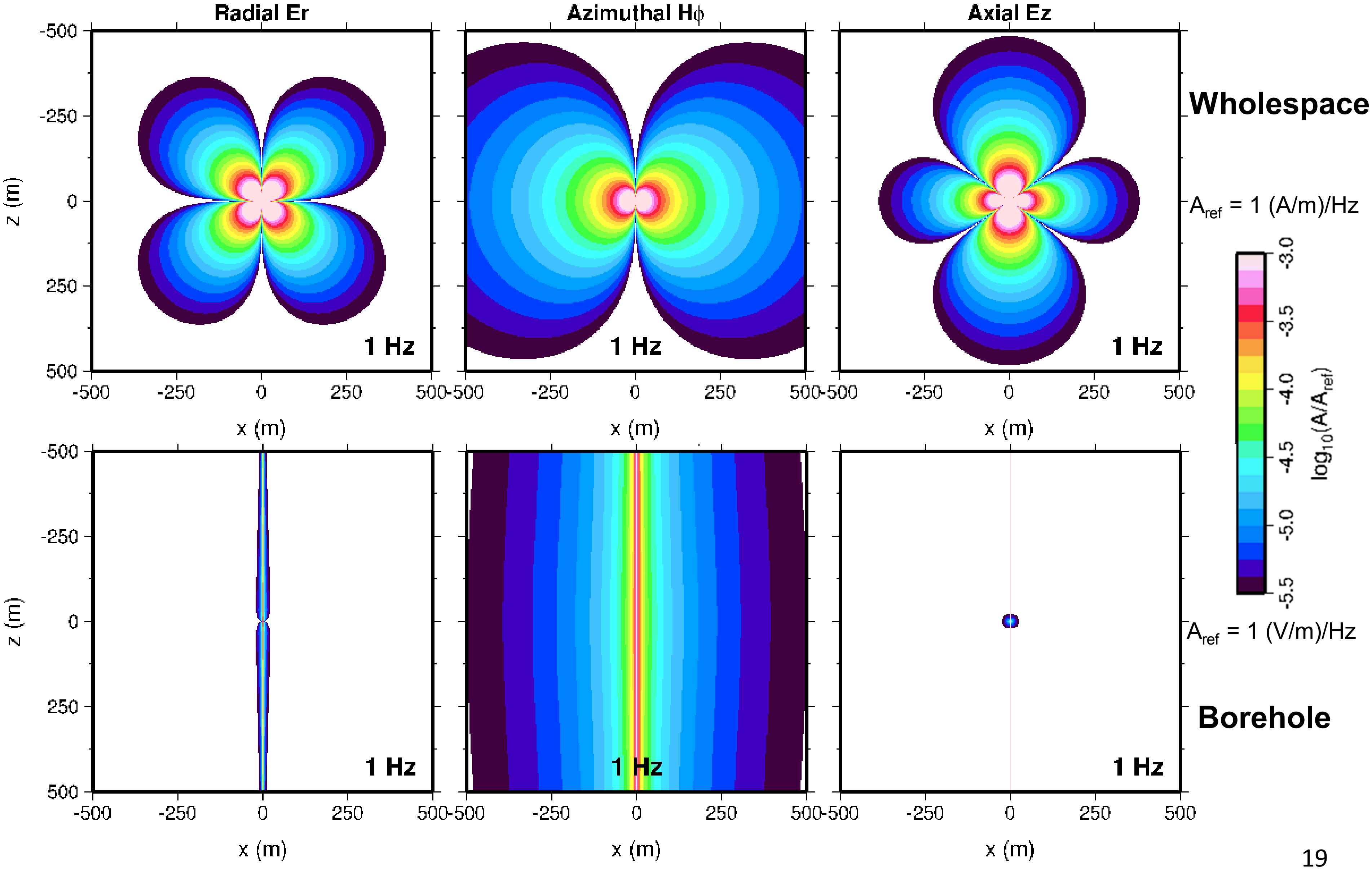

**Red**: Circular Electric System (CES) activated by a point vertical magnetic dipole (VMD).

**Blue**: Circular Magnetic System (CMS) activated by a point vertical electric dipole (VED).

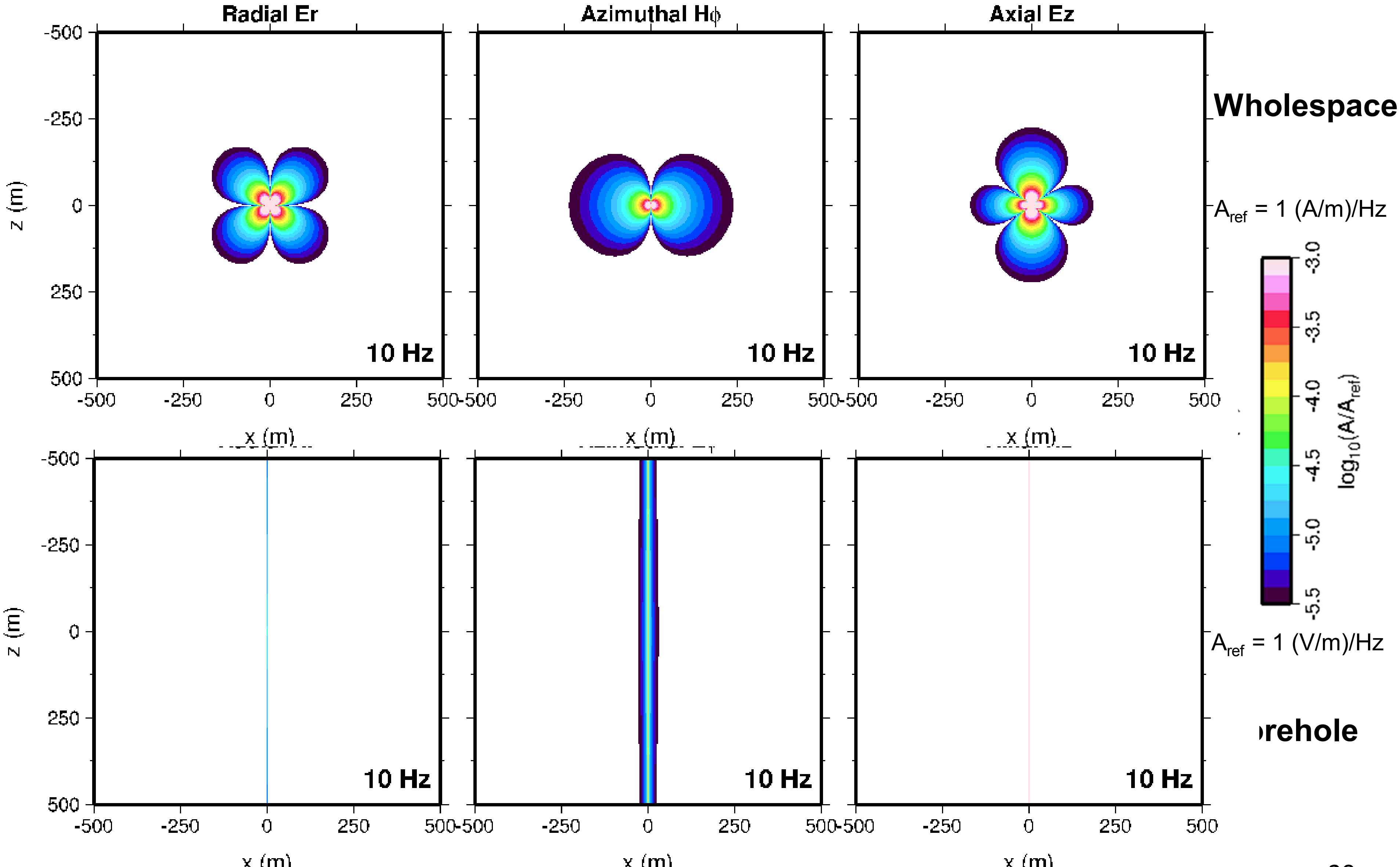
Filters are symmetric with respect to z = 0 m.

Amplitudes consistent with Cuevas (2014).

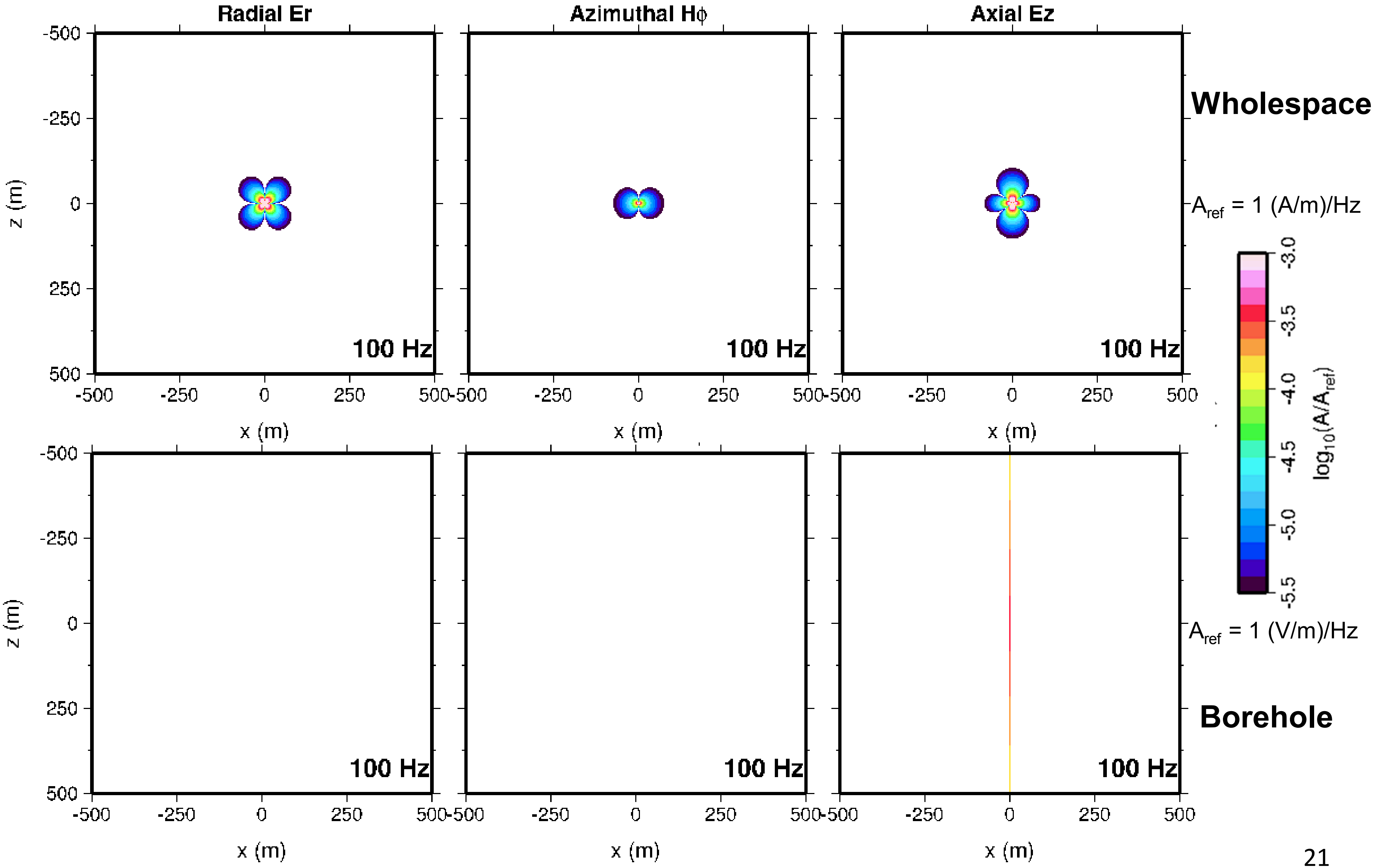
# Point Vertical Electric Dipole ($J_z$ = 1 A-m)



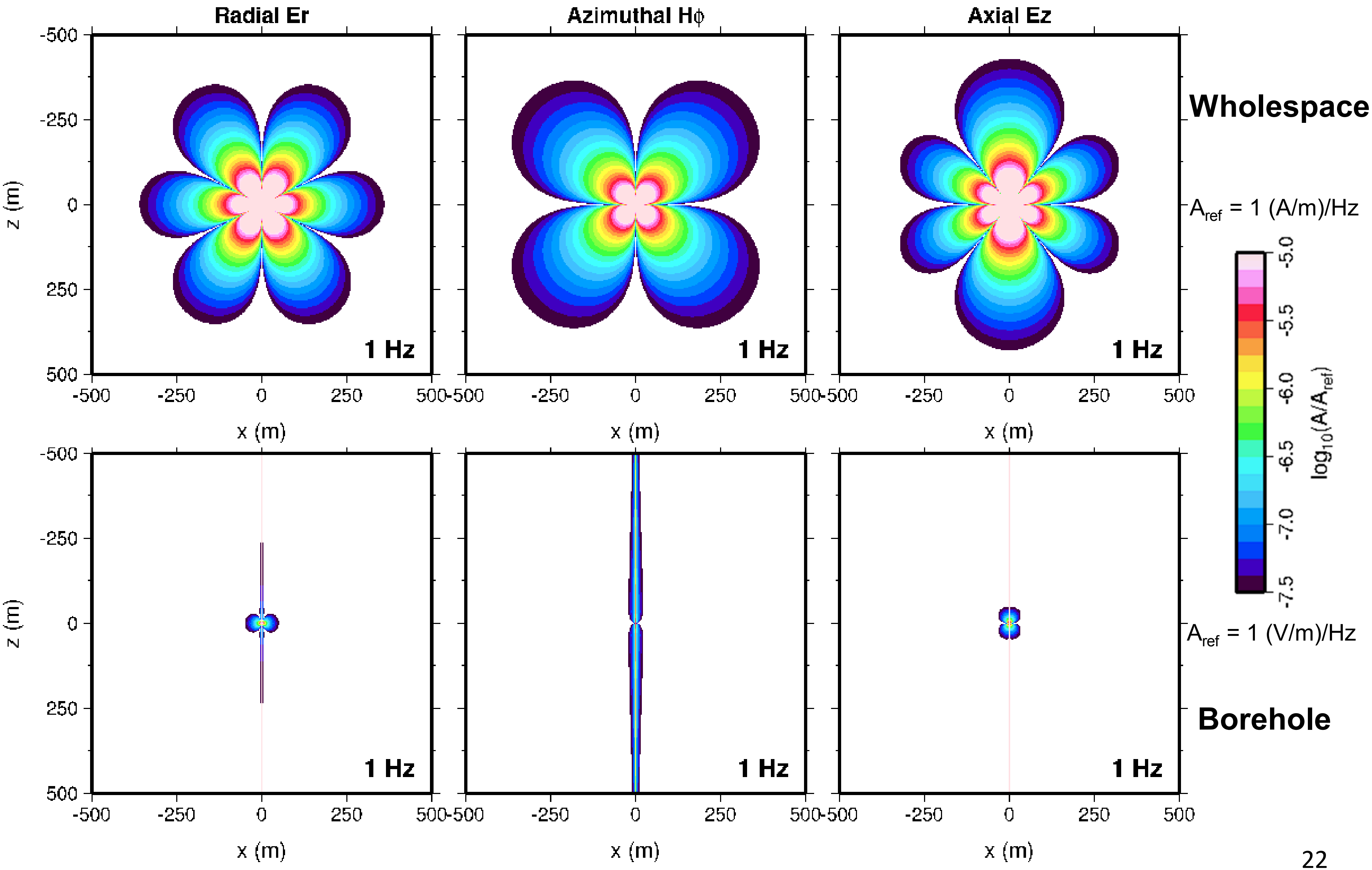
# Point Vertical Electric Dipole ($J_z$ = 1 A-m)



# Point Vertical Electric Dipole ($J_z$ = 1 A-m)



# Point Radial Ring Current ($M_r$ = 1 A-m$^2$)



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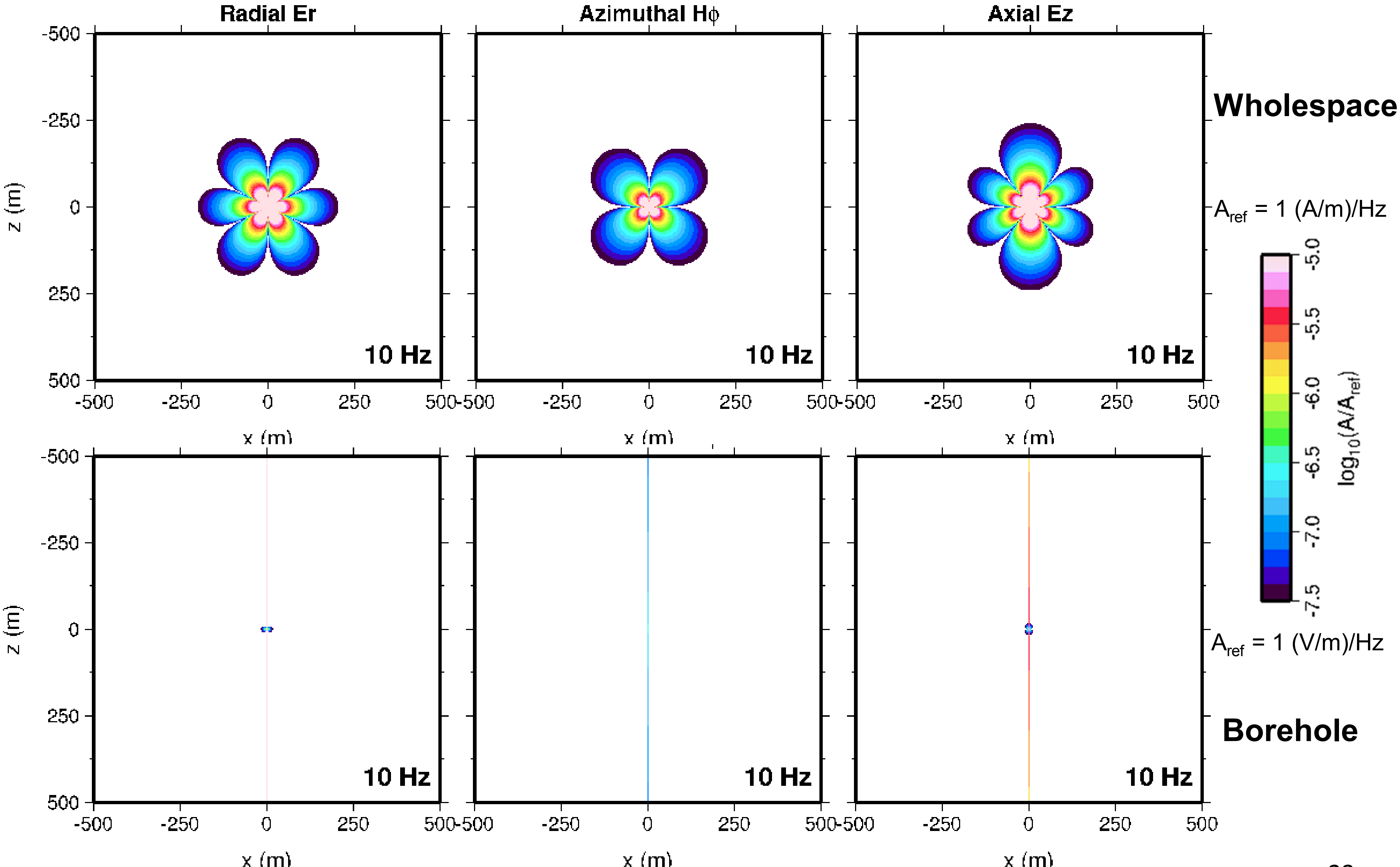
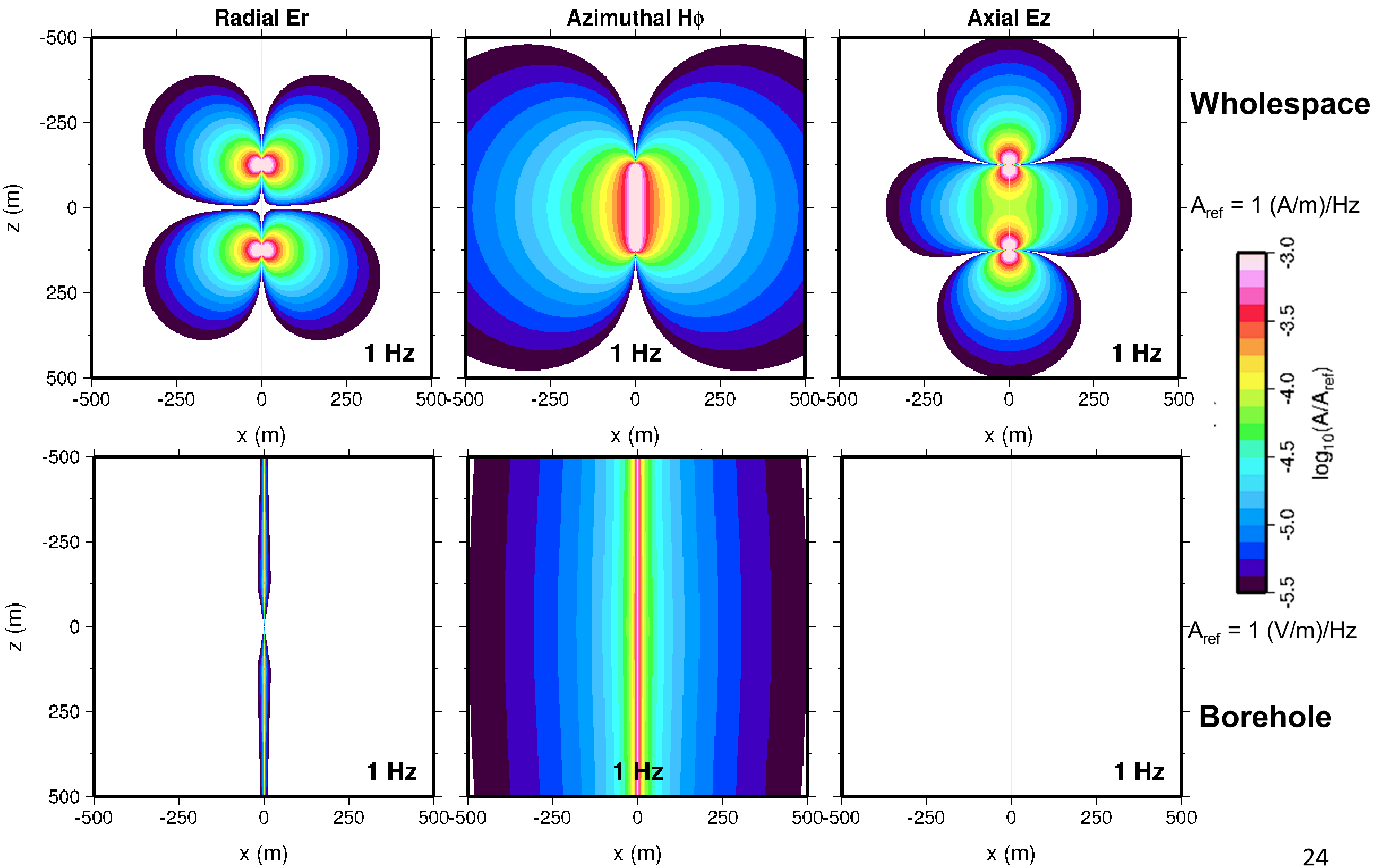


# Line (h = 250 m) Vertical Electric Dipole ($J_z$ = 1 A-m)



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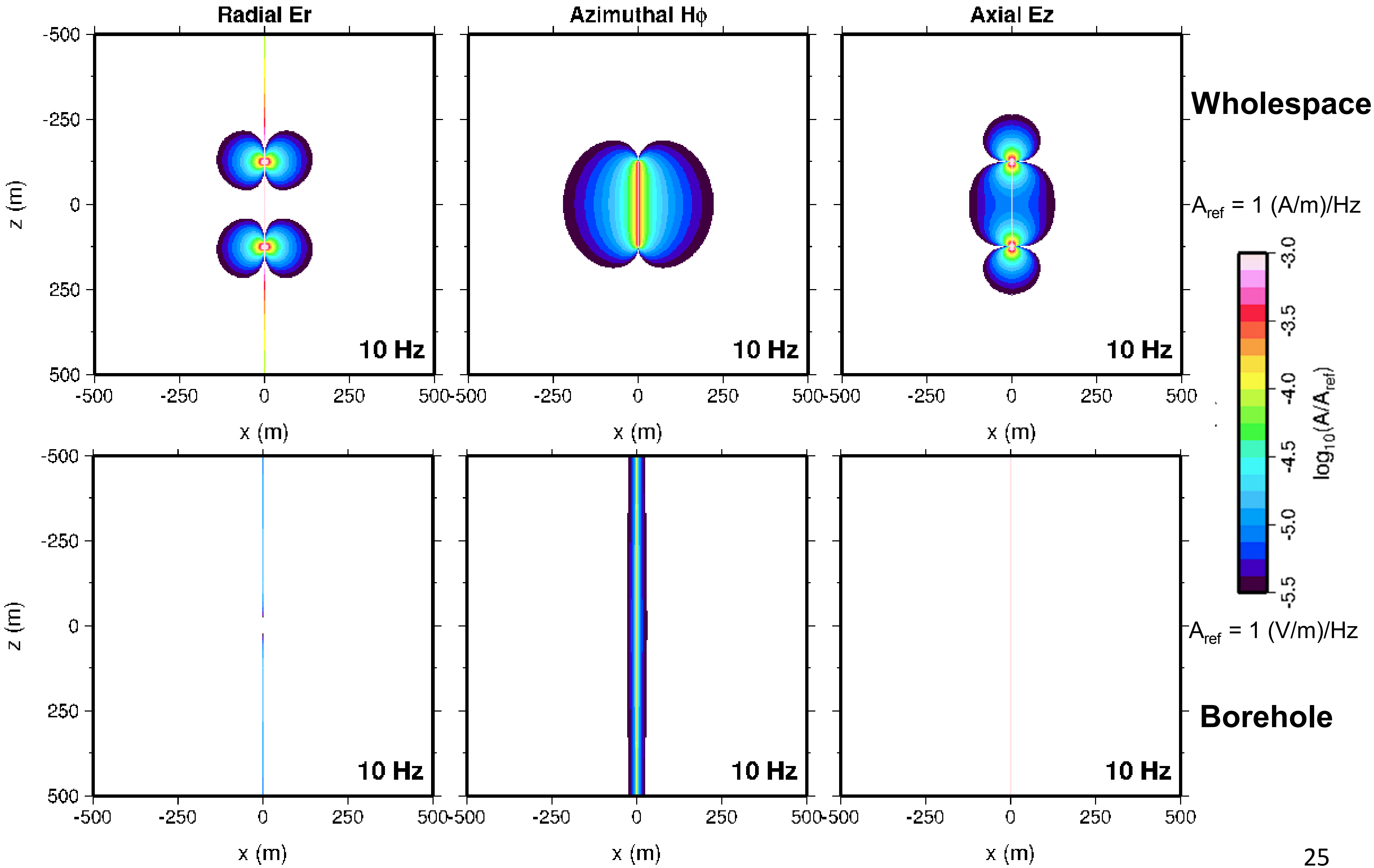

# Conclusions

**1)** Electrical transmission line model of a steel-cased geologic borehole has been developed. Requires:

1.1) azimuthal symmetry of borehole, medium, sourcing/grounding parameters.
1.2) contains free parameter $r_o$, the “outer radius” to electrical ground.

1.3) useful in numerical modeling for replacing fluid-filled, cased, and cemented borehole with a body source distribution of electric current.

**2)** Rigorous derivation of transmission line PDEs from Maxwell’s equations is worked.

2.1) “Borehole Filter” linking cased / cemented borehole EM responses to simpler wholespace responses is derived and analyzed.
2.1.1) works for either point or spatially-extended sources.
2.1.2) enables study of sensitivity of EM field to various system parameters.

# Acknowledgements

**Sandia National Laboratories** and **CARBO Ceramics Inc.**
are acknowledged for supporting this research and development effort.

This work is conducted under the auspices of CRADA (Cooperative Research and Development Agreement) SC11/01780.00 between CARBO Ceramics Inc. and Sandia National Laboratories.

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