Final Report:
Seaching for Physics Beyond the Standard Model: Strongly Coupled Theories at the Energy and Intensity Frontiers

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Syracuse component

P.I Simon Catterall, Syracuse University
smcatter@syr.edu, tel: 315 443 5978
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Physics summary

After close to thirty years of attempts to formulate lattice gauge theories in a manner that would preserve supersymmetry, this problem was finally solved during developments in the last decade. The key feature of the new approaches is to discretize a *topologically twisted* formulation of the original Yang-Mills theory. In flat space such a reformulation can be thought of as merely a change of variables and so the physical content of the theory remains the same.

The constraints inherent in the construction pick out uniquely both the form of action, the distribution of fermions and bosons over the lattice, the structure of gauge covariant difference operators and even the type of lattice itself - in the case of $\mathcal{N}=4$ Yang-Mills the so-called A_4^* lattice (a lattice whose fundamental basis vectors correspond to the weight lattice of $SU(5)$). The resultant lattice theory exhibits some remarkable features:

- Both fermions and bosons generically live on links not sites of the lattice. That gauge fields live on links is not unusual but the requirement that the theory is supersymmetric forces the fermions to also reside on links.
- Both fermions and bosons take their values in the algebra of the group. That fermions must live in the algebra is natural in a theory with adjoint fields but again exact supersymmetry forces the same feature to be true for bosons. For QCD such a procedure would break gauge invariance but in the context of these supersymmetric lattice theories this is avoided because the links must be complexified to accommodate the (twisted) scalar fields. The flat measure used in the path integral is nevertheless still gauge invariant since the Jacobians that arise under lattice gauge transformations cancel between the gauge field and its complex conjugate.
- No fermion doubling arises in these theories. It is easy to show that the boson action is free of doubles; supersymmetry then guarantees this is also true for the fermions. More prosaically it is easy to map the free link based fermion action into the site based (reduced) staggered fermion action familiar from lattice QCD. The factor of four degeneracy associated to such staggered quarks is precisely what is required of $\mathcal{N}=4$ super Yang-Mills whose fermion content comprises four Majorana fermions.

Most importantly the resulting lattice theory is invariant under a single exact scalar nilpotent supersymmetry at non zero lattice spacing. The presence of this supersymmetry together with lattice gauge invariance and an S^5 point group symmetry strongly constrain both the classical and quantum structure of the lattice theory. Specifically in a series of papers we have shown:

- The only *relevant* counterterms that can arise due to quantum corrections correspond to terms in the classical lattice action.

- The classical moduli space suffers no quantum corrections to all orders in perturbation theory [**?**]. In particular this feature prohibits the theory from developing a potential for the scalar fields - the latter remain massless even at strong coupling.
- The lattice beta function vanishes at one loop.
- Restoration of the remaining 15 supersymmetries requires at worst the tuning of a single marginal operator with coupling $c(g^2, L)$. The continuum limit is then approached by holding the gauge coupling fixed, while sending the lattice size $L \to \infty$ together with a potential tuning $\delta c = a(g^2) \ln L$. In practice this tuning can be accomplished by monitoring a series of simple Ward identities associated with discrete R symmetries of the continuum theory.

The availability of a supersymmetric lattice construction for this theory is clearly very exciting from the point of view of exploring the possible holographic connections between gauge theories and string/gravitational theories. In particular, lattice constructions allow new strategies to be employed such as strong coupling expansions and Monte Carlo simulation, which may lead to new insights and tools for extracting non-perturbative information.

While the lattice theory we have described appears somewhat exotic in comparison to say lattice QCD it can be simulated using the same dynamical fermion algorithms that have been developed in that case and we have developed an optimized, parallel code for simulations which utilizes the MILC codebase familar from lattice QCD. To stimulate further work by other groups we have made this available for download at `http://daschaich.github.com/susy`.

Using this code Catterall and collaborators have studied the phase diagram of $\mathcal{N} = 4$ Yang-Mills reporting results on the Wilson loops, Polyakov lines and static potential. Unlike conventional lattice gauge theories this work has revealed that the theory does *not* exhibit a phase with confinement and chiral symmetry breaking. Instead, good fits to the static quark potential can be achieved with just a Coulomb term. We have also developed a real space renormalization group transformation which preserves the exact lattice supersymmetry and which can be used to determine anomalous dimensions using a Monte Carlo renormalization group method. Computations are currently underway to extend these calculations to stronger coupling where the perturbative predictions should fail. For large enough numbers of colors one should be able to compare the lattice results to the planar results but for small N and large gauge coupling the lattice calculation would constitute the only reliable method for computing this quantity. It would be exciting to compare the lattice results with both the bootstrap bounds and the behavior expected on the basis of S duality.

One issue that emerged during the course of this work was the observation of instabilities in the lattice theory associated with the existence of $U(1)$ flat directions. The lattice construction requires the use of an $U(N)$ gauge group. In the continuum the $U(1)$ sector corresponds to free field theory and decouples from the $SU(N)$ sector. However, in the lattice theory the $U(1)$ gauge fields drive a transition to a monopole phase at strong coupling (this is well known in compact lattice QED). The presence of this phase blocks access to the propertries of the $SU(N)$ theory at strong coupling. To remove this lattice artifact phase a supersymmetry breaking term was added to the action that suppressed the fluctuations in the $U(1)$ sector. While this was successful in lifting the flat directions and removing the lattice monopoles it was observed to induce large lattice artifacts which slowed the approach to the continuum limit. Recently we have realized how to lift these $U(1)$ flat directions in a supersymmetric manner. As a bonus we have shown that the new lattice action is automatically $O(a)$ improved and the approach to a supersymmetric continuum limit is *much* faster even for large gauge coupling.

It is important to recognize that this lifting of the $U(1)$ flat directions is not merely a practical problem but a problem of principle; the lattice path integral is ill-defined until some method is found to remove this instability and until our work it was not known how to do this while retaining the exact lattice supersymmetry.

In addition, a significant amount of work has been completed on the question of whether the lattice theory exhibits a sign problem. The Pfaffian representing the effect of the fermion loops is generically complex in Euclidean space rendering the use of Monte Carlo methods potentially problematic. However it was shown that the magnitude of the phase fluctuations computed within the phase quenched ensemble was small $O(10^{-2})$ or smaller for 't Hooft couplings in the range $0 < \lambda < 2.0$. Furthermore the observed volume dependence of the phase was extremely weak unlike the exponential growth expected in a theory with a true sign problem. It is easy to show that the phase vanishes on the moduli space and our numerical results seem to indicate that this feature persists even as the coupling is increased. The absence of quantum corrections to the moduli space then clearly plays a role in suppressing phase fluctuations as the coupling is increased. Further work is clearly needed to understand this effect better but from a practical perspective it appears that there is no effective sign problem for the volumes and couplings that we currently exploring.

Completed work

- Hired a postdoc David Schaich to take the lead on code development. David joined the Syracuse group in September 2013 and was supported partially under this grant and partially with funds from Syracuse University. Departed end of August 2016 to take up a postdoc at the University of Bern, Switzerland to continue studies of lattice supersymmetry.
- Developed GPU and parallel (MILC based) code to simulate $\mathcal{N} = 4$ super Yang-Mills. Later refinements to code included developing supersymmetric deformation allowing for control of $U(1)$ modes which otherwise destabilize theory at strong coupling.
- Explored phase diagram of theory to provide evidence for conjectured conformality of theory at all couplings. Ensembles of lattices up $16^3 \times 32$ for $\lambda = 0.1 - 6.0$ generated where $\lambda = g^2 N$ the 't Hooft coupling. Code supports arbitrary numbers of colors and ensembles for $N = 2, 3, 4$ have been generated.
- Computed static potential, Ward identities and correlation functions of Konishi operator to compare with perturbative and conformal bootstrap predictions.
- Commenced work on establishing restoration of additional broken supersymmetries in the continuum limit.
- Code has been benchmarked on current USQCD hardware and has been made publically available.
- Code and physics program in lattice supersymmetry has been incorporated in USQCD hardware proposals and INCITE requests.

Papers

1. S. Catterall and A. Veernala, "Spontaneous supersymmetry breaking in two dimensional lattice super QCD," JHEP **1510**, 013 (2015) doi:10.1007/JHEP10(2015)013 [arXiv:1505.00467 [hep-lat]].
2. S. Catterall and D. Schaich, "Lifting flat directions in lattice supersymmetry," JHEP **1507**, 057 (2015) doi:10.1007/JHEP07(2015)057 [arXiv:1505.03135 [hep-lat]].

3. D. Schaich and T. DeGrand, “Parallel software for lattice N=4 supersymmetric YangMills theory,” Comput. Phys. Commun. **190**, 200 (2015) doi:10.1016/j.cpc.2014.12.025 [arXiv:1410.6971 [hep-lat]].

4. D. Schaich and S. Catterall, “Maximally supersymmetric Yang-Mills on the lattice,” arXiv:1508.00884 [hep-th].

5. G. Bergner and S. Catterall, “Supersymmetry on the lattice,” Int. J. Mod. Phys. A **31**, no. 22, 1643005 (2016) doi:10.1142/S0217751X16430053 [arXiv:1603.04478 [hep-lat]].

6. D. Schaich, S. Catterall, P. H. Damgaard and J. Giedt, “Latest results from lattice N=4 supersymmetric Yang–Mills,” PoS LATTICE **2016**, 221 (2016) [arXiv:1611.06561 [hep-lat]].

7. S. Catterall, J. Giedt, D. Schaich, P. H. Damgaard and T. DeGrand, “Results from lattice simulations of N=4 supersymmetric Yang–Mills,” PoS LATTICE **2014**, 267 (2014) [arXiv:1411.0166 [hep-lat]].

8. S. Catterall and J. Giedt, “Real space renormalization group for twisted lattice $\mathcal{N}$ =4 super Yang-Mills,” JHEP **1411**, 050 (2014) doi:10.1007/JHEP11(2014)050 [arXiv:1408.7067 [hep-lat]].

9. S. Catterall, D. Schaich, P. H. Damgaard, T. DeGrand and J. Giedt, “N=4 Supersymmetry on a Space-Time Lattice,” Phys. Rev. D **90**, no. 6, 065013 (2014) doi:10.1103/PhysRevD.90.065013 [arXiv:1405.0644 [hep-lat]].

10. D. J. Weir, S. Catterall and D. Mehta, “Eigenvalue spectrum of lattice $\mathcal{N} = 4$ super Yang-Mills,” PoS LATTICE **2013**, 093 (2014) [arXiv:1311.3676 [hep-lat]].

11. T. Appelquist *et al.*, “Lattice Gauge Theories at the Energy Frontier,” arXiv:1309.1206 [hep-lat].

12. S. Catterall, J. Giedt and A. Joseph, “Twisted supersymmetries in lattice $\mathcal{N} = 4$ super Yang-Mills theory,” JHEP **1310**, 166 (2013) doi:10.1007/JHEP10(2013)166 [arXiv:1306.3891 [hep-lat]].

Students and postdocs

- David Schaich. Supported in part by SciDAC grant. Took lead in software development and analysis. Currently a postdoc at University of Bern, Switzerland.
- Aarti Veernala. Received partial support under another DOE grant associated with High Energy Physics. Graduated in August 2015. Now a postdoc at Fermilab.
- Dhagash Mehta. Supported under another DOE HEP grant. Worked on the Physics program and now a postdoc at Notre Dame in the math department.
- Raghav Jha. Student receiving partial support from another DOE HEP grant. Currently working on aspects of $\mathcal{N} = 4$ Yang-MIlls connected to gauge-grabity duality. Projected graduation date August 2018.

Unexpended funds

There are no unexpended funds associated with this grant.