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Author(s): Langenbrunner, James R.
Makaruk, Hanna Ewa

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Temperature derivatives for fusion reactivity of D-D and D-T

J. R. Langenbrunner, H. E. Makaruk.
Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

Abstract

Deuterium-tritium (D-T) and deuterium-deuterium (D-D) fusion reaction rates are observable using leakage gamma flux. A direct measurement of γ-rays with equipment that exhibits fast temporal response could be used to infer temperature, if the detector signal is amenable for taking the logarithmic time-derivative, alpha. We consider the temperature dependence for fusion cross section reactivity.

I. Introduction

D-T fusion produces an excited ^{5}He nucleus, which de-excites via two high-energy gamma branches, while D-D capture fusion goes directly to the ^{4}He ground state:

$$D + T \rightarrow {}^5He^* \rightarrow {}^5He + \gamma_0 (16.75\,\text{MeV}) \quad (1)$$

$$D + T \rightarrow {}^5He^* \rightarrow {}^5He^* + \gamma_1 (\sim 13.5\,\text{MeV}) \quad (2)$$

$$D + D \rightarrow {}^4He \rightarrow {}^4He + \gamma_0 (23.8\,\text{MeV}) \quad (3)$$

These gammas can be measured in "current-mode" using gas Cherenkov γ-ray detectors with fast temporal responses and inherent energy thresholds, equivalent measuring flux of the gamma rays, for example [1]:

$$\dot{n}_\gamma \sim n_D n_T < \sigma v >_{DT} [T] \quad (4)$$

The reaction rate $< \sigma v >$ above is averaged over a Maxwellian (temperature *T*) distribution. We will apply the fitted model for the *T* dependence of the non-linear D-T reaction rate in the NRL Plasma Formulary, 2004 [2]. The fusion reactivity at low energy, below 25 keV, is expressed in the following form:

$$f \equiv < \sigma v > = A_0 T^{-2/3} \exp(-A T^{-1/3}) \quad (5)$$

with A_0 a constant and A = 19.94 for the D-T fusion reaction and A = 18.76 for the D-D fusion reaction. The scalar multiplier A_0 is equal to 3.68 x 10^{-12} for D-T fusion and 2.33 x 10^{-14} for D-D fusion in units of cm^3 sec^{-1}.

In fusion applications, the ion temperature changes, and this is one factor causing the flux to change as a function of time. The fundamental cross-sections for D-D and D-T fusion do not change as a function of time, but as a function of T. By application of the chain rule,

$$\frac{d}{dt} = \dot{T}\frac{d}{dT} \tag{6}$$

Understanding of the time dependence of gamma-flux $\dot{n}_\gamma$ requires knowledge of the temperature derivative of the reactivity:

$$\frac{df}{dT} = -\frac{2}{3}\frac{<\sigma v>}{T} + \frac{A}{3T^{1/3}}\frac{<\sigma v>}{T} \tag{7}$$

The fusion reactivity is frequently expressed as a power law,

$$f(T) \sim T^a \tag{8}$$

The temperature derivative of the power law expression is

$$\frac{df}{dT} = a\frac{<\sigma v>}{T} \tag{8}$$

Setting Eq. 6 and Eq. 8 equal produces an expression with which to estimate the power of T in Eq. (7). For D-T fusion, A = 19.94, such that for T = 1 keV, a in Eq. (10) below has the value of 6.

$$a = \frac{1}{3}\left(\frac{A}{T^{1/3}} - 2\right) \tag{10}$$

The constant a itself changes as a function of the ion temperature. If the measured flux of gammas from reactions Eq. 1 and Eq. 2 is of sufficient quality (high signal-to-noise, differentiable) then the logarithmic time-derivative of flux, alpha, defined as α, is observable, and is defined in Eq. (11).

$$\alpha \equiv \frac{d\ln(\dot{n}_\gamma)}{dt} = \frac{1}{\dot{n}_\gamma}\frac{d\dot{n}_\gamma}{dt} \sim a\frac{\dot{T}}{T} \tag{11}$$

The last proportionality holds if the time-rate-of-change change in the number of fusion reactants is negligible. Continuing our example, if it were known that the ion temperature of a D-T plasma at a point in time was 1 keV, then a measurement of alpha evaluated at that time is equivalent to the time-rate-of-change of the ion temperature at that point in time:

$$\dot{T} = \frac{\alpha(1\text{ keV})\cdot(1\text{ keV})}{6} \tag{12}$$

It is easily shown that measurement of the peak (extremum) of the observable alpha $d\alpha/dt$ = 0 is equivalent to:

$$\frac{d^2}{dt^2}(\dot{n}_\gamma) = \alpha^2\dot{n}_\gamma \tag{13}$$

If the fusion reaction rate is increasingly rapid (but below 25 keV), when $d\alpha / dt = 0$, it encounters a point at which the second time derivative in flux no longer exceeds the increase in $\alpha^2 \dot{n}_\gamma$. If flux and alpha could be evaluated at its peak, that is, at the instant in time with $d\alpha / dt = 0$, then quantities important for understanding the details of specific experimental conditions may likely be inferred. The timing and value of an extremum in α can sometimes be identified with greater certainty than other parts of current signals. This of course requires sufficient recording bandwidth, dynamic range and signal-to-noise ratios.

The study of complex fusion-plasma interactions require knowledge of fundamental fusion cross-sections, as well as the first, second and third derivatives of fusion reactivity with respect to temperature *T*.

II. Fusion D-T and D-D Reaction-rate Derivatives with respect to Temperature

Experimentally, if neutron-induced γ-ray (n-γ) backgrounds from the target are either negligible or could be excluded from the signal using energy thresholds, then a suitable gamma-ray detector of the flux would be sensitive to reaction history, Eq. (4). The following formula (Eq. (14)) was implemented in Mathematica ™ for expediency:

$$f[\mathrm{T\_}] := \mathrm{Ao} * \mathrm{Exp}[-A * T\wedge(-1/3)] * T\wedge(-2/3) \qquad (14)$$

The first derivative is written in Eq. (14). The constants and *A* = 19.94 for the D-T fusion reaction and *A* = 18.76 for the D-D fusion reaction were substituted into Eq. (15) for visualization in Figure 1, and the multiplication constant A_0 was set to one.

$$\frac{\mathrm{Ao}e^{-\frac{A}{T^{1/3}}}(A-2T^{1/3})}{3T^2} \qquad (15)$$

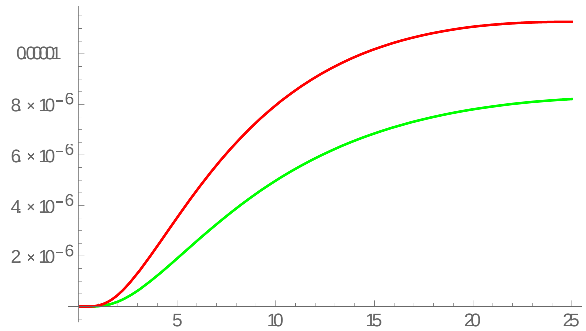

Figure 1: Mathematica ™ output plotted for the first derivative of the reaction rate with respect to *T* for D-T (green) and D-D (red). The plot does not include the scalar multipliers A_0. The horizontal axis is temperature in units of keV.

The second derivative is plotted in Figure 2, with the appropriate substitutions, and the constant A_0 set to one.

$$\frac{Aoe^{-\frac{A}{T^{1/3}}}(A^2-8AT^{1/3}+10T^{2/3})}{9T^{10/3}} \tag{16}$$

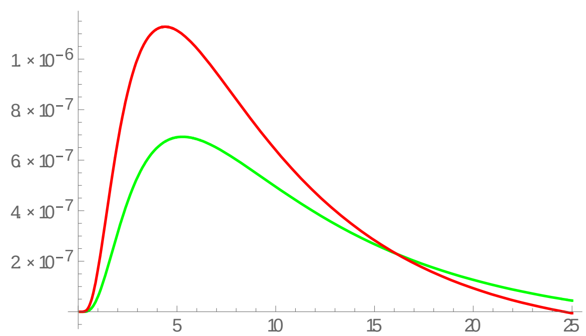

Figure 2: Mathematica ™ output plotted for the second derivative of the reaction rate with respect to *T* for D-T (green) and D-D (red). The plot does not include the scalar multipliers A_0. . The horizontal axis is temperature in units of keV.

III. Discussion

In ICF plasmas in thermal-equilibrium, T designates the burn-averaged ion temperature in units of keV . Consider a thought experiment wherein the time-rate-of-change of flux of gamma rays (Eq.'s (1-3)) ($\dot{n}_\gamma \sim n_D n_T < \sigma v >$) is produced by a region of steady burning plasma. If the flux depends mainly on the time-rate-of-change of *T*, then the condition $d\alpha / dt = 0$ occurs at a temperature determined by the peak of the curves shown in Figure 2. Those temperatures are 4.40 keV for D-D fusion and 5.29 keV for D-T fusion. This can be found by taking the third derivative of the reaction rates, as shown in Figure 3.

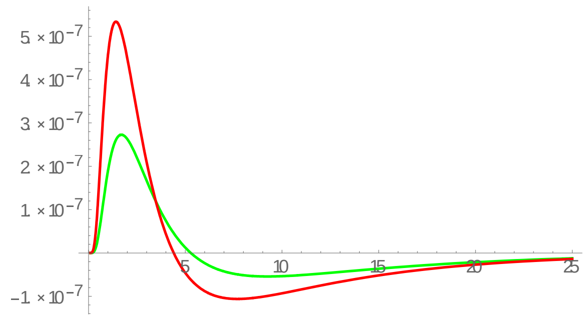

Figure 3: Mathematica ™ output plotted for the third derivative of the reaction rate with respect to *T* for D-T (green) and D-D (red). The plot does not include the scalar multipliers A_0. . The horizontal axis is temperature in units of keV. The zeros of these curves define the value of *T* for peaks of the functions shown in Figure 2.

References

[1] Y. Kim, *et al.* Phys. Plasmas 19, 056313 (2012)
[2] NRL Plasma Formulary, J.D. Huba, Naval Research Laboratory, NRL/PU/6790-040447, Revised 2004.