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Beam-Breakup Instability Penalties for Curing High-Voltage Breakdown in LIA Cells

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***Abstract*—The strength of the dangerous beam breakup (BBU) instability in linear induction accelerators (LIAs) is determined by the transverse coupling impedance $Z_\perp$ of the induction cell cavity. For accelerating gap width *w* much less than the beam pipe radius *b*, the transverse impedance is theoretically proportional to *w/b*, favoring narrow gaps to suppress BBU. On the other hand, cells with narrow gaps cannot support high accelerating gradients, because of electrical breakdown and shorting of the gap. Thus, there is an engineering trade-off between BBU growth and accelerating gradient that must be considered for next generation LIAs now being designed. In this article this tradeoff is explored, using a simple pillbox cavity as an illustrative example. For this model, widening the gap to reduce the probability of breakdown increases BBU growth, unless higher magnetic focusing fields are used to further suppress the instability.**

***Index Terms*— Accelerators, Electron beams, Instability, High-voltage breakdown**

I. Introduction

MODERN diagnostics of large hydrodynamic experiments driven by high explosives include linear induction accelerators (LIAs) that produce high-current, high-energy electron beams focused onto heavy metal targets to generate small source spots of bremsstrahlung radiation for stop-action, flash radiography [1, 2]. A virulent beam instability in high-current LIAs is the beam breakup (BBU) [3, 4, 5, 6], which is especially troublesome for flash radiography LIAs, because the high frequency motion is integrated over the beam pulse-width, thereby blurring the radiographic source spot.

BBU is caused by beam excitation of electromagnetic cavity modes that have a transverse magnetic field, in particular the TM_{1n0} modes. In an LIA the cavities are connected by lengths of beam pipe that form a waveguide beyond cutoff for these modes, so the cavities only communicate via RF oscillations of the beam centroid. This is known as cumulative BBU. It has been shown theoretically [4, 5, 6], through simulations [7], and experimentally [8, 9], that BBU growth depends exponentially on the transverse impedance, $Z_\perp$, which characterizes the strength of the interaction between the beam and the TM cavity modes. Since $Z_\perp$ is generally an increasing function of the width of the accelerating gap, w, it is usually thought one should reduce w to better suppress the BBU. However, high-voltage breakdown of the gap seriously constrains this approach; narrow gaps cannot support high accelerating voltages without breaking down and shorting. Thus, there is an engineering trade-off between gap breakdown and BBU mitigation. Moreover, it appears that there is always a BBU penalty for fixing breakdown problems.

For example, it may become necessary to prevent breakdown in an existing cavity design by reducing the electric field in the gap. This can be done in at least two different ways;

- Keep the accelerating voltage the same, and increase the gap width in a new cell design.
- Keep the cell design, decrease the accelerating voltage, and add cells to have the same final beam energy.

It would be useful to understand the trade-off between these two options with respect to the beam breakup instability.

The purpose of this article is to provide some insight into the problem of providing enough gap width to prevent high-voltage breakdowns, and the consequences for BBU growth.

II. BBU Growth Theory

The maximum amplitude of the BBU has been shown theoretically and experimentally to asymptote after a large number of cells (*N*) to $\max \xi(z) = \xi_0 \left[\gamma_0 / \gamma(z)\right]^{1/2} \exp\left(\Gamma_m\right)$ where subscript zero denotes initial conditions, and γ is the relativistic mass factor [4, 5, 6, 8, 9, 7]. Here, the number of amplitude e-foldings is

$$\Gamma_e = \Gamma_m - 0.5 \ln \gamma / \gamma_0 \tag{1}$$

where

$$\Gamma_m\left(z\right) = \frac{I\,N\,Z_\perp}{300}\left\langle \frac{1}{B} \right\rangle \quad , \tag{2}$$

in which *I* is the beam current in kA, $Z_\perp$ is the transverse coupling impedance in Ohms/cm, *B* is the solenoidal focusing field in kG, and $\langle\ \rangle$ indicates an average over the cells. Although theoretically derived for idealized conditions, these expressions have been experimentally validated on operational flash-radiography accelerators at Los Alamos National Laboratory [8, 9]. Since the last term in Eq. (1) is only of order unity in typical radiographic LIAs, Eq. (2) is a useful estimator of BBU growth in present and next generation machines.

In Eq. (2) $Z_\perp$ is understood to be the maximum value of the real part of the complex transverse impedance, which occurs at the resonant frequency. In general, $Z_\perp$ is a function of the cavity dimensions, and in particular the ratio of the gap width to the beam pipe radius, w/b.

It is instructive to recast Eq. (2) in terms of system requirements and engineering constraints. For example, at the LIA exit, the number of accelerating cells is related to the total energy gain required of the LIA by

$$\Delta KE = NV_g \tag{3}$$

where V_g is the accelerating potential of a single cell. This can be used to eliminate N in favor of the required ΔKE and the cell accelerating potential V_g;

$$\Gamma_m = \frac{I\,\Delta KE}{300V_g} Z_\perp \left\langle \frac{1}{B} \right\rangle \quad , \tag{4}$$

However, BBU cannot be reduced by increasing V_g beyond a limit determined by the constraint of electrical breakdown in the gap or across the insulating vacuum interface. Assume that the insulator can be located far enough back in an expanded cavity that vacuum breakdown across the narrow gap is the limiting factor. Then, one has the average electric field in the gap is $E_{av} = V_g / w$. Furthermore, due to field enhancement at gap edges, the maximum field in the gap is related to the average field by $E_{\max} = f\,E_{av}$, where $f(w/b)$ is a field enhancement factor that varies with the gap width. The maximum field, $E_{\max}$, is the high-voltage pulsed-power engineering constraint on BBU growth. Writing

$$V_g = E_{\max} w / f(w/b) \tag{5}$$

one has

$$\Gamma_m\left(w/b, z\right) = \left[\frac{I\,\Delta KE}{300E_{\max}}\right]\left[\frac{Z_\perp(w/b)}{w/\,f(w/b)}\right]\left\langle \frac{1}{B} \right\rangle \quad , \tag{6}$$

as a working model for BBU growth constrained by system requirements ($I, \Delta KE$) and high-voltage engineering best practices ($E_{\max}$). This equation can be used to estimate the BBU penalty incurred by any remediation of breakdown problems. With accurate calculations of $Z_\perp(w/b)$ and $f(w/b)$, it can also be used to evaluate trade-offs between various new designs. In this article, we illustrate the method by applying it to the simplest of cavity/gap designs; the pillbox.

Finally, if it is required to reduce the maximum field to lower the probability of breakdown, one can either increase the number of cells with lower drive voltage, or increase the gap width. Both approaches reduce $E_{\max}$ in the first term in Eq. (6), thereby increasing Γ_m and BBU growth. It follows that increased BBU is an unavoidable consequence of reducing the risk of electrical breakdown in the cells.

III. APPLICATION

In this section, we apply Eq. (6) to a simple pillbox cavity with outer radius R and width w terminated on both sides with a beam pipe of radius b. The cavity is assumed to be terminated at its outer radius by a shunt impedance, Z_s, and to have $R >> b$. Detailed calculations of $Z_\perp$ and f for a simple pillbox cavity are reviewed and applied to the theory of BBU growth given by Eq. (6). Transverse Impedance

For $w << b$ detailed analytic theory gives

$$Z_\perp = Z_0 \frac{w}{\pi b^2}\eta \quad , \tag{7}$$

where $Z_0 = 120\pi\ \Omega$ is the impedance of free space, and η is a non-dimensional form factor of order unity, and independent of w [11, 12]. Furthermore, the range of w/b has been established by direct calculation of wake potentials from the RF electromagnetic fields [13].

From the definition of the complex transverse impedance, it follows that its functional dependence on frequency and cavity dimensions is the same as that of the wake potential calculated in [14, 15, 13, 16]. For a pillbox with arbitrary dimensions it was found that the wake potential scaled as Eq.(7) for $w/b < 1$, but saturated to be independent of w for $w/b > 1$ [13, 16]. To show this, several values of the maximum of the imaginary part of the wake potential calculated in [16] are plotted in Fig. 1. (The real part of the impedance appearing in Eq. (6) governs instability growth, and is proportional to the imaginary part of the wake potential.) The w/b scaling noted in ref. [13] is clearly seen in this figure. Therefore, since it is unlikely that accelerating gaps in next generation LIAs will exceed half the tube radius (i.e., $w/b \le 0.5$), we use Eq. (7) as a model of impedance scaling for pillbox cavities, with the form factor a function of R/b and Z_s , but entirely independent of the gap width w. Thus, for a simple pillbox cavity, the growth of BBU given by Eq. (6) only depends on w through the field enhancement factor $f(w/b)$ and the unavoidable reduction of the maximum field.

$$\Gamma_m\left(w/b, z\right) = \frac{I\,\Delta KE}{300E_{\max}} \frac{Z_0\eta(Z_s, R/b)}{\pi b^2} f(w/b) \left\langle \frac{1}{B} \right\rangle \quad , \tag{8}$$

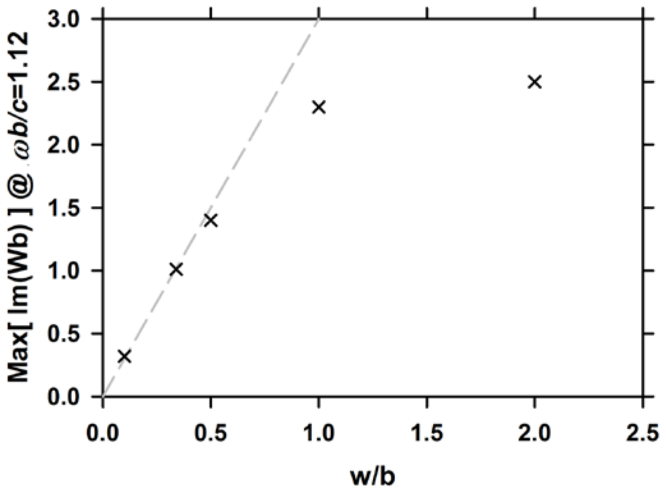

Fig. 1: Imaginary part of wake potential W (times the tube radius b) as a function of gap to tube radius ratio w/b (from ref. [16]) at $\omega_0 = 1.19c/b$. (The real part of the impedance, which governing instability growth, is proportional to the imaginary part of the wake potential.) For these calculations, $Z_s = Z_0$ and $R/b = 3.6$, giving the resonant frequency $\omega_0 = 4.3c/R$, which is somewhat greater than the resonant frwequency of a pure pillbox with no beam pipe. The dashed grey line through origin has been added to show the linear dependence on w/b for values less than ~0.5.

A. Electrical Breakdown

Simulations of the example pillbox gap were performed using the 2-D Estat finite-element code, which is a component of TriComp [17]. All simulations were performed with 250 kV applied to the 1.91-cm gap, which included rounded edges with an $r = 0.64$-cm radius to reduce field enhancement. Fig. 2 shows the shape of the gap, and the equi-potential contours of the accelerating field. As shown in Fig. 3, the maximum electric field occurring on the curved part of the cathode is much greater than $V_0 / w = 131$ kV/cm.

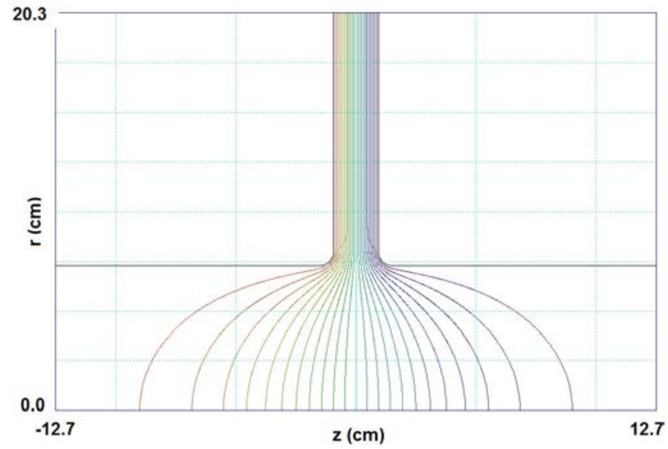

Fig. 2: Pillbox-cavity accelerating gap, showing equi-potential contours of the accelerating electric field.

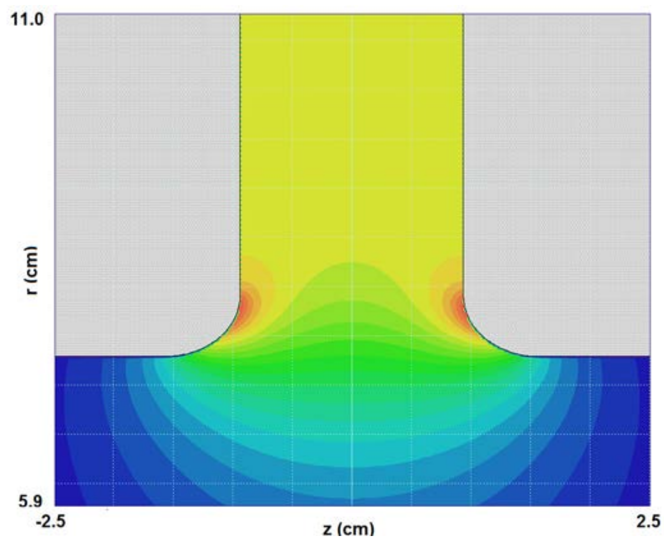

Fig. 3: Absolute value of electric field in the 1.91-cm wide pillbox gap as simulated with the Estat finite-element code. The left-hand is negatively charged (cathodic) to 250 kV and the right-hand side is ground. The maximum absolute field on the convex surfaces (shown in dark orange) is 170 kV/cm. The average field in the gap (shown in yellow) is 131 kV/cm.

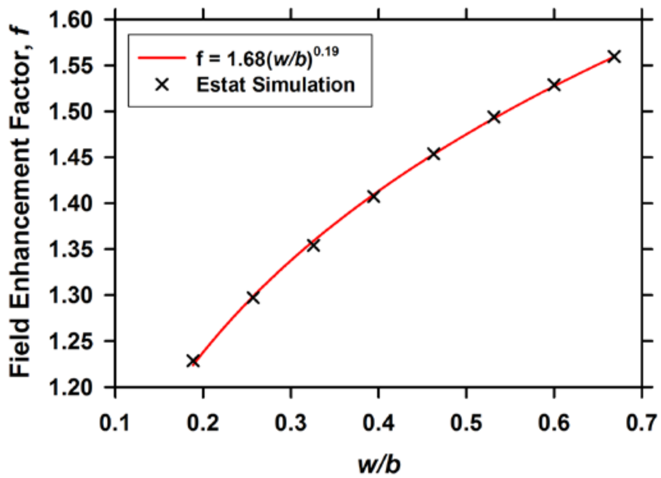

Fig. 4: Electric field enhancement factor, $f = E_{\max} / E_{av}$ as a function of gap width to tube radius ratio with a fixed corner radius. The symbols are results of Estat simulations, and the red line is a power-law fit.

If the corner radius is unchanged, the scaling of the field enhancement factor with increasing gap size is shown in Fig. 4. This shows that the simulations fit a power-law fit for the enhancement factor; $f \sim (w/b)^{0.2}$. Thus, according to Eq. (8), BBU growth is increased by more than what would result by simply reducing the drive voltage and adding more cells. Of course, the dependence of f on w/b can be reduced by permitting the corner radius to increase as the gap is increased.

Of course, it follows from Poisson's equation that the field enhancement factor is scale invariant. That is, the factor is invariant if all dimensions are inflated by a common factor. Thus, if every dimension of the pillbox gap problem is increased by the same factor as is the gap, then the enhancement factor will be unchanged. Unfortunately, this would require increasing the beam pipe size b, which may be fixed by other engineering constraints. However, for $w/b << 1$, the enhancement factor should be almost invariant if the ratio of gap-width to corner-radius, w/r, is held constant. To show this for the pillbox problem, a series of Estat simulations was performed with the gap-width to corner-radius ratio held constant at $w/r = 3$. The results plotted in Fig. 5 show that, with this tactic, the enhancement factor is constant to within 1% over a range of w/b that includes both modern LIAs [ref. Burns 1991] and possible next-generation designs. Therefore, using this tactic would facilitate reducing risk of breakdown without affecting BBU growth more than would result from simply lowering the drive voltage and adding cells to compensate.

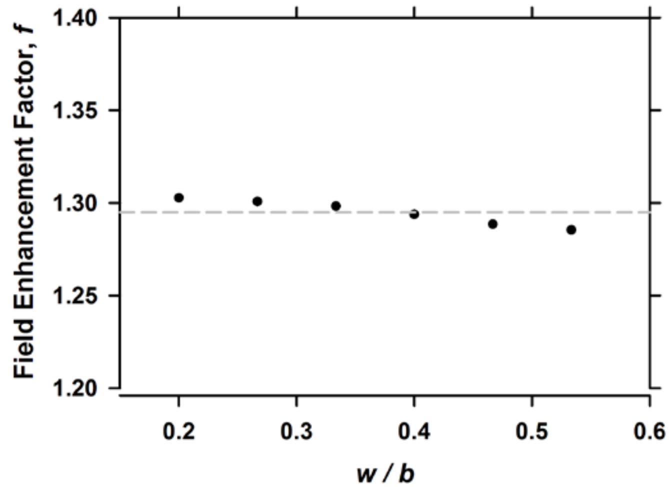

Fig. 4: Field enhancement factor variation with gap width for $w/b = 3$.

IV. Discussion

At face value, Eq. (2) argues for decreasing $Z_{\perp}$ as much as possible by decreasing w (through Eq. (7)), and increasing the external focusing field B as much as possible. However, there are practical constraints. For example, B cannot be increased indefinitely, because that also increases the growth of corkscrew motion [18], which is proportional to the total phase advance [19]. Furthermore, the gap size cannot be decreased without limit, because of electrical breakdown across the insulator, and/or emission from field enhancement at convex cathodic surfaces. This engineering tradeoff is summarized in Eq. (8), which shows that for the pillbox example and a fixed beam-pipe size, the number of BBU e-foldings increases as

$$\Gamma_m \propto (w/b)^{0.2} \left\langle \frac{1}{B} \right\rangle \tag{9}$$

Thus, doubling the gap width to reduce the chance of breakdown, only requires ~15% increase in magnetic field to compensate. The increased corkscrew due to this increase in phase advance can likely be corrected by use of dipole corrector magnets through application of the tuning-V algorithm [20, 21].

Finally, next generation LIAs will probably have much more complicated cavities than a simple pillbox, so their transverse impedance will probably not follow the simple w/b scaling law. Moreover, convoluted gaps will likely have a different field enhancement factor. Thus, it is imperative to have accurate knowledge of the functional forms of $Z_{\perp}(w/b)$ and $f(w/b)$ in order to ascertain the dependence of BBU growth on gap size under the engineering of electrical breakdown.

V. Conclusions

We have shown how the number of BBU e-foldings in an LIA depends on the accelerating gap width for a simple pillbox cavity. Wider gaps reduce the risk of breakdown, and it is vital to have accurate knowledge of impedance and field enhancement as a function of gap width when assessing the trade-off of breakdown vs BBU growth.

Should it become necessary to reduce $E_{\max}$, then the BBU growth will increase according to Eq. (6). If the transverse impedance scales as w/b, and the field enhancement is independent of gap width, then there is no advantage to either reducing the drive voltage or increasing the gap width; the increase in BBU growth revealed by Γ_m will be inversely proportional to the reduction of $E_{\max}$. On the other hand, if the field enhancement factor depends on gap width, there may be an advantage to reducing the drive voltage and increasing the number of cells instead of increasing the gap width.

Cavity designs for next generation LIAs will surely be more complicated than the simple pillbox considered herein. Moreover, practical cavities will include dielectric-vacuum interfaces that have their own breakdown criteria. Therefore, the calculations in this article must be revisited using a code such as AMOS [22] to accurately calculate the scaling of transverse impedance with w/b , and a code such as Estat [17] to calculate the field enhancement scaling. Of course, such calculations must be verified by experimental measurements of the cavity impedance, such as were done for the flash-radiography accelerators at Los Alamos [23, 24]

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