DOE-WISC-54813

Plasma Science and Innovation Center at Washington, Wisconsin, and Utah State

Final Scientific Report for the University of Wisconsin-Madison
March 1, 2005 – August 31, 2016

Principal Investigator at the University of Wisconsin-Madison: C. R. Sovinec
Lead Principal Investigator: T. R. Jarboe, University of Washington
Collaborating Principal Investigator: E. D. Held, Utah State University

University of Wisconsin-Madison
Madison, WI 53706-1609

November 28, 2016

THE U.S. DEPARTMENT OF ENERGY
OFFICE OF SCIENCE, FUSION ENERGY SCIENCES
AWARD NO. DE-FC02-05ER54813

1. Executive Summary

The University of Wisconsin-Madison component of the Plasma Science and Innovation Center (PSI Center) contributed to modeling capabilities and algorithmic efficiency of the Non-Ideal Magnetohydrodynamics with Rotation (NIMROD) Code [1], which is widely used to model macroscopic dynamics of magnetically confined plasma. It also contributed to the understanding of direct-current (DC) injection of electrical current for initiating and sustaining plasma in three spherical torus experiments: the Helicity Injected Torus-II (HIT-II) [2], the Pegasus Toroidal Experiment [3], and the National Spherical Torus Experiment (NSTX) [4]. The effort was funded through the PSI Center's cooperative agreement with the University of Washington and Utah State University over the period of March 1, 2005 - August 31, 2016. In addition to the computational and physics accomplishments, the Wisconsin effort contributed to the professional education of four graduate students and two postdoctoral research associates. The modeling for HIT-II and Pegasus was directly supported by the cooperative agreement, and contributions to the NSTX modeling were in support of work by Dr. Bickford Hooper, who was funded through a separate grant.

Our primary contribution to model development is the implementation of detailed closure relations for collisional plasma. Postdoctoral associate Adam Bayliss implemented the temperature-dependent effects of Braginskii's parallel collisional ion viscosity [5]. As a graduate student, John O'Bryan added runtime options for Braginskii's models and Ji's K2 models [6] of thermal conduction with magnetization effects and thermal equilibration. As a postdoctoral associate, O'Bryan added the magnetization effects for ion viscosity. Another area of model development completed through the PSI-Center is the implementation of Chodura's phenomenological resistivity model [7]. Finally, we investigated and tested linear electron parallel viscosity, leveraged by support from the Center for Extended Magnetohydrodynamic Modeling (CEMM).

Work on algorithmic efficiency improved NIMROD's element-based computations. We reordered arrays and eliminated a level of looping for computations over the data points that are used for numerical integration over elements. Moreover, the reordering allows fewer and larger communication calls when using distributed-memory parallel computation, thereby avoiding a data starvation problem that limited parallel scaling over NIMROD's Fourier components for the periodic coordinate. Together with improved parallel preconditioning, work that was supported by CEMM, these developments allowed NIMROD's first scaling to over 10,000 processor cores. Another algorithm improvement supported by the PSI Center is nonlinear numerical diffusivities for implicit advection. We also developed the Stitch code to enhance the flexibility of NIMROD's preprocessing.

Our simulations of HIT-II considered conditions with and without fluctuation-induced amplification of poloidal flux, but our validation efforts focused on conditions without amplification. A significant finding is that NIMROD reproduces the dependence of net plasma current as the imposed poloidal flux is varied [8]. The modeling of Pegasus startup from localized DC injectors predicted that development of a tokamak-like configuration occurs through a sequence of current-filament merger events [9]. Comparison of experimentally measured and numerically computed cross-power spectra enhance confidence in NIMROD's simulation of magnetic fluctuations [10]; however, energy confinement remains an open area for further research. Our contributions to the NSTX study include adaptation of the helicity-injection boundary conditions from the HIT-II simulations [11,12] and support for linear analysis and computation of 3D current-driven instabilities [13].

2. Goals and Accomplishments

The overarching goal of the PSI Center is to develop numerical simulation tools that are capable of predicting the performance of smaller "emerging-concept" (EC) plasma confinement experiments before they are constructed. While the Center cannot claim that this high-level objective has been achieved, it has taken many significant steps toward that objective. The Wisconsin component focused on development and application of the previously functioning, hence "workhorse," NIMROD code. This section compares what we have accomplished with tasks that were put forth in five proposals over the eleven-year period of the funded cooperative agreement. For convenience the tasks are organized into the topical areas of model development, algorithm improvement, and applications.

2.1 Model Development Goals

- Collisional closure development
    - Implement collisional temperature dependence in ion viscosity coefficients: Accomplished.
    - Implement a semi-implicit algorithm for nonlinear viscosity computation: This was superseded by a fully implicit formulation.
    - Implement collisional electron parallel viscosity: The linear contributions were implemented and verified. The full nonlinear implementation was not completed, given the development of the more accurate drift kinetic approach by our colleagues at Utah State.
    - Modify transport coefficients for temperature to account for magnetization effects: Magnetization effects for separate electron and ion temperature equations have been implemented, tested, and applied. Temperature-dependent thermal equilibration effects have also been implemented.
    - Implement magnetization effects in ion viscosity: Accomplished.
- Implement the phenomenological Chodura resistivity model: Accomplished.
- Help the Utah State group formulate a method of implementing general-moment relations: The Utah State group produced a hierarchy of moment equations that are suitable for implementation but decided to pursue integral closure relations as a more tractable alternative.

2.2 Algorithm Improvement Goals

- Efficiency improvements
    - Modify the numerical integration for finite elements to be a matrix-matrix multiplication: The efficiency of NIMROD's numerical integration was improved through loop reordering, and the matrix-matrix multiplication approach was not used.
    - Optimize the finite-element operations through changes to data storage: Accomplished.
    - Examine the efficiency and accuracy of performing pseudospectral operations at data-node locations instead of at numerical quadrature points: For a period of time, this

was used for plasma pressure, but concerns regarding accuracy led us back to keeping pressure computations at integration points within elements.

- Implicit advection
  - Temporally centering all advective terms in the advance of each physical field is a numerical stability requirement for NIMROD's two-fluid computations. While the task of making advective terms implicit was not specifically listed in a proposal, it was completed through partial support from the PSI Center.
  - Nonlinear numerical diffusivities were added to the implicit advection to avoid overshoot errors. This task was also not listed in a proposal, but it facilitated computations of field-reversed configuration (FRC) acceleration and merging experiments.

2.3 Application Goals

- Provide support for external users simulating innovative confinement concepts: We have provided boundary-condition modifications, geometries, and other forms of simulation support for external users modeling NSTX, the Levitated Dipole Experiment (LDX), the Compact Toroidal Hybrid (CTH), the Illinois compact torus facility, and other spheromaks. We have also provided support within the PSI Center for modeling the Helicity Injected Torus-Steady Induction (HIT-SI), other spheromaks, and FRC experiments.
- Simulate low-temperature magnetized plasmas, testing efficiency improvements and two-fluid developments: Our simulations of HIT-II were facilitated by implicit advection, and they contributed validation information. Our simulations of Pegasus apply two-fluid and two-temperature modeling.
- Perform simulations of EC experiments that exhibit magnetic relaxation using resistive-MHD, Hall-MHD, and kinetic-MHD models: Our simulations of non-inductive startup in Pegasus compare resistive-MHD and Hall-MHD model predictions. We have not yet attempted to model kinetic effects in this application, which is already computationally challenging with two-fluid modeling.
- Simulate plasma requiring both collisional and kinetic transport effects: We helped our Utah State collaborators compare collisional fluid modeling and kinetic integral-closure modeling of energy transport in the Sustained Spheromak Physics Experiment (SSPX).
- Apply NIMROD to study current drive from localized sources: We modeled localized current drive in Pegasus, and our studies predicted current-ring formation as the relaxation mechanism for building tokamak-like states from helical current-density filaments. Our modeling also contributed validation information for our implementation of collisional closures with spatially varying and dynamic magnetization effects.
- Provide support for modeling SSX: A postdoctoral research associate was supported for a fraction of one year to modify boundary conditions in the HiFi code [14] to model helicity injection. The RA made some progress but did not have the modifications functioning well enough to run SSX simulations with them before he left his position with us.

3. Project Summary

In this section, we present a summary of our key findings and their implications in the three areas of model development, algorithm improvement, and applications.

3.1 Model Development

EC experiments do not have the extreme separation of scales between macroscopic dynamics and transport effects that is typical of large tokamak and stellarator experiments. While this makes integrated simulation of dynamics more tractable, it also means that effects are not decoupled, as is often assumed for large experiments. In fact, our previous experience modeling SSPX shows that transport and macroscopic dynamics can have synergies that lead to important characteristics of EC behavior [15,16]. Our efforts for the PSI-Center cooperative agreement, therefore, focused on a relevant tractable model for whole-device EC simulation: fluid-like moment equations with collisional closure relations for thermal conduction and viscous stress. Prior to the start of the PSI Center, the NIMROD code had the Braginskii temperature dependencies for thermal conduction. However, the implementation assumed the high-magnetization limit, $x_s = \Omega_s \tau_s >> 1$ where Ω_s is the gyrofrequency of species s and τ_s is the effective collision time. The conductivity parallel to the magnetic field (**B**) used 3D temperature and Ω_s information, but the perpendicular conductivity used a simplified computation with the toroidally symmetric part of these fields. At the time, the viscous stress had the parallel-perpendicular anisotropy but only with fixed coefficients.

With support from the PSI Center, postdoctoral associate Adam Bayliss added the temperature dependence to parallel viscous stress. However, the majority of the code-development work was completed by graduate student and then postdoctoral associate John O'Bryan. He implemented two collisional models of thermal conduction and viscous stress in NIMROD. The first model is the classical Braginskii model [5], and the second is Ji's K2 model, which has more accurate transport coefficients, as described in Ref. [6]. O'Bryan included the effects of spatially varying and temporally evolving temperature and magnetization for the local parallel and perpendicular transport effects of each model [10]. The user selects between those models or simplified models at runtime. NIMROD uses finite Fourier series for the periodic coordinate, and O'Bryan's fully 3D implementation accounts for both explicit and implicit coupling among Fourier components during the advances of plasma flow velocity and of electron and ion temperature. O'Bryan also implemented temperature- and density-dependent effects in thermal equilibration modeling between electron and ion species. Using simplified computations of the transport coefficients, including computations based on averages over the periodic coordinate, remains an option.

O'Bryan verified his implementation by computing ion-acoustic (parallel) and magneto-acoustic (perpendicular) waves and comparing the frequencies and damping rates with analytical dispersion relations. Figure 1 shows this comparison for the Braginskii thermal-conduction implementation in conditions that maximize damping [10, Appendix A]. While the wave evolution is linear, i.e. described in the small-amplitude limit, the computations are run nonlinearly to test the closure implementation. Results with NIMROD's older high-magnetization relations are also shown for comparison. Implementing magnetization effects for thermal conduction proved to be important for modeling non-inductive startup in Pegasus, which is described in Sect. 3.3.

Friction between electron and ion species leads to electrical resistivity, and friction for large-scale dynamics results from electron-scale turbulence, in addition to particle collisions. One model for electron-scale turbulence is Chodura's resistivity model [7],

$$\eta_{ch} = \frac{m_e \nu_{ch}}{ne^2}, \quad \nu_{ch} = C_c \omega_{pi} \left[1 - \exp\left(-f \left| {}^{v_d}\!/_{v_s} \right| \right) \right] , \tag{1}$$

where v_d is the relative drift velocity between electrons and ions, ω_{pi} is the ion plasma frequency, v_s is the ion acoustic speed, and f and C_c are free parameters. We implemented this model for NIMROD's implicit advance of magnetic field using local (in 3D) information for ω_{pi}, v_s, and v_d. The drift speed depends on **B** through electrical current density and uses temporally lagged information from the start of each implicit advance. This model of resistivity has been applied by our PSI Center colleagues in simulations of FRCs.

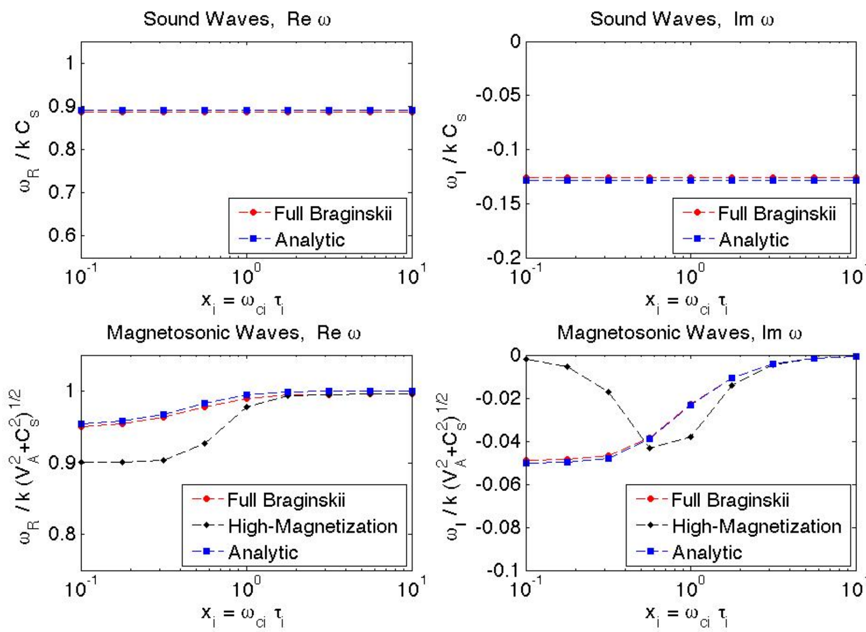

Figure 1. Verification tests of nonlinear thermal conductivity coefficients in the implementation of the Braginskii model. As magnetization is varied, the sound wave (top) frequency and damping rate are unaffected, but the magnetoacoustic wave (bottom) transitions from high to low damping. [From Ref. 10, Appendix A.]

3.2 Algorithm Improvement

Implicit element-based simulation codes, like NIMROD, need to perform different types of computations. Data at numerical integration points within elements are interpolated from basis-function expansions; floating-point operations stemming from the model are performed at the integration points; numerical integration is used to project the algebraic system for updating coefficients of the dependent fields; and algebraic solvers solve the resulting large system. Having all necessary computations perform efficiently is challenging, given that the optimal memory layout differs among the different operations and that communication is required for parallel computation.

We improved NIMROD's efficiency for simulations of EC experiments that tend to burden the element-related operations more than the algebraic operations, at least relative to computations for large devices. Instead of calling the integrand routines once for each integration point within an element, the revised approach calls an integrand routine once per integration. This required reordering data storage and collapsing two array indices into one to avoid a significant rewrite of each integrand routine. Tests on a large internal-kink computation show that the data and loop changes, themselves, reduce the total CPU time by approximately 20%. More importantly, the changes also allow NIMROD to use fewer parallel communication calls per step, where each call transfers a larger amount of data. This reduces the amount of time lost to communication latency, and it allows us to subdivide the Fourier expansion to a much greater extent. Together with improvements to our preconditioner, development supported through CEMM, the efficiency improvements allowed us to scale NIMROD to more than 10,000 processor cores for the first time. This is shown in Fig. 2, which presents weak-scaling results obtained on Franklin, NERSC's flagship supercomputer at the time.

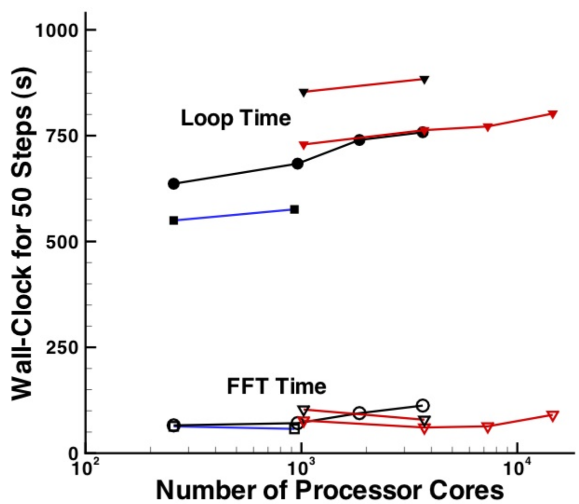

Figure 2. Weak scaling (fixed subdomain size per core) timings from Franklin with 512 elements (blue), 1024 elements (black), and 2048 elements (red) for total time and FFT/collective communication time for 50 steps. The red symbol traces show results with two cores per node. Traces with black symbols show results with four cores per node. Each trace represents changing the number of Fourier components with processor core count.

NIMROD's implicit leapfrog algorithm needs time-centered implicit advection (the terms with $\mathbf{V}\cdot\nabla$) in each of the separate field advances [17]. This avoids numerical dissipation, which is generally beneficial for modeling high-temperature plasma. However, it is problematic when advecting large gradients of number density *n* and temperature *T*, which are never negative, physically. To mitigate dispersive numerical errors that tend to violate this constraint, we developed ad hoc nonlinear diffusive fluxes for the number density and temperature advances. For number density, for example, the extra numerical flux density is

$$-f\left(\frac{\Delta t\,\mathbf{V}\cdot\nabla n}{n}\right)^2\left(\frac{A_e}{\Delta t}\right)\frac{\mathbf{VV}}{V^2}\cdot\nabla n \quad , \tag{2}$$

where A_e is the 2D element area, Δt is the timestep, and f is a coefficient of order unity or less. This provides smoothing along the streamline direction where the gradient is large. As shown in Fig. 3, it helps avoid negative values, as intended.

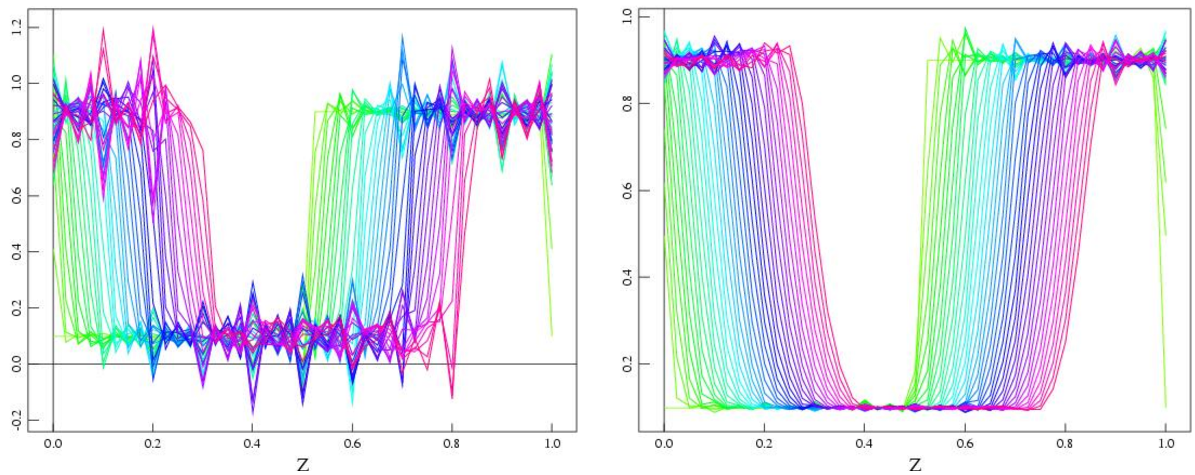

Figure 3. Passive advection of a square pulse without (left) and with (right) the nonlinear diffusivity. There are ten biquartic finite elements along the direction of inhomogeneity, and the computation is run 40 timesteps at a flow CFL of unity. Colors ranging from green to red indicate the evolution over time.

Development of the Stitch code is another computational activity that was supported by PSI Center funding. NIMROD's mesh of elements is organized into blocks for geometric flexibility and for domain decomposition for parallel processing. Element organization within each block is structured, but blocks can be assembled without structured organization, as long as elements along adjacent boundaries conform. The interactive Stitch code was developed to assemble relatively simple regions into complex shapes. An example assembly is shown in Fig. 4, which is a mesh that has been used for the new, small compact torus experiment at the University of Illinois, Champaign-Urbana. Stitch is part of the suite of preprocessing codes that are used for NIMROD.

3.3 Applications

The physics and validation-related studies of the Wisconsin group centered on three experiments where DC helicity injection is used for current drive: HIT-II, Pegasus, and NSTX. Postdoctoral associate Adam Bayliss and the Wisconsin PI conducted the investigation of HIT-II. John O'Bryan conducted the study of localized injection in Pegasus under the supervision of the PI. The NSTX study was led by Dr. Bickford Hooper of Lawrence Livermore National Laboratory and through Woodruff Scientific after his retirement from LLNL. The PI facilitated Hooper's numerical computations and contributed to a stability analysis.

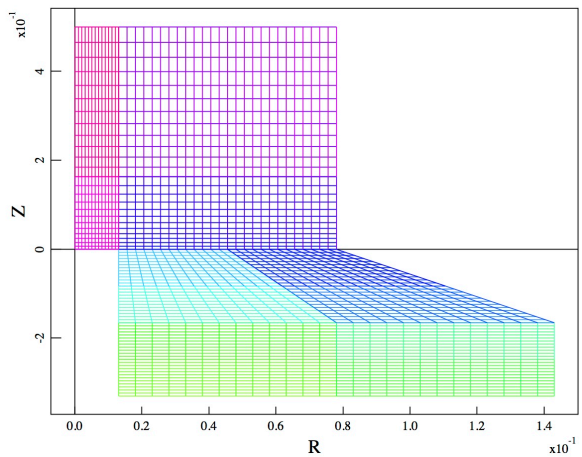

Figure 4. Finite-element assembly created with Stitch for modeling the Univ. of Illinois compact torus experiment. Three regions are assembled: the straight annular section at the bottom (adjacent to the insulating gap), the conical annular region, and the cylindrical region at top that is downstream of the inner electrode.

The HIT-II study considered conditions with and without significant relaxation and primarily focused on conditions without relaxation. Unlike coaxial helicity injection (CHI) in spheromaks, where there is no physical structure along the geometric axis, spherical torus (ST) configurations need a second (absorber) gap, in addition to the injector gap where voltage is applied. We developed appropriate combinations of absorber and injector boundary conditions to simulate CHI in STs. We then conducted parameter scans of steady conditions where the toroidal field is sufficiently large as to avoid asymmetric MHD activity. Figure 5 compares simulation results with the HIT-II experimental database [18] for plasma and injected current in separate scans of the toroidal field and the injector flux. The simulations reproduce the experimental trends of net plasma current being independent of toroidal field and increasing linearly with injected poloidal flux [8]. In this work, we also reinterpreted the "bubble-burst" criterion as a sharp transition in MHD equilibria with increasing injector current [8].

The DC current drive for Pegasus startup uses washer-gun plasma sources [19] to avoid large insulating breaks and to help control impurity sourcing. John O'Bryan, along with then undergraduate student Tom Bird in the early stages of this research, conducted a simulation-based study to understand relaxation in this configuration. Its uniqueness stems from the inherently 3D nature of the driven current streams, so it does not proceed through the typical paradigm of unstable axisymmetric state / development of asymmetric fluctuations / saturation and relaxation. The NIMROD simulations start from vacuum conditions and model evolution with a single injector mounted near the lower divertor. This is representative of the early Pegasus configuration described in Ref. [19] but not the outboard gun configuration that has been used in recent years [20], which has significant current drive from poloidal-field induction [21]. Our simulations model the injectors as spatially localized sources of heat and current drive. O'Bryan found that having thermal conduction depend on evolving magnetization, in addition to temperature, is critical for avoiding unphysical confinement of heat outside the simulated current stream.

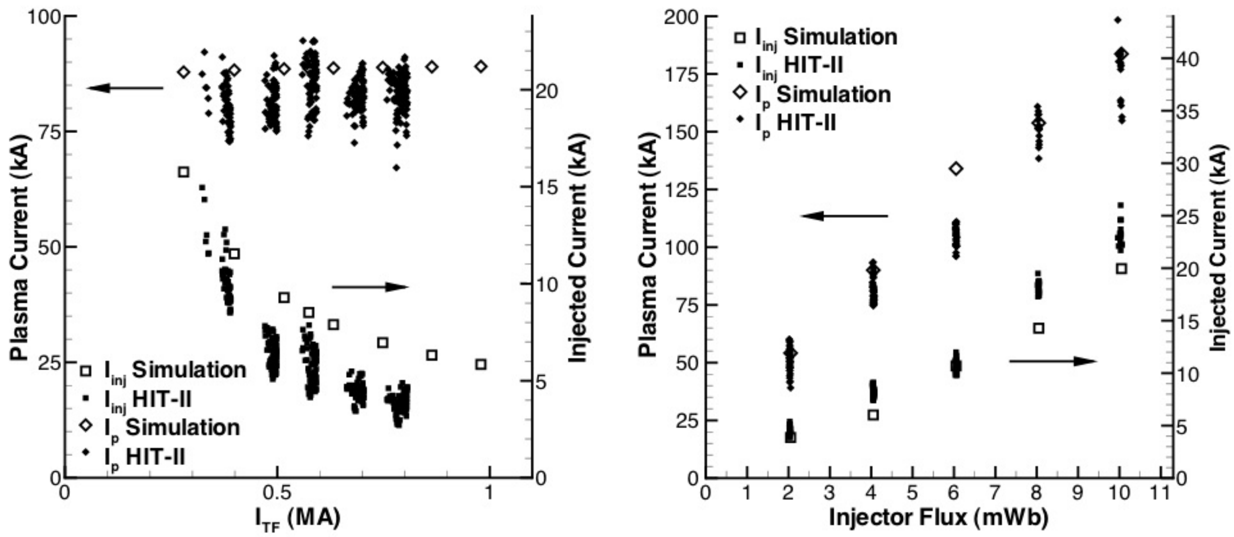

Figure 5. Comparison of NIMROD simulation results and HIT-II experimental results on plasma current as the toroidal field current is scanned (left) and as the injector flux is scanned (right). This figure is from Ref. [8].

The simulation study produced results on the dynamics of the driven current stream, on the development of a tokamak-like state, and on magnetic topology during flux-rope merger. From his simulations, O'Bryan found that relaxation proceeds via a sequence of flux-rope merger and reconnection events that release independent rings of current (Fig. 6). The free energy for merging different passes of the driven helical current stream is analogous to the island coalescence instability [22], but the Pegasus configuration is 3D and has open field lines. The poloidal flux associated with the current rings accumulates over many events, producing a tokamak-like configuration when averaged over the toroidal coordinate [9]. This finding represented a new paradigm for understanding how relaxation proceeds from localized current injection. In addition, after the DC injection is stopped, the asymmetries quickly dissipate, allowing the formation of topologically closed flux surfaces. Topological aspects of the study considered the existence of a quasi-separatrix layer (QSL) [23] by computing the squashing degree Q [24], a measure of magnetic field-line scattering, before and during computed reconnection events. We found that while reconnections sites contribute to large Q-values, bifurcation also occurs between current passes that are separating (not merging), and that behavior also leads to large Q-values [25].

O'Bryan's work also contributed validation information for fluid-based modeling of DC current drive. He implemented an array of synthetic magnetic probes at locations where probes have been mounted in Pegasus. Apart from camera images and globally integrated signals, such as plasma current, the magnetic probe array is the primary diagnostic for DC startup in Pegasus. A comparison of raw signals and cross-power "sonograms" from an early divertor gun experiment in Pegasus and from a two-fluid NIMROD simulation is shown in Fig. 7. The application of electrical potential is not modeled in detail in the simulation, but we note that the net plasma current as a function of time and all of the fluctuation activity and its consequences are results of the simulations; they are not prescribed. The simulations reproduce the 5% fluctuation level observed in the experiment, and the cross-power spectrum is dominated by 10 kHz activity, also similar to the Pegasus results. The simulations show that the 10 kHz activity results from Alfvén waves that are excited by the reconnection events and that lower-frequency signals represent rewinding of the helical channel between events [10]. O'Bryan's simulations also show that the differences between resistive-MHD and two-fluid modeling are localized to reconnection sites and tend to not affect overall relaxation.

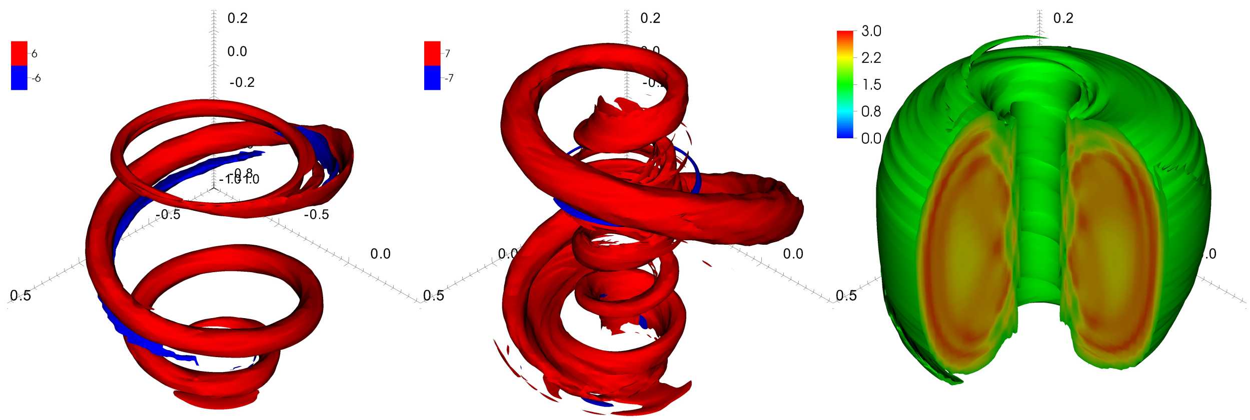

Figure 6. Isosurface images of simulated "parallel current," $\lambda = \mu_0 J_{\|}/B$ during early ring formation (left), the late driven phase (center), and after cessation of current drive (right). Images are taken from Ref. [25].

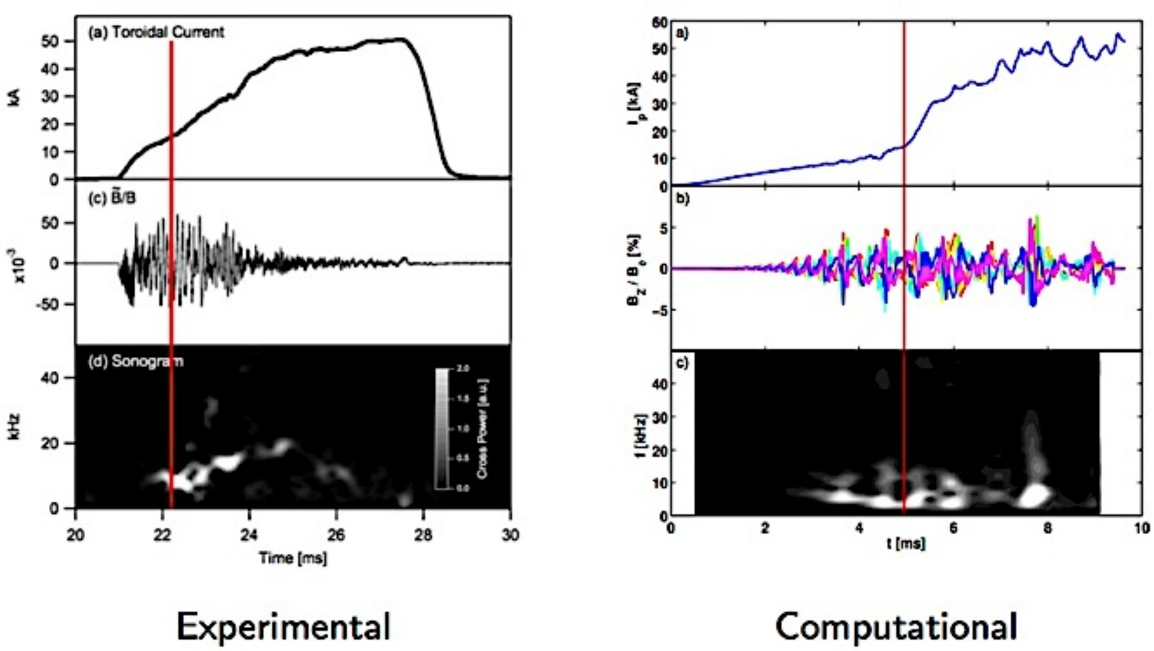

Figure 7. Comparison of net electrical current and magnetic fluctuation information from the Pegasus experiment (left, from Ref. [19]) and from a NIMROD simulation (right, from Ref. [10]). The cross-power sonogram of the experimental data has frequencies below a few kHz filtered.

For our collaboration on modeling transient CHI (TCHI) in NSTX, the PI helped Bick Hooper with problem-specific code modifications and with analysis of simulation results. Boundary conditions for the injector and absorber were adapted from the HIT-II study, but they were modified to allow coupling with Hooper's external circuit model that had previously been used for SSPX. We also implemented a computation of poloidal flux within NIMROD for the purpose of distinguishing regions within and downstream of the expanding flux bubble. Axisymmetric results show that impurity radiation has a role in keeping plasma temperature at the level observed in TCHI experiments in NSTX [11]. They also quantify the contribution of external poloidal-field induction in creating current in the confined region. A scenario of reduced impurity radiation leads to stronger drive of the flux bubble, which destabilizes the current sheet with respect to helical modes. Through linear computations and analysis, we found that the modes are current-gradient-driven tearing modes, which are related to plasmoid formation [13]. Their primary nonlinear effect is to broaden the bubble's current sheet; the fluctuations do not contribute to the formation of large-scale closed-flux regions. Finally, our developments for NSTX were provided to Dr. Fatima Ebrahimi, who has been using them in her numerical studies of NSTX.

4. Products

4.1 Journal Publications

1. A. I. D. Macnab, R. D. Milroy, C. C. Kim, and C. R. Sovinec, “Hall Magneto-hydrodynamics Simulations of End-Shorting Induced Rotation in Field-Reversed Configurations,” Physics of Plasmas **14**, 092503 (2007).
2. J.-Y. Ji, E. D. Held, and C. R. Sovinec, "Moment approach to deriving parallel heat flow for general collisionality," Physics of Plasmas **16**, 22312 (2009).
3. R. D. Milroy, C. C. Kim, and C. R. Sovinec, “Extended magnetohydrodynamic simulations of field reversed configuration formation and sustainment with rotating magnetic field current drive,” Physics of Plasmas **17**, 62502 (2010).
4. R. A. Bayliss, C. R. Sovinec, and A. J. Redd, “Zero-β Modeling of Coaxial Helicity Injection in the HIT-II Spherical Torus,” Physics of Plasmas **18**, 94502 (2011).
5. J. B. O’Bryan, C. R. Sovinec, and T. M Bird, “Simulation of Current-Filament Dynamics and Relaxation in the Pegasus Spherical Tokamak,” Physics of Plasmas **19**, 080701 (2012).
6. F. Ebrahimi, E. B. Hooper, C. R. Sovinec, and R. Raman, "Magnetic reconnection process in transient coaxial helicity injection," Physics of Plasmas **20**, 090702 (2013).
7. E. B. Hooper, C. R. Sovinec, R. Raman, F. Ebrahimi, and J. E. Menard, "Resistive MHD simulations of helicity-injected startup plasmas in NSTX," Physics of Plasmas **20**, 092510 (2013).
8. J. B. O'Bryan and C. R. Sovinec, "Simulated flux-rope evolution during non-inductive startup in Pegasus," Plasma Physics and Controlled Fusion **56**, 064005 (2014).
9. F. Ebrahimi, R. Raman, E. B. Hooper, C. R. Sovinec, and A. Bhattacharjee, "Physics of forced magnetic reconnection in coaxial helicity injection experiments in National Spherical Torus Experiment," Physics of Plasmas **21**, 056109 (2014).

10. E. B. Hooper and C. R. Sovinec, "A current-driven resistive instability and its effects in simulations of coaxial helicity injection in a tokamak," accepted for publication in Physics of Plasmas.

4.2 PhD Thesis

J. B. O'Bryan, "Numerical Simulation of Non-Inductive Startup of the Pegasus Toroidal Experiment," PhD Thesis, Department of Engineering Physics, University of Wisconsin-Madison, 2014. (http://gradworks.umi.com/36/24/3624567.html)

4.3 Conference Presentations

1. C. R. Sovinec, C. S. Carey, C. B. Forest, R. J. Fonck, and T. K. Fowler, "Numerical studies of line-tied kink modes," Innovative Confinement Conference Workshop, Austin, TX, Feb. 13-16, 2006.
2. R. A. Bayliss, N. W. Eidietis, T. R. Jarboe, B. A. Nelson, and C. R. Sovinec, "Nonlinear MHD simulation of helicity injection in spherical tokamaks," Innovative Confinement Conference Workshop, College Park, MD, Feb. 12-14, 2007.
3. R. A. Bayliss and C. R. Sovinec, "Nonlinear simulation of DC helicity injection in spherical tokamaks," 49th Annual Meeting of the APS Division of Plasma Physics, Orlando, FL, Nov. 12-16, 2007.
4. R. A. Bayliss and C. R. Sovinec, "A quantitative comparison of DC helicity injection in the HIT-II spherical tokamak and 3-D MHD simulation," International Sherwood Fusion Theory Conference, Boulder, CO, March 31-April 2, 2008.
5. C. R. Sovinec, E. C. Howell, J. R. King, and N. A. Murphy, "Computation for two-fluid relaxation physics," Innovative Confinement Concepts Workshop, Reno Nevada, June 24-27, 2008.
6. T. M. Bird, C. R. Sovinec, J. B. O'Bryan, D. J. Battaglia, "Nonlinear MHD computation of helicity injection and relaxation in spherical tokamaks," 50th Annual Meeting of the APS Division of Plasma Physics, Dallas, TX, Nov. 17-21, 2008.
7. J. B. O'Bryan, C. R. Sovinec, D. J. Battaglia, and T. M Bird, "Computational study of a non-Ohmic flux compression startup method for spherical tokamaks," 50th Annual Meeting of the APS Division of Plasma Physics, Dallas, TX, Nov. 17-21, 2008.
8. J. B. O'Bryan, C. R. Sovinec, D. J. Battaglia, and T. M. Bird, "Numerical simulation of non-inductive startup and flux compression in the Pegasus Toroidal Experiment," 51st Annual Meeting of the APS Division of Plasma Physics, Atlanta, GA, Nov. 2-6, 2009.
9. R. D. Milroy, C. C. Kim, and C. R. Sovinec, "NIMROD simulations of FRC formation with rotating magnetic field current drive," 51st Annual Meeting of the APS Division of Plasma Physics, Atlanta, GA, Nov. 2-6, 2009.
10. T. M. Bird, C. R. Sovinec, and J. B. O'Bryan, "Simulations of helical current-channel relaxation," International Sherwood Fusion Theory Conference, Seattle, WA, Apr. 19-21, 2010.
11. J. B. O'Bryan, C. R. Sovinec, and T. M. Bird, "Numerical simulation of non-inductive startup and flux compression in the Pegasus Spherical Toroidal Experiment," International Sherwood Fusion Theory Conference, Seattle, WA, Apr. 19-21, 2010.
12. J. B. O'Bryan, C. R. Sovinec, and T. M. Bird, "Numerical Simulation of non-inductive startup in the Pegasus Toroidal Experiment," 52nd Annual Meeting of the APS Division of Plasma Physics, Chicago, IL, Nov. 8-12, 2010.

13. J. B. O'Bryan, C. R. Sovinec, and T. M. Bird, "Numerical simulation of non-inductive startup in the Pegasus Toroidal Experiment," Innovative Confinement Concepts Workshop, Seattle, WA, Aug. 16-19, 2011.
14. C. R. Sovinec, E. B. Hooper, R. A. Bayliss, and A. J. Redd, "MHD modeling of coaxial helicity injection in HIT-II and NSTX," Innovative Confinement Concepts Workshop, Seattle, WA, Aug. 16-19, 2011.
15. J. B. O'Bryan and C. R. Sovinec, "Numerical simulation of non-inductive startup in the Pegasus Toroidal Experiment," 53rd Annual Meeting of the APS Division of Plasma Physics, Salt Lake City, UT, Nov. 14-18, 2011.
16. J. B. O'Bryan and C. R. Sovinec, "Simulation of current-filament dynamics and relaxation in the Pegasus ST," International Sherwood Fusion Theory Conference, Atlanta, GA, April 1-3, 2012.
17. J. B. O'Bryan and C. R. Sovinec, "Simulation of current-filament dynamics in the Pegasus ST," 54th Annual Meeting of the APS Division of Plasma Physics, Providence, RI, Oct. 29-Nov. 2, 2012.
18. B. Hooper, C. Sovinec, R. Raman, F. Ebrahimi, and J. Menard, "Flux-surface closure during resistive-MHD simulations of coaxial helicity injection (CHI) in NSTX," 54th Annual Meeting of the APS Division of Plasma Physics, Providence, RI, Oct. 29-Nov. 2, 2012.
19. J. B. O'Bryan and C. R. Sovinec, "Simulation of current-filament dynamics and relaxation in the Pegasus ST," International Sherwood Fusion Theory Conference, Santa Fe, NM, April 15-7, 2013.
20. F. Ebrahimi, B. Hooper, C. Sovinec, and R. Raman, "Physics of fast flux closure in coaxial helicity injection experiments in NSTX," International Sherwood Fusion Theory Conference, Santa Fe, NM, April 15-7, 2013.
21. J. B. O'Bryan and C. R. Sovinec, "Simulation of current-filament dynamics and relaxation in the Pegasus ST," 55th Annual Meeting of the APS Division of Plasma Physics, Denver, CO, Nov. 11-15, 2013.
22. J. B. O'Bryan and C. R. Sovinec, "Simulation of flux-rope merger and reconnection in the Pegasus spherical torkamak," International Sherwood Fusion Theory Conference, San Diego, CA, Mar. 24-26, 2014.
23. B. Hooper, C. Sovinec, and R. Raman, "Co-injection of helicity and plasma in CHI simulation of NSTX," Workshop on Exploratory Topics in Plasma and Fusion Research, Madison, WI, Aug. 5-8, 2014.
24. E. B. Hooper, C. R. Sovinec, and R. Raman, "Helicity and plasma evolution in CHI simulations of NSTX," 56th Annual Meeting of the APS Division of Plasma Physics, New Orleans, LA, Oct. 27-31, 2014.
25. E. B. Hooper and C. R. Sovinec, "Nonaxisymmetric effects in strongly driven coaxial helicity injection in simulations of NSTX," 57th Annual Meeting of the APS Division of Plasma Physics, Savannah, GA, Nov. 16-20, 2015.

5. Modeling

Documentation of the equations, methods, and verification of the NIMROD code, which has been used extensively in this work, is in Refs. [1, 17, 10] and at https://nimrodteam.org.

References

[1] C. R. Sovinec, A. H. Glasser, T. A. Gianakon, D. C. Barnes, R. A. Nebel, S. E. Kruger, D. D. Schnack, S. J. Plimpton, A. Tarditi, M. Chu, and the NIMROD Team, Journal of Computational Physics **195**, 355 (2004).
[2] A. J. Redd, B. A. Nelson, T. R. Jarboe, P. Gu, R. Raman, R. J. Smith, and K. M. McCollam, Physics of Plasmas **9**, 2006 (2002).
[3] N. W. Eidietis, R. J. Fonck, G. D. Garstka, E. A. Unterberg, and G. R. Winz, Journal of Fusion Energy **26**, 43 (2007).
[4] R. Raman, T. R. Jarboe, D. Mueller, B. A. Nelson, M. G. Bell, M. Ono, T. Bigelow, R. Kaita, B. LeBlanc, R. Maqueda, J. Menard, S. Paul, L. Roquemore, and the NSTX Research Team, Phys. Plasmas 14, 056106 (2007).
[5] S. I. Braginskii, Reviews of Plasma Physics **1**, 205 (1965).
[6] J.-Y. Ji and E. D. Held, Physics of Plasmas **20**, 042114 (2013).
[7] R. Chodura, Nuclear Fusion **15**, 65 (1975) and Ref. 7, thereof.
[8] R. A. Bayliss, C. R. Sovinec, and A. J. Redd, Physics of Plasmas **18**, 94502 (2011).
[9] J. B. O'Bryan, C. R. Sovinec, and T. M Bird, Physics of Plasmas **19**, 080701 (2012).
[10] J. B. O'Bryan, "Numerical Simulation of Non-Inductive Startup of the Pegasus Toroidal Experiment," PhD Thesis, Department of Engineering Physics, University of Wisconsin-Madison, May, 2014.
[11] E. B. Hooper, C. R. Sovinec, R. Raman, F. Ebrahimi, and J. E. Menard, Physics of Plasmas **20**, 092510 (2013).
[12] F. Ebrahimi, R. Raman, E. B. Hooper, C. R. Sovinec, and A. Bhattacharjee, Physics of Plasmas **21**, 056109 (2014).
[13] E. B. Hooper and C. R. Sovinec, Physics of Plasmas **23**, 102502 (2016).
[14] Vyacheslav S. Lukin, "Computational study of the internal kink mode evolution and associated magnetic reconnection phenomena". PhD thesis, Princeton (2008).
[15] C. R. Sovinec, B. I. Cohen, G. A. Cone, E. B. Hooper, and H. S. McLean, Physical Review Letters **94**, 35003 (2005).
[16] B. I. Cohen, E. B. Hooper, R. H. Cohen, D. N. Hill, H. S. McLean, R. D. Wood, S. Woodruff, C. R. Sovinec, and G. A. Cone, Physics of Plasmas **12**, 56106 (2005).
[17] C. R. Sovinec, J. R. King, and the NIMROD Team, Journal of Computational Physics **229**, 5803 (2010).
[18] A. J. Redd, T. R. Jarboe, B. A. Nelson, R. G. O'Neill, and R J Smith, Physics of Plasmas **14**, 112511 (2007).
[19] N. W. Eidietis, "Non-inductive startup of the Pegasus spherical torus using localized washer-gun current sources," PhD Thesis, Department of Engineering Physics, University of Wisconsin-Madison, 2007.
[20] D. J. Battaglia, M. W. Bongard, R. J. Fonck, and A. J. Redd, Nuclear Fusion **51**, 073029 (2011).
[21] J. L. Barr, "Inductive evolution and MHD in local helicity injection discharges on Pegasus," PhD Thesis, Department of Engineering Physics, University of Wisconsin-Madison, Nov. 2016.
[22] J. M. Finn and P. K. Kaw, Physics of Fluids **20**, 72 (1977).
[23] E. R. Priest and P. Démoulin, Journal of Geophysical Research **100**, 23443 (1995).
[24] V. Titov, G. Hornig, and P. Démoulin, Journal of Geophysical Research **107**, 1164 (2002).
[25] J. B. O'Bryan and C. R. Sovinec, Plasma Physics and Controlled Fusion **56**, 064005 (2014).