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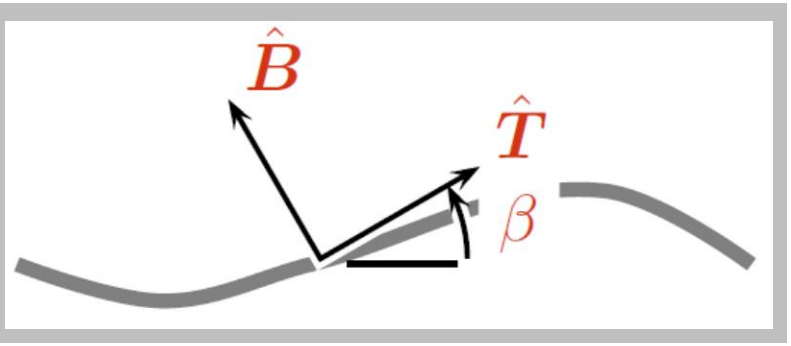

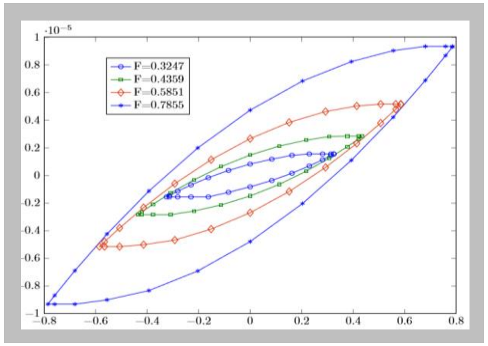

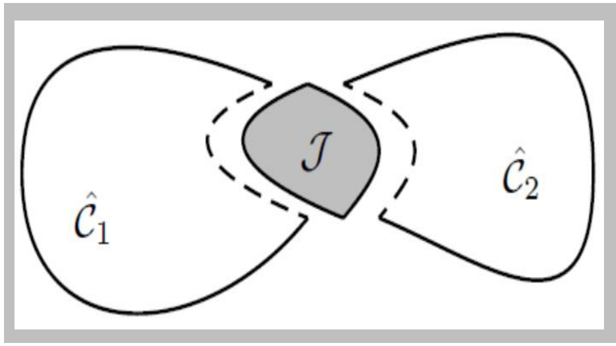

Continuum Shell Models for Structural Damping

Adam Brink – Sandia National Laboratories

Dane Quinn – University of Akron

Dan Segalman – Michigan State University

U.S. DEPARTMENT OF ENERGY · NNSA National Nuclear Security Administration

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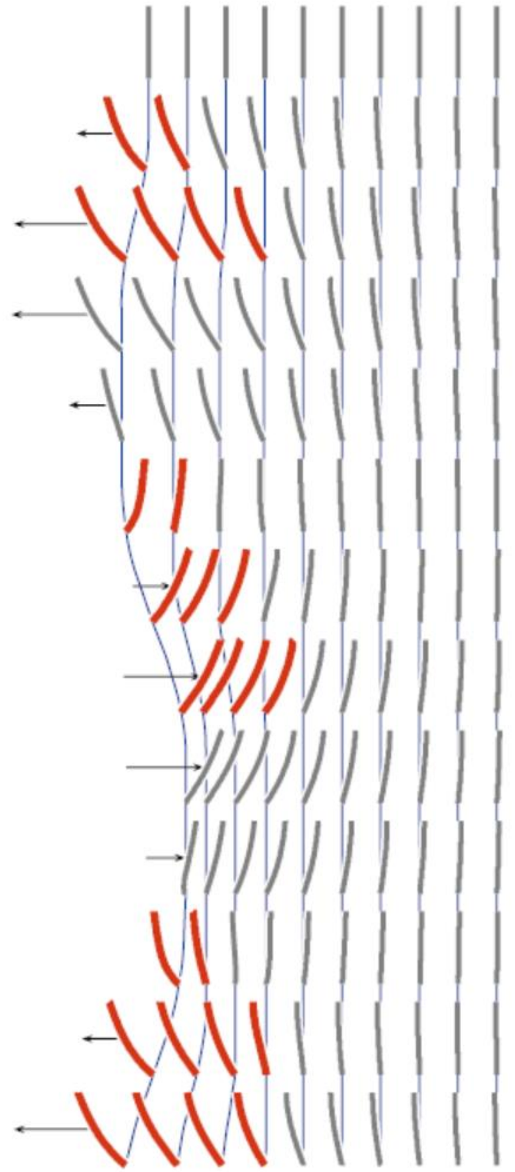
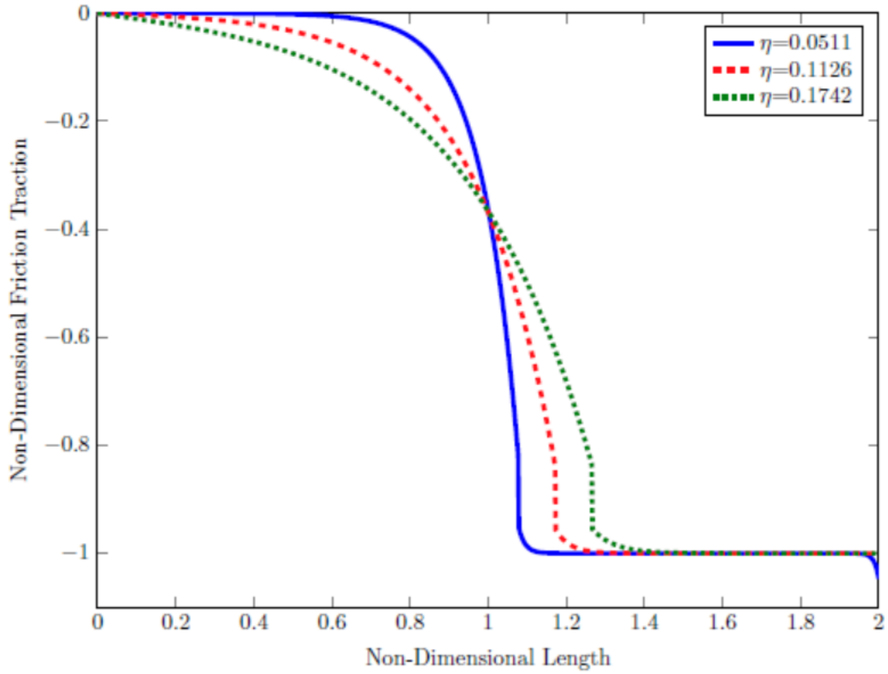
Research Motivation

- Develop shell models which capture structural damping and joint stiffness in a reduced manner.
  - Capture Micro- and Macro-Slip with physics incorporated directly into the shell.
  - Bar-like model developed by Quinn and Segalman.
  - Shearable beam-like model developed by Brink and Quinn.
- Develop a convenient framework to include the nonlinear joint into modal dynamics.
  - Quinn developed a method which incorporates joint forces into the modal equations of motion.

Importance of Including Shear

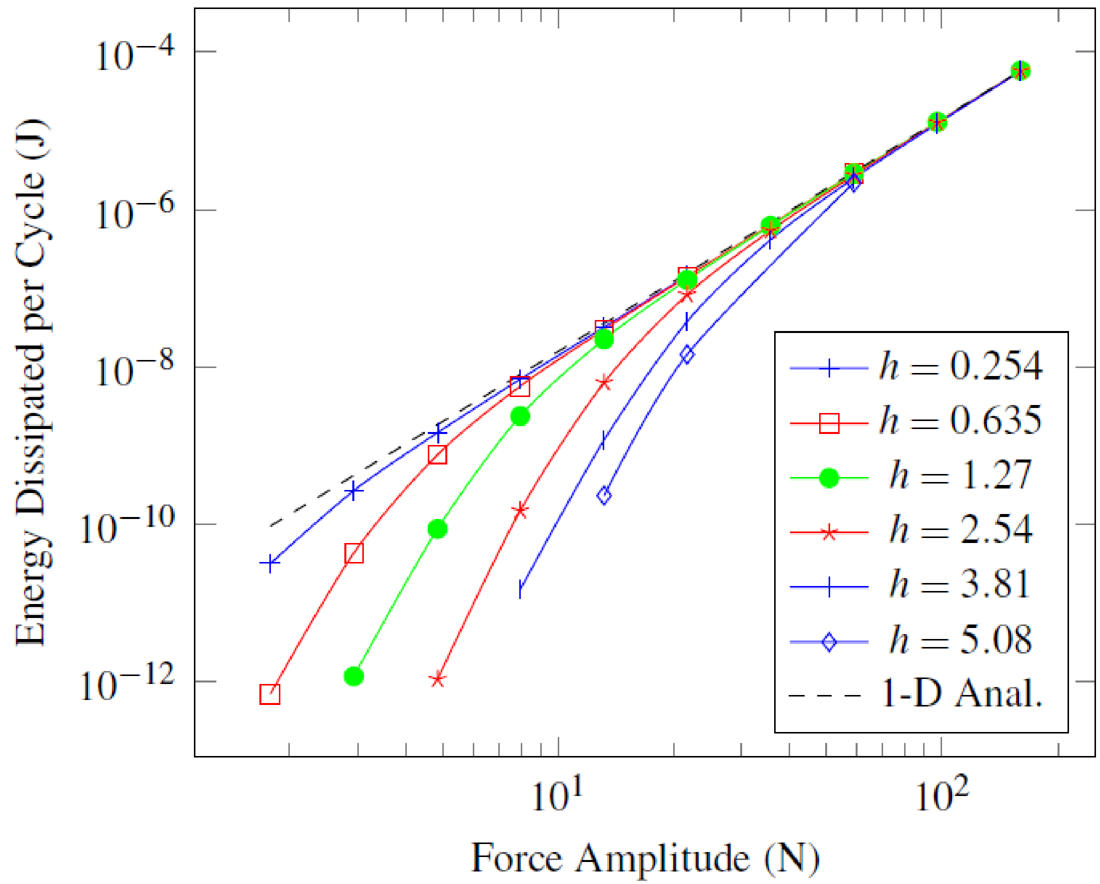




- Shearing allows forces to transmit passed the slip initiation length.
- The shorter the cross-section, the more bar-like the behavior becomes

Importance of Including Shear

- Shearing allows for a precipitous drop off from the cubic power law bar solution.



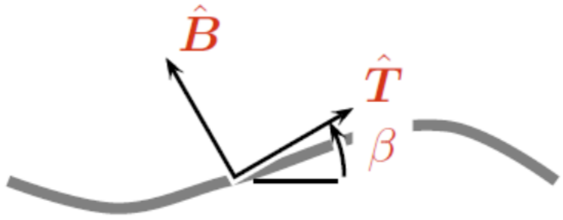
Shearable Shell Derivation

We use a geometrically exact formulation for shell theory presented by Libai and Simmonds (restricted here to quasi-static behavior)

Strains and Internal Forces

$$\mathbf{y}' = (1+e)\,\hat{\boldsymbol{T}} + g\,\hat{\boldsymbol{B}},$$

$$\beta' = k$$



$$\mathbf{m} \equiv \hat{\boldsymbol{k}} \times \mathbf{y}',$$

$$\mathbf{F} = N\,\hat{\boldsymbol{T}} + Q\,\hat{\boldsymbol{B}}$$

Linear and Angular Momentum Balance

$$\left(N\,\hat{\boldsymbol{T}} + Q\,\hat{\boldsymbol{B}}\right)' + \mathbf{p} = \mathbf{0}, \qquad M' + \mathbf{m}\cdot\left(N\,\hat{\boldsymbol{T}} + Q\,\hat{\boldsymbol{B}}\right) + \ell = 0.$$

Constitutive Laws

$$N = EHc_1\,e, \quad Q = EHc_2\,g, \quad M = EH^3c_3\,k$$

Shearable Shell Derivation

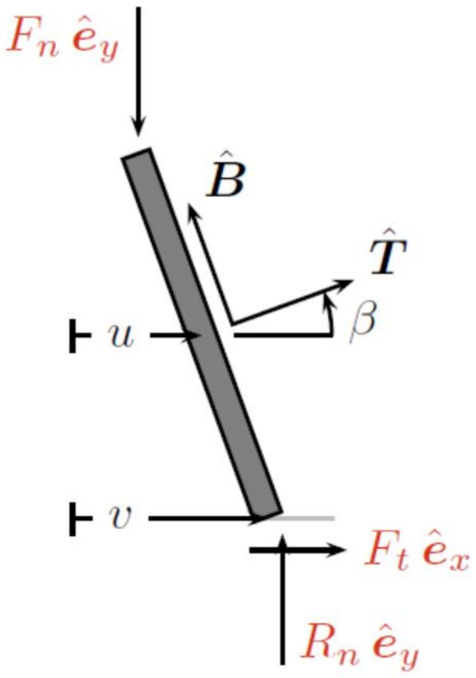
External forces and moments

$$\mathbf{p} = F_t\,\hat{\boldsymbol{e}}_x + (R_n - F_n)\,\hat{\boldsymbol{e}}_y,$$
$$\ell = h\,\left(F_t\,\cos\beta + (R_n + F_n)\,\sin\beta\right).$$

The contact force is assumed to follow Coulomb's law:

Stick: $u = v - h\,\sin\beta$

Slip: $F_t = \mu\,R_n\,\mathrm{sgn}(\dot{v})$



The equations of motion reduce to

$$\left(N' - \beta'\,Q\right) + F_t\,\cos\beta - (R_n - F_n)\,\sin\beta = 0,$$
$$\left(Q' + \beta'\,N\right) + F_t\,\sin\beta + (R_n - F_n)\,\cos\beta = 0,$$
$$M' + (1+e)\,N - g\,Q + h\,\left(F_t\,\cos\beta + (R_n + F_n)\,\sin\beta\right) = 0.$$

Shearable Shell Derivation

The equations of motion can be nondimensionalized and linearized in the deformation

$$\text{Linear} \qquad \kappa\, u'' + F_t = 0, \qquad -\kappa\, c_1\, \beta' + (R_n - F_n) = 0,$$

$$\text{Angular} \qquad \alpha\, \eta^2\, \beta'' + \kappa\, c_2\, \beta + \eta\, (F_t + (R_n + F_n)\, \beta) = 0.$$

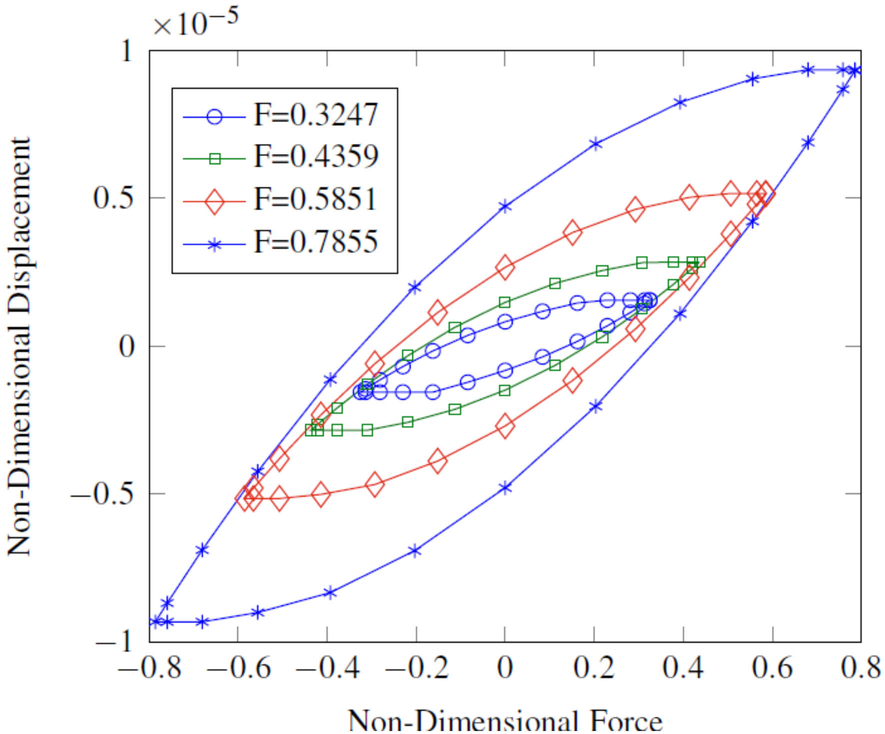
α: nondimensional bending stiffness

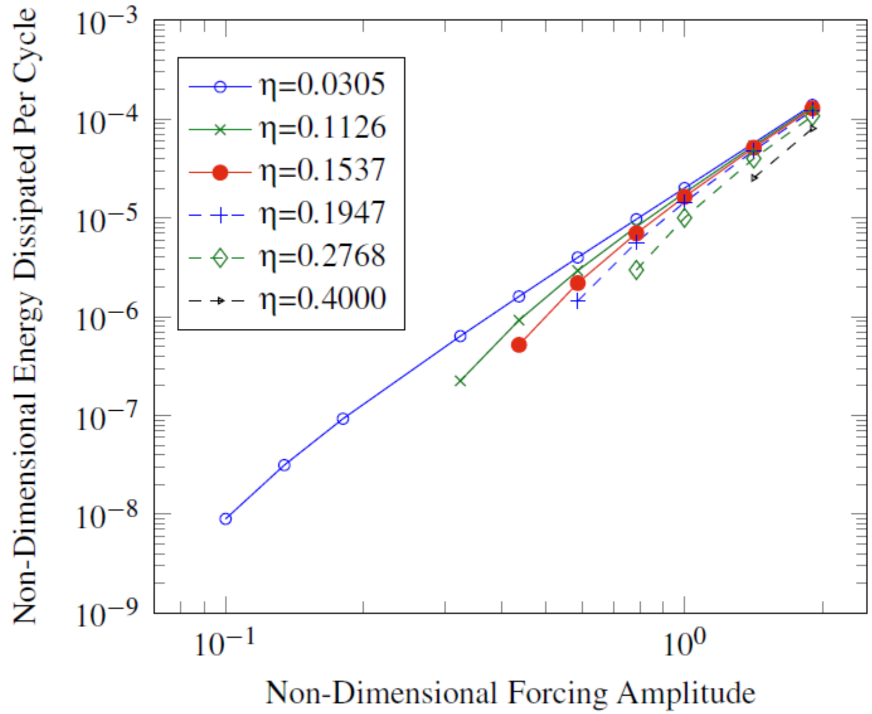
κ: nondimensional axial stiffness

η: nondimensional height

The external loading is prescribed at the boundaries

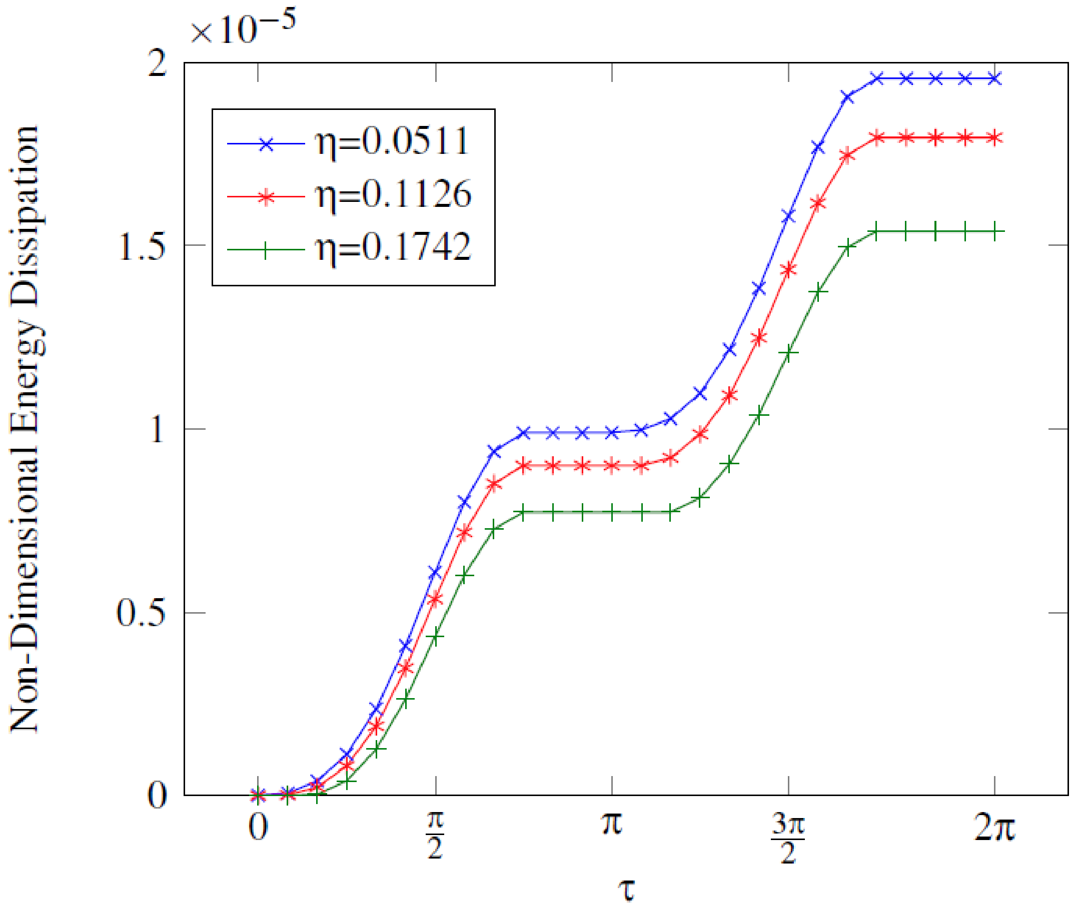
Some Shell Results





Hysteresis curves and energy dissipated per cycle versus forcing amplitude are as expected.

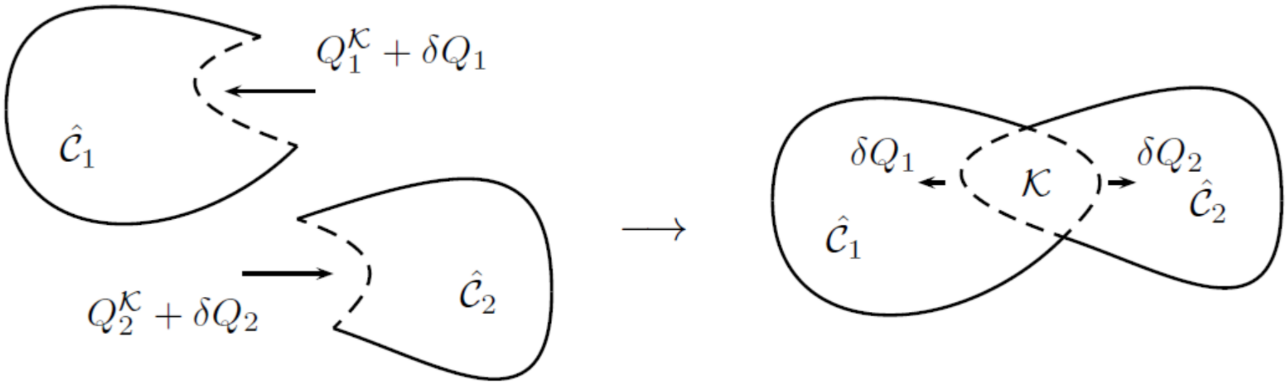
More Shell Results



Energy dissipated over a cycle shows expected flat spotting.

Modal Analysis

- Quinn solves modal equations of motion for a monolithic structure, then adds the effect of the joint back in.



$$\left[\int_{\mathcal{M}} \phi_i(x)\, \rho_{\mathcal{M}}(x)\, \phi_i(x)\, dx\right] \ddot{A}_i(t) + \left[\int_{\mathcal{M}} \frac{\partial \phi_i(x)}{\partial x}\, EA_{\mathcal{M}}(x)\, \frac{\partial \phi_i(x)}{\partial x}\, dx\right] A_i(t)$$

Monolithic Response

$$= -\left(\phi_i(s_1)\, \delta Q_1(t) + \phi_i(s_2)\, \delta Q_2(t)\right).$$

Forces Arising From Joint at Interfaces

Modal Analysis – Elastic Bar

Consider the response of an elastic rod with simply supported boundaries and a distributed interface.

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \quad x \in (0,1),$$

The response of the monolithic structure can be expressed as

$$u(x,t) = \sum_{k=1}^{\infty} A_k(t)\,\phi_k(x), \qquad \phi_k(x) = \sqrt{2}\,\sin(k\,\pi\,x)\,.$$

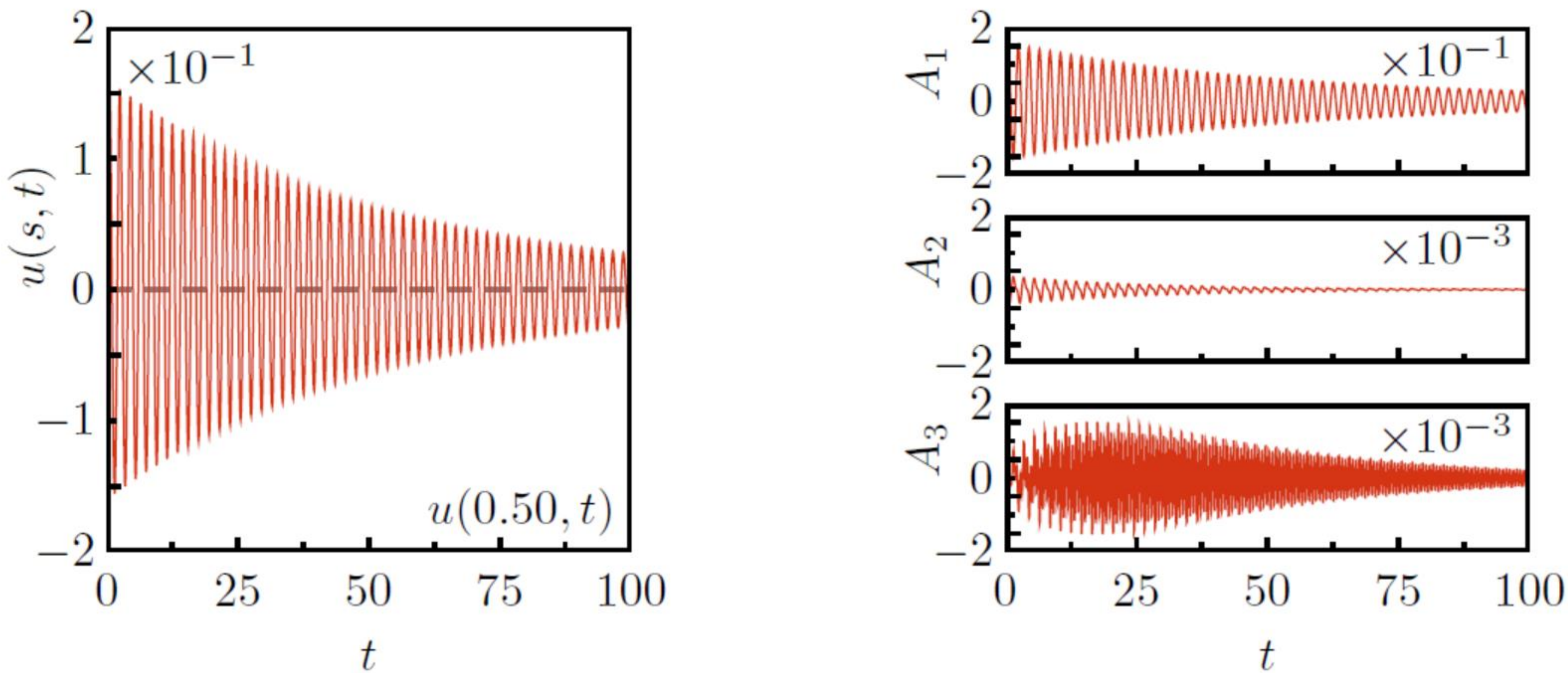
An N mode truncation

$$\ddot{A}_j + 2\,\zeta\,(j\,\pi)\,\dot{A}_i + (j\,\pi)^2\,A_j$$
$$+ (\phi_j(s_2) - \phi_j(s_1))\;\delta Q\left(\sum_{k=1}^{N} A_k(t)\;(\phi_k(s_2) - \phi_k(s_1))\right) = 0,$$

Modal Analysis – Elastic Bar

The interface is located between $s_1 = 0.20$ and $s_2 = 0.30$ and the initial conditions only excite the fundamental mode of the rod.

$$N = 5, \quad \zeta = 0.005, \quad \beta = 1.00, \quad \ell_0 = 1.00, \quad F_0 = 0.10.$$



Conclusions

- A nonlinear shearable elastic shell is developed which directly incorporates friction into its formulation.
- The shell is capable of modeling micro- and macro-slip phenomena.
- Quinn’s modal analysis method is a concise way to introduce joint nonlinearities into modal framework.

Future Work

- Compare the shearable elastic shell to the elastic bar using the modal analysis techniques.
- Introduce the reduced order shell models and Quinn's modal analysis techniques into a finite element framework.